View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by Open Archive Toulouse Archive Ouverte

Open Archive Toulouse Archive Ouverte (OATAO)

OATAO is an open access repository that collects the work of Toulouse researchers and makes it freely available over the web where possible.

This is an author-deposited version published in: http://oatao.univ-toulouse.fr/
Eprints ID: 5787

To link to this article: DOI: 10.1016/j.jfi.2008.09.002
URL: http://dx.doi.org/10.1016/j.jfi.2008.09.002

To cite this version: Dridi, Ramdan and Germain, Laurent *Noise and competition in strategic oligopoly.* (2009) Journal of financial intermediation, 18 (2). pp. 311-327. ISSN 1042-9573

Any correspondence concerning this service should be sent to the repository administrator: staff-oatao@inp-toulouse.fr

Noise and Competition in Strategic Oligopoly *

Ramdan Dridi† and Laurent Germain‡

June 2008

Abstract

In this paper, we propose a model where N strategic informed traders who are endowed with heterogeneous noisy signals with different precisions compete in a market with a single risky asset. We explicitly describe the unique linear equilibrium that exists in this setup and derive its properties. Moreover, we focus on the effects of noise on the competition between traders. We show that noise softens the competition between traders. In particular, for N exceeding three and for certain sets of noise in traders' signals, each trader's individual profit is greater than the one obtained in the case of perfect information.

Keywords: Competition, Optimal Noise, Price Manipulation.

JEL Classification: D43, D82, G14, G24.

*We are particularly indebted to Hervé Boco, Bruno Biais, Gilles Chemla, Gabrielle Demange, Marcus Brunnermeier, Antoine Faure-Grimaud, Mike Fishman, Jean-Claude Gabillon, André Grimaud, Thomas Mariotti, Jean-Charles Rochet, and especially to David Stolin as well as seminar participants at Toulouse, the London School of Economics, the American Finance Association Meeting (2001) in New Orleans, the Western Finance Association (2001) in Tucson for long conversations and very thoughtful comments which have greatly improved the quality of this paper. All remaining errors are our own.

†Toulouse Business School, Groupe de Finance, 20 boulevard Lascrosses, BP 7010, 31068 Toulouse cedex 7, France. Tel 33 5 61 29 49 43, Fax: 33 5 61 23 32 79, E-mail: r.dridi@esc-toulouse.fr

‡Corresponding author. Toulouse Business School, Groupe de Finance, SUPAERO and Europlace Institute of Finance, Toulouse Business School 20 boulevard Lascrosses, BP 7010, 31068 Toulouse cedex 7, France. Tel 33 5 61 29 49 43, Fax: 33 5 61 23 32 79, E-mail: l.germain@esc-toulouse.fr

1 Introduction

Collecting and exploiting information in financial markets is one of the main activities of institutions such as banks, securities houses, or hedge funds. Such institutions have different reasons for trading and therefore they collect information in different ways. Thus, a realistic theoretical framework for understanding competition between different informed traders would endow these agents with different signals stemming from different distributions. However, most models in the literature consider that traders' information is homogeneous and collected signals are the same or that signals are heterogeneous but following the same distributions. In our paper, the collected signals are heterogeneous and stem from different distributions. Thus it is plausible that different informed traders have different signals from different decisions.

More precisely, there are three types of models according to whether informed agents know perfectly the liquidation value of the asset, receive identical noisy signals or have different signals with similar precision. Kyle (1985) and Holden and Subramanyam (1992) consider traders who are perfectly informed about the liquidation value of the risky asset. These models show that an informed trader's information is gradually revealed in market prices. Admati and Pfleiderer (1988) develop a model in the vein of Kyle (1985) but have studied homogeneous as well as heterogeneous signals. Admati and Pfleiderer provide an explanation of the patterns of volume and price volatility in intraday transaction data. The purpose of our article is to contribute to the understanding of competition where traders have different information. Several articles study competition in asymmetric information markets. Grossman and Stiglitz (1980) examine under what conditions it is worthwhile to pay for information. Foster and Vishwanathan (1996) consider a multi-period discrete time model where strategic informed traders receive different initial correlated signals. Back, Cao and Willard (2000) consider the same framework in continuous time. In their model the average of agents' signals is the liquidation value of the asset. In these two setups, the authors show that the nature of competition between informed traders depends on the initial degree of correlation between insiders' signals. In contrast to Foster and Vishwanathan (1996) and Back, Cao and Willard (2000) we do not focus on the correlation of the competitors' signals. Indeed, in our setup all agents observe the liquidation value of the risky asset plus a noise term so that we only have a range of positive correlations.

We develop a model in which we assume that informed traders have heterogeneous information and where all signals can have different precisions. This leads agents to compete with each other less aggressively since they choose different trading strategies to optimize the use of their own specific information. We prove the existence of a linear equilibrium and derive several properties about market depth, the informativeness of prices and traders' profits. We explore also the particular case where all traders' signals have identical precision as in Admati and Pfleiderer (1988).

More specifically, we point out the tradeoff between competition and noise in agents' signals. Indeed, noise in signals has two effects. One effect is costly for the trader since a part of his trade is noisy, thus diminishing his profits. In this sense, the insider is partly acting as a noise trader. The other effect of noise in the traders' signals which has received scant attention till now is to weaken competition thus benefiting the traders. The weakening of competition stems from the fact that other traders trade less aggressively. Hence, we show that in some cases noise reduces the correlation between the signals of the various agents as in Back, Cao and Willard (2000) and so reduces competition.

In this article, we assume that there are N risk neutral informed traders who rationally and strategically exploit their private information. Each informed trader receives a signal, $\widetilde{S}_i = \widetilde{v} + \widetilde{\varepsilon}_i$ where $\widetilde{v}$ is the liquidation value of the asset and $\widetilde{\varepsilon}_i$ is a random disturbance term. In constrast to other papers the noises $\{\widetilde{\varepsilon}_1, \ldots, \widetilde{\varepsilon}_N\}$ are independent random variables with different variances. First, we analyze all findings in this general case then, for the sake of analytical tractability, we focus on the special case where all variances are identical as in Admati and Pfleiderer (1988). This allows us to develop the theory in more detail. In particular, we derive the following result for certain sets of information and a number of competitors in excess of three: each individual expected profit is greater than the one obtained in the perfectly informed case. Information can have a negative value in presence of competition.

The paper is organized as follows. We first lay out the general setup (unequal precisions in traders' signals) in Section 2. We show in Section 3 the existence and uniqueness of a linear equilibrium and characterize the equilibrium as well as the individual expected profits when traders do not have the same information. In Section 4, we focus on the case where all informed traders have the same precision in their signals. We derive the equilibrium properties and delineate the regions for which each informed trader is better off with respect to the perfectly informed case. We then discuss and interpret these results. We also focus on the liquidity, individual and aggregate profits, and informativeness properties at equilibrium. Finally, Section 5 states some concluding remarks.

2 The Setup

We consider a financial market with a risky asset whose liquidation value v is normally distributed $\widetilde{v} \rightsquigarrow \mathcal{N}(0, \sigma_v^2)$. There are three types of agents:

1. N risk neutral informed traders who observe in advance a signal $\widetilde{S}_i = \widetilde{v} + \widetilde{\varepsilon}_i$, where $\widetilde{\varepsilon}_i$ is a random disturbance term (the noise) such that $\widetilde{\varepsilon}_i \rightsquigarrow \mathcal{N}(0, \sigma_i^2)$ and $\{\tilde{v}, \widetilde{\varepsilon}_1, \ldots, \widetilde{\varepsilon}_N\}$ are mutually independent.

2. Liquidity traders who submit market orders $\widetilde{u} \rightsquigarrow \mathcal{N}(0, \sigma_u^2)$ such that $\{\widetilde{v}, \widetilde{\varepsilon}_1, \ldots, \widetilde{\varepsilon}_N, \widetilde{u}\}$ are

mutually independent.

3. Risk neutral market makers, who observe the aggregate volume $\widetilde{w}$ and rationally set the price in a Bayesian way.

Note that the independence assumption on $\{\widetilde{v}, \widetilde{\varepsilon}_1, \ldots, \widetilde{\varepsilon}_N\}$ does not mean that the signals $\left\{\widetilde{S}_1, \ldots, \widetilde{S}_N\right\}$ are independent. Indeed, there is a correlation due to the common information structure $\widetilde{v}$.
A strategy for the informed agent $i = 1, \ldots, N$ is a random variable: $\widetilde{X}_i : I\!R \longrightarrow I\!R$, determining his market order as a function of the observed signal S_i. For given strategies $\left(\widetilde{X}_1, \ldots, \widetilde{X}_N\right)$, let $\widetilde{x}_i = \widetilde{X}_i(\widetilde{S}_i)$. These strategies determine the aggregate order flow $\widetilde{w} = \sum_{i=1}^{N} \widetilde{x}_i + \widetilde{u}$.
Market makers observe the realization of the order flow, but not of any of its components, and are engaged in a competitive auction. The outcome of this competition is described by a random variable $\widetilde{P}(.)$. Given $\left(\widetilde{P}, \widetilde{X}_1, \ldots, \widetilde{X}_N\right)$, we denote $\widetilde{p} = \widetilde{P}(\widetilde{w})$ and let $\widetilde{\pi}_i = (\widetilde{v} - \widetilde{p})\, \widetilde{x}_i$ be the resulting trading profit of each insider $i = 1, \ldots, N$. The equilibrium conditions are that the competition in which market makers are involved drives their expected profits to zero conditional on the aggregate submitted volume, and that the informed traders choose their trading strategies so as to maximize their expected profits.

Definition 2.1 : $\left(\widetilde{P}, \widetilde{X}_1, \ldots, \widetilde{X}_N\right)$ *is an equilibrium if:*

$$E\left[\widetilde{v} - \widetilde{p} | \widetilde{w}\right] = 0, \tag{1}$$

and for all random variable $\widetilde{X}_i$, given the (rational) beliefs of the market makers, and the corresponding price function $\widetilde{P}(.)$, each informed trader chooses $\widetilde{X}_i$ to maximize his expected profits:

$$\widetilde{x}_i \in \underset{x \in I\!R}{Argmax}\, E\left[\left(\widetilde{v} - \widetilde{P}(x + \sum_{j \neq i} \widetilde{x}_j + \widetilde{u})\right) x | S_i\right]. \tag{2}$$

Definition 2.2 : $\left(\widetilde{P}, \widetilde{X}_1, \ldots, \widetilde{X}_N\right)$ *is a linear equilibrium if in addition there exists a scalar $\lambda \in I\!R_+$ such that:*

$$\widetilde{p} = E\left[\widetilde{v} | \widetilde{w}\right] = \lambda \widetilde{\omega}. \tag{3}$$

With λ the parameter of liquidity and we define $\frac{1}{\lambda}$ as the liquidity. We next derive the unique perfect Bayesian linear equilibrium of this game.

3 The Equilibrium and its Properties

We now state the equilibrium for the case of N informed agents (which embeds the well-known result for the monopolistic informed agent case):

Proposition 3.1 *There exists a unique linear equilibrium defined by* $i = 1, \ldots, N$, $\widetilde{x}_i = \beta_i^*(\sigma)\widetilde{S}_i$ *and* $\widetilde{p} = \lambda^*(\sigma)\widetilde{\omega}$ *where* $\sigma = (\sigma_1, ..., \sigma_N)$ *is a N-size vector representing the variances of the noises* ϵ_i *and where* $\beta_i^*(\sigma)$ *and* $\lambda^*(\sigma)$ *are given by:*

$$\beta_i^*(\sigma) = \frac{\sigma_u}{\sigma_v} \frac{1}{\left\{ \sum_{j=1}^{N} \frac{1+\tau_j}{(1+2\tau_j)^2} \right\}^{\frac{1}{2}}} \frac{1}{1+2\tau_i}, \quad i = 1, \ldots, N, \tag{4}$$

$$\frac{1}{\lambda^*(\sigma)} = \frac{\sigma_u}{\sigma_v} \frac{1}{\left\{ \sum_{j=1}^{N} \frac{1+\tau_j}{(1+2\tau_j)^2} \right\}^{\frac{1}{2}}} \left[1 + \sum_{j=1}^{N} \frac{1}{1+2\tau_j} \right], \text{ with } \tau_j = \frac{\sigma_j^2}{\sigma_v^2}, j = 1 \ldots, N\cdot$$

In equilibrium the expected profit $\pi_i^*(\sigma)$ *for agent* i *is:*

$$\pi_i^*(\sigma) = \frac{\sigma_v \sigma_u \frac{1+\tau_i}{(1+2\tau_i)^2}}{\left\{ \sum_{j=1}^{N} \frac{1+\tau_j}{(1+2\tau_j)^2} \right\}^{\frac{1}{2}} \left\{ 1 + \sum_{j=1}^{N} \frac{1}{1+2\tau_j} \right\}}, \quad i = 1, \ldots, N. \tag{5}$$

.

Proof *: See Appendix A.2.*

One may note that each individual informed trader's expected profit is the expected profit achieved by the Kyle (1985) monopolistic trader in case of perfect information times a non-linear function of the amount of noise in the signal[1]. For $\sigma = (0, ..., 0)$, we obtain the result of Holden and Subramanyam (1992)[2]. In particular, as in Holden and Subramanyam (1992), the competition leads informed agents' profits to converge to zero quickly. By studying the non-linear function when $\sigma \neq (0, ..., 0)$ we will derive the behavior of the individual and aggregate profits with respect to the level of noise in all cases. It is worth noting that the market depth increases very quickly and tends to infinity with the number of insiders in the Holden and Subramanyam (1992) model since the liquidity is given by $\frac{(N+1)}{\sqrt{N}} \frac{\sigma_u}{\sigma_v}$. We will show that due to the non-linear form of the liquidity when insiders receive heterogeneous signals with different precisions, it is possible to slow the speed with which the liquidity tends to infinity.

[1]Recall that in Kyle (1985) this profit is given by the following formula : $\pi^*(0,1) = \frac{1}{2}\sigma_u\sigma_v$.
[2]The individual expected profit in this model is: $\pi_i^*(0,N) = \frac{1}{(N+1)\sqrt{N}}$.

Proposition 3.2 *: In equilibrium, the individual reaction $\beta_i^*(\sigma)$ of agent i is decreasing with each other competitor's precision of information (i.e. increasing with σ_{-i}), and increasing with his own information precision, (i.e. decreasing with σ_i).*
The individual profit $\pi_i^*(\sigma)$ is an increasing function of σ_{-i} and a decreasing function of σ_i.

Proof *: See Appendix B.1.*

This result is intuitive. Informed traders react all the more to their private information when the other traders' information is more noisy. Indeed, given one trader, since the total amount of noise in the other informed traders signals increases, the trader can camouflage his information more and his reaction becomes stronger. As a consequence, the individual profit increases with the noise in the other competitors' signals.

Proposition 3.3 *: Let us define*

$$b(\tau_{-i}) = 1 - \sum_{j \neq i} \frac{3 + \tau_j}{(1 + 2\tau_j)^2} \quad \textit{and} \quad c(\tau_{-i}) = 2 + \sum_{j \neq i} \frac{2\tau_j - 1}{(1 + 2\tau_j)^2}.$$

The aggregate profit is equal to the inverse of the liquidity parameter times the variance of the noise trading:

$$\pi^* = \frac{\sigma_v^2}{\lambda^*(\tau)} = \sigma_v \sigma_u \frac{\left[\frac{1}{8}(1 + c(\tau_{-i}) - 3b(\tau_{-i})) + \frac{1+\tau_i}{(1+2\tau_i)^2}\right]^{\frac{1}{2}}}{\frac{1}{2}\left(1 + c(\tau_{-i}) - b(\tau_{-i})\right) + \frac{1}{1+2\tau_i}}$$

In equilibrium, the liquidity $\lambda^{*^{-1}}$ can be either an increasing or a decreasing function of $\tau_i = \dfrac{\sigma_i^2}{\sigma_v^2}$ for a fixed τ_{-i}. It is increasing if:

- $b(\tau_{-i}) \geq 0$ *or*
- $b(\tau_{-i}) \leq 0$, $c(\tau_{-i}) \geq 0$, *and* $\tau_i \leq -\dfrac{c(\tau_{-i})}{2b(\tau_{-i})}$.

It is decreasing if:

- $c(\tau_{-i}) \leq 0$ *or*
- $b(\tau_{-i}) \geq 0$, $c(\tau_{-i}) \geq 0$, *and* $\tau_i \geq -\dfrac{c(\tau_{-i})}{2b(\tau_{-i})}$.

Thus, the aggregate expected profit is a non-monotonic function of information precision in the market.

Proof : *See Appendix B.2.*

The interpretation of Proposition 3.3 is as follows. We can distinguish three zones:

- Zone A: for small levels of noise in the signals ($c(\tau_{-i}) \leq 0$) price pressure and aggregate expected profit decrease with the precision of the information collected by the trader, all else being equal. This situation approaches the perfectly informed case where $\widetilde{S} = \widetilde{v}$, as in Holden and Subrahmanyam (1992). The competition between traders is very fierce and the aggregate profit decreases in the information precision.

- Zone B: if the amount of noise in the market is small enough but not too small ($b(\tau_{-i}) \leq 0$, $c(\tau_{-i}) \geq 0$) and $\tau_i \leq -\dfrac{c(\tau_{-i})}{2b(\tau_{-i})}$, price pressure and aggregate expected profit increase with the precision of the information collected by a given trader, all else being equal. Otherwise ($\tau_i > -\dfrac{c(\tau_{-i})}{2b(\tau_{-i})}$) price pressure and aggregate expected profit decrease with the precision of the information collected by the trader.

- Zone C: if there is a sufficient amount of noise in the market ($b(\tau_{-i}) \geq 0$), then the more precise is the information collected by an insider, the smaller the depth and the bigger the aggregate expected profit. The more precise is his information compared to others, the better he can camouflage his trade and the greater is his profit. The insider's profit increases more than the decreasing of the other traders' profits and therefore, the aggregate profit increases. In Foster and Vishwanathan (1996) and Back, Cao and Willard (2000) the aggregate expected profit of the informed traders depends only on the correlation between the initial signals and the number of informed traders. This profit is more important when there is some positive correlation between the signals than when the signals are uncorrelated. On the other hand, the correlation at which this profit is maximized diminishes as the number of informed traders increases.

Those three kinds of behavior are intuitive since the noise has two opposing effects. The negative effect is that for a part of his trade, the trader is acting as a noise trader. The positive effect is that the noise weakens the competition. As a matter of fact, other traders trade less aggressively and the competition is then weaker than before. Adding noise into signals weakens the competition because it decreases the correlation between the traders' signals and also reduces the amount of information available as in Back, Cao and Willard (2000). To summarize the previous effects, a small (large) amount of noise in the market allows (does not allow) the competition to soften, so that the competition effect dominates (is superseded by) the cost of noise in the signal. For intermediate amounts of noise in the market, we observe different regions for which one effect dominates the other.

FIGURE 1 GOES HERE

Figure 1 shows the aggregate expected profit of 11 traders in the specific case for which we have one trader with his own level of noise τ_1 and ten traders each with the same level of noise τ_2. To distinguish the three zones, we have marked each of them with a different mesh.

- Zone A: For small levels of noise in the signals - $b(\tau_{-i}) \leq 0$ and $c(\tau_{-i}) \leq 0$ - which is fulfilled here for τ_2 between 0 and 0.265, the aggregate expected profit is always increasing with the level of noise of the τ_1 trader.
- Zone B: for an amount of noise τ_2 in the signals of others which fulfills $b(\tau_{-i}) \leq 0$ and $c(\tau_{-i}) \geq 0$, here for τ_2 between 0.265 and 5.354, the aggregate expected profit, as a function of τ_1, decreases when $\tau_1 \geq \frac{c(\tau_2)}{2b(\tau_2)}$ and increases when $\tau_1 \leq \frac{c(\tau_2)}{2b(\tau_2)}$.
- Zone C: for a sufficient level of noise τ_2, here with $\tau_2 > 5.354$ the aggregate expected profit is always decreasing with the level of noise of the τ_1 trader.

These new patterns – namely the non-monotonicity of both liquidity and expected profits – lead us to characterize sets of information for which the presence of heterogeneous noise in signals can generate greater profits than the ones achieved with perfect information.

In our opinion, this result is a key contribution of the paper because it pinpoints the tradeoff between noise and competition. In Holden and Subramanyam (1992) the expected profits decrease quickly with the number of traders. The traders cannot hide their private information. In Foster and Vishnathan (1996) and Back, Cao and Willard (2000), the initial correlation between the signals determines the kind of competition. If the correlation is positive but not perfect there are two phases over time. First, the rat race, when competition is fierce. Second, the waiting game when traders are on opposite sides of the market. This creates an incentive to wait to trade. Therefore, in this second phase competition becomes less fierce. In our model, since it is a one period game, we have the same effect as in the rat race.

4 The Symmetric Case

We focus in this section on the symmetric heterogeneous signals case, that is the setup when all informed agents are endowed with signals of the same precision, $\sigma_i = \sigma_1$ for $i = 1, \ldots, N$.
We still maintain the mutual independence of $\{\widetilde{\varepsilon}_1, \ldots, \widetilde{\varepsilon}_N, \widetilde{v}, \widetilde{u}\}$. Then we can state the following proposition as a particular case of the previous propositions $3.1 - 3.3$.

Proposition 4.1 *: There exists a unique perfect linear equilibrium defined as in proposition*

3.1. *In this particular case* $(\beta_1^*, \ldots, \beta_N^*, \lambda^*)(\sigma_1, N)$ *is characterized by:*

$$\beta_i^*(\sigma_1, N) = \beta^*(\sigma_1, N) = \frac{\sigma_u}{\sigma_v} \frac{1}{\sqrt{N}\sqrt{1 + \frac{\sigma_1^2}{\sigma_v^2}}}, \quad i = 1, \ldots, N,$$

$$\frac{1}{\lambda^*(\sigma_1, N)} = \frac{\sigma_u}{\sigma_v} \frac{N + 1 + 2\frac{\sigma_1^2}{\sigma_v^2}}{\sqrt{N}\sqrt{1 + \frac{\sigma_1^2}{\sigma_v^2}}}.$$

In equilibrium, the individual profit $\pi_i^*(\sigma_1, N)$*, for* $i = 1, \ldots, N$ *is given by:*

$$\pi_i^*(\sigma_1, N) = \sigma_v \sigma_u \frac{\sqrt{1 + \frac{\sigma_1^2}{\sigma_v^2}}}{\sqrt{N}\left(N + 1 + \frac{2\sigma_1^2}{\sigma_v^2}\right)}, \tag{6}$$

and the aggregate profit $\pi^*(\sigma_1, N)$ *is:*

$$\pi^*(\sigma_1, N) = \sigma_v \sigma_u \frac{\sqrt{N}\sqrt{1 + \frac{\sigma_1^2}{\sigma_v^2}}}{N + 1 + \frac{2\sigma_1^2}{\sigma_v^2}}. \tag{7}$$

FIGURE 2 GOES HERE

Figure 2 presents the normalized aggregate profit as a function of the level of noise in the signals and the number of insiders. The normalization is performed with respect to the profit achieved by the monopoly in Kyle (1985) setup ($\pi^*(0, 1) = \frac{1}{2}\sigma_u \sigma_v$).

By setting $\sigma_1 = 0$, we get the Holden and Subramanyam (1992) and the Admati and Pfleiderer (1986) results, notably that the aggregate profit is close to zero when the number of insiders is large. We provide in the following proposition a detailed characterization of the parameters which determine profits.

Proposition 4.2 *i) For a number of informed traders* $N \geq 4$*, individual profits are increasing with the noise in the insider's information for* σ_1 *smaller than* $\sigma_v\sqrt{\frac{N-3}{2}}$*, and individual profits are maximized at* $\sigma_1^*(N) = \sigma_v\sqrt{\frac{N-3}{2}}$*. This is related to the two effects (beneficial and costly ones) of the noise in the collection of information.*

ii) For a small number of informed traders ($N \leq 3$)*, the aggregate profit is always decreasing with the level of noise in the signals.*

iii)In equilibrium, the informed traders' aggregate profit evaluated at the optimal level of noise is:

$$\pi^*_{informed\ traders}(N) = \pi^*(N) = \frac{\sigma_v \sigma_u}{2\sqrt{2}} \frac{1}{\sqrt{1 - \frac{1}{N}}}.$$

iv) For all $N \geq 4$, the insiders make a greater profit than in the perfectly informed case

since σ_1 is smaller than $\frac{\sigma_v \sqrt{N+1}\sqrt{N-3}}{2}$.

Proof *See Appendix C.1.*

Proposition 4.2 states that for the entire interval $(0, B_N)$ with $B_N = \frac{\sigma_v \sqrt{N+1}\sqrt{N-3}}{2}$, noise in signals is preferable. Of course, there is a particular value $\sigma_1^{*^2}(N)$ for which the expected profits are maximized. It is important to note that we do not describe how to implement the optimal level of noise. However, we derive all the properties of the model for this optimal value. We do not formally present ways for traders to implement this value in their strategies. For example, traders could collude or put in place contracts for the sale of information as in Admati and Pfleiderer (1986) or Germain (2005).
Moreover, it is important to note that as N increases, the interval for which noise and signals is preferable widens and at the limit extends to the whole positive real line. In this respect, we can argue that the value of information becomes more and more negative as N increases. Moreover, this result relies solely on the heterogeneous signals framework. Indeed, we already know from Admati and Pfleiderer (1988a), that in case of homogeneous signals (i.e. same signals), the value of information does not become more negative as N increases.

FIGURE 3 GOES HERE

Figure 3 shows the individual profit of the informed traders as a function of both the level of noise in the signals and of the number of insiders for more than four insiders. We have plotted in bold the individual profit for the optimal amount of noise as a function of the number of insiders.

The expected aggregate profit $\pi^*(N) = N\pi_i^*(N)$ does not go to zero when N is large, but instead tends to a finite positive value $\frac{\sigma_v \sigma_u}{2\sqrt{2}}$. In the literature Admati and Pfleiderer (1988a) have focused on homogeneous signals. In our framework of heterogeneous signals two cases have to be distinguished. When the precision of information is fixed and different from the optimal level of noise, the expected aggregate profits go to zero with the number of informed traders. Otherwise, for the optimal level of noise in signals, the expected aggregate profit goes to a finite

positive limit. In contrast to the Holden and Subramanyam (1992) model, the competition between informed traders may not destroy all their profits. In Foster and Vishnathan (1996) and Back, Cao and Willard (2000) the fierce competition causes almost all private information to be released in the earlier periods and therefore the aggregate expected profit tends to zero; this convergence is faster when the initial correlation between the informed traders' signals is high.

In Figure 4, the aggregate expected profit is represented as a function of the number of insiders for both the optimal noise and the perfect information. When traders have perfect information, they reveal it immediately, and the aggregate expected profit tends to zero when the number of informed traders is large. However, Figure 4 shows that they can substantially reduce the negative consequences of competition by using an optimal level of noise in their signal.

FIGURE 4 GOES HERE

As the liquidity is the inverse of the aggregate profit times σ_v^2, one can show that when insiders have perfect information, the liquidity tends to infinity with the number of informed traders. However, at the optimal level of noise the liquidity tends to a finite limit $\lambda_\infty^{-1} = \frac{\sigma_u}{\sigma_v} 2\sqrt{2}$ when the number of insiders is large.

In order to fully understand this result, we need to focus on the informativeness of prices and also on the process of aggregation of the collection of information. This is the purpose of the next proposition.

Proposition 4.3 : *For each level of signal precision and number of informed traders (σ_1, N), the informativeness of prices $I(\sigma_1, N)$ is given in equilibrium by:*

$$I(\sigma_1, N) = \{Var\,(\widetilde{v}|\widetilde{p})\}^{-1} = \{Var\,(\widetilde{v}|\widetilde{w})\}^{-1} = \frac{1}{\sigma_v^2} \frac{N + 1 + 2\frac{\sigma_1^2}{\sigma_v^2}}{1 + 2\frac{\sigma_1^2}{\sigma_v^2}}, \tag{8}$$

The informativeness of prices $I^*(N)$ for the optimal level of noise in the signals $\sigma_1^*(N)$ at equilibrium is given by:

$$I^*(N) = Var\,(\widetilde{v}|\widetilde{w})^{-1} = \frac{1}{\sigma_v^2} \frac{2(N-1)}{N-2}, \tag{9}$$

and when N is large, $I^*(N)$ is equivalent to:

$$I^*(N) = Var\,(\widetilde{v}|w)^{-1} \underset{N \to +\infty}{\sim} \frac{2}{\sigma_v^2}\,. \tag{10}$$

If we denote $\overline{S} = \frac{1}{N} \sum_{i=1}^{N} S_i$, then:

- *if* σ_1^2 *is fixed,*
  - $\overline{S} \underset{N\to+\infty}{\overset{L_2}{\longrightarrow}} v$,
  - $\widetilde{p} \underset{N\to+\infty}{\overset{L_2}{\longrightarrow}} v$
- *if* $\sigma_1^2 = \sigma_1^{*^2}(N)$,
  - $\overline{S} - v \underset{N\to+\infty}{\overset{\mathcal{L}}{\longrightarrow}} \frac{\sigma_v}{\sqrt{2}}\overline{X}$, *with* $\overline{X} \rightsquigarrow \mathcal{N}(0,1)$,
  - $\widetilde{p} - \frac{1}{2}v \underset{N\to+\infty}{\overset{\mathcal{L}}{\longrightarrow}} \frac{\sigma_v}{2}\overline{Y}$, *with* $\overline{Y} = \frac{1}{\sqrt{2}}\left(\overline{X} + \frac{u}{\sigma_u}\right) \rightsquigarrow \mathcal{N}(0,1)$,
  - $\widetilde{p} - \overline{S} \underset{N\to+\infty}{\overset{\mathcal{L}}{\longrightarrow}} -\frac{1}{2}v + \frac{\sqrt{3}}{4}\sigma_v\overline{Z}$, *with* $\overline{Z} = -\frac{1}{\sqrt{3}}\overline{X} + \sqrt{\frac{2}{3}}\frac{u}{\sigma_u} \rightsquigarrow \mathcal{N}(0,1)$

Proof *See Appendix C.2.*

Proposition 4.3 shows that the informativeness of prices tends to infinity for a fixed variance of the signals, whereas there is a finite limit for the case of optimal noise in the signals or more generally when the noise in the signals is of the same order as the optimal one. This shows that in the latter case efficiency is not reached in this market. This is confirmed by the results on the aggregation of information. When the variance is fixed, both the statistic $\overline{S}$ and the price $\widetilde{p}$ aggregate to the actual value v. For the case of optimal noise in the signals, the statistic $\overline{S}$ appears at the limit as the sum of the actual value v plus some error term. Thus, the collection of signals $\overline{S}$ does not aggregate to the final liquidation value of the risky asset. Nor does the price $\widetilde{p}$ aggregate to the value v. Indeed, it equals $\frac{1}{2}v$ plus an error term. Moreover, the price $\widetilde{p}$ does not aggregate the statistics $\overline{S}$.

This relies on both heterogeneous signals and optimal noise in signals or more generally on noise in signals being of the same order as the optimal noise. In this context, we can understand why at the limit the expected aggregate profits do not go to zero for large N. This is essentially due to the imperfect aggregation of information. As a result, in this economy as the number of competitors becomes larger, the market is not strongly efficient[3] With perfect information, the competition between informed traders cancels almost all uncertainty in the market. In this case, the market is strongly efficient as shown in Holden and Subramanyam (1992). However, in Foster and Vishwanathan (1996) as well as in Back, Cao and Willard (2000) competition may or may not lead to greater efficiency of prices. There is more or less uncertainty left depending on how the information is distributed among the agents. Moreover, in their model they show that the market is more efficient when the information is possessed by a single trader than when

[3]The efficiency of the price in our paper is the same concept as the informativeness of the price which is measured by the quantity $\{Var(\widetilde{v}|\widetilde{p})\}^{-1}$. Thus, a strongly efficient price is a price which reveals all the information about the fundamental value of the risky asset: v.

several traders share information by means of correlated signals. This result is similar to the result we obtain with the optimal level of noise.

Proposition 4.4 : *In equilibrium, the aggregate profit, where $\pi^*(N, \sigma_1)$ is maximized for the value $N = N^*(\sigma_1)$.*

$$\begin{aligned}
&N^*(\sigma_1) = 1 + 2\frac{\sigma_1^2}{\sigma_v^2},\ if\ 1 + 2\frac{\sigma_1^2}{\sigma_v^2} \quad \textit{is a positive integer,} \\
&N^*(\sigma_1) = \underset{N \in (\widetilde{N}_1(\sigma_1), \widetilde{N}_2(\sigma_1))}{argmax} \pi^*(N, \sigma_1),\ if\ 1 + 2\frac{\sigma_1^2}{\sigma_v^2} \quad \textit{is not a positive integer,} \\
&and\ \widetilde{N}_1(\sigma_1) = int\left(1 + 2\frac{\sigma_1^2}{\sigma_v^2}\right), \\
&\widetilde{N}_2(\sigma_1) = \widetilde{N}_1(\sigma_1) + 1.
\end{aligned} \tag{11}$$

Proof : *Straightforward.*

Proposition 4.4 states that for a given level of available information precision σ_1, there exists an optimal size of the market $N^*(\sigma_1)$ for which the aggregate profit is maximized. Moreover, the greater the level of noise in the signals the bigger the market. One implication of this result is that for the case of direct sale of information as in Admati and Pfleiderer (1986-1990), a seller of information should choose $N^*(\sigma_1)$ participants.[4]

5 Concluding Remarks

We determine the nature of the competition according to the distribution of information among insiders. Firstly, we show that each insider reacts all the more to her private information as the other informed traders' information is more noisy. More generally, we point out a tradeoff between the noise and the competition. We show that noise weakens the competition between traders so that they compete less aggressively. We delineate several regions of level of noise which lead to different profiles of liquidity and profits and compute the optimal level of noise. Under certain conditions, we point out regions of level of noises for which the expected profits of the insiders increase in the level of noise and are higher than the ones when insiders know perfectly the liquidation value of the risky asset and expected profits decrease slowly with the number of insiders. This differentiates us from Holden and Subramanyam (1992) where profits decrease quickly with the number of insiders. A possible extension of our model would be to study the tradeoff between noise and competition in a dynamic setting. Boco and Germain (2007) study this dynamic tradeoff in the case of direct sales of information.

[4] Boco and Germain (2007) study optimal selling of information in a dynamic setting.

Appendices

A.1. Proof of Proposition 3.1

Agent i maximizes his conditional expected profit ($N \geq 2$):

$$\underset{x \in I\!R}{max}\, E\left\{ x\left[\widetilde{v} - \lambda\left(x + \sum_{j \neq i} \widetilde{x}_j + \widetilde{u}\right)\right] \mid S_i \right\}.\ i = 1, \ldots, N,$$

Since $\widetilde{u}$ and $\widetilde{S}_i$ are independent, we have:

$$\underset{x \in I\!R}{max}\, \frac{1}{1+\tau_i} S_i x - \lambda x^2 - \lambda x \sum_{j \neq i} E\left(\widetilde{x}_j \mid S_i\right),\ i = 1, \ldots, N\cdot$$

It follows that:

$$\widetilde{x}_i = \frac{1}{2\lambda}\frac{1}{1+\tau_i} S_i - \frac{1}{2}\sum_{j \neq i} E\left(\widetilde{x}_j \mid S_i\right) = 0,\ i = 1, \ldots, N, \tag{12}$$

Let $\{x_i^*,\ i = 1, \ldots, N\}$ be a particular solution to (12). Then if we define:

$$\widetilde{y}_i = \widetilde{x}_i - x_i^*,\ i = 1, \ldots, N,$$

$\{\widetilde{x}_i,\ i = 1, \ldots, N\}$ is a solution to (12) if and only if $\{\widetilde{y}_i,\ i = 1, \ldots, N\}$ is a solution to (13):

$$\widetilde{y}_i = -\frac{1}{2}\sum_{j \neq i} E\left(\widetilde{y}_j \mid S_i\right),\ i = 1, \ldots, N\ \cdot \tag{13}$$

We now show that equation (13) has exactly one solution : $\widetilde{y}_i$=0 for $i = 1, \ldots, N$ almost surely.

Define $\widetilde{y}_i + \frac{1}{2}\sum_{j \neq i} \widetilde{y}_j = \widetilde{\eta}_i$for $i = 1, \ldots, N$. From (13) we have that $E\left(\widetilde{\eta}_i / S_i\right) = 0$.

We consider the linear system:

$$\begin{cases} \widetilde{y}_1 + \frac{1}{2}\sum_{j \neq 1} \widetilde{y}_j = \widetilde{\eta}_1 \\ \vdots \\ \widetilde{y}_n + \frac{1}{2}\sum_{j \neq n} \widetilde{y}_j = \widetilde{\eta}_n \end{cases}$$

By inverting we obtain that

$$\text{for } i = 1, \ldots, N, \quad \widetilde{y}_i = \frac{2}{N+1}\left\{N\widetilde{\eta}_i - \sum_{j \neq i} \widetilde{\eta}_j\right\}\cdot$$

If we substitute the latter relation into (13), we obtain that $\widetilde{y}$ is a solution to (13) if and only if there exists $\{\widetilde{\eta}_i,\ i=1,\ldots,N\}$ such that $E[\widetilde{\eta}_i|S_i]=0$ and

$$(1)\ \widetilde{y}_i = \frac{2}{N+1}\left\{N\widetilde{\eta}_i - \sum_{j\neq i}\widetilde{\eta}_j\right\},\ i=1,\ldots,N,$$

$$(2)\ \frac{2}{N+1}\left\{N\widetilde{\eta}_i - \sum_{j\neq i}\widetilde{\eta}_j\right\} = -\frac{1}{2}\sum_{j\neq i}E\left\{\frac{2}{N+1}\left[N\widetilde{\eta}_j - \sum_{k\neq j}\widetilde{\eta}_k\right] \mid S_i\right\},$$

$$\iff\ (1)\ and\ N\widetilde{\eta}_i - \sum_{j\neq i}\widetilde{\eta}_j = -\frac{1}{2}E\left\{N\sum_{j\neq i}\widetilde{\eta}_j - \sum_{j\neq i}\sum_{k\neq j}\widetilde{\eta}_k/ \mid S_i\right\},$$

$$\iff\ (1)\ and\ N\widetilde{\eta}_i - \sum_{j\neq i}\widetilde{\eta}_j = -\frac{1}{2}E\left\{N\sum_{j\neq i}\widetilde{\eta}_j - (N-1)\widetilde{\eta}_i - (N-2)\sum_{k\neq i}\widetilde{\eta}_k/ \mid S_i\right\},$$

$$\iff\ (1)\ and\ N\widetilde{\eta}_i = \sum_{j\neq i}\widetilde{\eta}_j - E\left[\sum_{j\neq i}\widetilde{\eta}_j \mid S_i\right],\ i=1,\ldots,N,\quad \text{since } E[\widetilde{\eta}_i|S_i]=0\cdot$$

From the last equation, we deduce that $E(\widetilde{\eta}_i) = 0,\ i=1,\ldots,N$ since $E(\widetilde{\eta}_i) = \frac{1}{N}E\{\widetilde{z} - E[\widetilde{z} \mid S_i]\} = 0$ where $\widetilde{z} = \sum_{j\neq i}\widetilde{\eta}_j$.

$$\begin{aligned} Var(\widetilde{\eta}_i) &= \frac{1}{N^2}Var\{\widetilde{z} - E[\widetilde{z} \mid S_i]\} = \frac{1}{N^2}E[Var(\widetilde{z} \mid S_i)], \\ &\leq \frac{1}{N^2}Var(\widetilde{z}) = \frac{1}{N^2}Var\left(\sum_{j\neq i}\widetilde{\eta}_j\right) \leq \frac{1}{N^2}\left[\sum_{j\neq i}\sqrt{Var(\widetilde{\eta}_j)}\right]^2, \end{aligned}$$

$$Var\left(\sum_{j\neq i}\widetilde{\eta}_j\right) = \sum_{j\neq i}Var(\widetilde{\eta}_j) + 2\sum_{\substack{j,k\neq i\\ k<j}}Cov(\widetilde{\eta}_j,\widetilde{\eta}_k) \leq \sum_{j\neq i}\left[\sqrt{Var(\widetilde{\eta}_j)}\right]^2 +$$

$$+2\sum_{\substack{j,k\neq i\\ k<j}}\sqrt{Var(\widetilde{\eta}_j)}\sqrt{Var(\widetilde{\eta}_k)} = \left[\sum_{j\neq i}\sqrt{Var(\widetilde{\eta}_j)}\right]^2.$$

$$\Longrightarrow \quad \forall i = 1, \ldots, N, \; Var\left(\widetilde{\eta}_i\right) \leq \left(\frac{N-1}{N}\right)^2 \underset{j=1,\ldots,N}{max} \left\{Var\left(\widetilde{\eta}_j\right)\right\},$$

$$\Longrightarrow \quad \underset{i=1,\ldots,N}{max} Var\left(\widetilde{\eta}_i\right) \leq \left(\frac{N-1}{N}\right)^2 \underset{j=1,\ldots,N}{max} \left\{Var\left(\widetilde{\eta}_j\right)\right\},$$

$$\Longrightarrow \quad \left[1 - \left(1 - \frac{1}{N}\right)^2\right] \underset{i=1,\ldots,N}{max} Var\left(\widetilde{\eta}_i\right) \leq 0,$$

$$\Longrightarrow \quad \underset{i=1,\ldots,N}{max} Var\left(\widetilde{\eta}_i\right) = 0\cdot$$

Thus we have for $i = 1, \ldots, N$:

$$\left.\begin{array}{l} E\left(\widetilde{\eta}_i\right) = 0, \\ \\ Var\left(\widetilde{\eta}_i\right) = 0, \end{array}\right\} \Longrightarrow \widetilde{\eta}_i \overset{a.s.}{=} 0\cdot$$

Therefore the only solution to (13) is $\widetilde{y} = 0 \in I\!R^N$.

We now show the existence of a linear equilibrium $\{x_i^*, i = 1, \ldots, N\}$ which is, in addition, the unique linear equilibrium. We thus look for a particular solution to (13) of the form $\{x_i^* = \beta_i^* S_i, \; i = 1, \ldots, N\}$. We introduce the following quantities:

$$a_i = \frac{1}{1+\tau_i} < 1, \; i = 1, \ldots, N,$$

$$b_i = \frac{1}{2\lambda} a_i\cdot$$

Thanks to the normality assumption and the mutual independence of $\{\widetilde{v}, \widetilde{\varepsilon}_1, \ldots, \widetilde{\varepsilon}_N\}$, we have $E\left(\widetilde{S}_j \mid S_i\right) = a_i S_i$ for $j \neq i$. Using (12), we have

$$\beta_i S_i = b_i S_i - \frac{1}{2} \sum_{j \neq i} E\left[\beta_j \widetilde{S}_j \mid S_i\right] = b_i S_i - \frac{1}{2} \sum_{j \neq i} a_i \beta_j \widetilde{S}_j, \; i = 1, \ldots, N,$$

or equivalently

$$\beta_i = b_i - \frac{a_i}{2} \sum_{j \neq i} \beta_j, \; i = 1, \ldots, N,$$

$$\beta_i \left(\frac{1}{2} + \tau_i\right) = \frac{1}{2\lambda} - \frac{1}{2} \sum_{j=1}^{N} \beta_j, \; i = 1, \ldots, N,$$

$$\beta_i = \frac{a}{1+2\tau_i} \Longleftrightarrow \frac{1}{\lambda} = a \left[1 + \sum_{j=1}^{N} \frac{1}{1+2\tau_j}\right], \tag{14}$$

with a independent of i. We also know that, at equilibrium, $\widetilde{p} = E\left(\widetilde{v}|w\right)$. In this particular equilibrium, we have $\widetilde{w} = \sum_{i=1}^{N} \beta_i \widetilde{S}_i + \widetilde{u}$. From the mutual independence of $\{\widetilde{v}, \widetilde{\varepsilon}_1, \ldots, \widetilde{\varepsilon}_N, \widetilde{u}\}$ and

normality of each components, it follows that $\widetilde{p} = \lambda w$ and that therefore we obtain a linear equilibrium with

$$\lambda = \frac{Cov\left(\widetilde{v}, \widetilde{w}\right)}{Var\left(\widetilde{w}\right)} = \frac{Cov\left(\widetilde{v}, \sum_{j=1}^{N} \beta_j \widetilde{S}_j + \widetilde{u}\right)}{Var\left(\sum_{j=1}^{N} \beta_j \widetilde{S}_j + \widetilde{u}\right)} = \frac{\sum_{j=1}^{N} \beta_j \sigma_v^2}{\left(\sum_{j=1}^{N} \beta_j\right)^2 \sigma_v^2 + \sum_{j=1}^{N} \beta_j^2 \sigma_j^2 + \sigma_u^2},$$

so

$$\frac{1}{\lambda} = a \sum_{j=1}^{N} \frac{1}{1+2\tau_j} + a \frac{\sum_{j=1}^{N} \tau_j \frac{1}{(1+2\tau_j)^2}}{\sum_{j=1}^{N} \frac{1}{1+2\tau_j}} + \frac{\sigma_u^2}{\sigma_v^2} \frac{1}{a \sum_{j=1}^{N} \frac{1}{1+2\tau_j}} \cdot \tag{15}$$

Using (14) and (15), we obtain:

$$a \left\{ 1 - \frac{\sum_{j=1}^{N} \frac{\tau_j}{(1+2\tau_j)^2}}{\sum_{j=1}^{N} \frac{1}{1+2\tau_j}} \right\} = \frac{\sigma_u^2}{\sigma_v^2} \frac{1}{a \sum_{j=1}^{N} \frac{1}{1+2\tau_j}},$$

$$\text{so that} \quad a = \frac{\sigma_u}{\sigma_v} \frac{1}{\left\{ \sum_{j=1}^{N} \frac{1+\tau_j}{(1+2\tau_j)^2} \right\}^{\frac{1}{2}}},$$

Inserting the above value of a into (12) yields the formula for $\beta_i^*(\sigma)$ and $\frac{1}{\lambda^*(\sigma)}$ in the proposition.

Following algebraic manipulation, we obtain the formula of the expected individual profit at the equilibrium:

$$\pi_i^*(\sigma) = \frac{\sigma_v \sigma_u}{\left[1 + \sum_{j=1}^{N} \frac{1}{1+2\tau_j}\right] \left\{ \sum_{j=1}^{N} \frac{1+\tau_j}{(1+2\tau_j)^2} \right\}^{\frac{1}{2}}} \frac{1+\tau_i}{[1+2\tau_i]^2}.$$

This ends the proof of Proposition 3.1.

B.1. Proof of proposition 3.2

We calculate the derivative of β_i^* with respect to τ_j and then with respect to τ_i. After simplification, we obtain

$$\frac{\partial \beta_i^*}{\partial \tau_j}(\tau) = \frac{1}{2}\frac{\sigma_u}{\sigma_v}\frac{3+2\tau_j}{\left\{\sum_{j=1}^{N}\frac{1+\tau_j}{(1+2\tau_j)^2}\right\}^{\frac{3}{2}}(1+2\tau_i)(1+2\tau_j)^3} > 0, \quad \text{for } j \neq i, \ so$$

$$\frac{\partial \beta_i^*}{\partial \tau_i}(\tau) = -\frac{1}{2}\frac{\sigma_u}{\sigma_v}\frac{4(1+2\tau_i)\sum_{j\neq i}\frac{1+\tau_j}{(1+2\tau_j)^2}+1}{\left\{\sum_{j=1}^{N}\frac{1+\tau_j}{(1+2\tau_j)^2}\right\}^{\frac{3}{2}}(1+2\tau_i)^3} < 0\cdot$$

We do the same with the function $\ln(\pi_i^*(\tau))$

$$\frac{\partial \ln \pi_i^*}{\partial \tau_j}(\tau) = \frac{1}{2}\frac{3+2\tau_j}{\sum_{k=1}^{N}\frac{1+\tau_k}{(1+2\tau_k)^2}(1+2\tau_j)^3} + \frac{2}{\left[1+\sum_{k=1}^{N}\frac{1}{1+2\tau_k}\right](1+2\tau_j)^2} > 0$$

$$\Longrightarrow \pi_i^*(\sigma) \quad \text{is increasing in } \sigma_{-i}, therefore$$

$$\begin{aligned}\frac{\partial \ln \pi_i^*}{\partial \tau_i}(\tau) &= \frac{1}{2}\frac{3+2\tau_i}{(1+2\tau_i)^3\sum_{k=1}^{N}\frac{1+\tau_k}{(1+2\tau_k)^2}} + \frac{2}{(1+2\tau_i)^2\left[1+\sum_{k=1}^{N}\frac{1}{1+2\tau_k}\right]} + \frac{1}{1+\tau_i} - \frac{4}{1+2\tau_i}, \\ &= \frac{1}{2(1+2\tau_i)^3\left[\sum_{k=1}^{N}\frac{1+\tau_k}{(1+2\tau_k)^2}\right]\left[1+\sum_{k=1}^{N}\frac{1}{1+2\tau_k}\right](1+\tau_i)} g_i(\tau),\end{aligned}$$

where

$$\begin{aligned}g_i(\tau) &= (3+2\tau_i)(1+\tau_i)\left[1+\sum_{k=1}^{N}\frac{1}{1+2\tau_k}\right] + 4(1+2\tau_i)\left[\sum_{k=1}^{N}\frac{1+\tau_k}{(1+2\tau_k)^2}\right](1+\tau_i) \\ &\quad -2(3+2\tau_i)(1+2\tau_i)^2\sum_{k=1}^{N}\frac{1+\tau_k}{(1+2\tau_k)^2}\left[1+\sum_{k=1}^{N}\frac{1}{1+2\tau_k}\right].\end{aligned}$$

The sign of $\frac{\partial \ln \pi_i^*}{\partial \tau_i}(\tau)$ is the sign of $g_i(\tau)$. To determine the sign of $g_i(\tau)$, we rewrite it as

$$-2\left(1+\tau_i\right)^2-\left(3+2\tau_i\right)\left(1+\tau_i\right)\sum_{k\neq i}\frac{1}{1+2\tau_k}-8\left(1+2\tau_i\right)\left(1+\tau_i\right)^2\sum_{k\neq i}\frac{1+\tau_k}{\left(1+2\tau_k\right)^2}$$
$$-2\left(3+2\tau_i\right)\left(1+2\tau_i\right)^2\sum_{k\neq i}\frac{1+\tau_k}{\left(1+2\tau_k\right)^2}\sum_{k\neq i}\frac{1}{1+2\tau_k}.$$

Since $g_i(\tau)<0$ it follows that $\pi_i^*(\sigma)$ is decreasing in $\tau_i=\dfrac{\sigma_i^2}{\sigma_v^2}$.
This ends the proof of Proposition 3.2.

B.2. Proof of Proposition 3.3

$$\frac{\partial\lambda^{*^{-1}}}{\partial\tau_i}(\tau)=\frac{\sigma_u}{\sigma_v}\left\{\frac{1}{2}\frac{\left(3+2\tau_i\right)\left[1+\sum_{j=1}^{N}\frac{1}{1+2\tau_j}\right]}{\left\{\sum_{j=1}^{N}\frac{1+\tau_j}{\left(1+2\tau_j\right)^2}\right\}^{\frac{3}{2}}\left(1+2\tau_i\right)^3}-\frac{2}{\left\{\sum_{j=1}^{N}\frac{1+\tau_j}{\left(1+2\tau_j\right)^2}\right\}^{\frac{1}{2}}\left(1+2\tau_i\right)^2}\right\},$$

$$\frac{\partial\lambda^{*^{-1}}}{\partial\tau_i}(\tau)=\frac{1}{2}\frac{\sigma_u}{\sigma_v}\frac{1}{\left\{\sum_{j=1}^{N}\frac{1+\tau_j}{\left(1+2\tau_j\right)^2}\right\}^{\frac{3}{2}}\left(1+2\tau_i\right)^3}h_i(\tau),\ \textit{with}:\ h_i(\tau)=2\tau_i b(\tau_{-i})+c(\tau_{-i}),$$

where $b(\tau_{-i})$ and $c(\tau_{-i})$ are defined in Proposition (3.3). It is worth noting that $c(\tau_{-i})>b(\tau_{-i})$, indeed $c(\tau_{-i})-b(\tau_{-i})=1+\sum_{j\neq i}\dfrac{2}{1+2\tau_j}$. This ends the proof of Proposition 3.3.

If we define Σ_N as

$$\Sigma_N=\{\sigma=(\sigma_1,\ldots,\sigma_N)'\in I\!R_+^N\mid\forall i=1,\ldots,N\quad\pi_i^*(\sigma_1,\ldots,\sigma_N)\geq\pi_i^*(0,\ldots,0),$$
$$\textit{and}\ \ \exists i_\circ\in(1,\ldots,N)\ \ \textit{with}\ \ \pi_{i_\circ}^*(\sigma_1,\ldots,\sigma_N)>\pi_{i_\circ}^*(0,\ldots,0)\},$$

we have

- for $N\geq 4$ Σ_N is non-empty. In other words, noise in the collection of signals is ex–ante optimal;
- for $N\leq 3$, Σ_N is empty.

In order to show the result of the non-emptiness of Σ_N for $N\geq 4$, we just need to focus on the symmetric case where $\sigma_1=\ldots=\sigma_N$ and this is the purpose of proposition 4.2. We now show that Σ_N is empty for $N\leq 3$.

Proof for N=2:

(As the proofs for N=2 and N=3 are almost identical, we leave this proof to the reader.)

Proof for N=3:

$$\pi_i(\tau) = \sigma_u \sigma_v \frac{\dfrac{1+\tau_i}{(1+2\tau_i)^2}}{\left\{\displaystyle\sum_{j=1}^{3} \frac{1+\tau_j}{(1+2\tau_j)^2}\right\}^{\frac{1}{2}} \left\{1 + \displaystyle\sum_{i=1}^{3} \frac{1}{1+2\tau_i}\right\}}.$$

We again denote $x_i = \dfrac{1+\tau_i}{(1+2\tau_i)^2}$, $i = 1,2,3$. For $\tau_i \neq 0$, $x_i < 1$, $y_i = \dfrac{1}{1+2\tau_i} = \dfrac{\sqrt{1+8x_i}-1}{2}$.
Therefore, for $i = 1,2,3$ we have

$$\pi_i(\tau) = \sigma_v \sigma_u \frac{2x_i}{\left(\displaystyle\sum_{j=1}^{3} x_j\right)^{\frac{1}{2}} \left(\displaystyle\sum_{j=1}^{3} \sqrt{1+8x_j} - 1\right)},$$

$$\pi_i(0) = \frac{\sigma_v \sigma_u}{4\sqrt{3}}.$$

We already know that for $\tau_1 = \tau_2 = \tau_3 \neq 0$, $\pi_1(\tau)$, $\pi_2(\tau)$ and $\pi_3(\tau)$ are strictly smaller than $\pi_i(0) = \dfrac{\sigma_v \sigma_u}{4\sqrt{3}}$ and equal if and only if $\tau_1 = \tau_2 = \tau_3 = 0$. We thus focus on cases where $\exists\ (i,j) \in \{1,2,3\}^2$ such that $\tau_j \neq \tau_i \iff x_j \neq x_i$. Without any loss of generality, we will suppose (up to a reordering) that:

a) either $x_1 > x_2 \geq x_3$,

b) or $x_1 \geq x_2 > x_3$.

Now assume that $\pi_i(\tau) \geq \dfrac{\sigma_v \sigma_u}{4\sqrt{3}}$, $i = 1,2,3$.
a)If $x_1 > x_2 \geq x_3$:

$$\textit{then} \quad x_3 \geq \frac{1}{8\sqrt{3}} \left(\sum_{j=1}^{3} x_j\right)^{\frac{1}{2}} \left(\sum_{j=1}^{3} \sqrt{1+8x_j} - 1\right)$$

$$\implies \quad 8\sqrt{x_3} \geq 3\sqrt{1+8x_3} - 1.$$

Let f be:

$$f(x) = 8\sqrt{x} - 3\sqrt{1+8x} + 1,$$

$$where\ 0 \leq x,\ so$$

$$f'(x) = \frac{4(1-x)}{\sqrt{x}\sqrt{1+8x}\left[\sqrt{1+8x} + 3\sqrt{x}\right]}.$$

f is increasing on $[0,1]$, decreasing on $[1,+\infty[$ and $f(1) = 0$. Therefore $\forall x \geq 0,\ f(x) \leq 0$ and $f(x) = 0 \Longrightarrow x = 1$.

$$8\sqrt{x_3} \geq 3\sqrt{1+8x_3} - 1 \Longrightarrow x_3 = 1,$$

which is ruled out.

b) $x_1 \geq x_2 > x_3$: the proof is analogous.

The above cases imply that Σ_3 is empty.

C.1. Proof of Proposition 4.2

$$\begin{aligned}
N \geq 4, \quad & \pi_i^*(\sigma_1) > \pi_i^*(0), \\
\Longleftrightarrow \quad & \frac{\sqrt{1+\tau}}{\sqrt{N}\,(N+1+2\tau)} > \frac{1}{\sqrt{N}\,(N+1)}, \quad where\ \tau = \frac{\sigma_1^2}{\sigma_v^2} > 0 \\
\Longleftrightarrow \quad & \sigma_1 < \frac{\sigma_v}{2}\sqrt{N-3}\sqrt{N+1}. \\
\pi_i^*(\tau) \quad & = \sigma_v \sigma_u \frac{\sqrt{1+\tau}}{\sqrt{N}\,(N+1+2\tau)}, \\
\frac{\partial \ln \pi_i^*}{\partial \tau}(\tau) \quad & = \frac{\partial}{\partial \tau}\left\{\frac{1}{2}\ln(1+\tau) - \ln(N+1+2\tau)\right\}, \\
& = \frac{1}{2\,(1+\tau)} - \frac{2}{N+1+2\tau} = \frac{N-3-2\tau}{2\,(1+\tau)\,(N+1+2\tau)}.
\end{aligned}$$

Therefore $\pi_i^*(\tau)$ is maximized for $\tau = \dfrac{N-3}{2}$. This ends the proof of Proposition 4.2.

C.2. Proof of Proposition 4.3

$$\begin{pmatrix} \widetilde{v} \\ \\ \widetilde{w} \end{pmatrix} \rightsquigarrow \mathcal{N}\left[\begin{pmatrix} 0 \\ \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_v^2 & \dfrac{\sqrt{N}\sigma_u\sigma_v}{\sqrt{1+\dfrac{\sigma_1^2}{\sigma_v^2}}} \\ \\ \dfrac{\sqrt{N}\sigma_u\sigma_v}{\sqrt{1+\dfrac{\sigma_1^2}{\sigma_v^2}}} & \sigma_u^2\left[1+\dfrac{N+\frac{\sigma_1^2}{\sigma_v^2}}{1+\frac{\sigma_1^2}{\sigma_v^2}}\right] \end{pmatrix}\right],$$

$\begin{pmatrix} \widetilde{v} \\ \\ \widetilde{u} \end{pmatrix}$ is normal $\Longrightarrow \begin{pmatrix} \widetilde{v} \\ \\ \widetilde{w} \end{pmatrix}$ is normal. Moreover, we have:

- $Cov\left(\widetilde{v}, \widetilde{w}\right) = Cov\left(\widetilde{v}, N\beta^*(\sigma_1, N)\widetilde{v} + \sum_{i=1}^{N}\beta^*(\sigma_1, N)\widetilde{\varepsilon}_i + \widetilde{u}\right) = N\beta^*(\sigma_1, N)\sigma_v^2 = \dfrac{\sqrt{N}\sigma_u\sigma_v}{\sqrt{1+\frac{\sigma_1^2}{\sigma_v^2}}}\cdot$

- $Var\left(\widetilde{w}\right) = Var\left(N\beta^*(\sigma_1, N)\widetilde{v} + \sum_{i=1}^{N}\beta^*(\sigma_1, N)\widetilde{\varepsilon}_i + \widetilde{u}\right) = N^2{\beta^*}^2(\sigma_1, N)\sigma_v^2 + N{\beta^*}^2(\sigma_1, N)\sigma_1^2 + \sigma_u^2,$

  $$Var\left(\widetilde{w}\right) = \frac{N\sigma_u^2}{1+\frac{\sigma_1^2}{\sigma_v^2}} + \frac{\sigma_u^2\frac{\sigma_1^2}{\sigma_v^2}}{1+\frac{\sigma_1^2}{\sigma_v^2}} + \sigma_u^2 = \sigma_u^2\left[1+\frac{N+\frac{\sigma_1^2}{\sigma_v^2}}{1+\frac{\sigma_1^2}{\sigma_v^2}}\right],$$

- $Var\left(\widetilde{v} \mid w\right) = Var\left(\widetilde{v}\right) - \dfrac{\left[Cov\left(\widetilde{v}, \widetilde{w}\right)\right]^2}{Var\left(\widetilde{w}\right)} = \sigma_v^2\left[1 - \dfrac{N}{N+1+2\frac{\sigma_1^2}{\sigma_v^2}}\right]\cdot$

The rest of Proposition 4.3 is proved using the central limit theorem and is therefore omitted.

Fig. 1. Aggregate profit of the traders as a function of both the level of noise in the signal of one trader and the level of noise in the signal of the other traders.

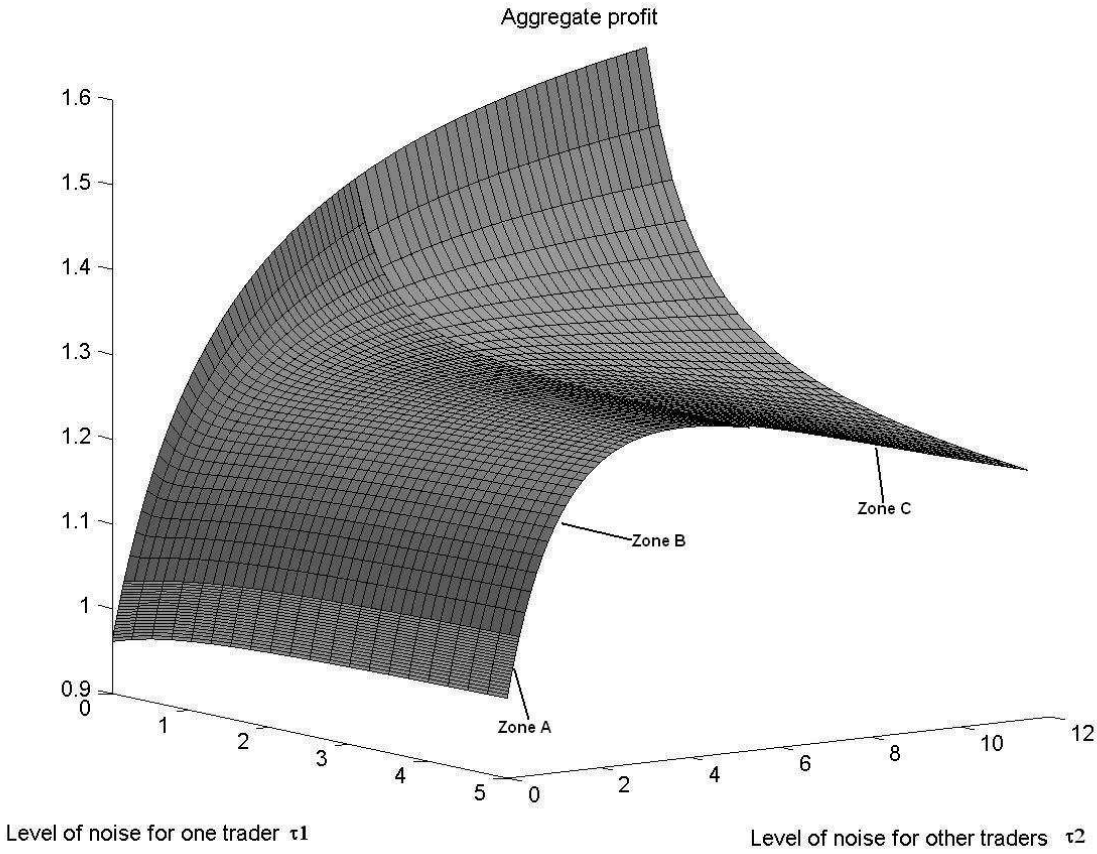

Figure 1:

This figure shows the aggregate expected profit of 11 traders in the specific case for which we have on the one hand a trader with his own level of noise τ_1 and on the other hand 10 traders with the same level of noise τ_2.

Fig. 2. Normalized aggregate profit as a function of both the level of noise in the signals and the number of insiders.

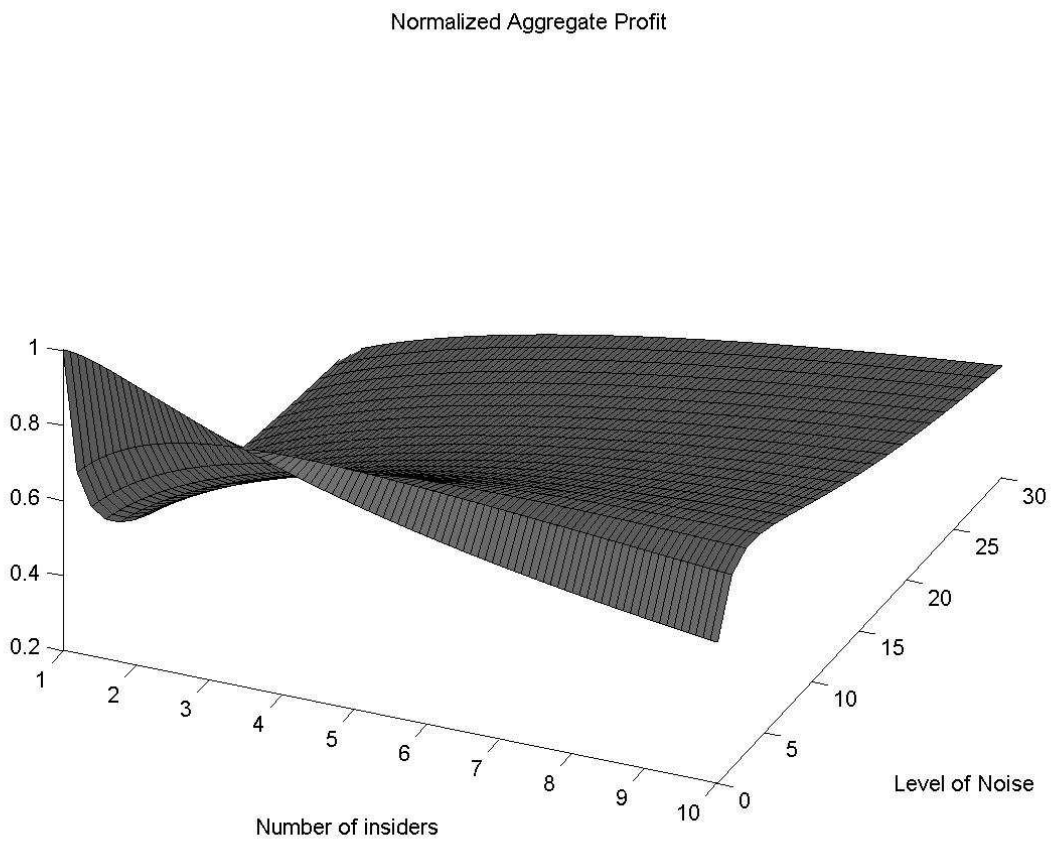

Figure 2: This figure represents the normalized aggregate profit as a function of the level of noise in the signals and the number of insiders. The normalization is performed with respect to the profit achieved by the monopoly in Kyle's (1985) setup ($\pi^*(0,1) = \frac{1}{2}\sigma_u\sigma_v$).

Fig. 3. Individual profit as a function of both level of noise in the signals and the numbers of insiders with a number of insiders higher than 4.

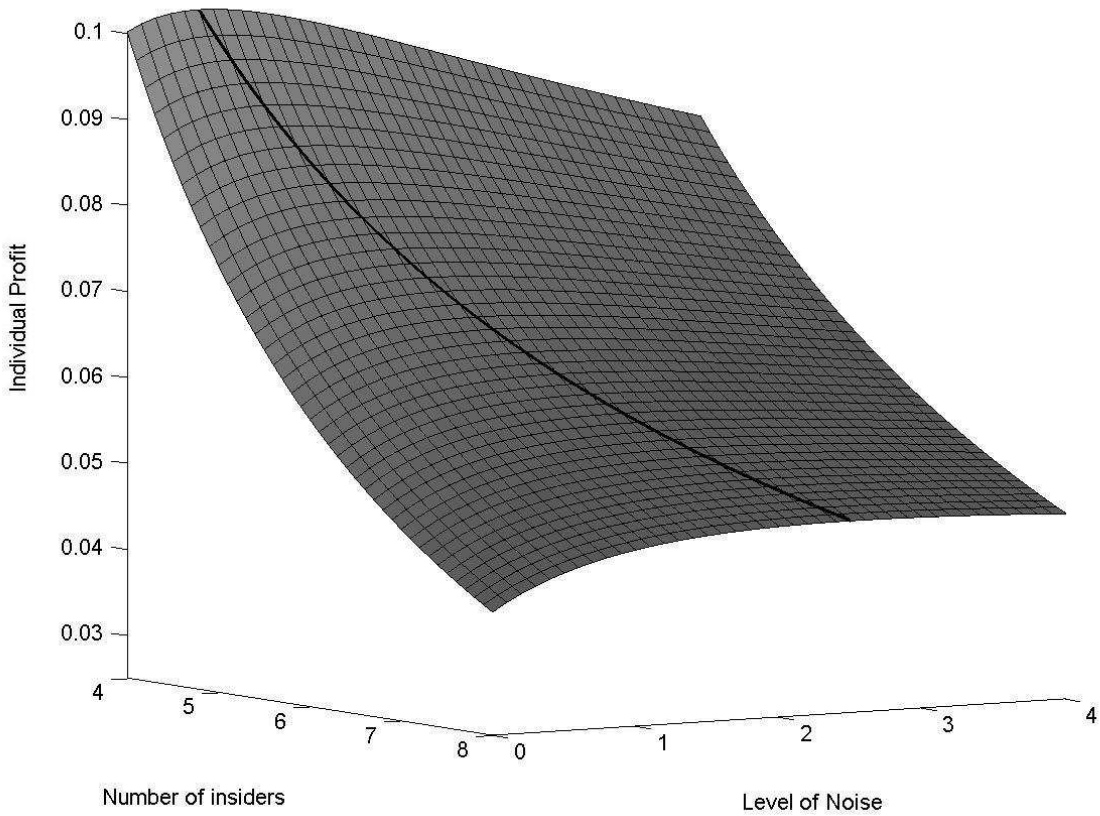

Figure 3: This figure shows the individual profit of the informed traders as a function of both the level of noise in the signals and the number of insiders for more than four insiders. We have plotted in bold the optimal amount of noise as a function of the number of insiders.

Fig. 4. Aggregate profit: Optimal symmetric noise versus perfect information.

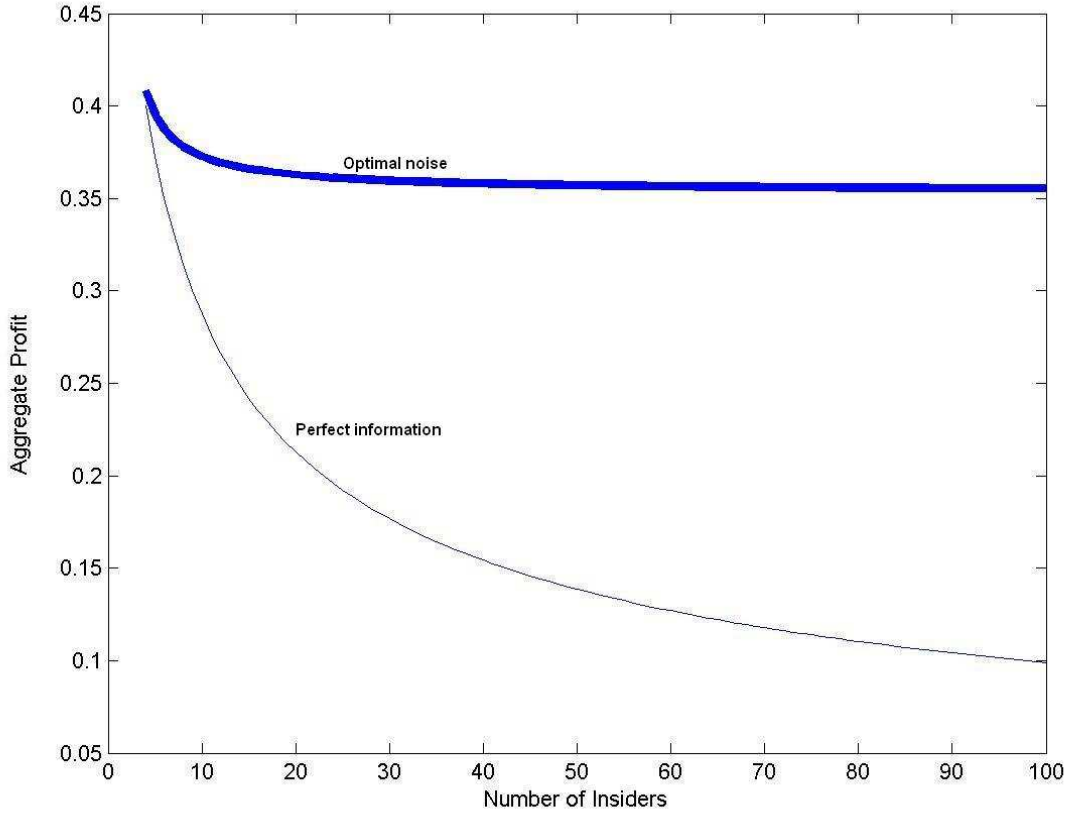

Figure 4: This figure shows the aggregate profit as a function of the number of insiders for both the optimal noise and the perfect information cases.

References

Admati, A., and Pfleiderer, P., 1986. A Monopolistic Market for Information. J. Econ. Theory, 39, 400-38.

Admati, A., and Pfleiderer, P., 1988a. Selling and Trading on Information in Financial Markets, American Econ. Rev., 78, 1165-1183.

Admati, A., and Pfleiderer, P., 1988b. A Theory of Intraday Patterns: Volume and Price Variability. Rev. Finan. Stud, 1, 3-40.

Admati, A., and Pfleiderer, P., 1990. Direct and Indirect Sale of Information. Econometrica 58, 901-928.

Back, K., Cao, K., and Willard, A., 2000. Imperfect Competition Among Informed Traders. J. Finance 5, 2117-2155.

Biais, B., and Germain, L., 2002. Incentive Compatible Contracts for the Sale of Information. Rev. Finan. Stud 4, 987-1004.

Biais, B., Martimort, D., and Rochet, J. C., 2000. Competing Mechanisms in a Common Value Environment. Econometrica 4, 799-837.

Boco, H., and Germain, L., 2007 Optimal Market Size for Selling on Information. forthcoming.

Foster, F. D., and Viswanathan, S., 1994. Strategic Trading with Asymmetrically Informed Traders and Long-lived Information. *J. Fin. and Quant.*, 29, 499-518.

Foster, F. D., and Viswanathan, S. 1996. Strategic Trading When Agents Forecast the Forecasts of Others. J. Finance 51, 1437-78.

Germain, L., 2007. Strategic Noise in Competitive Markets for the Sale of Information. forthcoming J. Finan. Intermediation.

Grossman, S. J., 1976. On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information. J. Finance 31, 573-585.

Grossman, S. J., and Stiglitz, J. E., 1980. On the Impossibility of Informationally Efficient Markets. American Econ. Rev 70, 393-408.

Hellwig, M. F., 1980. On the Aggregation of Information in Competitive Markets. J. Econ. Theory 22, 477-498.

Holden, C. W., and Subrahmanyam, A., 1992. Long-Lived Private Information and Imperfect Competition. J. Finance 47, 247-270.

Holden, C. W., and Subrahmanyam, A., 1994. Risk Aversion, Imperfect Competition and Long-lived Information. Econ. Letters 44, 181-190.

Kyle, A., 1984. Market Structure, Information, Futures Markets and Price Formation in International Agricultural Trade. *Advanced Readings in Price formation, Market Structure, and Price Instability* ed. Story, Schmitz and Sarris (Westview Press, Boulder and London).

Kyle, A., 1985. Continuous Auctions and Insider Trading. Econometrica 53, 1315-1335.

Rochet, J. C., and Villa, J. L., 1994. Insider Trading without Normality. Rev. Econ. Stud 61, 131-152.