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H. WELEMANE & F. CORMERY

Abstract. A three-dimensional model of damage by microcrack growth is proposed to account for the mechanical behavior of quasi-brittle materials (especially for concrete and rocks). The emphasis is put on the induced anisotropy and on the elastic moduli dependence on the opening and closure of microcracks (unilateral effect). This formulation is based first on a damage characterization through the microcrack density distribution, and secondly avoids the use of spectral decompositions generally adopted in literature and which induce some major inconsistencies.

1. INTRODUCTION

Microcracking plays a crucial role in the mechanical response of a large class of materials (called quasi-brittle materials), which includes notably concrete and some rocks. Owing to the specific nature of such defects, the macroscopic behavior of these materials is complex and generally anisotropic due to the preferential orientation taken by microcracks under loading. Even in a fixed damage state, their stress-strain response may be non linear because of the unilateral contact of the lips of microcracks that can be open but not interpenetrate.

The representation of the salient features of such behavior within a consistent macroscopic model is a difficult task. In particular, the simultaneous description of the induced anisotropy and the unilateral effect still remains an open research field, since the existing theories exhibit either major internal inconsistencies (cf. Chaboche [1], Carol & Willam [2], Cormery & Welemane [3]), or a lack of physical justification (cf. Chaboche [4], Welemane & Cormery [5]).

This paper aims to introduce a novel three-dimensional model for damage by microcrack growth, which is based on a damage characterization through the microcrack density distribution. This formulation makes it possible to account for the induced anisotropy and also for the recovery phenomenon observed at the closure of microcracks (cf. Reinhardt [6], Ikogou [7]), while avoiding the use of spectral decompositions generally employed and whose limits have been previously pointed out.

The formulation is developed within the theoretical framework of irreversible thermodynamics with internal variables. The presentation is restrained here to the thermodynamic potential of the model, namely the free energy function (total strain tensor $\boldsymbol{\varepsilon}$ is chosen as the observable variable), which is formulated for rate independent, isothermal and small transformations and for a single dissipative mechanism, namely the generation and growth of microcracks.

Usual intrinsic notations are employed throughout. In particular, the term $\mathbf{a}^{\otimes i} = \mathbf{a} \otimes \mathbf{a} .. \otimes \mathbf{a}$ denotes the i^{th} tensor product power of any tensor $\mathbf{a}$, $\lfloor \mathbf{b} \rfloor$ the irreducible part of any tensor $\mathbf{b}$, $\bullet$ the complete contraction, 0_i the zero tensor of order i, $\mathbf{I}$ the second-order identity tensor, and ds the infinitesimal surface element on the unit sphere $S = \{\mathbf{n} \in \mathbb{R}^3, \mathbf{n} \cdot \mathbf{n} = 1\}$.

2. DAMAGE DESCRIPTION

We consider that the main information to characterize the damage state of a medium weakened by a set of plane microcracks is the microcrack density distribution. The term density in a direction refers here to a scalar and adimensional measure which accounts for the extent of microcracks orthogonal to this direction.

This distribution can be approximated by an orientation function ρ defined by (cf. Kanatani [8]) :

$$\rho(\mathbf{n},\mathbf{d}) = d_0 + d_2 : \lfloor \mathbf{n}^{\otimes 2} \rfloor + d_4 :: \lfloor \mathbf{n}^{\otimes 4} \rfloor + \ldots + d_{\mathrm{p}} \bullet \lfloor \mathbf{n}^{\otimes \mathrm{p}} \rfloor \tag{1}$$

where $\mathbf{d} = (d_0, d_2, d_4, ..d_{\mathrm{p}})$ denotes a set of irreducible tensors respectively of order (0,2,4,..p), with p an even integer. The set of tensors **d** allows to characterize the microcrack density in any direction of the space, and constitutes then appropriate internal damage variables.

For the moment, the approximation order p is not specified. Note however that we consider in what follows that $p \geq 2$, in order to make the representation of oriented damage possible.

3. THERMODYNAMIC POTENTIAL

3.1 General presentation

We assume the existence of a thermodynamic potential W (free energy), depending on the state variables $(\boldsymbol{\varepsilon},\mathbf{d})$ and verifying the following properties :

- W is objective and of class C^1. With the first condition, the fundamental principle of space isotropy is satisfied (Boehler [9]) ; the second one ensures in particular the existence and the continuity of the stress :

$$\boldsymbol{\sigma} = \frac{\partial \mathrm{W}}{\partial \boldsymbol{\varepsilon}} \tag{2}$$

  and of the thermodynamic forces relevant to damage :

$$\mathbf{F}^{\mathrm{d}} = (F^{d_0}, F^{d_2}, F^{d_4}, .., F^{d_{\mathrm{p}}}) \quad \text{with} \quad F^{d_0} = -\frac{\partial \mathrm{W}}{\partial d_0}, F^{d_2} = -\frac{\partial \mathrm{W}}{\partial d_2}, F^{d_4} = -\frac{\partial \mathrm{W}}{\partial d_4}, .., F^{d_{\mathrm{p}}} = -\frac{\partial \mathrm{W}}{\partial d_{\mathrm{p}}} \tag{3}$$

- W is positively homogeneous of degree two with respect to **ε**, that is :

$$\mathrm{W}(\alpha\,\boldsymbol{\varepsilon},\mathbf{d}) = \alpha^2\,\mathrm{W}(\boldsymbol{\varepsilon},\mathbf{d}), \quad \forall\ \alpha \geq 0 \tag{4}$$

  This condition implies that the stress defined by (2) is positively homogeneous of degree one with respect to **ε** :

$$\boldsymbol{\sigma}(\alpha\,\boldsymbol{\varepsilon},\mathbf{d}) = \alpha\,\boldsymbol{\sigma}(\boldsymbol{\varepsilon},\mathbf{d}), \quad \forall\ \alpha \geq 0 \tag{5}$$

  Consequently, the stress-strain response of the material will be linear during a reversible proportional process, with asymmetrical behaviors between tensile and compressive loading (Marigo [10]).

We postulate that the free energy of the material can be divided in two parts :

$$\mathrm{W}(\boldsymbol{\varepsilon},\mathbf{d}) = \mathrm{W}_0(\boldsymbol{\varepsilon}) + \mathrm{W}_{\mathrm{d}}(\boldsymbol{\varepsilon},\mathbf{d}) \tag{6}$$

where $\mathrm{W}_0(\boldsymbol{\varepsilon})$ denotes the free energy of the undamaged material, supposed to be isotropic and linear elastic (with λ_0 and μ_0 the Lamé coefficients), and $\mathrm{W}_{\mathrm{d}}(\boldsymbol{\varepsilon},\mathbf{d})$ represents the contribution due to damage. For a current state $(\boldsymbol{\varepsilon},\mathbf{d})$, the latter is defined by :

$$W_d(\varepsilon,\mathbf{d}) = \frac{1}{4\pi}\int_S w(\rho(\mathbf{n},\mathbf{d}),\mathbf{n},\varepsilon)\,ds \tag{7}$$

with $w(\rho(\mathbf{n},\mathbf{d}),\mathbf{n},\varepsilon)$ the elementary energy relevant to the microcrack density $\rho(\mathbf{n},\mathbf{d})$ in the direction of unit normal $\mathbf{n}$. Therefore, the definition of the thermodynamic potential W reduces to the determination of the elementary energy function w.

3.2 Elementary energy and unilateral effect

The elementary energy $w(\rho(\mathbf{n},\mathbf{d}),\mathbf{n},\varepsilon)$ related to the microcracks of normal $\mathbf{n}$ is taken here linear in the density $\rho(\mathbf{n},\mathbf{d})$ of these microcracks :

$$w(\rho(\mathbf{n},\mathbf{d}),\mathbf{n},\varepsilon) = \rho(\mathbf{n},\mathbf{d})\,h(\mathbf{n},\varepsilon) \tag{8}$$

which corresponds implicitly to the hypothesis of non-interacting microdefects (Kachanov [11]). The main advantage of the formulation postulated in (8) is to make it possible to specify, by means of the function h, the nature of the energetic contribution of the microcrack density in each direction of the space. This function must especially account for the unilateral effect, that is the special feature of microcracks to contribute differently depending on whether they are open or closed, and describe in particular the elastic properties recovery at the closure of microcracks.

To this end, we assume the following recovery condition motivated by micromechanics : *for any direction of unit vector* $\mathbf{n}$ *so that* $g(\mathbf{n},\varepsilon) = \delta_1\,\mathrm{tr}\,\varepsilon + \delta_2\,\mathbf{n}\cdot\varepsilon\cdot\mathbf{n} \leq 0$ (δ_1 *and* δ_2 *two constants), the microcrack density* $\rho(\mathbf{n},\mathbf{d})$ *does not contribute to the degradation of the elongation modulus* $L(\mathbf{n})$ *related to the direction of* $\mathbf{n}$*, neither to the volumetric modulus* $\kappa(\mathbf{m})$ *related to any direction of unit vector* $\mathbf{m}$*.*

For a given state ε, microcracks with normal $\mathbf{n}$ will be referred to as open if $g(\mathbf{n},\varepsilon) > 0$ and, on the contrary, referred to as closed if $g(\mathbf{n},\varepsilon) \leq 0$; thus, the relation $g(\mathbf{n},\varepsilon) = 0$ represents in the model the opening- closure criterion for microcracks of unit normal $\mathbf{n}$. In view of the previous recovery condition, the closure of microcracks will then only induce a partial damage deactivation, since the density of closed microcracks still contributes to the degradation of the elongation moduli related to directions different from their normal one ([5]).

Consequently, the application h takes two different forms whether microcracks are open or closed, that is :

$$h(\mathbf{n},\varepsilon) = \begin{cases} h_1(\mathbf{n},\varepsilon), & \text{if } g(\mathbf{n},\varepsilon) > 0 \\ h_2(\mathbf{n},\varepsilon), & \text{if } g(\mathbf{n},\varepsilon) \leq 0 \end{cases} \tag{9}$$

According to the conditions imposed to the potential W and to the properties of ρ and g, the scalar functions h_1 and h_2 must be objective, radially symmetric with respect to $\mathbf{n}$ (that is so that $h_i(\mathbf{n},\varepsilon) = h_i(-\mathbf{n},\varepsilon)$, $\forall\ \mathbf{n}$, $\forall\ i = 1,2$), and positively homogeneous of degree two with respect to ε. Representation theorems for isotropic scalar-valued functions give then (cf. Boehler [9]) :

$$h_i(\mathbf{n},\varepsilon) = \alpha_i\,\mathrm{tr}(\varepsilon\cdot\varepsilon) + \beta_i\,\mathrm{tr}^2\varepsilon + \chi_i\,\mathrm{tr}\,\varepsilon\,\mathrm{tr}(\varepsilon\cdot\mathbf{n}^{\otimes 2}) + \varphi_i\,\mathrm{tr}(\varepsilon\cdot\varepsilon\cdot\mathbf{n}^{\otimes 2}) + \psi_i\,\mathrm{tr}^2(\varepsilon\cdot\mathbf{n}^{\otimes 2}) \quad , \ \forall\ i = 1,2 \tag{10}$$

where the coefficients $(\alpha_i,\beta_i,\chi_i,\varphi_i,\psi_i)_{i=1,2}$ are supposed to be constant.

Since W must be of class C^1, and since h_1 and h_2 are at least of class C^2, the function h must be of class C^1 in every point $(\mathbf{n},\varepsilon)$ so that $g(\mathbf{n},\varepsilon) = 0$. A generalization of the works of Curnier et al. [12] on the continuity of multilinear functions (cf. Welemane [13]) shows that this property is satisfied if and only if (s a constant) :

$$\forall\ (\mathbf{n},\varepsilon),\ \ g(\mathbf{n},\varepsilon) = 0, \qquad \frac{\partial^2[h_1 - h_2]}{\partial\varepsilon^2}(\mathbf{n},\varepsilon) = s\ \frac{\partial g}{\partial\varepsilon}(\mathbf{n},\varepsilon)\otimes\frac{\partial g}{\partial\varepsilon}(\mathbf{n},\varepsilon) \tag{11}$$

which leads to the following conditions on the model parameters :

$$\begin{cases} \alpha_1 - \alpha_2 = 0 \\ 2(\beta_1 - \beta_2) = s\,\delta_1^2 \\ \chi_1 - \chi_2 = s\,\delta_1\delta_2 \\ \varphi_1 - \varphi_2 = 0 \\ 2(\psi_1 - \psi_2) = s\,\delta_2^2 \end{cases} \tag{12}$$

We need now to account for the elastic properties restoration postulated in the recovery condition, that is to specify the expression h_2 of the function h when microcracks are closed. The elongation modulus $L(\mathbf{v})$ and the volumetric modulus $\kappa(\mathbf{v})$ of the material related to the direction of unit vector $\mathbf{v}$ are defined by (He & Curnier [14]) :

$$L(\mathbf{v}) = \frac{\mathbf{v}\cdot\boldsymbol{\sigma}\cdot\mathbf{v}}{\mathbf{v}\cdot\boldsymbol{\varepsilon}\cdot\mathbf{v}} \quad , \quad \kappa(\mathbf{v}) = \frac{\mathrm{tr}\,\boldsymbol{\sigma}}{\mathrm{tr}\,\boldsymbol{\varepsilon}} \tag{13}$$

when the material is submitted to a pure elongation test $\boldsymbol{\varepsilon} = \bar{\varepsilon}\,\mathbf{v}^{\otimes 2}$. In the present model, relevant calculations give rise to the following expressions :

$$\begin{aligned} L(\mathbf{v}) &= L_0 + \frac{1}{4\pi\bar{\varepsilon}} \int_S \rho(\mathbf{n},\mathbf{d})\, \mathbf{v}\cdot\frac{\partial h}{\partial \boldsymbol{\varepsilon}}(\mathbf{n},\bar{\varepsilon}\,\mathbf{v}^{\otimes 2})\cdot\mathbf{v}\, ds \\ \kappa(\mathbf{v}) &= \kappa_0 + \frac{1}{4\pi\bar{\varepsilon}} \int_S \rho(\mathbf{n},\mathbf{d})\, \mathrm{tr}[\frac{\partial h}{\partial \boldsymbol{\varepsilon}}(\mathbf{n},\bar{\varepsilon}\,\mathbf{v}^{\otimes 2})]\, ds \end{aligned} \tag{14}$$

where L_0 and κ_0 represent respectively the elongation and the volumetric moduli for the undamaged material. The recovery condition is then verified if and only if :

$$\begin{cases} \mathbf{v}\cdot\dfrac{\partial h_2}{\partial \boldsymbol{\varepsilon}}(\mathbf{v},\bar{\varepsilon}\,\mathbf{v}^{\otimes 2})\cdot\mathbf{v} = 0 \\ \mathrm{tr}[\dfrac{\partial h_2}{\partial \boldsymbol{\varepsilon}}(\mathbf{n},\bar{\varepsilon}\,\mathbf{v}^{\otimes 2})] = 0 \quad , \quad \forall\, \mathbf{v} \end{cases} \tag{15}$$

which imposes consequently :

$$\begin{cases} \alpha_2 + \beta_2 + \chi_2 + \varphi_2 + \psi_2 = 0 \\ 2\alpha_2 + 6\beta_2 + \chi_2 = 0 \\ 3\chi_2 + 2\varphi_2 + 2\psi_2 = 0 \end{cases} \tag{16}$$

3.3 Final expression of the potential

For any state $\boldsymbol{\varepsilon}$, let us note $S_1(\boldsymbol{\varepsilon}) = \{\mathbf{n}\in S,\, g(\mathbf{n},\boldsymbol{\varepsilon}) > 0\}$ and $S_2(\boldsymbol{\varepsilon}) = \{\mathbf{n}\in S,\, g(\mathbf{n},\boldsymbol{\varepsilon}) \le 0\}$ the domains of the unit sphere S corresponding to vectors normal to respectively open and closed microcracks. The thermodynamic potential is then given by :

$$\begin{aligned} W(\boldsymbol{\varepsilon},\mathbf{d}) &= W_0(\boldsymbol{\varepsilon}) + \frac{1}{4\pi}\int_S \rho(\mathbf{n},\mathbf{d})\, h(\mathbf{n},\boldsymbol{\varepsilon})\, ds \\ &= W_0(\boldsymbol{\varepsilon}) + \frac{1}{4\pi}\int_{S_1(\boldsymbol{\varepsilon})} \rho(\mathbf{n},\mathbf{d})\, h_1(\mathbf{n},\boldsymbol{\varepsilon})\, ds + \frac{1}{4\pi}\int_{S_2(\boldsymbol{\varepsilon})} \rho(\mathbf{n},\mathbf{d})\, h_2(\mathbf{n},\boldsymbol{\varepsilon})\, ds \end{aligned} \tag{17}$$

This expression of the free energy can also be expressed as a sum of three terms :

$$\mathrm{W}(\boldsymbol{\varepsilon}, \mathbf{d}) = \mathrm{W}_0(\boldsymbol{\varepsilon}) + \mathrm{W}_1(\boldsymbol{\varepsilon}, \mathbf{d}) + \mathrm{W}_2(\boldsymbol{\varepsilon}, \mathbf{d}) \tag{18}$$

which makes appear the damage contribution $\mathrm{W}_1(\boldsymbol{\varepsilon},\mathbf{d})$ when all microcracks are open (the expression of the coefficients α, β, χ, φ and ψ is given in table 1) :

$$\begin{aligned} \mathrm{W}_1(\boldsymbol{\varepsilon}, \mathbf{d}) &= \frac{1}{4\pi} \int_S \rho(\mathbf{n}, \mathbf{d})\, \mathrm{h}_1(\mathbf{n}, \boldsymbol{\varepsilon})\, \mathrm{ds} \\ &= \alpha\, \mathrm{tr}(\boldsymbol{\varepsilon} \cdot \boldsymbol{\varepsilon})\, d_0 + \beta\, \mathrm{tr}^2 \boldsymbol{\varepsilon}\, d_0 + \chi\, \mathrm{tr}\, \boldsymbol{\varepsilon}\, \mathrm{tr}(\boldsymbol{\varepsilon} \cdot d_2) + \varphi\, \mathrm{tr}(\boldsymbol{\varepsilon} \cdot \boldsymbol{\varepsilon} \cdot d_2) + \psi\, \boldsymbol{\varepsilon} : d_4 : \boldsymbol{\varepsilon} \end{aligned} \tag{19}$$

and a correction term $\mathrm{W}_2(\boldsymbol{\varepsilon},\mathbf{d})$ accounting for the eventual closure of some of the microdefects :

$$\begin{aligned} \mathrm{W}_2(\boldsymbol{\varepsilon}, \mathbf{d}) &= -\frac{1}{4\pi} \int_{S_2(\varepsilon)} \rho(\mathbf{n}, \mathbf{d})\, [\mathrm{h}_1 - \mathrm{h}_2](\mathbf{n}, \boldsymbol{\varepsilon})\, \mathrm{ds} \\ &= -\frac{\mathrm{s}}{2}\, \boldsymbol{\varepsilon} : \frac{1}{4\pi} \int_{S_2(\varepsilon)} \rho(\mathbf{n}, \mathbf{d})\, [\delta_1\, \mathbf{I} + \delta_2\, \mathbf{n}^{\otimes 2}]^{\otimes 2}\, \mathrm{ds} : \boldsymbol{\varepsilon} \end{aligned} \tag{20}$$

In comparison with existing formulations which account for the opening-closure effects only in the principal directions of the state variables, we obtain consequently an enriched description of this phenomenon, since all the directions of the space are concerned here. In addition to the solution of mathematical problems, this approach makes then it possible to represent the unilateral effect in a more realistic way. The modular character of the formulation should also be noted, since the degree of accuracy p is chosen *a posteriori* by the users depending on the desired applications.

Table 1. Expression of the coefficients α, β, χ, φ and ψ.

p	α	β	χ	φ	ψ
p = 2	$\frac{15\alpha_1 + 5\varphi_1 + 2\psi_1}{15}$	$\frac{15\beta_1 + 5\chi_1 + \psi_1}{15}$	$\frac{2(7\chi_1 + 2\psi_1)}{105}$	$\frac{2(7\varphi_1 + 4\psi_1)}{105}$	0
p ≥ 4					$\frac{8\psi_1}{315}$

4. APPLICATIONS

Let us now illustrate the predictive ability of the formulation through some simple examples. We consider here the approximation order p = 4, that is $\mathbf{d} = (d_0, d_2, d_4)$, and we suppose reversible processes.

4.1 Example 1

Consider first an isotropic microcracked medium (the damage state is then represented by the damage variables $\mathbf{d} = (d_0, 0_2, 0_4)$), submitted to a hydrostatic test $\boldsymbol{\sigma} = \sigma_0 \mathbf{I}$, first under tensile loading ($\sigma_0 > 0$) and then under a compressive one ($\sigma_0 \leq 0$). The bulk modulus K of the material takes the following form :

$$\mathrm{K} = \begin{cases} \mathrm{K}_0 + \frac{2}{3}(\alpha + 3\beta)\, d_0 \;, & \text{si} \quad \sigma_0 > 0 \\ \mathrm{K}_0 \;, & \text{si} \quad \sigma_0 \leq 0 \end{cases} \tag{21}$$

with K_0 the bulk modulus of the undamaged material.

Under the condition $\alpha+3\beta < 0$, the model is then able to represent two specific aspects of the experimental behavior of quasi-brittle materials ([7]) :

- first the degradation of the bulk modulus K under tensile loading, that is when all microcracks are open ($S_2(\boldsymbol{\sigma}) = \varnothing$),
- and secondly, the recovery of this modulus under compressive loading, when this time microcracks are all closed ($S_2(\boldsymbol{\sigma}) = S$).

4.2 Example 2

Let us examine now the case of concrete weakened by an isotropic distribution of microcracks ($\mathbf{d} = (d_0, 0_2, 0_4)$), and submitted to two uniaxial tests with unit axis $\mathbf{m}$ ($\boldsymbol{\sigma} = \overline{\sigma}\,\mathbf{m}^{\otimes 2}$), under tensile ($\overline{\sigma} > 0$) and compressive ($\overline{\sigma} \leq 0$) loading. These two loading paths correspond to mixed states of opening-closure of microcracks, with a larger domain $S_2(\boldsymbol{\sigma})$ for the compressive test. Figure 1 shows the uniaxial response of the material for these both tests.

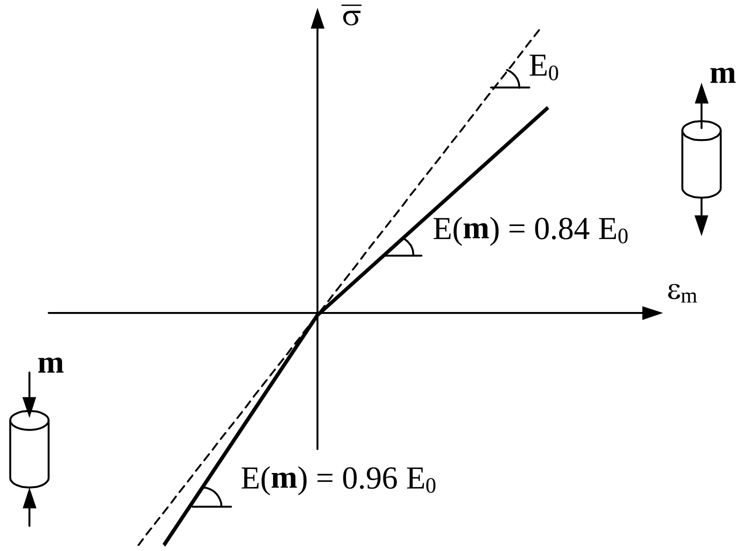

Figure 1. Uniaxial tensile and compressive tests on concrete weakened by an isotropic distribution of microcracks ($d_0 = 0.1$; $\lambda_0 = 9722$ MPa, $\mu_0 = 14583$ MPa, $\alpha_1 = 2.\ 10^4$ MPa, $\beta_1 = -1.\ 10^4$ MPa, $\chi_1 = 2.\ 10^4$ MPa, $\varphi_1 = -1{,}04\ 10^5$ MPa, $\psi_1 = -7.\ 10^4$ MPa).

One can note the degradation of the axial Young modulus E($\mathbf{m}$) of concrete in comparison with its value in the undamaged state during the tensile test, and its partial recovery under compressive conditions, according to experimental results ([6]). This example shows the ability of the model to represent the asymmetry of the behavior of quasi-brittle materials between tensile and compressive loading ([6],[7]), and more generally its capacity to account for a non linear response induced by unilateral effect.

4.3 Example 3

Consider finally the case of Vosges sandstone weakened by an isotropic transverse microcrack distribution with unit axis $\mathbf{m}$ represented by the damage variables $\mathbf{d} = (d_0, \gamma\lfloor \mathbf{m}^{\otimes 2}\rfloor, \eta\lfloor \mathbf{m}^{\otimes 4}\rfloor)$ (γ and η two constants). Figure 2 shows the roses of the elastic moduli L($\mathbf{v}$) and κ($\mathbf{v}$) of the material related to a direction of unit vector $\mathbf{v}$, when microcracks are open or closed (vector $\mathbf{v}$ is defined by spherical angles θ and ϕ in the orthonormal basis ($\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 = \mathbf{m}$)).

This figure clearly illustrates the ability of the formulation to account for the elastic moduli dependence upon the space direction (induced anisotropy) and upon the opening-closure of microcracks (unilateral effect). This example points also out the partial character of the damage deactivation through the partial recovery of the elongation moduli.

$$\frac{L(\mathbf{v})}{L_0} = \frac{L(\phi)}{L_0} \qquad \frac{\kappa(\mathbf{v})}{\kappa_0} = \frac{\kappa(\phi)}{\kappa_0}$$

Figure 2. Roses of the normalized elongation and volumetric moduli for Vosges sandstone weakened by an isotropic transverse distribution of microcracks with unit axis **m** : —○— open microcracks, —●— closed microcracks, ---- unit circle (d_0 = 0.001, γ = 0.002, η = 0.02 ; λ_0 = 3250 MPa, μ_0 = 4875 MPa, α_1 = -9. 10^3 MPa, β_1 = 4,5 10^3 MPa, χ_1 = -9. 10^3 MPa, φ_1 = -5,55 10^4 MPa, ψ_1 = -5,25 10^4 MPa) :

5. CONCLUSION

The objective of this work was to associate a particular damage mechanism (by microcrack growth) with the damage activation-deactivation phenomenon in a consistent model able to predict the complex mechanical behavior of quasi-brittle materials. The paper puts forward a new theoretical solution which makes it possible to describe the main consequences of these phenomena on the elastic properties of the materials, in particular the induced anisotropy and the elastic moduli dependence upon the opening-closure of these defects. This approach is rigorous and exhibits a deep physical justification, owing to the damage characterization chosen and to the introduction of some micromechanical considerations. The first comparisons with experimental data already give very interesting results. Further investigations must now be conducted in order to complete the model, that is to establish a damage evolution law in accordance with the physical mechanisms involved. Such research is currently in process.

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