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Strategic Noise in Competitive Markets for the Sale of Information

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Abstract

This paper shows how informed financial intermediaries can reduce their trading competition by designing optimal incentive compatible contracts for the sale of information. With fund management contracts – indirect sale of information – banks can credibly commit to collaborate and add noise into prices. This is a way to circumvent the Grossman and Stiglitz (1980) paradox: when information is costly, by committing to add noise, the banks can recover the cost of collecting information and enter the market. By contrast, when information is costless, even with a large number of sellers of information entering the market prices are not fully informative.

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1. Introduction

Financial intermediaries such as commercial banks, investment banks, securities houses and rating agencies all invest in the collection of information. In fact, producing information and selling it is considered a *raison d'être* for financial intermediation (see Allen (1990) and Ramakrishnan and Thakor (1984)). Another purpose of information generation is trading. In particular, intermediaries use information i) for their proprietary trading activities, ii) for managing funds on behalf of their clients (indirect sale of information), and iii) for giving trading recommendations (direct sale of information)¹. We know quite a bit about when information collection will be profitable and when it will not. In the classical literature on competition and aggregation of information, Kyle (1984) and Admati and Pfleiderer (1988) have shown that in the case of imperfect competition prices become efficient as the number of traders goes to infinity. Grossman and Stiglitz (1980) have shown that when a large number of perfectly competitive agents trade in the market, prices fully reveal their private information. Hence, no profit can be earned on collecting costly information, unless prices represent a noisy signal of the information of the informed agents.

Financial institutions collect information, and some are simultaneously engaged in proprietary trading activities and in indirect sales of information.

¹The latter two cases constitute sales of information. Admati and Pfleiderer (1986) have defined two ways of buying information: by subscribing to newsletters or bulletins (direct sale of information), or by signing a contract with a financial institution which manages an investment fund on behalf of its client (indirect sale of information)

This duality in the use of the information inside the firm could tempt financial intermediaries to add noise to the signal they sell in order to enhance the profitability of their own trades. A potential conflict of interest thus arises between the bank and the client who delegates his money to the fund manager.²

Competition among intermediaries may temper these incentives, but it is not clear that it will eliminate them. The interesting question this raises is: what kinds of contracts will emerge to resolve this conflict of interest in a competitive financial intermediation market?

In this paper we address this question by considering competitive financial intermediaries who trade at the same time for their own account and for their clients. This is in contrast to Biais and Germain (2002) who consider contracts in the case of a monopolistic information seller. The intended contribution of this paper is to show that fund management contracts³ allow financial intermediaries to collude and commit to add noise into prices to increase their profits. We show then how bankers can diminish competition

²Indeed, recent financial scandals have shown how financial intermediaries can use information in a strategic way, for example when financial analysts issued systematically bullish recommendations on stocks of companies that are clients of the banks: Michaely and Womack (1999) provide empirical evidence of such a type of conflict.

³In this paper we assume that contracts are public information. It is indeed the case that contracts proposed by bankers to the clients for fund management are observable. Fund management contracts are usually standardized and the type of contract proposed by fund managers or hedge funds are in most cases known by the market. This information is usually not private.

between their trading divisions via fund management contracts.

There are not many empirical papers documenting collusion in finance. Marsh (1998, 1999) gives empirical evidence of implicit collusion between investment banks for the underwriting of new issues. He shows that this financial service is overpriced and sub-underwriting investors make excess profits. As in fund management activities, contracts are public information.⁴ Christie and Schultz (1994) also document collusion but between the market makers on the NASDAQ. We show that in the case of fund management, the introduction of public contracts allows the banks to collude and to reduce competition and diminish the informativeness of prices. The banking industry is then able to create an endogenous noise in the market in addition to the exogenous noise of the liquidity traders.

In particular, we show that there is a set of contracts which: i) allow financial institutions to commit to add an optimal level of noise in the market, ii) increase the profitability of the combined trades of the proprietary trading and the funds managed and iii) address the competition problem between these institutions by increasing their aggregate profit. In this model, without sales of information, prices are fully revealing as soon as two competitors are trading in the market, which makes their individual expected profits zero. In this case, as in Grossman and Stiglitz (1980), prices fully reveal the private information, and if collection of information is costly no-

⁴These arguments have been taken seriously by the Office of Fair Trading and the Monopolies and Mergers Commission in the UK, see Research Paper 6, Office of Fair Trading.

body will do it ; this is the same result as the Grossman and Stiglitz (1980) paradox but with imperfectly competitive agents. Introducing the possibility of indirectly selling information is a way to circumvent the paradox because financial institutions can now credibly commit to add noise. Pareto optimality is restored due to the contracts used, and banks can collect costly information. Selling information is thus a way to strategically endogenize the amount of noise in the market.

Different papers are related to this study. Fishman and Hagerty (1995) have analyzed competition if information is sold directly, not through a fund like in this paper. They show that by selling information the informed agent can commit to trade aggressively and extract a larger part of the profit. Admati and Pfleiderer (1986) analyze also direct sales of information and show that the informed party adds noise to the signals he sells in order to reduce the sensitivity of prices to trades. They assume that the precision of the signal is contractible, and analyze contracts whereby the fees paid to the seller of information are a function of this precision. This is different from this paper where the remuneration of the information seller is contingent on the profits of the fund. Allen (1990) studies the reliability problem when information is sold. He shows how this creates an opportunity for intermediation as in Leland and Pyle (1977). In this paper we do not address the reliability problem as the ability to collect information and the ability to use it are assumed to be the same for the banks. Ramakrishnan and Thakor (1984) analyze the sufficiency conditions for coalitions for information producers

to arise endogenously in a competitive equilibrium. Our paper is similar to Ramakrishnan and Thakor (1984) in that we also consider situations in which it is optimal to collaborate and sell information. The difference is that, unlike Ramakrishnan and Thakor (1984), we also consider proprietary trading by intermediaries. In a different setup Zurita (2004) shows that the existence of intermediaries in financial markets is necessary to overcome adverse selection problems when utility traders and speculators trade in the market.

In Section 2 we study the competition between N banks and derive the optimal contracts. In Section 3 we study competition when the number of banks is large and information is costless. In Section 4 we endogenize the amount of noise and the number of banks when information is costly. In Section 5 we study the robustness of our model. Finally in Section 6 we derive some concluding remarks. All proofs are in the Appendix.

2. Competition among N sellers

2.1. The model

There is one risky asset \tilde{v} and one riskless asset normalized to zero. There are two equiprobable states of the world, $u = \bar{v} + \varepsilon$ and $d = \bar{v} - \varepsilon$, where ε is a positive real number. There are three types of agents in the market:

- N risk neutral informed traders, perfectly informed, who submit an

order $X_j(\tilde{v}) = \omega_j, j = 1, 2, \dots, N,$

- noise traders who transmit equiprobable orders $\omega_l = (L, 0, -L),$
- competitive risk neutral market makers, who observe the orders $\omega = (\omega_1, \dots, \omega_N, \omega_l),$ where orders are anonymous.⁵ Because of anonymity the market-makers cannot ascertain the origin of the different orders they receive. Market-makers quote:

$$P(\omega) = E(\tilde{v}|\omega)$$

The microstructure is different from Kyle (1985). Indeed, the market makers do not observe the sum of all the orders but the different orders coming from the informed and the uninformed traders. This approach is similar to that of Dow and Gorton (1997) where market makers observe orders stemming from the informed and uninformed hedgers. This structure is different from Glosten and Milgrom (1985) because the informed and the uninformed traders are simultaneously present in the market.⁶

⁵One could modify the structure of the model considering that the banks could transmit non anonymous orders. In this case we could assume that the banks transmit non informative orders (brokerage) w_{l_i} and informative orders w_i (Germain and Vanhems (2003) study this type of microstructure in an another context). This would lead to the same type of result.

⁶It can be shown that qualitatively similar results are obtained in the Glosten and Milgrom (1985) set-up when the informed trader and the liquidity trader are not simultaneously present in the market.

2.2. Equilibrium without sales of information

2.2.1. Pure strategies equilibrium

We analyze Perfect Bayesian Equilibria. In equilibrium, given the rational beliefs of the market makers, and the corresponding price function, each insider chooses X_j to maximize his expected profits:

$$X_j(v) \in \text{Argmax}_{x_j} E[(\tilde{v} - P(\omega_1, \omega_2, \dots, \omega_N, \omega_l))x_j | v].$$

To avoid being spotted, the insider chooses $X_j(v)$ in $(L, 0, -L)$. The choice is straightforward: in state u he buys L , and sells $-L$ in state d . Because there are more than two informed agents, market makers observe either (L, L, \dots, L) , $(L, L, \dots, 0)$ or $(L, \dots, L, -L)$ in state u , and $(-L, -L, \dots, -L, L)$, $(-L, -L, \dots, 0)$ or $(-L, -L, \dots, -L, -L)$ in state d . Therefore, the price is $u = \bar{v} + \varepsilon$ in state u and $d = \bar{v} - \varepsilon$ in state d . The expected profit of each informed agent is 0 because market makers can always infer the presence of an informed trader in the market. Out-of-equilibrium beliefs, which support this equilibrium, are that market makers quote u if they observe a positive order different from L and d if they observe a negative order different from $-L$. This leads to the following proposition:

Proposition 1 *There is no Perfect Bayesian Equilibrium with pure strategies in the game with costly collection of information and with at least two traders who collect information and trade in the financial market.*

This proposition asserts that it is not worth collecting costly information. This result is similar to the one obtained by Grossman and Stiglitz (1980), except for the fact that, in this case, traders are not competitive but strategic, as in Kyle (1985). This leads to the Grossman and Stiglitz (1980) paradox: if information is costly, prices are no longer informative because no informed trader wishes to trade.⁷ We show that the introduction of contracts can circumvent this paradox.

2.2.2. Adding noise and collaboration

How can the Grossman and Stiglitz (1980) paradox be overcome? One natural way to do it would be to increase the level of noise in the market. However, because the level of noise is exogenous and constant, this is not feasible. Another way would be for the informed j to trade randomly and commit ex ante to a certain strategy. More precisely, in state u with probability $\alpha_j(N)$ the N insiders trade 0 and with probability $1 - \alpha_j(N)$, they trade L (it is symmetric in state d). In doing so, the insiders mimic the noise traders and add noise into prices. We mention this case only to highlight the benefits derived from adding noise. It is not plausible in practice that a large informed trader can commit to add noise. As Laffont and Maskin (1991) point out:

”...it is difficult to see how the large trader can commit himself

⁷In this model, because the number of liquidity traders is constant and equal to one, there is not enough noise to hide the informed demand and make profits. A constant number of liquidity traders is equivalent to a constant amount of exogenous noise.

to a pricing strategy beforehand. To begin with there is no "beforehand." Often, a trader is in the market only because he has acquired private information. Before obtaining this information, he may not foresee his participation and so cannot contemplate what his strategy will be."

The following proposition shows that without any coordination it is not optimal to add noise.

Proposition 2 *If the informed traders $j = 1, 2, \dots, N$ could commit to a trading strategy $\alpha_j(N)$ to play 0 with probability $\alpha_j(N)$ in the states u and d , and L (or $-L$) with probability $1 - \alpha_j(N) > 0$ in state u (or d). At the equilibrium, they would set:*

$$\alpha_j(N) = 0.$$

If the informed trader could commit ex ante to a symmetric trading strategy where $\alpha_j(N) = \alpha(N)$ the aggregated expected profit is:

$$NE(\pi_j) = 1/3\alpha(N)^{N-1}(1 - \alpha(N))\frac{4\alpha(N) + N(1 - \alpha(N))}{N(1 - \alpha(N)) + 2\alpha(N)}L\varepsilon$$

each of the N informed traders plays $\alpha^(N)$, the unique solution in $[0, 1]$ of the following cubic equation:*

$$\alpha^3(-n^3 + 3n^2 + n - 3) + \alpha^2(3n^3 - 4n^2 - 5n - 6) + \alpha(-3n^3 - n^2 + 3n + 1) + (n^3 + 2n^2 + n) = 0 \quad (1)$$

where $n = N - 1$.

In the presence of competition, informed traders never wish to randomize their trades. So, it is not optimal to add noise. Indeed, if any trader plays 0 the others would free ride in the noise added by the randomization. Again, we face the Grossman and Stiglitz (1980) paradox because prices fully reveal the private information of the two traders. The result of this proposition contrasts with Biais and Germain (2002) where ex ante the monopolist informed trader wishes to add noise ; there exists an $\alpha^* > 0$ that maximizes the expected profit of the monopolistic trader. This is due to the competition effect which drives the competitors to play the pure strategy of choosing L with probability 1.⁸ Nevertheless, in the next proposition, we show that each trader would be better off if traders could collaborate. As in a prisoner's dilemma, the best outcome would be the collaborative one. In that case, we would have an implicit collusion or complicity between financial institutions which would maximize the joint profit of the banking industry. Assuming that the firms are symmetric and enter the market, we derive the optimal symmetric randomization strategy for each of them.

When informed traders could commit ex ante to a trading strategy, playing 0 involves camouflage because market makers do not know if the null trade stems from an informed or a noise trader. Thus, they create an endogenous noise effect which can be measured by the probability to play 0, which is the probability to mimic liquidity trades and to strategically decrease the reaction to the private information. Moreover, this manipulation

⁸Playing 0 with probability 1, which is equivalent to not entering the market is an equilibrium too.

is costless and the noise added is the maximum camouflage for the oligopoly in the financial market. ⁹ As a consequence, if the traders were able to collaborate by committing to this costless noisy strategy, then it would be optimal for them to play α^* , which maximizes the profit of the coalition. But even if they could collaborate, it is not ex post optimal to play 0. This mixed strategy is not an equilibrium because: i) informed traders are not indifferent between playing 0 or L in state u , or playing 0 or $-L$ in state d ; and ii) there is no credible commitment. The only way to give rise to optimal mixed strategy and to credible commitment is to introduce contracts for the sale of information. In the presence of competition, we will show that the banks can use public contracts signed with their clients to commit to a certain strategy.

2.3. Sale of information

2.3.1. The contract

Now we consider the possibility of selling information. There are N sellers of information who manage N different funds. Each client delegates his money to a manager. Each fund manager makes a take-it-or-leave-it offer and afterwards receives a perfect information. The trading information collected by the fund managers is short-lived, so renegotiation of the contract

⁹Biais and Germain (2002) show that it is never optimal to trade $-L$ in state u because this way of adding noise is costly. The trader playing in u as if he were in d gives him good prices but this manoeuvre is too costly.

after having received the information is physically impossible.¹⁰ Moreover, the contracts are public, so each competitor can observe his competitor's contract at zero cost. In the model, all the banks are symmetric: i) their ability to collect information is the same ii) they have access to clients who go randomly to fund managers.¹¹

There are N informed sellers of information. The contract specifies: i) the trading volume of each fund j , y_j , authorized by the client on the risky asset; and ii) the compensation of the seller of information, $t(\pi_j^f)$, contingent on fund's profits where π_j^f are each j fund's profits and $t(\cdot)$ the function which maps the profits of the funds into transfers. Each client receives:

$$\pi_j^f - t(\pi_j^f)$$

and the individual rationality condition of the client is:

$$E(\pi_j^f - t(\pi_j^f)) = k \tag{2}$$

In this model we suppose that the client accepts the offer if it gives rise to expected net profits greater than his (known) reservation level k . As in Biais and Germain (2002), k can be interpreted as reflecting the bargaining power of the customer in her negotiation with the informed agent.

¹⁰Nevertheless, Caillaud Jullien and Picard (1995) and Dewatripont (1988) show pre-commitment effects with public contracts even in the case of a secret renegotiation.

¹¹Note that in this model, all the managers have the same ability to collect perfect information. Moreover, there are no reputation effects. See Benabou and Laroque (1992) for a reputation-building model.

2.3.2. The trading game

We consider now that fund managers trade also for their own account (proprietary trading) and transmit to risk neutral competitive market-makers the sum of their own orders X_j and those on behalf of their clients Y_j equal to ω_j :

$$(\omega_1, \omega_2, \dots, \omega_N) = (X_1 + Y_1, X_2 + Y_2, \dots, X_N + Y_N)$$

As we know from Proposition 2, the strategies are symmetric, and it is optimal to play 0 with probability $\alpha^*(N)$, and L with probability $1 - \alpha^*(N)$ in state u , they play 0 with probability $\alpha^*(N)$ and $-L$ with probability $1 - \alpha^*(N)$ in state d .

Market makers observe the order flow $\omega = (\omega_1, \omega_2, \dots, \omega_N, \omega_l)$, where ω_l is the volume transmitted by the noise traders and equal to L with probability $\frac{1}{3}$, 0 with probability $\frac{1}{3}$ and $-L$ with probability $\frac{1}{3}$. Orders are anonymous. Market makers quote:

$$P = E(\tilde{v}|\omega).$$

The profits of the funds are:

$$\pi_j^f = Y_j(\tilde{v} - P) \quad \text{for } j = 1, 2, \dots, N.$$

The profits of the proprietary trading activities are:

$$\pi_j^i = X_j(\tilde{v} - P) \quad \text{for } j = 1, 2, \dots, N.$$

2.3.3. Equilibrium strategy

Anonymity of orders guarantees that market makers do not know who is trading. In order to avoid being discovered by the market makers, financial institutions j send orders $L = X_j + Y_j$ or $0 = X_j + Y_j$ in state u .

Therefore, each bank j follows the strategy (α_1^j, α_2^j) to randomize its trades, where:

- α_1^j is the probability that the informed in state u buys $Y_j = y_j$ for the fund and $X_j = L - y_j$ for his own account.
- α_2^j , is the probability that the informed trader buys $Y_j = -y_j$ for the fund and $X_j = y_j$ for his own account.
- $(1 - \alpha_1^j - \alpha_2^j)$ is the probability that the informed buys $Y_j = -y_j$ for the fund and $X_j = L + y_j$ for his own account.¹²

2.3.4. The optimal contract

The financial institutions can adopt a collaborative behavior and offer the contract which maximizes the profit of the coalition of the banks. There are multiple equilibria and we focus on the symmetric case, which gives

¹²In Biais and Germain (2002), the banker can report any trade for the client ex post. Without loss of generality, we consider here, as is usually the case in practice, that the bank account of the client is separated from the bank account of the proprietary trading. Therefore, there is no problem of misreporting and no need to consider ex post deviations as the profits or the losses of the client are deduced from what is actually traded.

rise to the optimal $\alpha^*(N) = \alpha_2^*$. In this subsection we derive the optimal symmetric contract:

$\forall j = 1, 2, \dots, N$ we have,

$$X_j = X, \quad Y_j = Y, \quad \pi_j^f = \pi^f, \pi_j^i = \pi^i \quad \text{and} \quad (\alpha_1^j, \alpha_2^j) = (\alpha_1, \alpha_2).$$

The expected profits of the client are:

$$E(\pi^f) = k + E(t(\pi^f)). \quad (3)$$

The seller's expected profit is:

$$E(\pi^i + t(\pi^f)). \quad (4)$$

Substituting (2.3) in (2.4), the seller's expected profit is:

$$E(\pi^i + \pi^f) - k.$$

The program maximized by each of the N coalitions of informed traders and their clients is (where $-j$ characterizes the strategies of all agents but j):

$$\text{Max}_{t(\cdot)} E((X_j + Y_j)(\tilde{v} - P(X_j + Y_j, X_{-j} + Y_{-j}, \omega_l))) \quad (5)$$

with

$$(X_j, Y_j) \in \text{Argmax}_{X_j, Y_j} E(X_j(\tilde{v} - P(X_j + Y_j, X_{-j} + Y_{-j}, \omega_l)) + t(\pi_j^f) | v) \quad (6)$$

subject to:

$$E(\pi^f - t(\pi^f)) = k. \quad (7)$$

This program seeks a compensation function $t(\cdot)$ for the information seller (bank) that maximizes the total profit of the coalition (client and fund management), as expressed in (5), subject to the incentive compatibility constraint (6) that the bank will choose (X_j, Y_j) to maximize the expected profit from proprietary trading, and the client's individual rationality (participation) constraint (7). X_j represents the quantity traded by the fund manager on behalf of the client and Y_j represents the quantity traded by the bank, and X_{-j} and Y_{-j} represent all the quantities traded by all the funds and the banks except the fund j and the bank j .

The optimal contract gives the set of transfers $t(\cdot)$ which make the fund manager indifferent between the different states of the world. The contract specifies: i) a set of transfers $t(\cdot)$ from the funds to the banks and, ii) y the quantity traded on behalf of the clients. Moreover, the optimal contract gives rise to the optimal level of noise for the duopoly:

$$\alpha_2^*(N).$$

The profits of this game are presented in the following Table.

INSERT TABLE 1 HERE.

Proposition 3 *The optimal contract is characterized by a set of transfers t_1, t_2, t_3 which make the fund manager indifferent to trade in the following three states:*

- buying y for the fund and $L - y$ for proprietary trading,
- buying $-y$ for the fund and y for proprietary trading,
- buying $-y$ for the fund and $L + y$ for proprietary trading.

The quantity y traded on behalf on the clients is:

$$y = \frac{L\alpha_2^*((3-n)\alpha_2^* + n + 1)}{(\alpha_2^*(n^2 - 6n + n) + \alpha_2^*(-2n^2 + 4n + 2) + n^2 + n)}$$

The transfers are defined as follows:

$$\begin{aligned} \forall \pi_f \leq 0, \quad t(\pi_f) &= -t_1 \leq 0 \\ \text{If } \pi_f &= y\varepsilon \frac{2\alpha_2^*}{2\alpha_2^* + (n+1)(1-\alpha_2^*)} \quad \text{then } t(\pi_f) = t_2 \\ \text{If } \pi_f &\geq y\varepsilon \quad \text{then } t(\pi_f^j) = t_3 \end{aligned}$$

The optimal contract can be implemented by the following appropriate option: selling K bullish vertical spreads where K is the number of options necessary to get the points $(0, t_1)$ and $(0, t_2)$ aligned.

- The fund manager buys a call option with a zero exercise price and a premium equal to t_1 (paid by the informed to the client when the fund's profit are received).
- The fund manager sells a call with an exercise price equal to $\frac{2\alpha_2^*y\varepsilon}{N(1-\alpha_2^*) + 2\alpha_2^*}$ and a premium equal to zero.

This proposition shows that it is possible to derive optimal incentive-compatible contracts for the sale of information in the presence of compe-

tition. The optimal contract is a function of the number of sellers of information. Like in a prisoner's dilemma, without collaboration (in this model commitment), the competitors would make zero profits. Being able to collaborate by proposing fund management contracts to their clients, financial institutions can make profits. In the prisoner's dilemma, there is no way for the two prisoners to collaborate because of the competition between them and the divergence between the individual interest and the collective one. In this model, the banks face the same dilemma. However, they can commit ex ante to another strategy through their fund management activities. Moreover, this circumvents the Grossman and Stiglitz (1980) paradox because the traders can create some additional noise to recover the cost of collecting information. The features of this optimal contract are the following:

- For negative profits of the fund, the manager is not punished too much, which creates incentives to fund managers to add the optimal level of noise.
- For profits in the range $[0, y\varepsilon \frac{2\alpha_2^*}{2\alpha_2^* + (n+1)(1-\alpha_2^*)}]$, the fund manager's compensation increases with the profits of the fund. This gives incentives to the fund manager to reveal his private information to his client.
- For higher profits, the fund manager's remuneration remains constant in order to take into account a limited liability constraint (the manager can not take more money from the fund than the fund's profits).

It is worth noting that linear contracts are not optimal in this setting.¹³ Moreover, there is an infinite number of incentive compatible contracts – we focus on this one because it is easy to implement in practice.

In this model we assume that the financial institution can bundle its proprietary trades with the trades for its clients. This netting is legal in the NYSE (see NYSE rule 390 and SEC Rule 19c 3) and customary on the NASDAQ¹⁴).

3. The case where information is costless

The following proposition defines the characteristics of the market, where a large number of financial intermediaries collect information at zero cost.

Proposition 4 *When the number of financial intermediaries N increases, the optimal amount of endogenous strategic noise $\alpha^*(N)$ strictly increases (see Fig. 1) and goes to unity. The aggregate profit $NE(\pi^i + \pi^f)(\alpha^*(N))$*

¹³Indeed, in Biais and Germain (2002), the bank account of the fund management and the proprietary trading activities are not separated. As a consequence, there is a problem of misreporting the client's trade ex post, which creates additional constraints and leads to an optimal linear contract. Of course, we can show that optimal contracts are also linear in the case where the banker could report ex post any trade.

¹⁴However, if the Chinese walls in those financial institutions are too thick, say because the trades of both parts of the banks cannot be coordinated, we can show that the model is still robust. Indeed, in that case, to give rise to the optimal trading strategy of trading 0 with probability $\alpha^*(N)$, the financial institution just sets up a fund (indirect sale of information) and draws up a contract stipulating that the manager of the fund can commit not to trade with probability $\alpha^*(N)$.

decreases with N (see Fig. 2)¹⁵ and is equal at the limit to:

$$\frac{1}{3} \frac{4+c}{2+c} e^{-c} L \varepsilon > 0$$

where c is a constant defined in the proof.

FIGURE 1 AND 2 GO HERE.

The interpretation of this proposition is as follows. When a very large number of sellers of information enter the market they can endogenize the amount of noise in such a way that they still not reveal their private information. This suggests that fund management activities create an endogenous noise on top of the noise stemming from the liquidity trades. Usually, as in Kyle (1984) and Admati and Pfleiderer (1988), the informativeness of prices is an increasing function of the number of informed traders, which implies that the aggregate profit is decreasing.¹⁶ In this model, the aggregate profit is slightly decreasing (see Fig. 2).¹⁷

In an oligopoly *à la* Cournot, like in Kyle (1984) or Admati and Pfleiderer (1988), the aggregate profit is decreasing and always smaller than the monopolistic trader's profit. This result holds in this model too. But in the limit, contrary to Kyle (1984) or Admati and Pfleiderer (1988), the aggregate profit is not going to zero and prices do not reveal all the private

¹⁵In all the figures $\varepsilon = L = 1$.

¹⁶In those two models, in the limit, the aggregate profit goes to 0 and prices are fully revealing.

¹⁷In Fig. 2, we see clearly that the aggregate profit is decreasing very slowly to become almost constant. This highlights the fact that the informativeness of prices reaches a limit very fast.

information. Indeed, in the limit, the competition effect and the commitment effect have the same magnitude. Even with an infinite number of sellers of information, the financial market is not efficient.

In the following lemma we characterize the optimal contract in the limit.

Lemma 1 *If the number of sellers of information goes to infinity, the optimal contract goes to:*

$$y_\infty = \frac{4+c}{c^2+6c+2}L \quad (\text{see Fig. 3})$$

$$t_{1\infty} = k + y_\infty \frac{1}{3} e^{-c} \frac{c^2 + 7c + 6}{c + 2} \quad (\text{see Fig. 4})$$

$$t_{2\infty} = t_{3\infty} = y_\infty - t_{1\infty} \quad (\text{see Fig. 5})$$

where c is the real solution of the following cubic equation:

$$c^3 + 5c^2 + 4c - 8 = 0.$$

FIGURE 3 AND 4 AND 5 GO HERE.

In fact, the more informed sellers there are, the larger are the quantities traded on behalf of their clients as well as the transfers to the fund managers.

4. The case where information is costly

When information is costly, informed traders do not enter the market freely, but pay C to observe u or d . They do so if and only if the benefits of being an informed trader selling his information are at least equal to the cost of being informed. Hence, there is an equilibrium number of sellers of information corresponding to the case where the equality holds. The

following proposition characterizes the number of sellers at the equilibrium.

Proposition 5 *If C is the information cost then there is only one number N^* of sellers such that:*

$$3\frac{C}{L} = \alpha^{N^*-1}(1-\alpha)\frac{4\alpha + N(1-\alpha)}{N(1-\alpha) + 2\alpha}L\varepsilon \quad (8)$$

See Fig. 2 for the monotonicity of the aggregate profit.

The cheaper the information, the more prices are informative and the more people collect information. With costly collection of information, there is a finite number of sellers of information who can enter the market and prices partially reveal their private information.

5. Robustness

5.1. Robustness to assumptions on the distribution of liquidity trades

One might question to what extent our results reflect the highly stylized nature of our assumptions regarding the distribution of the liquidity trades. The economic point we want to make, and which we believe is robust to variations in the parametric assumptions, is the following: commitment enables the informed trader to reduce the aggressiveness of his trades, and thus to increase his trading profits by reducing information revelation. Randomization in the trading strategy of the informed agent is a way to thus reduce the aggressiveness of his trades.

To shed some light on this issue, we now consider a generalization of our model whereby the liquidity trades can take, with equal probability, one of the following five values: $\{-2L, -L, 0, L, 2L\}$. In this slightly more general context, we show that it is indeed still the case that commitment enables informed traders to enhance their profits by reducing their responsiveness to their signals. Moreover, we show that the optimal contract is such that the quantity y traded on behalf of the client depends on the structure of the liquidity trading and can take different values.

Lemma 2 *Consider the case where there are N informed agents and one liquidity trader sending their orders to the market makers. If the informed agents can commit to a trading strategy, then in state u it is optimal for them to give a positive probability to the three non-negative trades: $0, L$ and $2L$. Moreover, there exists, as in the previous case, an optimal incentive compatible contract where y_0 is the quantity traded by the fund when the bank transmits an order of the size 0 , y_1 is the quantity traded by the fund when the bank transmits an order of the size L , and y_2 is the quantity traded by the fund when the bank transmits an order of the size $2L$. The optimal contract can be implemented by selling K bullish vertical spreads also.*

It is worth noting that in a continuous framework, more complex contracts would have to be designed and there would be a continuum of possible trades: $[-y, y]$ where $-y$ and $+y$ can be interpreted as the maximum risky positions that the fund manager is allowed to take. Such bounds are actually

observed in practice. However, this proposition shows that we can obtain more complex and realistic contracts with a richer set of liquidity trades.

5.2. Generalization of the structure of information

We have maintained the assumption that an informed trader is always informed before trading. In this subsection, we relax this assumption by assuming that the informed trader receives a perfect signal with probability π and has no information with the complementary probability.

First of all, we show that above a certain probability π^* it is always optimal to add some noise in the market:

Lemma 3 *Let $\alpha^*(N)$ be the former value computed for a game with perfect information and which maximizes the profits of the coalition. Let π be the probability for an informed agent to be informed and $\alpha^{**}(N)$ the new probability which maximizes the expected profits:*

- *for $\pi > 1 - \alpha^*(N)$, there exists an $\alpha^{**}(N)$ in $(0,1)$ (mixed strategy) which maximizes the profits of the coalition.*
- *for $\pi < 1 - \alpha^*(N)$, $\alpha^{**}(N) = 1$.*

This lemma shows that for π large enough it is still optimal to commit to add noise as in Proposition 2.

Now let us consider the possibility for informed traders to sell their information. Following the same methodology used in the previous case, it can be shown that there exists optimal contracts allowing the possibility to

play 0 in the case in which there is no information.

Lemma 4 *The optimal contracts are formally the same as the ones described in the case of perfect information.*

6. Conclusion

We have addressed the broad question of competition between informed financial institutions and shown that public contracts permit banks to commit to trade less aggressively and recover part of their cost of collection of information. In fact, optimal contracts: i) give rise to optimal mixed strategies and ii) are public, which makes credible the competitors' commitment. Those contracts are incentive compatible because: i) they allow the banks to generate the optimal level of noise for the banking industry and ii) they respect the individual rationality constraint of the clients. Optimal contracts restore Pareto optimality. In particular, competitors can enter the market with costly information and partially reveal their private information which avoids the Grossman and Stiglitz (1980) paradox.

There are very few empirical studies investigating the impact of fund management activities on stock prices. It would be interesting, for example, to test the informational content of fund management trades versus the informational content of proprietary trading activities. One empirical prediction of this model is that proprietary trading activities are more profitable than fund management activities. Declerck (2003) shows that

expected profits of proprietary trading activities are higher than expected profits of agents' orders (fund management activities and retail customers). Moreover, she shows that proprietary trading orders obtain better execution and are more aggressive. It would be also interesting to try to disentangle the orders stemming from fund managers from those coming from retail orders to measure their informational content and test part of the empirical implications of our model. Another empirical prediction is that the level of noise observed in prices is in part stemming from fund management activities and also that an increase in the bank competition level does not diminish the level of noise in the prices.

Table 1
Profits in state u if N sellers of information play the strategy α_1, α_2

Y^i	X^i	ω_{-N}	L	Profits
y	$L - y$	if $L \in \omega_{-j}$	$L, 0, -L$	0
y	$L - y$	$\omega_{-j} = (0, \dots, 0)$	L	0
y	$L - y$	$\omega_{-j} = (0, \dots, 0)$	0	$\frac{2\alpha_2}{N(1-\alpha_2)+2\alpha_2}\varepsilon$
y	$L - y$	$\omega_{-j} = (0, \dots, 0)$	$-L$	ε
$-y$	y	if $L \in \omega_{-j}$	L	0
$-y$	y	if $(L, L) \in \omega_{-j}$	$0, -L$	0
$-y$	y	$\omega_{-j} = (L, 0, \dots, 0)$	L	0
$-y$	y	$\omega_{-j} = (L, 0, \dots, 0)$	0	$\frac{2\alpha_2}{N(1-\alpha_2)+2\alpha_2}\varepsilon$
$-y$	y	$\omega_{-j} = (L, 0, \dots, 0)$	$-L$	ε
$-y$	y	$\omega_{-j} = (0, \dots, 0)$	L	$\frac{2\alpha_2}{N(1-\alpha_2)+2\alpha_2}\varepsilon$
$-y$	y	$\omega_{-j} = (0, \dots, 0)$	0	ε
$-y$	y	$\omega_{-j} = (0, \dots, 0)$	$-L$	$\frac{2N(1-\alpha_2)+2\alpha_2}{N(1-\alpha_2)+2\alpha_2}\varepsilon$
$-y$	$L + y$	$L \in \omega_{-j}$	$L, 0$	0
$-y$	$L + y$	$(L, L) \in \omega_{-j}$	$-L$	0
$-y$	$L + y$	$(0, \dots, 0) = \omega_{-j}$	L	0
$-y$	$L + y$	$(0, \dots, 0) = \omega_{-j}$	0	$\frac{2\alpha_2}{N(1-\alpha_2)+2\alpha_2}\varepsilon$
$-y$	$L + y$	$(0, \dots, 0) = \omega_{-j}$	$-L$	$\frac{2N(1-\alpha_2)+2\alpha_2}{N(1-\alpha_2)+2\alpha_2}\varepsilon$

Note. Profits of the sellers of information. The tables gives the profits of N banks in state u when they play y for the fund and $L - y$ for their own account with probability α_1 , $-y$ for the fund and y for their own account with probability α_2 and $-y$ for the fund and $L + y$ for their own account with probability $1 - \alpha_1 - \alpha_2$.

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Appendix

Proof of Proposition 1 and 2

N informed sellers $j = (1, \dots, N)$ transmit an order of size 0 with probability $\alpha_j(N)$ and of size L with probability $1 - \alpha_j(N)$. Probabilities are computed by market makers using Bayes' rule. Market makers observe the anonymous volumes $\omega = (\omega_1, \omega_2, \dots, \omega_N, \omega_l)$ where ω_l is L, 0 or -L with prob-

ability $(1/3, 1/3, 1/3)$. Conditional probabilities for each observed volume in state u are:

$$\begin{aligned}
p(u|L, L, \dots) &= 1 \\
p(u|L, 0, \dots, 0) &= \frac{1 + \sum_{i=1}^N \frac{(1 - \alpha_i)}{\alpha_i}}{2 + \sum_{i=1}^N \frac{(1 - \alpha_i)}{\alpha_i}} \\
p(u|-L, 0, \dots, 0) &= 1 - p(u|L, 0, \dots, 0) \\
p(u|0, \dots, 0) &= \frac{1}{2} \\
p(u|L, -L, 0, \dots, 0) &= \frac{1}{2}
\end{aligned}$$

Probabilities in state d are straightforward as $p(u|\omega) + p(d|\omega) = 1$. The profits of the informed j is:

$$(u - d)(1 - p(u|\omega_j, \omega_{-j}, \omega_l))$$

where ω_{-j} characterizes all the volumes transmitted by the traders except for the j th informed. Let $\alpha_i(N) = \alpha_i$. The expected profit of each agent is:

$$\pi_i(\alpha) = 1/3(1 - \alpha_i) \left[\prod_{j \neq i} \alpha_j \right] \left(3 - \frac{2 \left[\prod_{j \neq i} \alpha_j \right] + \sum_{j=1}^N (1 - \alpha_j) \left[\prod_{j \neq i} \alpha_j \right]}{2 \left[\prod_{j \neq i} \alpha_j \right] + \sum_{j=1}^N (1 - \alpha_j) \left[\prod_{j \neq i} \alpha_j \right]} \right) \varepsilon L.$$

This is equivalent to:

$$\pi_i(\alpha) = 1/3(1 - \alpha_i) \left[\prod_{j \neq i} \alpha_j \right] \left(\frac{4 + \sum_{j=1}^N \frac{1 - \alpha_j}{\alpha_j}}{2 + \sum_{j=1}^N \frac{1 - \alpha_j}{\alpha_j}} \right) \varepsilon L.$$

Indeed, the *ith* agent makes profits when he plays L with probability $1 - \alpha_j$ and the other agents play 0 with probability α_j . In that case the profits are $(u - d)p(u|\omega)$ i.e $(2\varepsilon)[1/3](0 + 1 + [1 - p(u|L, 0, \dots, 0)])$ where 0, $1/3$ and $1/3(1 - p(u|L, 0, \dots, 0))$ are the expected profits for each state of the world.

We consider the one-to-one change in variable:

$$(0, 1) \rightarrow (0, \infty)$$

$$\alpha_i \rightarrow \beta_i = \frac{1 - \alpha_i}{\alpha_i}$$

Define $\beta = (\beta_1, \beta_2, \dots, \beta_N)'$.

$$\pi_i(\beta) = 1/3\beta_i \prod_{j=1}^N \frac{1}{1 + \beta_j} \frac{4 + \sum_{j=1}^N \beta_j}{2 + \sum_{j=1}^N \beta_j}$$

The program of each agent is now equivalent to maximizing the trading profit:

$$\max_{\beta_i} \pi_i(\beta),$$

which is equivalent to:

$$Max \ln(\pi_i(\beta)) = -\ln 3 + \ln \beta_i - \sum_{j=1}^N \ln(1 + \beta_j) + \ln(4 + \sum_{j=1}^N \beta_j) - \ln(2 + \sum_{j=1}^N \beta_j).$$

The first-order condition (FOC) associated with the i th agent program is $\forall i = 1, \dots, N$:

$$\frac{1}{\beta_i} - \frac{1}{1 + \beta_i} + \frac{1}{4 + \sum_{j=1}^N \beta_j} - \frac{1}{2 + \sum_{j=1}^N \beta_j} = 0.$$

This is equivalent for $i \neq j$ to:

$$\beta_i(1 + \beta_i) = \beta_j(1 + \beta_j).$$

This implies either $\forall i = 1, \dots, N \quad \beta_i = \beta_j$ or $\beta_i + \beta_j + 1 = 0$. But this is impossible since $\beta_i > 0$.

Let β^* be this common value. We have:

$$2\beta^*(1 + \beta^*) = (2 + N\beta^*)(4 + N\beta^*).$$

The β s solutions of the former equation are:

$$\beta_1 = \frac{-(3N - 1) - \sqrt{(N^2 - 6N + 17)}}{N^2 - 2},$$

$$\beta_2 = \frac{-(3N - 1) + \sqrt{(N^2 - 6N + 17)}}{N^2 - 2}.$$

If we look at the condition under which $\beta^* > 0$, we have the condition $\sqrt{N^2 - 6N + 17} > 3N - 1$, which implies that $N < 1$. Therefore, there is no local extremum in the interior.

Hence, $\pi_i(\alpha_i) = 0$ for $\alpha_i = 1$ or $\alpha_i = 0$. This proves that the maximum profit is 0 and reached either for $\alpha_i = 0$ or $\alpha_i = 1$. Therefore, it is not optimal to add noise for two or more traders engaged in Nash competition. Moreover, if information is costly, they cannot recover the cost of collection of information.

Maximizing the aggregate profit without imposing the constraint that each agent participate obviously leads to the monopolistic trader case. We turn to the following program, where the N traders collaborate and maximize their joint profit:

$$\text{Max}_{\beta} \sum_{i=1}^N N\pi_i(\beta)$$

with $\beta = b(1, 1, \dots, 1)'$.

$$\text{Max}_{\beta} N/3b \left(\frac{1}{1+b} \right)^N \frac{4+Nb}{2+Nb}$$

The FOC is:

$$N^2(N-1)b^3 + b^2N(5N-4) + 4b(N-2) - 8 = 0.$$

We will now show the existence for $b > 0$ of a maximum $\alpha \in [0, 1]$.

Define $g(N, b) = N^2(N-1)b^3 + N(5N-4)b^2 + 4b(N-2) - 8$ then, $g(N, 0) = -8 < 0$, and $\lim_{b \rightarrow \infty} g = \infty$ (for $N > 1$).

Since g is continuous, there exists a solution to the equation $g(N, b) = 0$ such that for $b > 0$, there exists a $\alpha \in (0, 1)$ which maximizes:

$$\text{Max} \sum_{i=1}^N \pi_i(\alpha)$$

$$s.t \ \alpha = \alpha(1, \dots, 1)'$$

The uniqueness follows from:

$$\frac{dG}{dB} = 3N^2(N-1)b^2 + 2N(5N-4)b + 4(N-2).$$

Because $3N^2(N-1)b^2 + 2N(5N-4)b + 4(N-2) = 0$ has two negative roots for $N \geq 2$, the sign of $dg/db > 0, \forall b > 0$.

This proves the uniqueness for $N \geq 2$.

Expressing the probabilities and expected profit when the traders maximize the profit of the coalition as a function of N and $\alpha(N)$ leads to:

$$\begin{aligned} p(u|L, L, \dots) &= 1 \\ p(u|L, 0, \dots, 0) &= \frac{N(1-\alpha) + \alpha(N)}{N(1-\alpha(N)) + 2\alpha(N)} \\ p(u|-L, 0, \dots, 0) &= \frac{\alpha(N)}{N(1-\alpha(N)) + 2\alpha(N)} \\ p(u|0, \dots, 0) &= \frac{1}{2} \\ p(u|L, -L, 0, \dots, 0) &= \frac{1}{2} \end{aligned}$$

The expected profit of the informed j is:

$$(u-d)(1 - p(u|\omega_j, \omega_{-j}, \omega_l))$$

where ω_{-j} are the quantities transmitted by informed traders except j .

It is equal to:

$$(u-d)(1 - p(u|\omega_j, \omega_{-j}, \omega_l)) = \frac{1}{3}\alpha^{N-1}(1-\alpha)\frac{4\alpha + N(1-\alpha)}{N(1-\alpha) + 2\alpha}\varepsilon L.$$

Expressing the FOC with $n = N - 1$ leads to:

$$\alpha^3(-n^3+3n^2+n-3)+\alpha^2(3n^3-4n^2-5n-6)+\alpha(-3n^3-n^2+3n+1)+n^3+2n^2+n = 0.$$

We have proved that this equation admits only one real solution in $[0,1]$

which is a maximum.

QED

Proof of Proposition 3

Incentive Compatibility Condition of the informed agent

To randomize the informed has to be indifferent between:

- buying y for the fund and $L - y$ for proprietary trading,
- buying $-y$ for the fund and y for proprietary trading,
- buying $-y$ for the fund and $L + y$ for proprietary trading.

Expected profits in these three cases are:

$$E\pi_0^i = \frac{1}{3}\alpha_2^n \frac{\alpha_2(3-n) + n + 1}{\alpha_2(1-n) + n + 1} (L - y)\varepsilon + \frac{1}{3}\alpha_2^n (t_2 + t_3 + 2t_1) - t_1 \quad (A.1)$$

$$E\pi_1^i = \frac{1}{3}[n(1 - \alpha_2)\alpha_2^{n-1} \frac{\alpha_2(3-n) + n + 1}{\alpha_2(1-n) + n + 1} + \alpha_2^n]\varepsilon y - t_1 \quad (A.2)$$

$$E\pi_2^i = \frac{1}{3}\alpha_2^n \frac{\alpha_2(3-n) + n + 1}{\alpha_2(1-n) + n + 1} \varepsilon (y + L) - t_1 \quad (A.3)$$

$$(A.2) = (A.3) \Leftrightarrow y = L \frac{\alpha_2(\alpha_2(3-n) + n + 1)}{\alpha_2^2(n^2 - 5n) + \alpha_2(2 + 4n - 2n^2) + n + n^2} \quad (A.4)$$

$$(A.1) = (A.3) \Leftrightarrow t_2 + t_3 + 2t_1 = 2y\left(\frac{\alpha_2(3-n) + n + 1}{\alpha_2(1-n) + n + 1}\right)\varepsilon L \quad (A.5)$$

Individual rationality condition of the buyer:

$$\begin{aligned} & \alpha_1\left[\frac{1}{3}\alpha_2^n\left(\frac{\alpha_2(3-n) + n + 1}{\alpha_2(1-n) + n + 1}\right)y\varepsilon + t_1 - \frac{1}{3}\alpha_2^n(t_2 + t_3 + 2t_1)\right] \\ & - \alpha_2\left[\left(\frac{1}{3}\left(\frac{\alpha_2(3-n) + n + 1}{\alpha_2(1-n) + n + 1}\right)n(1 - \alpha_2\alpha_2^{n-1} + \alpha_2^n)y\varepsilon - t_1\right)\right] \\ & - (1 - \alpha_1 - \alpha_2)\left[\frac{1}{3}\alpha_2^n\left(\frac{\alpha_2(3-n) + n + 1}{\alpha_2(1-n) + n + 1}\right)y\varepsilon - t_1\right] = k \end{aligned} \quad (A.6)$$

Note that k can't be arbitrarily large because the expected profit of an agent is positive. Thus, k must have an upper bound. In fact, the agent and the client have to share their common profits in the coalition. Therefore, the maximum value of k decreases rapidly to zero with $\frac{1}{N}$ when N goes to infinity (see Proposition 4).

Substituting (A.4) and (A.5) in (A.6) we have:

$$\begin{aligned} t_1 = k + \frac{-\frac{1}{3}y\varepsilon}{\alpha_2(1-n) + n + 1} \alpha_2^{*n} [\alpha_2^2((3-n)(1+n) - \\ 3(1-n)) + \alpha_2(n+1)(2n-5) - (1+n)^2] \end{aligned} \quad (A.7)$$

Equations (A.4), (A.5) and (A.7) define the optimal contract for all N . $\alpha_2 = \alpha^*(N)$ is the solution of equation (1) which gives the optimal level of noise for a given number of banks.

We now argue that the optimal contract is stable. If the agents observe that a certain competitor is not engaged in fund management activities, they retaliate by not signing a contract either. In this case the expected profit

of the all agents is zero. Therefore, there are no incentives to be out of the coalition. In fact, the presence of public contracts allows the competitors to check any deviations.

The Bullish Vertical Spread

To characterize the bullish vertical spread we substitute $t_2 = t_3$ in (A.5):

$$t_1 + t_2 = y \frac{\alpha_2(3 - n) + n + 1}{\alpha_2(1 - n) + n + 1} \quad (A.8)$$

Substituting (A.7) in (A.8) one finds t_2 .

Let K be the slope of the line for the points $(0, -t_1)$, $(y(u - P(L, 0, \dots, 0)), t_2)$ to be aligned:

$$t_2 + t_1 = K \left(\frac{2\alpha_2 y \varepsilon}{(n + 1)(1 - \alpha_2) + 2\alpha_2} \right) \quad (A.9)$$

Substituting (A.8) in (A.9) one finds that:

$$K = \frac{\frac{y\alpha_2(3 - n) + n + 1}{\alpha_2(1 - n) + n + 1}}{\frac{2\alpha_2 y \varepsilon}{(n + 1)(1 - \alpha_2) + 2\alpha_2}} \quad (A.10)$$

K is the number of options to be sold and bought by the fund manager. $(y(u - P(L, 0, \dots, 0))) = \frac{2\alpha_2 y}{(n + 1)(1 - \alpha_2) + 2\alpha_2} \varepsilon$ is the exercise price of the option.

The optimal contract

For each α_2 we define a contract which makes the banker indifferent between playing 0 and L in state u or 0 and $-L$ in state d . To get the contract optimal, we showed in Proposition 2 that it is sufficient to replace α_2 by α_2^* in the above equations.

QED

Proof of Proposition 4

We first compute an equivalent of $\alpha^*(n)$ when n goes to ∞ .

Since $\alpha(n)$ is solution of the cubic equation (1), we expand $\alpha^*(n)$ in the following polynomial form in $\frac{1}{n}$:

$$\alpha^*(n) = 1 - \frac{c}{n^\delta} + o\left(\frac{1}{n^\delta}\right)$$

where c is a positive real number and δ an integer.

Plugging this form into (1) we obtain:

$$\frac{c^3}{n^{3\delta}} + o\left(\frac{1}{n^{3\delta}}\right) + 5\frac{c^2}{n^{2\delta+1}} + o\left(\frac{1}{n^{2\delta+1}}\right) + 4\frac{c}{n^{\delta+2}} + o\left(\frac{1}{n^{\delta+2}}\right) - \frac{8}{n^3} + o\left(\frac{1}{n^{\delta+2}}\right) = 0.$$

This implies $\delta = 1$, and

$$c^3 + 5c^2 + 4c - 8 = 0,$$

which admits a unique solution in $[0, 1]$. Indeed, it is easy to show that if $\delta \neq 1$, then the equality cannot hold. With no loss of generality, let suppose

that $\delta < 1$. Multiplying the left-hand and the right-hand sides by n^3 , we have:

$$c^3 + \varepsilon_n = 0$$

where $\lim_{n \rightarrow \infty} \varepsilon_n = 0$. This implies that $c = 0$, which is impossible.

Moreover (1) \Leftrightarrow

$$\begin{aligned} & (-\alpha^{*3}3 + 3\alpha^{*2}2 - 3\alpha^* + 1) + \frac{1}{n}(3\alpha^{*3} - 4\alpha^{*2} - \alpha^* + 2) + \\ & \frac{1}{n^2}(\alpha^{*3}3 - 5\alpha^{*2} + 3\alpha^* + 1) + \alpha^* \frac{1}{n^3}(-3\alpha^{*2} - 6\alpha^* + 1) = 0 \end{aligned}$$

\Leftrightarrow

$$\begin{aligned} P_n(\alpha^*) &= (1 - \alpha^*)^3 + \frac{(1 - \alpha^*)^2}{n}(3\alpha^* + 2) + \\ & \frac{(\alpha^* - 1)}{n^2}(\alpha^{*2} - 4\alpha^* - 1) + \frac{\alpha^*}{n^3(-3\alpha^{*2} - 6\alpha^* + 1)} = 0 \end{aligned}$$

Because α is in $[0, 1]$, when $n \rightarrow \infty$ one shows that $P_n(\alpha) \rightarrow (1 - \alpha)^3$, and so $\alpha \rightarrow 1$.

To obtain the limit of the aggregated profit, substitute $\alpha^*(n)$ by $\alpha^*(n) = 1 - \frac{c}{n} + o(\frac{1}{n^2})$ in the aggregate profit, which is equal to:

$$\frac{1}{3}N\alpha^*(N)^{N-1}(1 - \alpha^*(N)) \frac{4\alpha^*(N) + N(1 - \alpha^*(N))}{N(1 - \alpha^*(N)) + 2\alpha^*(N)} L\varepsilon.$$

We find the limit of the expected profit:

$$\frac{1}{3}c \frac{4+c}{2+c} \left[\left(1 - \frac{c}{n}\right)^n \right] L\varepsilon = \frac{1}{3}c \frac{4+c}{2+c} e^{-c} L\varepsilon$$

because $\lim_{n \rightarrow \infty} (1 - \frac{c}{n})^n = e^{-c}$.

The aggregate profit, a function of α^* , is a decreasing function of n . This result is characterized graphically in Fig. 2. Consequently, the informativeness of prices increases and also goes to a finite limit.

QED

Proof of Lemma 1

Replacing α^* by $1 - \frac{c}{n} + o(\frac{1}{n^2})$ one finds $y_\infty, t_{1\infty}, t_{2\infty} + t_{3\infty}$. QED

Proof of Proposition 5

The aggregate profit is strictly decreasing with $\alpha^*(N)$, so there is only one α^* for a level of aggregate profit. Moreover, $\alpha^*(N)$ is strictly increasing with N , so there is only one N^* for one α^* . QED

Proof of Lemma 2

Consider the case where in state u the insider trades 0 with probability α_0 , L with probability α_1 and $2L$ with probability α_2 . In state d the insider trades 0 with probability α_0 , $-L$ with probability α_1 and $-2L$ with probability α_2 . One can easily see that sales in state u or purchases in d are not optimal. First we show that there exists (α_1^*, α_2^*) which maximizes the

expected profits of the informed. In this context, the updated probabilities for the market makers are the following:

$$\begin{aligned}
p(u|2L, 0\dots 0) &= \frac{n\alpha_2 + \alpha_0}{n\alpha_2 + 2\alpha_0} \\
p(u|2L, 0\dots - L) &= \frac{\alpha_2}{\alpha_2 + \alpha_1} \\
p(u|2L, 0\dots - 2L) &= \frac{1}{2} \\
p(u|L, 0\dots 0) &= \frac{n\alpha_1 + \alpha_0}{n\alpha_1 + 2\alpha_0} \\
p(u|L, 0\dots - L) &= \frac{1}{2} \\
p(u|L, 0\dots - 2L) &= \frac{\alpha_1}{\alpha_2 + \alpha_1}
\end{aligned}$$

In state u , his expected profit if he trades $2L$ is:

$$E(\pi_2) = 2\varepsilon\left(\frac{2}{5}\frac{\alpha_0^N}{N\alpha_2 + 2\alpha_0} + \frac{2}{5}\frac{\alpha_0^{N-1}\alpha_1}{\alpha_1 + \alpha_2} + \frac{1}{5}\alpha_0^{N-1}\right).$$

While his expected profit from trading L is:

$$E(\pi_1) = 2\varepsilon\left(\frac{1}{5}\frac{\alpha_0^N}{N\alpha_1 + 2\alpha_0} + \frac{1}{5}\frac{\alpha_0^{N-1}\alpha_2}{\alpha_2 + \alpha_1} + \frac{1}{10}\alpha_0^{N-1}\right).$$

Therefore, his total expected profit is:

$$\begin{aligned}
E\pi &= \left(\frac{1}{5}(1 - \alpha_1 - \alpha_2)^n 2\varepsilon \frac{1 - \alpha_1 - \alpha_2}{2 - 2\alpha_1 - 2\alpha_2 + (n+1)\alpha_2}\right. \\
&+ \frac{1}{5}(1 - \alpha_1 - \alpha_2)^n 2\varepsilon \frac{\alpha_1}{\alpha_1 + \alpha_2} + \frac{1}{5}(1 - \alpha_1 - \alpha_2)^n \varepsilon) \alpha_2 2L \\
&+ \left(\frac{1}{5}(1 - \alpha_1 - \alpha_2)^n 2\varepsilon \frac{1 - \alpha_1 - \alpha_2}{2 - 2\alpha_1 - 2\alpha_2 + (n+1)\alpha_1}\right)
\end{aligned}$$

$$+\frac{1}{5}(1-\alpha_1-\alpha_2)^{n_2}\varepsilon\frac{\alpha_2}{\alpha_1+\alpha_2}+\frac{1}{5}(1-\alpha_1-\alpha_2)^n\varepsilon)L\alpha_1$$

We define f such that:

$$f(x) = \begin{cases} 0 & \text{for } x = (0, 0) \\ E\pi & \text{for } x \in ([0, 1] \times [0, 1]) \cap \{(\alpha_1, \alpha_2)/\alpha_1 + \alpha_2 \leq 1\} - (0, 0) \end{cases}$$

This function of two variables (α_1, α_2) is defined on the compact:

$$([0, 1] \times [0, 1]) \cap \{(\alpha_1, \alpha_2)/\alpha_1 + \alpha_2 \leq 1\}.$$

$E\pi(\cdot)$ is a continuous function because it is a sum of rational functions with denominators different from zero on its domain of definition. Moreover, the limit of $E\pi(\cdot)$ when α_1 and α_2 go to zero is equal to zero. In conclusion, f is continuous. Because the considered domain is a compact, applying Heine's theorem, f has got a maximum and it is reached on its domain. Two cases are possible:

- α_1^*, α_2^* are inside the domain,
- the maximum is reached on one of the frontiers.

We will prove that the second assertion is not possible. We will check the cases $\alpha_1^* = 0$ or $\alpha_2^* = 0$. For $\alpha_1 + \alpha_2 = 1$ when $N \geq 2$ the market maker will immediately deduce from the trades the true state of the world.

Let show that on the frontier $\alpha_1 = 0$ the maximum is not reached.

Note that f is a continuously differentiable function on the domain $([0, 1] \times [0, 1]) \cap \{(\alpha_1, \alpha_2)/\alpha_1 + \alpha_2 \leq 1\} - (0, 0)$. For a given α_2' on the considered frontier we consider the mixed strategies (α_1, α_2) such that $\alpha_1 + \alpha_2 = \alpha_2'$.

The differential of $E\pi(\cdot)$ in $(0, \alpha'_2)$ along the line defined by the equation $\alpha_1 + \alpha_2 = \alpha'_2$ is equal to:

$$\begin{aligned} & \frac{1}{5}\epsilon(1-A)^n \left(2\left(\frac{2A}{2(1-A) + (n+1)\alpha_2} + A\right) + \right. \\ & 2\alpha_2 \left(-\frac{2(n+1)A\Delta(\alpha_2)}{(2(1-A) + (n+1)\alpha_2)^2} + \frac{2\Delta(\alpha_1)}{A} \right) + \\ & \left. \frac{2A}{2(1+A)} + \frac{2\alpha_2}{A} + A \right) \end{aligned}$$

with $A = (1 - \alpha_1 - \alpha_2)^n$, with $\Delta(\alpha_1) > 0$ and $\Delta(\alpha_2) < 0$.

Note that all the terms are positive. Therefore, the differential is also positive. We conclude that $\alpha_1 = 0$ is not a maximum.

The same proof on the frontier $\alpha_2 = 0$ leads to the same conclusion.

We conclude that the global maximum(s) of f is (are) reached inside the domain, and therefore that the possible strategies maximizing the profits are mixed strategies. Numerically it can be shown that this maximum is unique (see Fig. 6).

FIGURE 6 GOES HERE

The optimal contract

Incentive Compatibility Condition of the informed agent

To randomize the informed has to be indifferent between:

- buy y_2 for the fund and $2L - y_2$ for proprietary trading (with a probability α_4).

- buy $-y_2$ for the fund and $2L + y_2$ for proprietary trading (with a probability $\alpha_2 - \alpha_4$).
- buy y_1 for the fund and $L - y_1$ for proprietary trading (with a probability α_3).
- buy $-y_1$ for the fund and $L + y_1$ for proprietary trading (with a probability $\alpha_1 - \alpha_3$).
- buy $-y_3$ for the fund and y_3 for proprietary trading (with a probability $1 - \alpha_2 - \alpha_1$).

Expected profits in these cases are:

$$\begin{aligned}
E\pi_{21}^i &= \left(\frac{1}{5}(1 - \alpha_1 - \alpha_2)^n 2\varepsilon \frac{1 - \alpha_1 - \alpha_2}{2 - 2\alpha_1 - 2\alpha_2 + (n+1)\alpha_2} + \frac{1}{5}(1 - \alpha_1 - \alpha_2)^n 2\varepsilon \frac{\alpha_1}{\alpha_1 + \alpha_2} + \right. \\
&\quad \left. \frac{1}{5}(1 - \alpha_1 - \alpha_2)^n \varepsilon (2L - y_2) + \frac{1}{5}(1 - \alpha_1 - \alpha_2)^n (t_{22} + t_{32} + t_{42}) \right. \\
&\quad \left. - t_1 \left(1 - \frac{3}{5}(1 - \alpha_1 - \alpha_2)^n\right) \right) \tag{A.11}
\end{aligned}$$

$$\begin{aligned}
E\pi_{22}^i &= \left(\frac{1}{5}(1 - \alpha_1 - \alpha_2)^n 2\varepsilon \frac{1 - \alpha_1 - \alpha_2}{2 - 2\alpha_1 - 2\alpha_2 + (n+1)\alpha_2} + \frac{1}{5}(1 - \alpha_1 - \alpha_2)^n 2\varepsilon \frac{\alpha_1}{\alpha_1 + \alpha_2} + \right. \\
&\quad \left. \frac{1}{5}(1 - \alpha_1 - \alpha_2)^n \varepsilon (2L + y_2) - t_1 \right) \tag{A.12}
\end{aligned}$$

$$\begin{aligned}
E\pi_0^i &= (n\frac{1}{5}(1-\alpha_T)^{(n-1)}\alpha_2 2\varepsilon \frac{1-\alpha_T}{2(1-\alpha_T)+(n+1)\alpha_2} + n\frac{1}{5}(1-\alpha_T)^{(n-1)}\alpha_2 2\varepsilon \frac{\alpha_1}{\alpha_1+\alpha_2} + \\
& n\frac{1}{5}(1-\alpha_T)^{(n-1)}\alpha_2 2\varepsilon + n\frac{1}{5}(1-\alpha_T)^{(n-1)}\alpha_1 2\varepsilon \frac{1-\alpha_T}{2(1-\alpha_T)+(n+1)\alpha_1^*} + \\
& n\frac{1}{5}(1-\alpha_T)^{(n-1)}\alpha_1^* 2\varepsilon \frac{\alpha_2}{\alpha_1^*+\alpha_2} + n\frac{1}{5}(1-\alpha_T)^{(n-1)}\alpha_1 2\varepsilon + \\
& \frac{1}{5}(1-\alpha_T)^n \frac{(1-\alpha_T)+(n+1)\alpha_1}{2(1-\alpha_T)+(n+1)\alpha_1} + \frac{1}{5}(1-\alpha_T)^n \frac{(1-\alpha_T)+(n+1)\alpha_2}{2(1-\alpha_T)+(n+1)\alpha_2} + \\
& \frac{1}{5}(1-\alpha_T^n \varepsilon)(y_3) - t_1 \tag{A.13}
\end{aligned}$$

$$\begin{aligned}
E\pi_{11}^i &= (\frac{1}{5}(1-\alpha_1-\alpha_2)^n 2\varepsilon \frac{1-\alpha_1-\alpha_2}{2-2\alpha_1-2\alpha_2+(n+1)\alpha_1} + \frac{1}{5}(1-\alpha_1-\alpha_2)^n 2\varepsilon \frac{\alpha_2}{\alpha_1+\alpha_2} + \\
& \frac{1}{5}(1-\alpha_1-\alpha_2)^n \varepsilon)(L-y_1) + \frac{1}{5}(1-\alpha_1-\alpha_2)^n (t_{21}+t_{31}+t_{41}) \\
& -t_1(1-\frac{3}{5}(1-\alpha_1-\alpha_2)^n) \tag{A.14}
\end{aligned}$$

$$\begin{aligned}
E\pi_{12}^i &= (\frac{1}{5}(1-\alpha_1-\alpha_2)^n 2\varepsilon \frac{1-\alpha_1-\alpha_2}{2-2\alpha_1-2\alpha_2+(n+1)\alpha_1} + \frac{1}{5}(1-\alpha_1-\alpha_2)^n 2\varepsilon \frac{\alpha_2}{\alpha_1+\alpha_2} + \\
& \frac{1}{5}(1-\alpha_1-\alpha_2)^n \varepsilon)(L+y_1) - t_1 \tag{A.15}
\end{aligned}$$

The individual rationality of the client is given by the following equation:

$$\begin{aligned}
\alpha_4 & (\frac{2}{5} \frac{((1-\alpha_1-\alpha_2)^n \varepsilon (1-\alpha_1-\alpha_2))}{2-2\alpha_1-2\alpha_2+(n+1)\alpha_2} + \frac{2}{5} \frac{(1-\alpha_1-\alpha_2)^n \varepsilon \alpha_1}{\alpha_1+\alpha_2} + \frac{1}{5} (1-\alpha_1-\alpha_2)^n \varepsilon) y_2 \\
& - \frac{1}{5} (t_{24}+t_{23}+t_{24})(1-\alpha_1-\alpha_2)^n + t_1 (1-\frac{3}{5}(1-\alpha_1-\alpha_2)^n) + \\
& (\alpha_2-\alpha_4)(t_1-y_2) (\frac{2}{5} \frac{(1-\alpha_1-\alpha_2)^n \varepsilon (1-\alpha_1-\alpha_2)}{2-2\alpha_1-2\alpha_2+(n+1)\alpha_2} + \frac{2}{5} \frac{(1-\alpha_1-\alpha_2)^n \varepsilon \alpha_1}{\alpha_1+\alpha_2})
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{5}(1 - \alpha_1 - \alpha_2)^n \epsilon) + \\
& (\alpha_1 - \alpha_3)(t_1 - y_1 \left(\frac{2}{5} \frac{(1 - \alpha_1 - \alpha_2)^{(n+1)} \epsilon}{2 - 2\alpha_1 - 2\alpha_2 + (n+1)\alpha_1} + \frac{2}{5} \frac{(1 - \alpha_1 - \alpha_2)^n \epsilon \alpha_2}{\alpha_1 + \alpha_2} \right. \\
& \quad \left. + \frac{1}{5}(1 - \alpha_1 - \alpha_2)^n \epsilon) \right) + \\
& \alpha_3 \left(\left(\frac{2}{5} \frac{(1 - \alpha_1 - \alpha_2)^{(n+1)} \epsilon}{2 - 2\alpha_1 - 2\alpha_2 + (n+1)\alpha_1} + \frac{2}{5} \frac{(1 - \alpha_1 - \alpha_2)^n \epsilon \alpha_2}{\alpha_1 + \alpha_2} \right) + \right. \\
& \quad \left. \frac{1}{5}(1 - \alpha_1 - \alpha_2)^n \epsilon \right) y_1 - \frac{1}{5}(t_{12} + t_{13} + t_{14})(1 - \alpha_1 - \alpha_2)^n + t_1 \left(1 - \frac{3}{5}(1 - \alpha_1 - \alpha_2)^n \right) \\
& + (1 - \alpha_1 - \alpha_2)(t_1 - y_3 \epsilon \left(\left(\frac{1}{5} n (1 - \alpha_1 - \alpha_2)^{(n-1)} (\alpha_2 + \alpha_1) + \frac{1}{5}(1 - \alpha_1 - \right. \right. \\
& \left. \left. \alpha_2)^n \right) + 4 \frac{n(1 - \alpha_1 - \alpha_2)^{(n-1)} (\alpha_1) \alpha_2}{\alpha_1 + \alpha_2} + \frac{1}{5} n (1 - \alpha_1 - \alpha_2)^{(n-1)} (\alpha_2 + \alpha_1) \right) \\
& + \frac{2}{5} \frac{n(1 - \alpha_1 - \alpha_2)^n \alpha_1}{2(1 - \alpha_1 - \alpha_2) + (n+1)\alpha_1} + \frac{1}{5} \frac{(1 - \alpha_1 - \alpha_2)^n ((1 - \alpha_1 - \alpha_2) + (n+1)\alpha_2)}{2(1 - \alpha_1 - \alpha_2) + (n+1)\alpha_2} \\
& \quad + \frac{1}{5} \frac{((1 - \alpha_1 - \alpha_2) + (n+1)\alpha_1)(1 - \alpha_1 - \alpha_2)^n)}{2(1 - \alpha_1 - \alpha_2) + (n+1)\alpha_1} = k
\end{aligned}$$

To define a bullish vertical spread we need only to fix two transfers as the other are defined by the form of the contract. We have to find y_1, y_2, y_3 as well the two transfers t_{ij} . We have five equations for five unknown values. After cumbersome computations, we can find solutions with $y_1 \neq y_2 \neq y_3$.

The following tables give numerical values for the optimal contract as a function of n (computed with $\epsilon = 1$ and $L = 1$):

n	α_1	α_2	y_1	y_2	y_3
2	0.2	0.13	11.84	8.66	6.49
10	0.055	0.035	11.65	8.49	6.51
50	0.015	0.01	11.16	8.24	5.36
100	0.0075	0.005	11.12	8.2	5.39
500	0.002	0.002	-0.82	11.51	16.02

n	α_1	α_2	t_{min}	t_{max}	K
2	0.2	0.13	-1.18	17.7	26.98
10	0.055	0.035	-2.09	17.88	26.88
50	0.015	0.01	-1.91	17.32	25.60
100	0.0075	0.005	-1.80	17.27	25.53
500	0.002	0.002	-0.82	11.51	16.02

The same form of the contract can be achieved in this case as well.

QED

Proof of Lemma 3

It is straightforward that the game with a probability of being informed is strictly equivalent to the former one by replacing the probability α by the probability $1 - \pi(1 - \alpha)$. For a game with perfect information, we show that there exists a single α^* in $(0, 1)$ such that the aggregate profits of the coalition increase on $(0, \alpha^*)$ and decrease on $(\alpha^*, 1)$. Therefore, there are

two cases depending on the value of π :

- for $\pi > 1 - \alpha^*$ there exists α^{**} in $(0, 1)$ (mixed strategy) such that $\alpha^{**} = 1 - \pi(1 - \alpha^*)$ and maximizes the expected profits of the coalition,
- for $\pi < 1 - \alpha^*$ we have $\alpha^* < 1 - \pi(1 - \alpha) < 1$. In a game with perfect information, the aggregate profits decreases on $(\alpha^*, 1)$ with α . Therefore $1 - \pi(1 - \alpha)$ has to be the smallest possible (the closest from α^*) to maximize the aggregate profits. As the function $\alpha \mapsto 1 - \pi(1 - \alpha)$ increases on $(0, 1)$, the expected profit is maximized for $\alpha^{**} = 0$.

Note that in both cases this maximum is unique.

Proof of Lemma 4

With probability $1 - \pi$, the agents have no information. In this case, it is straightforward that the informed will not send an aggregate order different from zero. It is worth noticing that contracts where the banks do not trade are not robust. Indeed, in this case banks could pretend not to be informed and trade alone for their own account. Therefore, at the equilibrium banks have to be indifferent between trading quantities a or $-a$.

In this case, as the banks have no information, they have to be indifferent between buying $y' = a$ or selling $-y' = a$ for the fund buying $y = -a$ or selling $-y = -a$ for their own account whatever the state of the world. As a consequence we obtain the following system of equations:

$$E\pi_0^i = \frac{1}{3}\alpha_2^n \frac{\alpha_2(3-n) + n + 1}{\alpha_2(1-n) + n + 1} (L - y)\varepsilon + \frac{1}{3}\alpha_2^n (t_2 + t_3 + 2t_1) - t_1 \quad (A.16)$$

$$E\pi_1^i = \frac{1}{3}[n(1 - \alpha_2)\alpha_2^{n-1} \frac{\alpha_2(3-n) + n + 1}{\alpha_2(1-n) + n + 1} + \alpha_2^n]\varepsilon y - t_1 \quad (A.17)$$

$$E\pi_1^{t_i} = -\frac{1}{3}[n(1 - \alpha_2)\alpha_2^{n-1} \frac{\alpha_2^*(3-n) + n + 1}{\alpha_2(1-n) + n + 1} + \alpha_2^n]\varepsilon y' + \frac{1}{3}(\alpha_2^n(t_2 + 2t_3) + n(1 - \alpha_2)\alpha_2^{n-1}(t_2 + t_3 - t_1)) \quad (A.18)$$

$$E\pi_2^i = \frac{1}{3}\alpha_2^n \frac{\alpha_2(3-n) + n + 1}{\alpha_2(1-n) + n + 1} \varepsilon(y + L) - t_1. \quad (A.19)$$

And the rationality condition of the buyer is:

$$\begin{aligned} & \alpha_1 \left[\frac{1}{3}\alpha_2^n \left(\frac{\alpha_2(3-n) + n + 1}{\alpha_2(1-n) + n + 1} \right) y\varepsilon + t_1 - \frac{1}{3}\alpha_2^n (t_2 + t_3 + 2t_1) \right] \\ & - \alpha_2 \left[\frac{1}{2} \left(\frac{1}{3} \left(\frac{\alpha_2(3-n) + n + 1}{\alpha_2(1-n) + n + 1} n(1 - \alpha_2^*)\alpha_2^{n-1} + \alpha_2^n \right) y'\varepsilon - t_1 \right) \right. \\ & + \frac{1}{2} \left(-\frac{1}{3}[n(1 - \alpha_2)\alpha_2^{n-1} \frac{\alpha_2(3-n) + n + 1}{\alpha_2(1-n) + n + 1} + \alpha_2^n]\varepsilon y' + \frac{1}{3}(\alpha_2^n(t_2 + 2t_3) + n(1 - \alpha_2)\alpha_2^{n-1}(t_2 + t_3 - t_1)) \right) \\ & \left. - (1 - \alpha_1 - \alpha_2) \left[\frac{1}{3}\alpha_2^n \left(\frac{\alpha_2(3-n) + n + 1}{\alpha_2(1-n) + n + 1} \right) y\varepsilon - t_1 \right] \right] = k \end{aligned} \quad (A.20)$$

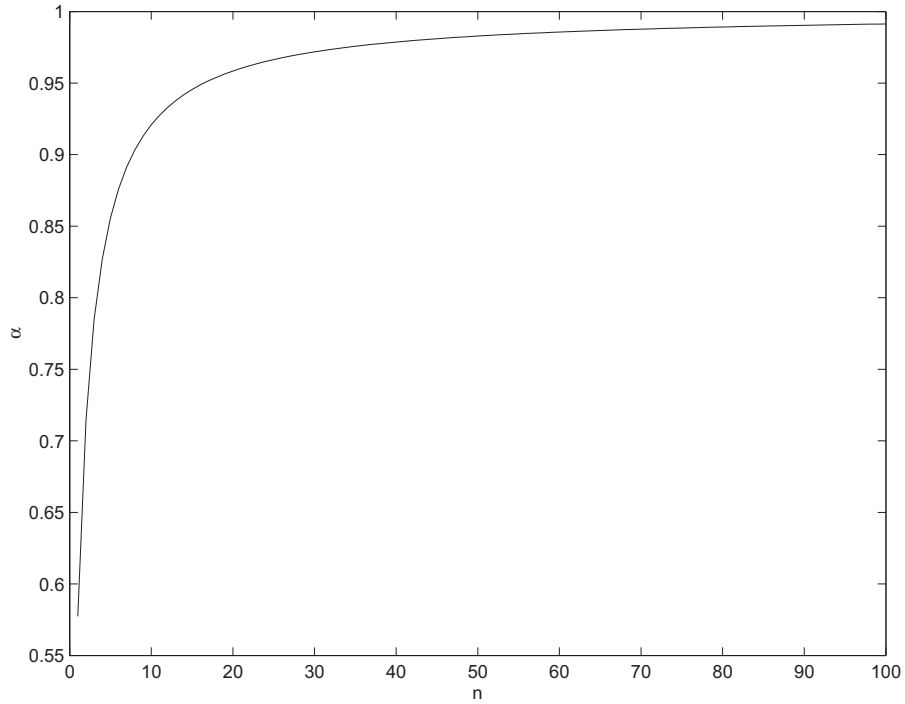
After solving this system, we find that $y = y'$ with y defined by the following equation:

$$y = y' = \frac{\alpha_2(-3\alpha_2 + \alpha_2 n - n - 1)L}{(2\alpha_2^2 - 3n(1 - \alpha_2)\alpha_2 + n^2(1 - \alpha_2)\alpha_2 - n^2(1 - \alpha_2) - n(1 - \alpha_2))} \quad (A.21)$$

In conclusion, we obtain the same type of contract as the one computed with perfect information.

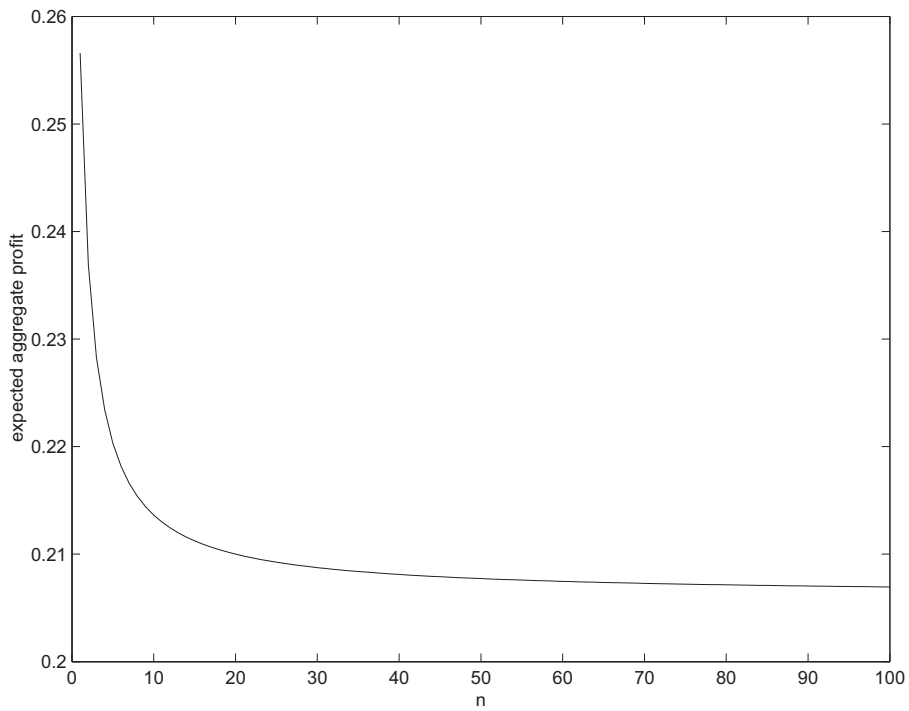
QED

Fig. 1. Probability of trade as a function of the number of informed traders.



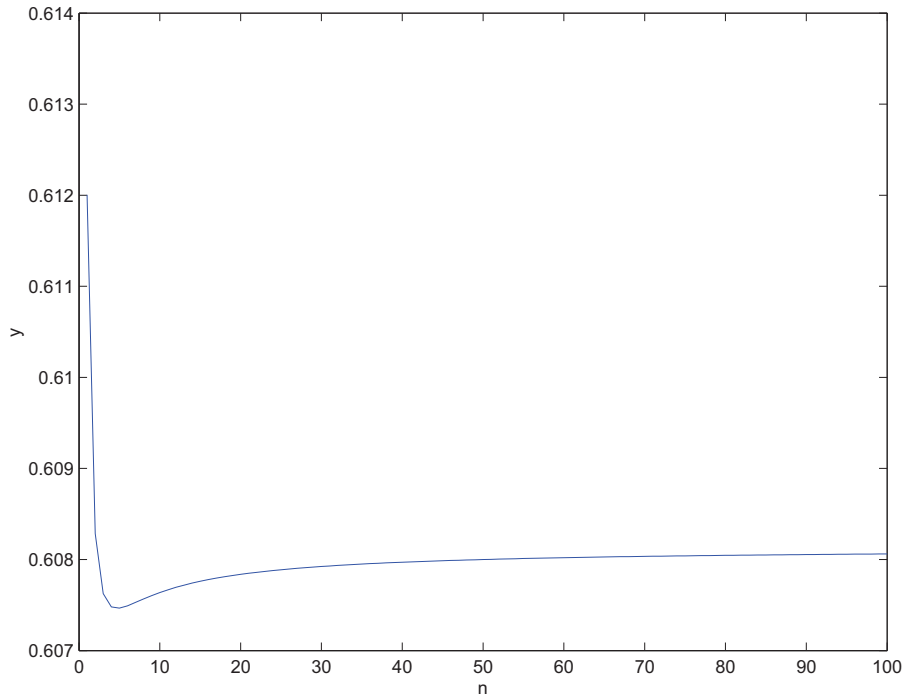
The figure shows α the probability to trade zero as a function of $n = N - 1$ where N is the number of informed traders.

Fig. 2. Expected aggregate profit as a function of the number informed traders.



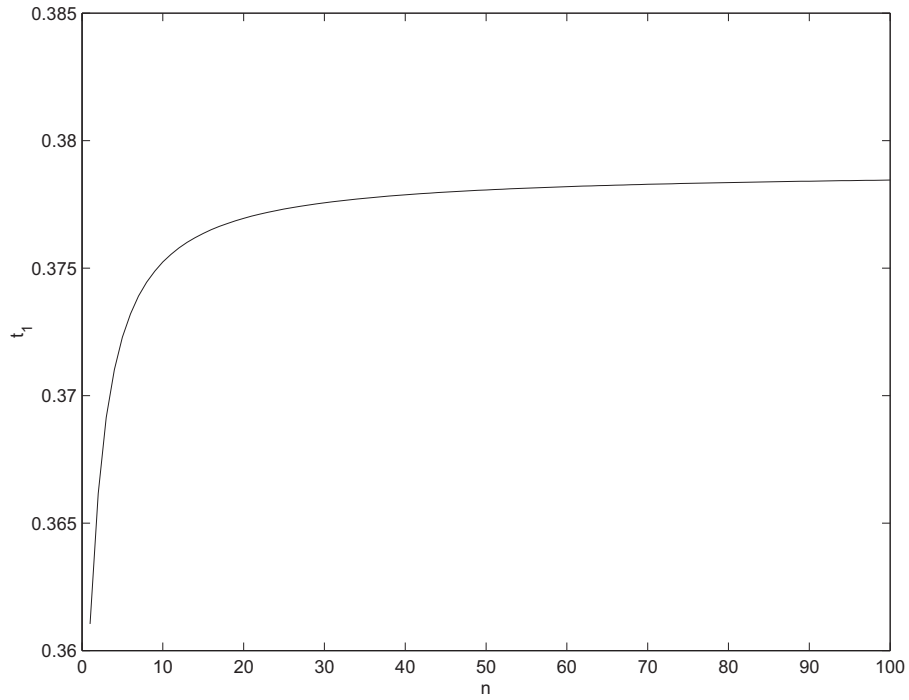
The figure shows the expected aggregate profit as a function of $n = N - 1$ where N is the number of informed traders and k , the reservation level of the client, is equal to 0.

Fig. 3. Quantity traded by the financial intermediary on behalf of clients as a function of the number of informed traders.



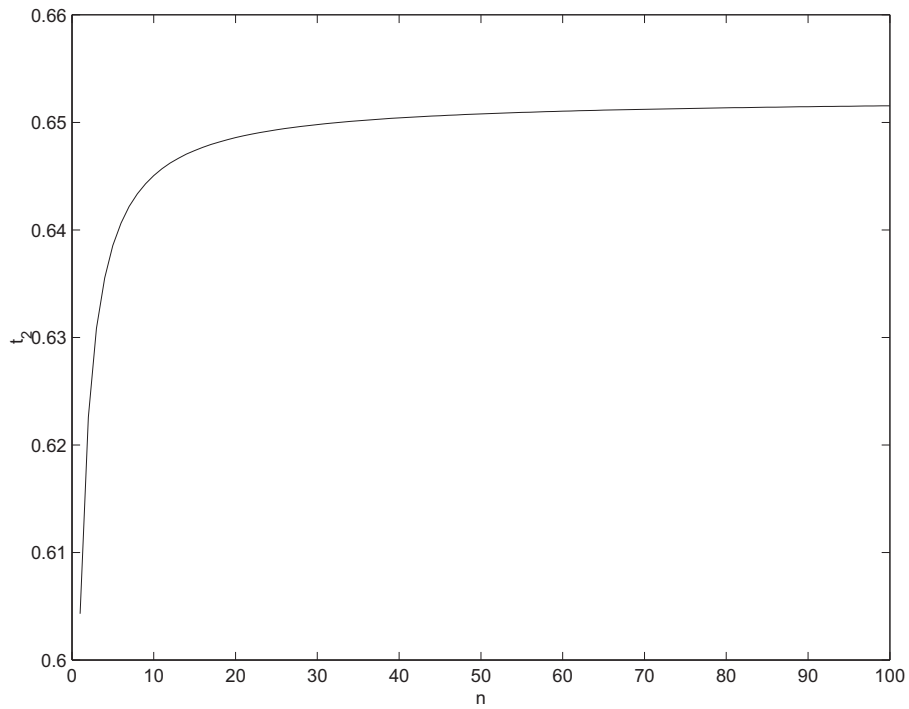
The figure shows the quantity y traded on behalf on the clients as a function of $n = N - 1$ where N is the number of informed traders and k , the reservation level of the client, is equal to 0.

Fig. 4. The negative transfer function paid by the clients to the banks in case of losses of the funds.



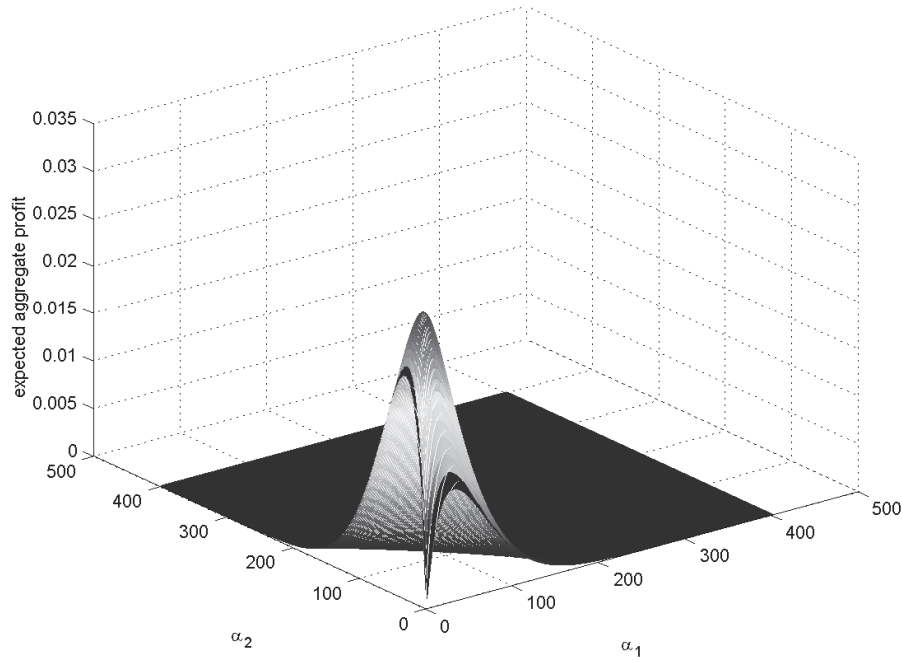
The figure shows the transfer t_1 as a function of $n = N - 1$ where N is the number of informed traders and k , the reservation level of the client, is equal to 0.

Fig. 5. The positive transfer functions paid by the clients to the banks in case of profits of the funds.



The figure shows the transfer $t_2 = t_3$ as a function of $n = N - 1$ where N is the number of informed traders and k , the reservation level of the client, is equal to 0.

Fig. 6. Expected aggregate profits as a function of the number of informed traders when the banks can trade multiple positive quantities.



The figure shows the expected aggregate profit as a function of α_1 the probability that the informed in state u buys y for the fund and $L - y$ for his own account, α_2 the probability that the informed trader buys $-y$ for the fund and y for his own account and $1 - \alpha_1 - \alpha_2$.

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