

# APPLICATION OF THE STABILITY LOBES THEORY TO MILLING OF THIN WORKPIECES, EXPERIMENTAL APPROACH

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## **Abstract :**

*The optimisation of cutting conditions in High Speed Machining (HSM) requires the use of a vibratory approach in order to avoid a fast deterioration of the tool and of the spindle, as well as a loss of quality of the surface roughness. We suggest a transposition of the method of stability lobes to the case of the milling thin parts, which is very typical from the aeronautical manufacturing context. After having modelled the dynamic behaviour of a blade and of the cutting efforts in side milling, we describe the zones of machining instability. An experimental validation permits us to emphasise the transition from stability to instability, in accordance to our theoretical results. The experimental profile is then compared with a computed profile. A decomposition of the different situations of contact between the tool and the part permits to show the influence of back cutting in the model. Tests of machining permit then to quantify its role. The objective of these works is the definition of a quick methodology for determining the optimal cutting conditions in a given industrial machining configuration.*

**Keywords : Stability lobes, milling, chatter**

## **1 Introduction**

The optimisation of the cutting conditions for the machining of turbo machine blades in High Speed Machining (HSM) requires to set under control the vibratory phenomenon. Vibrations of machines, that withstand important efforts and are optimised in term of mass, can be controlled, notably by the appropriate filter. Vibrations of the part-tool couple are more difficult for eliminate. Many research publications aimed at searching of an efficient strategy in order to avoid tool vibrations, but the proposed solutions have not yet been completely validated on industrial applications. Our objective is to apply and to experimentally validate the approach of stability lobes to the case of milling of thin walled parts with a constant surface roughness and an optimised depth cut, and to identify the back cut effect.

## **2 Theoretical approach**

The analysis of the chatter during the machining of metals has shown the mechanism of regenerative vibration to be the main cause of vibrations. The analytic model resulting from these works shows the stable and unstable zones. Studies concerning the nonlinear aspects of the process have allowed to better understand the limits of the model and have shown the nonlinearities to be mainly located in the tool-part contact. Initially developed for turning machining, this approach has been largely used for many types of milling. Stability lobes have been largely validated experimentally, although through simple experiences [1]. In order to

enrich the initial analytic model, many numerical models have been suggested [2] [3], and have notably allowed to make the connection between stability lobes and step by step time simulation.

The elimination of regenerative chatter is obtained using various methods :

- The diminution of the chip section (feed reduction for example) provides an elementary way for obtaining stable conditions, and is therefore the most employed technique when information lacks.

- The modification of cutting conditions, in consideration with the lobes of stability of the considered machining, allows to preserve a short machining time, but requires experimentation or preliminary modelling.

- The utilisation of variable tooth pitch milling also allows to eliminate vibrations for tools including several teeth, and for given cutting conditions [4].

It is also possible to perform a real time compensation of the vibrations, which requires the detection, the processing then the performance of the action of compensation itself. The detection can be made using an accelerometer, by effort measurement, by displacement measurement, by microphone [5] or by monitoring the intensity of the spindle driving [6]. The data processing, either realised by spectral analysis or time analysis, either sophisticated or very simple, has in any case to be calibrated and very quickly performed in order to prevent too strong vibrations to appear. The action of compensation can be performed by speed modification of feed [7] or of the frequency of rotation [8].

Others strategies aim at eliminating the source of vibrations itself, i.e. to obtain a better compliance between the vibrations of the tool-part couple and the spindle frequency. In that purpose, the sinusoidal variation of the spindle speed [9] is a solution according to given cutting conditions. The multilevel random variation [10] functions may also provide a solution for a range of cutting conditions.

Several axes of research are particularly active :

It has been shown in the 3S laboratory of Grenoble (France), with which the authors have collaborated for this work, that numerical modelling allows to better and better predict the dynamic effects of machining :

- Approaches based on real time compensation require to take in account the problem of regenerative vibrations, by allowing sophisticated variations around the nominal cutting conditions [10].

- CAD-CAM systems can presently take into account some forced vibration models and dynamic machine capacities [11]; they can also in some cases take into account regenerative vibrations. Finally, others processes such as high speed sawing by blade or by disc also suffer from vibrations of regenerative type [12].

As a first step, we have applied the theory of stability lobes to the side milling of our flexible parts, the tool being considered as rigid.

We consider the milling process as defined in figure 1, one consider a tool with  $p$  teeth, turning to  $N$  revolutions by minute with an feed by tooth of  $h$  and a width of cut  $b$ .

Only effects related to the degree of freedom of the system are here taken in account

The part is characterized by its stiffness  $k$ , its damping  $\zeta$  and its natural frequency  $\omega_0$ .

We are first looking for the conditions in which the oscillations of tooth  $n+1$  have an more or less important amplitude in comparison with tooth  $n$ .

We considers that the effort of cut is linked to the width of cut  $b$  and to the specific cut coefficient in the direction of vibration  $K_s$  as equation (1).

$$F_n = -b K_s (U_n - U_{n-1}) \quad (1)$$

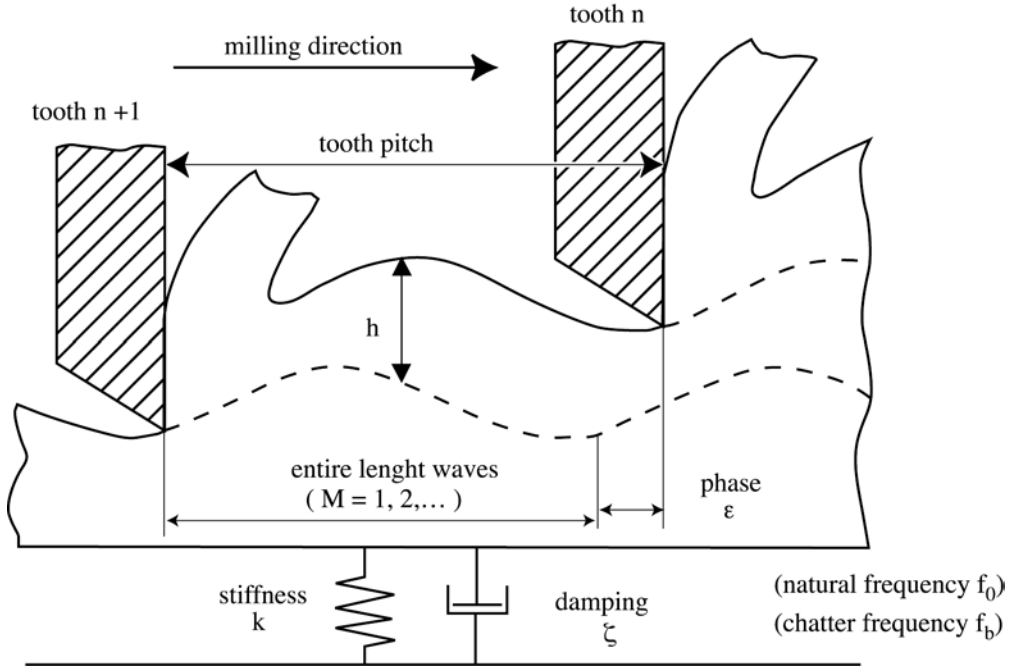


Fig 1 : Cutting process with regenerative vibrations

In a permanent regime and in the linear area, an harmonic vibratory regime can be observed. In the complex domain, the displacement of tooth  $n$  depends on the cutting force and on the dynamic stiffness of the system :

$$U_n = Z_{(f_b)} F_n \quad (2)$$

The two previous equations allow to express :

$$U_{n+1} = \frac{b K_s}{Z^{-1} + b K_s} U_n \quad (3)$$

The condition of stability limit can be written as follows :

$$|U_{n+1}| = |U_n| \quad (4)$$

By taking account the previous equation, one can obtain :

$$Re \{ b_{limite} K_s Z \} = -\frac{1}{2} \quad (5)$$

Let us consider now that the dynamic stiffness of the system can be written as :

$$Z_{(f_b)}^{-1} = k \left( 1 - \left( \frac{f_b}{f_0} \right)^2 + 2 i \zeta \frac{f_b}{f_0} \right) \quad (6)$$

The axial depth at the limit of stability can then be expressed as :

$$b_{limit} = -\frac{k \left( 1 - \left( \frac{f_b}{f_0} \right)^2 \right)^2 + 4 \zeta^2 \left( \frac{f_b}{f_0} \right)^2}{2 K_s \left( 1 - \left( \frac{f_b}{f_0} \right)^2 \right)} \quad (7)$$

We have now to express it according to the frequency of spindle  $N$ . The phase between the tooth frequency and the chatter frequency shows  $M$  entire chatter periods, and a remaining fraction denoted  $\varepsilon$  :

$$\frac{f_b}{N p} = M + \frac{\varepsilon}{2\pi} \quad (8)$$

The phase between the two consecutive teeth passage, can be directly expressed as follows :

$$\varepsilon = \text{Arg}(U_{n+1}) - \text{Arg}(U_n) = \pi - 2 \text{arcTan} \left[ \frac{2\zeta \frac{f_b}{f_0}}{1 - \left(\frac{f_b}{f_0}\right)^2} \right] \quad (9)$$

We can then deduce the link between the spindle frequency and the chatter frequency :

$$\frac{f_b}{N p} = M + \frac{1}{2\pi} \left( \pi - 2 \text{arcTan} \left[ \frac{2\zeta \frac{f_b}{f_0}}{1 - \left(\frac{f_b}{f_0}\right)^2} \right] \right) \quad (10)$$

The determination of the chatter frequency according to the speed of spindle by the previous equation and the previous expression of the depth limits allows to obtain the stability lobes.

As a second step, we have used a software developed in the 3S laboratory [13]. This software is dedicated to the time domain study of the vibratory behaviour of the milling system. The only phenomenon taken in account is the phenomenon of self-maintained vibration. Only a macroscopic scale of modeling of the cutting process is considered and used for simulation. At each instant is calculated the tool-workpiece interaction. An implicit time integration scheme is used to solve the nonlinear system. The mechanical behaviour of the whole workpiece-machine system is modeled by truncated modal base, whose coefficients have been determined by spectral analysis and by finite element. The calculation of cutting forces uses two models for taking into account vibratory aspects. In effective cut, the model of [14] is applied; during back cut, the model described in [15] is applied. The simulation is performed on side milling of flexible parts with a deformable tool, the part being considered as non deformable on the totality of the volume of milling, all solid motions being allowed. The main results provided by the software are the state of the manufactured surface and the vibrations of the part and of the tool. We have represented in figure 5 and figure 6 the results obtained on our experience.

### 3 Experimental approach

#### 3.1 Experimental procedure

In order to check for the consistence between our study and a given industrial case, we have chosen to manufacture a flexible part having a behaviour similar to a blade and allowing the milling of several rails. By using the configuration given figure 2 and the technique described in [1], we can limit movements of the part to an single degree of freedom.

In order to experimentally cover the different zones of stability lobes, we use a mill with a diameter of 40 mm (4 teeth,  $\kappa = 90$  and  $\gamma_p = 0$ ) considered as infinitely rigid in comparison of the part (see figure 2). The absence of helical allows us a simplified modelling and does not require the utilisation of the different planes of tools [3]. The maximal regime of rotation of the

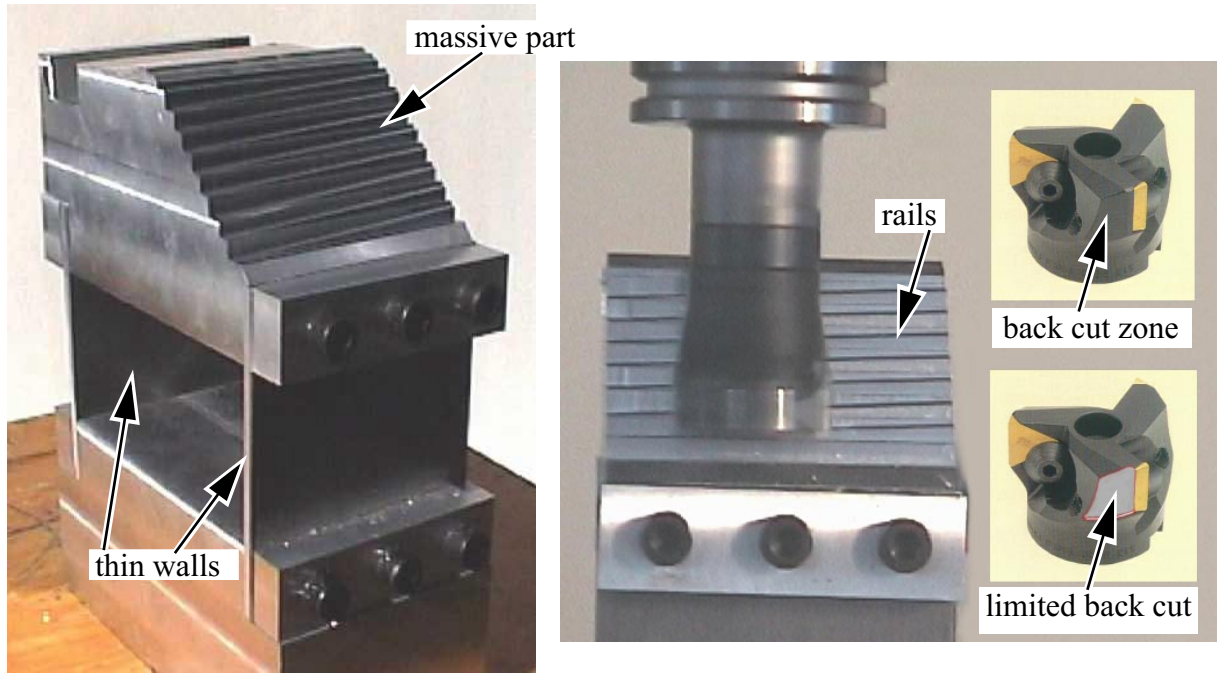


Fig 2 : Tested parts, machined rails and tool drills

tool is 8000 rev/min. The test part has been dimensioned for obtaining a first natural frequency of around 180 Hz. The first natural frequency of the tool-spindle system is approximately of 1700 Hz, what does not have to affect our results.

The machined rail allows to make vary the depth of cut from 0 to 5.5 mm and allows to identify the stability-instability transition, keeping the other cutting conditions constant (width of work  $A_r = 2$  mm and feed  $f_z = 0.15$  mm/tooth). Each rail corresponds to a different spindle speed. Tests are firstly performed with the tool in configuration of strong back cutting effect (initial tool), then in configuration of limited back cutting (figure 2), corresponding to a reduction of the friction surface of the tool, limited to the thickness of the tip.

### 3.2 Experimental results on the stability transition-instability

After performing the tests, machined surfaces which are function of the vibratory behaviour of the part are obtained. For some rails, two zones clearly appear: the stable area, where the surface roughness is consistent with the cutting process, and the unstable area where the surface can be of poor quality (see figure 3). In this case, we can notice defects of 2<sup>nd</sup> and 3<sup>rd</sup> order, randomly distributed or consequence of a periodic alternation of irregular undulations, and an increase of the roughness due to changing cutting conditions. On the other hand, the case of milling with strong back cut effect appears more problematical than the case with limited back cut. It seems actually that the back cut provokes here an increase of efforts transmitted to the part and therefore a more immediate vibratory phenomenon with an important amplitude.

We are more particularly interested in the depth of cut limits under which the machining process seems stable. Using a 3D roughness tester, we can identify this parameter and draw these measures on the theoretical layout of stability lobes (figure 4). For tests with limited back cut, the experimental interpretation is consistent with the theory of stability lobes. Generally speaking, the areas deduced from theoretical transitions have a good correspondence with the

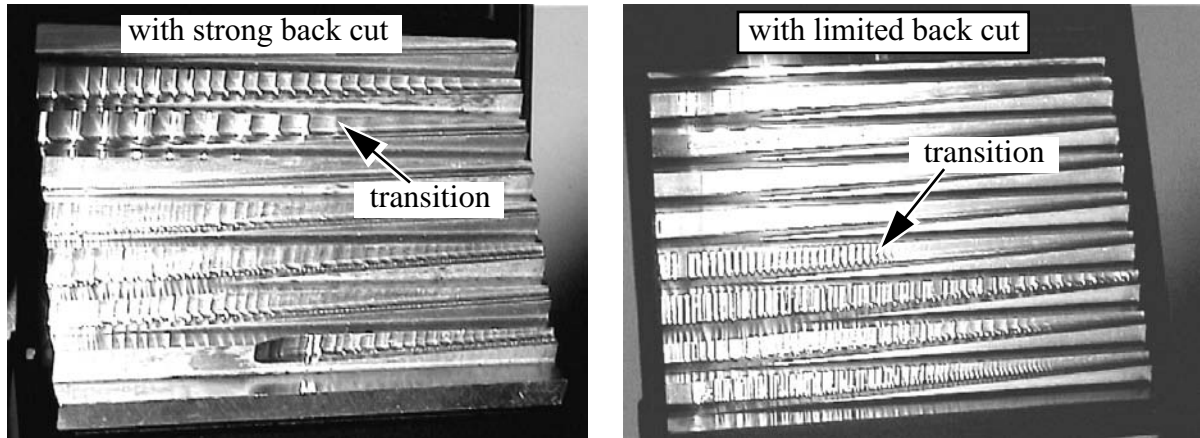


Fig 3 : Stability-instability transition and influence on machined surface

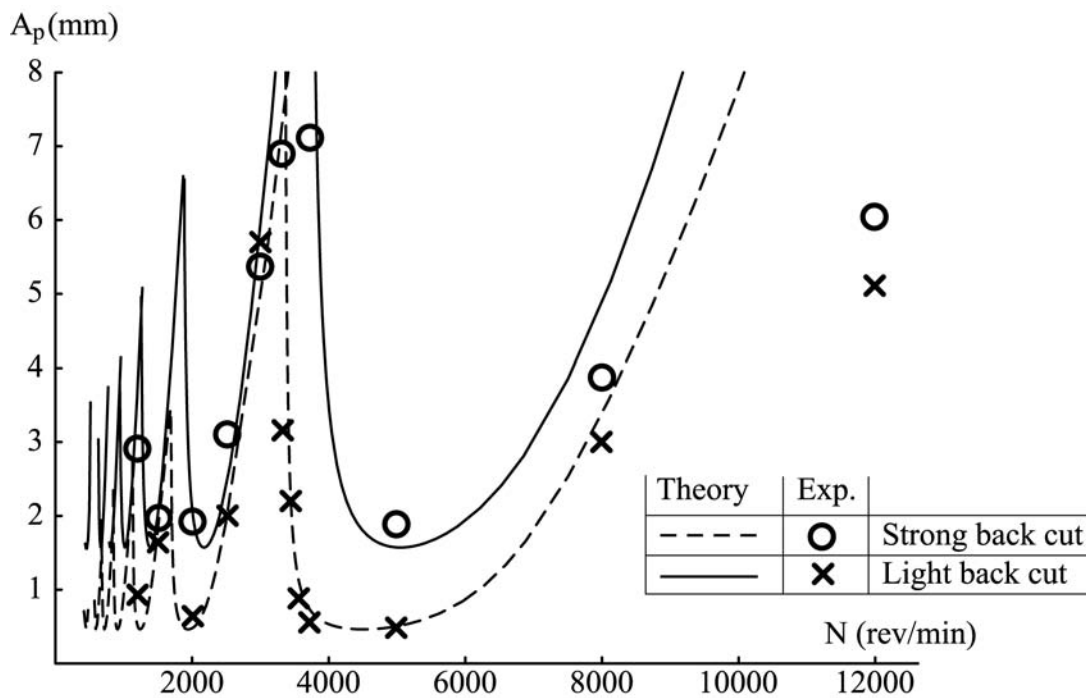


Fig 4 : Theoretical and experimental cut depth limit

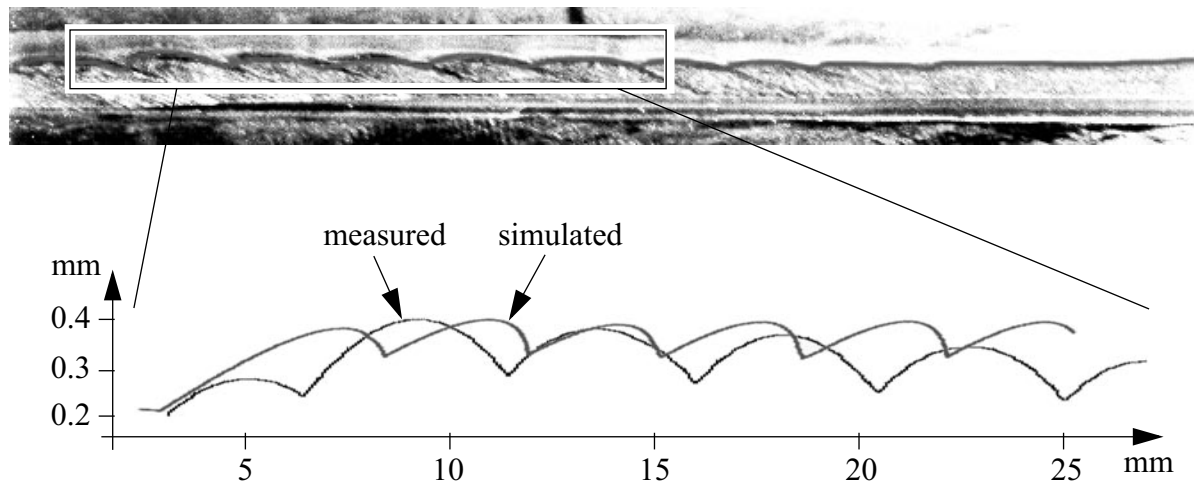
depths of cut noticed during tests. Only the rail obtained at 12000 rev/min does not satisfy the layout of the lobes, which can be explained by cutting conditions not recommended by the tool manufacturer.

Tests with a strong back cut show a comparable dynamic behaviour but a modification of stability lobes parameters can yet be noticed. In order to set our tests in correspondence with the layout of the lobes, it is necessary to modify the specific cutting coefficient  $K_s$  in the direction of the degree of freedom of 250 MPa to 415 MPa. The damping coefficient  $\zeta$  during cutting is then around 0.02 instead of 0.04 initially. This can be explained by an increase of the efforts of the body of the tool, consequence of the friction. The interpretation of a reduction of  $\zeta$  is more

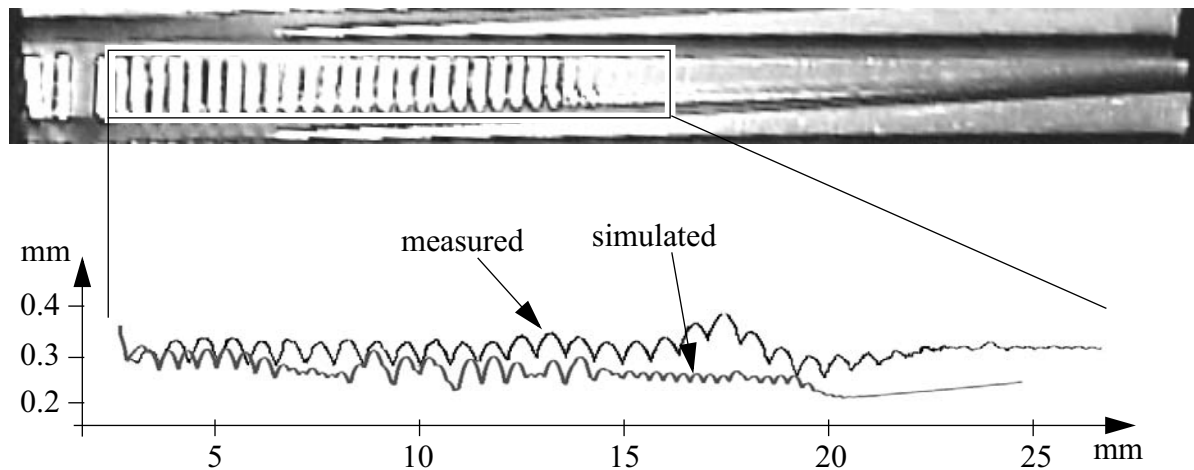
difficult to provide. An accurate modelling of the back cut phenomenon will be required for validating the new values of these parameters. Similarly to the case of the limited back cut, the test with  $N = 12000$  rev/min is difficult to interpret.

### 3.3 Confrontation of theoretical and manufactured profiles

The example of figure 5 and figure 6 shows a good coherence between the numerical



*Fig 5 : simulated and measured profile for strong back cut*



*Fig 6 : simulated and measured profile for slight back cut*

simulation of the profile and its measure on the roughness tester. The transition zone between stability and instability is identical, but the amplitude and the period of roughness defects vary from 10 % to 40 %. This imprecision can only be observed in the unstable area and can be due to the simplification of the cutting and back cutting model. Globally, tests with strong back cut confirm its bad influence on the surface roughness, this configuration being naturally harmful.

The engaged length of the tool is much superior to the advance by tooth. It is therefore difficult to directly interpret the vibratory behaviour of the part from the manufactured profile since the generated surfaces are almost completely re-manufactured by the passage of the next teeth.

The criteria of quality of a surface roughness (typical  $R_a$ ) hardly allow to validate the stability of the milling process. We have chosen the occurrence of a visible jump on the profile for characterising the vibratory behaviour change. In this way, it is possible to determine rapidly the optimal cut depth on the manufactured rail.

#### 4 Perspectives

The adaptation of the methodology to slim blade requires to take into account other phenomenon. The effects of edges can no longer be neglected and it is therefore necessary to integrate the modes of twist of the part in the model of stability lobes. The tool with a diameter of 16 mm is notably adapted to end milling, and is characterised by a weak helix angle: the distribution of cutting efforts is therefore different. On the other hand, its deformation is no longer negligible and must be taken into account in the model. The first tests on a blade with a thickness of 6 mm (figure 7) with the adequate tool shows consistent results. Further experi-

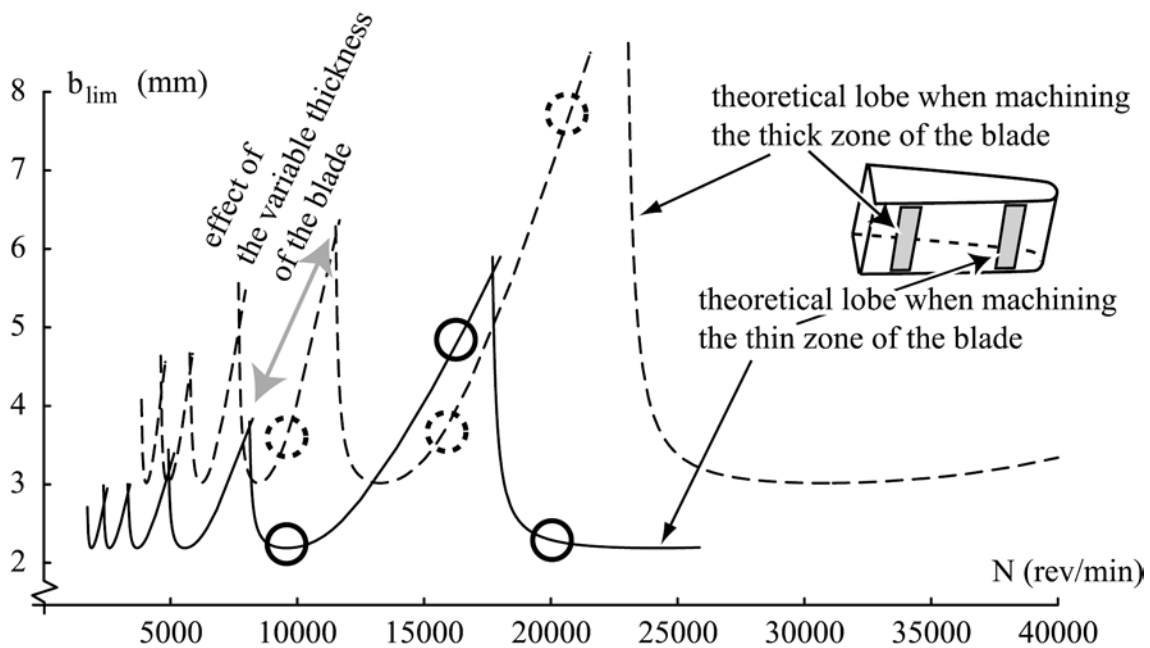


Fig 7 : Typical behaviour of the 6 mm thick blade

ments are in progress in order to model 3D stability lobes in order to take into account the modification of the natural frequency of the part during the manufacturing process.

The experimental approach based on rails machining can provide here an interesting alternative to the complexity of the theoretical models due to the multiplicity of parameters. A global model using several vibration modes (part and tool) could guide an experimental procedure depending on the expected shape of rails.



In the longer term, the integration of this methodology (numerical modelling and experimental refitting) in a CAD-CAM system could allow to better define the cutting conditions, on the base of a depth of cut limited according to the geometry of the part. Moreover, the comparison of the different results, according to the configuration and the mode of milling, could lead to suggest a strategy of elaboration of the programmed tool path insuring the quality of the manufactured surface.

## 5 Conclusion

Experimental results show a good correlation with calculations based on the theory of stability lobes in the case of rails by side milling on a flexible part. This technique could allow to determine very rapidly and very easily the vibratory characteristics of a tool-part couple. The back cutting process is interpreted in the model of lobes as an increase of the cutting efforts, which provokes an earlier instability. The numerical simulation allows to quantify the influence of several parameters and to guide more precise experimentation. The case of the blade makes obvious the limit of our initial modelling and shows the necessity for taking into account new parameters. This approach will so have to evolve towards an integration of the methodology in CAD-CAM systems, for the determination of the cutting and strategy conditions of an optimal manufacturing process.

## References

- [1] S.A. JENSEN, Y.C. SHIN, “*Stability analysis in face milling operations*”, Journal of Manufacturing Science and Engineering, N°121, 1999, pp. 600-614.
- [2] J. TLUSTY, F. ISMAIL, “*Basic non-linearity in machining chatter*”, Annals of the CIRP, N°30, 1981, pp. 299-304.
- [3] Y. ALTINTAS, E. BUDAK, “*Analytical prediction of stability lobes in milling*”, Annals of the CIRP, N°44, 1995, pp. 357-362.
- [4] H. FU, R. DEVOR, S. KAPPOR, “*The optimal design of tooth spacing in face milling via dynamic force model*”, Proc. of the 12th NAMRC, 1984, pp. 291-297.
- [5] T. DELIO, S. SMITH, J. TLUSTY, “*Use of Audio Signals chatter detection and control*”, Journal of Engineering for Industry, N°114, 1992, pp. 146-157.
- [6] S. SOLIMAN, F. ISMAIL, “*Chatter detection by monitoring spindle drive current*”, International Journal of Advanced Manufacturing Technology, N°13, 1997, pp. 27-34.
- [7] J. TLUSTY, M. ELBESTAWI, “*Analysis of transients in a adaptive control servomechanism for milling with constant force*”, Transaction of the ASME, 1976, N°76-WA/prod-31.
- [8] S. SMITH, J. TLUSTY, “*Stabilizing chatter by automatic spindle speed regulation*”, Annals of the CIRP, N°41, 1992, pp. 433-436.
- [9] R. RADULESCU, S. KAPOOR, R. DEVOR, “*An investigation of variable spindle speed face milling for tool-work structures with complex dynamics*”, Journal of Engineering for Industry, N°119, 1987, pp. 266-280.
- [10] E. I. AL-REGIB, “*Machining systems stability analysis for chatter suppression and detection*”, Ph.D. Dissertation, 2000, University of Michigan (USA).
- [11] A. DUGAS, J.J. LEE, J.Y. DASCOËT, “*Feed rate and tracking errors simulation in high speed milling*”, Proc. of the 4th ESAFORM conf., 2001, pp. 667-670.
- [12] A.G. ULISOY, C.D. Jr MOTE, “*Vibration of wide band saw blades*”, Journal of Engineering for Industry, N°104, 1982, pp.71-78.

- [13] G. PEIGNÉ, H. PARIS, D. BRISSAUD, “*Simulation of non-linear vibrations and surface generation in flank milling : application to HSM*”, Proc. of the IDMME 2002, Clermond Ferrand, France, 2002, p.126.
- [14] W.A. KLINE, R.E. DEVOR, J.R. LINBERG, “*The prediction of cutting forces in end milling with application to cornering cuts*”, International journal of machine tool, 1982, N°22/1, pp. 7-22.
- [15] D. W. WU, “*A new approach of formulating the transfer function for dynamic cutting process*”, Journal of Engineering for Industry, N°111, 1989, pp. 37-47.