

Open Archive Toulouse Archive Ouverte (OATAO)

OATAO is an open access repository that collects the work of Toulouse researchers and makes it freely available over the web where possible.

This is an author-deposited version published in: <u>http://oatao.univ-toulouse.fr/</u> Eprints ID: 5785

> **To link to this article**: DOI:10.1017/S0022109000003240 URL: <u>http://dx.doi.org/10.1017/S0022109000003240</u>

To cite this version: Dridi, Ramdan and Germain, Laurent *Bullish-Bearish strategies of trading: A non-linear equilibrium.* (2004) Journal of financial and quantitative analysis (JFQA), 39 (4). pp. 873-886. ISSN 0022-1090

Any correspondence concerning this service should be sent to the repository administrator: <u>staff-oatao@inp-toulouse.fr</u>

Bullish-Bearish Strategies of Trading: A Non-linear Equilibrium *

Ramdan Dridi[†] and Laurent Germain[‡]

First Draft: May 1999 This Draft March 2003

Abstract

In this paper, we study a financial market where risk neutral traders are endowed with a signal which is perfectly revealing of the direction (but not the exact amount) of the liquidation value of a normally distributed risky asset. This type of information is known as bullish or bearish. When the signal is positive (negative) the traders buy (sell) the asset. This type of information is different with the type of information which is classically considered in the literature where informed traders are endowed with a perfect or a noisy signal. In this model, since the optimal trading strategy is not linear, the pricing schedule is also a non-linear function of the volumes. The main results are the following i) the price function is a non-linear Sigmoïd-shaped function. ii) A monopolistic bullish-bearish type trader makes nearly thirty six percent of the profits she would have made with a perfect signal in a linear model à la Kyle (1985). iii) In the presence of competition, the market reveals his private information quicker than in a noisy informed strategic oligopoly. Moreover, liquidity is no longer a monotonic increasing function of the number of competitors.

Keywords: Non-linear equilibrium, recommendations of trading, asymmetric information, investment banks.

JEL Classification: D43, D82, G14, G24.

[†]Ecole Supérieure de Commerce de Toulouse, Groupe de Finance, 20 boulevard Lascrosses, BP 7010, 31068 Toulouse cedex 7, France. Tel 33 5 61 29 49 43, Fax: 33 5 61 23 32 79, E-mail: r.dridi@esc-toulouse.fr

^{*}We are particularly indebted to Bruno Biais, Giovanni Cespa, Gilles Chemla, Gabrielle Demange, Antoine Faure-Grimaud, Mike Fishman, Jean-Claude Gabillon, André Grimaud, Thomas Mariotti, Purnendu Nath, Monique Pontier, Jean-Charles Rochet, David Stolin and seminar participants at Toulouse, at the Asymmetric Information in Financial Markets Conference in Perpignan, at the World Econometric Congress (2000) in Seatle, at the TMR Conference at Pompeu Fabra for long conversations and very thoughtful comments which have greatly improved the quality of this paper and we also address special thanks to Purnendu Nath and Herve Boco for great research assistance. All remaining errors are our own. Part of this article was written while Germain was at the London Business School.

[‡]Ecole Supérieure de Commerce de Toulouse, Groupe de Finance and SUPAERO 20 boulevard Lascrosses, BP 7010, 31068 Toulouse cedex 7, France. Tel 33 5 61 29 49 43, Fax: 33 5 61 23 32 79, E-mail: l.germain@esctoulouse.fr

1 Introduction

How do people collect information? What type of information is used by traders on financial markets? Most of the literature in finance has focused on the study of a particular type of information: either traders collect a perfect signal equal to the liquidation value of the risky asset or traders collect a noisy signal equal to the liquidation value of the risky asset plus some noise. This is the case, in particular, in Kyle's (1985) seminal paper, Kyle (1984, 1989), Grossman (1976), Grossman and Stiglitz (1980), Foster and Vishwanathan (1990,1994,1996), Subrahmanyam (1991), Holden and Subrahmanyam (1992), Admati and Pfleiderer (1988a,b), Vives (1995), Dridi and Germain (1999) and others in the case where the liquidation value of the risky asset is normally distributed. Several other models such as Glosten and Milgrom (1985). Easley and O'Hara (1987,1992) study the case where the risky asset and/or the signal follow a two point (high/low) distribution. In our model, the liquidation value is normally distributed but the signal can only take two values. Indeed, one of the most popular types of information is a **buy or a sell signal**. In effect, this type of information is well known to practitioners and is referred to as bullish versus bearish trading recommendations. Traders can use this type of information either because they bought a trading signal (through direct sale of information),¹ or because the type of information they collected as insiders is of the bullish/bearish form. For example, if the firm receives a new order this is clearly "good news", but quantifying this information further is a challenge.² Most of the recommendations given by financial analysts or banks' research departments are of the bullish versus bearish variety.³ Nevertheless, in certain cases the clients are not only delivered the trading direction of the security but also a magnitude such as the target price or an upper bound for the price. This type of information is different from the classical "true value plus noise" assumption, as in such cases the traders know with certainty the direction of the stock price, which is not the case when the signal is noisy. In this article we propose a theoretical framework to understand price formation when market information is

¹Sales of information have been extensively studied in financial economics. Admati and Pfleiderer (1990) have defined two ways of buying information, by subscribing to newsletters or bulletins (direct sale of information), or by signing a contract with a financial institution which manages an investment fund on behalf of its client (indirect sale of information). In direct sale of information, the client can observe the information which is transmitted by the informed, whereas in indirect sale of information the client can only infer the information of the informed through the fund's profits.

²In other cases the insider can obtain the exact value v or v + noise.

³It is worth mentioning at this stage that there are a myriad of web sites devoted to financial recommendations.

A common feature of many of these web sites is that they deliver buy or sell signals without any other refinement.

such that traders know only the trading direction. The predictions of this type of information structure are very different from those entailed by the true value plus noise assumption, since at the equilibrium the reaction of the traders to their information is no longer a linear function of the signal and the equilibrium price is not a linear function of the order flow. In this paper, however, we only study the case of "bullish", "hold" or "bearish" recommendations. One could of course argue that what is more often observed in a financial market is a multi-level structure such as "strong buy", "buy", "hold", "sell", "strong sell". However, we do believe that the simpler version "buy", "hold", and "sell" intrinsically contains the main relevant features of the phenomenon.⁴

Moreover, empirical studies such as Womack (1996) or Kim, Lin and Slovin (1997) show that this type of signal is commonly collected and used by the market.⁵ Thus it seems natural to consider the issue raised by bullish/bearish informed type of agents trading in a market. In this paper we explore from the theoretical point of view the consequences of this type of information on prices. This corresponds to a genuine new problem that cannot be treated within the standard microstructure literature. In effect, most previous research has focused on linear equilibria. In the standard Kyle (1985) framework a monopolistic perfectly informed trader submits a market order to market makers. After having observed the total order flow generated by the informed and the noise traders, competitive market makers set the price in a Bayesian way. In this framework it is shown that there exists a unique linear equilibrium if we assume linearity of prices in order flow. In this equilibrium, strategies are linear - the quantity bought is linear in the signal - and the price function is a linear function of the order flow no matter how large the order imbalance. In a setting differing from the standard Kyle (1985) model where a monopolistic informed trader submits limit orders - or equivalently observes the realization of the noise trading - Rochet and Vila (1994) demonstrate existence and uniqueness of the equilibrium regardless of the distribution of the risky asset and that of the noise trading (see Back (1992) for a continuous time version of the Kyle (1985) model). In another paper, Cho and El Karoui (1997) relax the assumption of normality and derive a quasi-linear equilibrium in the Kyle framework. Bhattacharya and Spiegel (1991) and Genotte and Leland (1990) consider other types of non-linearities, e.g. stemming from the hedging needs of the traders. We take

⁴We also want to stress that the computational burden for developing the theory within the framework of multi-level signals (this is notably the question of best response functions defined implicitly through elliptical forms) is such that we have decided to restrict ourself to the simpler version - namely bullish versus bearish signals.

⁵In this paper we study the impact of a buy or a sell signal as private information. This is different from the empirical studies in the literature which address the question of the impact of these signals as public information.

here another route. In our framework, the ex-ante distribution of the risky financial asset as well as noise trading are normally distributed. On the other hand, the process of collection of information is different. But, when the ex-ante asset value is a gaussian random variable, the strategy of trading of a bullish/bearish trader is not linear. Hence, such an issue cannot be studied in the standard Kyle's (1985) model.⁶ Moreover, a number of financial empirical studies have shown that the linear set-up is counterfactual. The link between the information content of the price and the order flow has been empirically investigated by Hasbrouk (1988, 1991) and Algert (1990) among others. They conclude that the relation is increasing in a nonlinear fashion and is concave (convex) for positive (negative) order flow i.e. when the quantity of buy orders is greater (smaller) than the quantity of sell orders. ⁷ In addition, in recent empirical work, Kempf and Korn (1999) show that the price function is marginally less sensitive for large order flow, which entails concavity (convexity) of the shape of the price function for positive (negative) order flows. Related strands of the literature, one focusing on transaction prices (see for instance Hausman, Lo and Mac Kinlay, 1992, DeJong, Nijman and Röell, 1995) and the other on the order book (Biais, Hillion and Spatt, 1995) lead to the same conclusions. Further, as pointed out by O'Hara (1995), it is now widely acknowledged that "a linear pricing rule is, at best, an approximation of the actual price behavior".⁸

Our model is set up as follows. N risk neutral informed traders observe a buy or sell signal S about the liquidation value v of the risky asset which is assumed to be normally distributed $\tilde{v} \sim \mathcal{N}(0, \sigma_v^2)$. ⁹ Uninformed noise traders submit normally distributed orders $\tilde{u} \sim \mathcal{N}(0, \sigma_u^2)$ independent of \tilde{v} . The informed traders place orders X(S) to risk neutral market makers who supply liquidity at prices equal to the expectation of the final value of the asset conditionally on the total order flow generated by informed and uninformed traders.

Our results are summarized below. At the equilibrium, the informed buys a positive quantity γ^* after having observed a bullish signal (resp. sells a negative quantity $-\gamma^*$ after having

⁶It is important to notice that setting a linear price is not efficient in this framework.

⁷Seppi (1990) provides justification for the concave relationship between order size and price impact on the structure of the market (upstairs versus downstairs). Indeed, he shows that it can be optimal to execute a trade as a block rather than to break it up into smaller trades.

⁸The nonlinearity of the price function can also be caused by the non-linearity of the distribution of the random variables in the economy - see Foster and Viswanathan (1993a) and Nöldeke and Tröger (1998), who characterize larger classes of distributions for which the linear set-up still holds.

⁹Without any loss of generality we assume that the mean is zero. Extension to the case where the mean is different from zero is straightforward and does not change the results.

observed a bearish signal.) ¹⁰ The equilibrium price functions are Sigmoid-shaped. In other words, there is a monotone increasing and concave (convex) relation between prices and order flows for positive (negative) order flows (see Figure 1). Defining the liquidity parameter λ (as the price sensitivity to the order flow) we find that λ is a bell-shaped curve.¹¹ The trader's profits amount to nearly 36% of the profits made by a monopolistic informed trader who knows exactly the liquidation value of v in a linear setup as in Kyle (1985). In the presence of competition, revelation of private information is faster than in the linear case and the expected individual profit (as well as the expected aggregate profit) is decreasing at an exponential rate. When N, the number of informed agents is large, the pricing schedule approaches a step-shaped function. ¹² The market makers are in that case able to infer the strongly efficient price. Namely, if S = 1 then $p = p^* = \frac{2\sigma_v}{\sqrt{2\pi}}$ (and if S = -1 then $p = -p^*$). Intuitively, the value $\frac{2\sigma_v}{\sqrt{2\pi}}$ corresponds to the mean of the truncated normal distribution on the positive real line. In other words, it is the value separating the positive real axis into two equally weighted regions of one half probability each. The liquidity parameter is no longer constant as a function of the order flow for a given N^{13} .

The empirical predictions of our theoretical model are not in contradiction with the empirical findings (namely the non linear aspects of both price function and price pressure) such as in those of Kempf and Korn (1999) who analyze through a battery of parametric and robust non-parametric econometric methods the shape of the equilibrium price as a function of the order imbalance.¹⁴

¹⁰This is analogous to the Glosten and Milgrom (1985) model where the informed traders buy or sell a fixed quantity.

 12 This result is analogous to the result e.g. of Easley and O'Hara (1987, 1992) where in a sequential trade model as the fraction of informed traders increases, the speed of convergence to the liquidation value of the asset increases.

¹³In Kyle (1985), the value of λ is independent of the order flow.

¹⁴However, in order to compare the predictions of our model with the empirical evidence on the market, one needs studies that focus on the link between prices and the nature of information collected by informed traders. Existing empirical studies do not show the effect of information release on the shape of the price function. According to our model, the non-linearities of the price function should be enhanced by the bullish/bearish type of information. It would therefore be interesting to attempt to identify this effect in market data.

¹¹In other words, "Liquid markets are generally viewed as those which accomodate trading with the least effect on price ... in the Kyle (1985) model, λ is a measure of liquidity [and it] measures the order flow needed to move prices one unit" (O'Hara, 1995, p. 216). Therefore, the smaller is λ , the more liquid is the market.

The paper is organized as follows. We first lay out in Section 2 the general setup and the considered model. We extensively analyze in Section 3 the properties of the equilibrium (profits, liquidity and price). Finally, Section 4 makes some concluding remarks.

2 The Model

We consider a financial market with a risky asset whose value is normally distributed $\tilde{v} \rightsquigarrow \mathcal{N}(0, \sigma_v^2)$. We denote the final liquidation value v. There are three types of agents:

1. N risk neutral informed traders who observe in advance the direction of v. In other words, they are each endowed with a signal \tilde{S} which fully reveals whether the state of the world is bullish, bearish or neutral ¹⁵:

$$\begin{array}{rrrr} S = & -1 & if \ v < 0, \\ S = & 0 & if \ v = 0, \\ S = & 1 & if \ v > 0. \end{array}$$

The strategy of the informed agent i = 1, ..., N is a Lebesgue measurable function: $X_i: \{-1, 0, 1\} \longrightarrow \mathbb{R}$, determining his market order as a function of the observed signal S. For given strategies $(X_1, ..., X_N)$, let $\tilde{x}_i = X_i(\tilde{S})$.

- 2. Liquidity traders who submit market orders $\widetilde{u} \sim \mathcal{N}(0, \sigma_u^2)$ independent of \widetilde{v} .
- 3. Competitive risk neutral market makers, who observe the aggregate order flow $\tilde{w} = \sum_{i=1}^{N} \tilde{x}_i + \tilde{u}$ set the price equal to the expectation of the value of the security conditional on the order flow.

The outcome of the competition between market makers is described by a Lebesgue measurable price function: $P(w) : \mathbb{R} \longrightarrow \mathbb{R}$.

Given (P, X_1, \ldots, X_N) , we denote $\tilde{p} = P(\tilde{w})$ and let $\tilde{\pi}_i = (\tilde{v} - \tilde{p})\tilde{x}_i$ be the resulting trading profit of each insider $i = 1, \ldots, N$.

The equilibrium conditions are that the competition in which market makers are involved drives their expected profits to zero and that the informed traders choose their trading strategies so as to maximize their expected profits.

¹⁵The neutral state occurs with zero probability.

Definition 2.1 : $(P, X_1, \ldots, X_N) \in L_2^{N+1}$ is an equilibrium if:

$$E\left[\widetilde{v} - \widetilde{p}|\widetilde{w}\right] = 0, \tag{2.1}$$

and for all $X_i \in L_2$, given the (rational) beliefs of the market makers, and the corresponding price function P(.), each informed chooses X_i to maximize his expected profits:

$$X_{i} \in \underset{x \in I\!\!R}{\operatorname{Argmax}} E\left[\left(\widetilde{v} - P(x + \sum_{j \neq i} \widetilde{x}_{j} + \widetilde{u})\right) x | S\right] \cdot$$

$$(2.2)$$

Again, we note here that we are in a framework where insiders behave strategically, so that the informed trader takes into account her impact onto prices. We will now derive the unique perfect Bayesian equilibrium of this game.

Proposition 2.1 : There exists a unique equilibrium $(P, X_1, ..., X_N)$ defined for trading strategies $X_i(\cdot), i = 1, ..., N$ by

$$\widetilde{x}_{i} = X_{i}(S) = -\gamma^{*}(N) = -\frac{\sigma_{u}}{\sqrt{N}}, \text{ if } S = -1,$$

 $\widetilde{x}_{i} = X_{i}(S) = 0, \text{ if } S = 0,$
(2.3)

$$\widetilde{x}_i = X_i(S) = \gamma^*(N) = \frac{\sigma_u}{\sqrt{N}}, \ if \ S = 1.$$

The equilibrium price function is

$$\tilde{p} = P(w) = \frac{\sigma_v}{\sqrt{2\pi}} \left[K_2^*(w, N) - K_1^*(w, N) \right]$$

where the functions $K_1^*(\cdot, \cdot)$ and $K_2^*(\cdot, \cdot)$ are defined as:

(2.4)

$$K_1^*(w,N) = \frac{2\varphi\left(\frac{w}{\sigma_u} + \sqrt{N}\right)}{\varphi\left(\frac{w}{\sigma_u} - \sqrt{N}\right) + \varphi\left(\frac{w}{\sigma_u} + \sqrt{N}\right)},$$

$$K_2^*(w, N) = 2 - K_1^*(w, N),$$

where $\varphi(\cdot)$ corresponds to the probability distribution function of the standard normal variable $\mathcal{N}(0,1)$.

Proof : See Appendix A.

Therefore the equilibrium is a symmetric one, each informed agent chooses the same trading strategy and submits the same volume. Even though the setup is non-linear, the reaction function is again, as in the standard Kyle (1985) model, proportional to the level of noise trading measured by σ_u . Thus the higher the level of noise trading, the more the informed trader can camouflage his trade. However it should be pointed out that this is as an unexpected result in this non-linear framework.

It is also possible to reinterpret the pricing schedule in terms of the probability which market makers attach to the two non-trivial states of the world and their associated strongly efficient prices $(p^* = \frac{2\sigma_v}{\sqrt{2\pi}})$ if bullish and $-p^* = -\frac{2\sigma_v}{\sqrt{2\pi}}$ if bearish). The probabilities are

$$P_u(w,N) = \frac{K_2^*(w,N)}{2} = \frac{\varphi\left(\frac{w}{\sigma_u} - \sqrt{N}\right)}{\varphi\left(\frac{w}{\sigma_u} - \sqrt{N}\right) + \varphi\left(\frac{w}{\sigma_u} + \sqrt{N}\right)} = \frac{1}{1 + \exp\left(-\frac{2w\sqrt{N}}{\sigma_u}\right)},$$
 (2.5)

$$P_d(w, N) = 1 - P_u(w, N), (2.6)$$

where $0 < P_u(w, N) < 1$ ($0 < P_d(w, N) < 1$) corresponds to the probability that the market makers rationally assign to the event of a bullish (bearish) state of the nature having observed the aggregate volume w. Therefore, the price set by the market makers equals:

$$\widetilde{p} = P_u(w, N)p^* + P_d(w, N)\left(-p^*\right).$$
(2.7)

We note the following properties of the equilibrium price function. First, the quoted price always lies between the floor price $-p^*$ and the ceiling price p^* . The interpretation is that informed agents know at best the trading direction i.e the bearish or bullish state of the risky asset. Second, the price function is a Sigmoid-shaped curve. Price pressure is not constant and decreases with order imbalance. This is consistent with the empirical evidence, see Kempf and Korn (1999).

The price function is non linear because at the equilibrium, the greater is the order flow, the greater is the probability of the presence of informed traders and the closer is the price to the ceiling price. In standard linear models, this is not the case as the quantity bought by the informed traders is increasing linearly with the collected signal. Hence, price is also increasing linearly with aggregate volume.

In all the forthcoming graphs, we have normalized the parameters σ_u and σ_v to 1. The variable N corresponds to the number of informed traders. The legend on the horizontal axis "order flow" refers to the aggregate submitted order both by the informed traders and the noise traders.

Figure 1 and figure 2 represent the market makers' beliefs and pricing schedules, respectively, for different oligopoly sizes. The effect of competition is that the strength of the beliefs of the market makers is increasing with the number N of competitors. This is essentially due to the fact that although the individual orders $\gamma^{*(N)}$ is decreasing with N the aggregate informed order $N\gamma^{*(N)}$ is increasing with N. We also see that competition brings about more efficient prices. This result is similar with the results found by Foster and Viswanathan (1996) and Holden and Subrahmanyam (1992) that the more competitive is a market the quicker market efficiency is achieved.

Note that the empirical literature (see Saar, 2001, and the references therein) argues that response of prices to buys and sells is asymmetric. While in our model the bullish and bearish regions are symmetric around zero, any asymmetry would in turn lead to an asymmetric price response.



Figure 1: Market Makers' Beliefs About the Probability of the Bullish State.



Figure 2: Pricing Schedule.

3 Properties of the equilibrium

3.1 Expected profits

In this subsection, we are interested in the expected profit of each informed agent. Futhermore, we analyze the behavior of the individual and the aggregate expected profit when the number of informed traders is large.

Proposition 3.1 :

The expected profit of each agent $\pi_i^*(N) = E(\tilde{\pi}_i^*)$, i = 1, ..., N, is a decreasing function of the number N of informed agents and tends towards 0 when N is large:

$$\lim_{N \to +\infty} \pi_i^*(N) = \lim_{N \to +\infty} \frac{\sigma_u \sigma_v}{N} \exp\left(-\frac{N}{2}\right) = 0.$$
(3.1)

The aggregate expected profit $N\pi_i^*(N)$ is also a decreasing function of the number N of informed agents and tends towards 0 when N is large:

$$\lim_{N \to +\infty} N\pi_i^*(N) = \lim_{N \to +\infty} \sigma_u \sigma_v \exp\left(-\frac{N}{2}\right) = 0.$$
(3.2)

Proof : See Appendix B.



Figure 3: Individual Expected Profit: a Comparison.

One way to measure the reward of a bullish/bearish signal is to compare the expected profit of a bullish/bearish monopoly with the expected profit of the Kyle (1985) monopoly. We find that a bullsh/bearish monopoly achieves 36% of the expected profits of a trader with a perfect signal. Whether it is worthwhile to collect more refined information depends on the cost structure of the information collection process. As a side note, Brav and Lehavy (2003, footnote 5) observe that certain brokerage houses that issue recommendations have either a formal or an informal policy barring the issuance of price targets. In figure 3, we have plotted the expected individual profit expressed as a proportion of the standard Kyle (1985) profit as a function of the number of informed traders for the bullish-bearish oligopoly (BB) and compare it with two other models. First, the noisy informed oligopoly as developed by Admati and Pfleiderer (1988b) and where to facilitate the comparison, σ_{ε}^2 is set to be such that the monopoly in this case achieves thirty six percent of the Kyle (1985) monopoly profits (AP). Second, optimally noisy oligopoly as developed by Dridi and Germain (1999) ¹⁶ for $N \geq 4$ (ON).

As a standard result, Figure 3 and Proposition 3.1 show that in the bullish/bearish model competition destroys the expected individual profit which goes to zero when the number of informed agents is large. This result is similar to the one we could expect to obtain in a Cournot

¹⁶Dridi and Germain (1999) have shown that with linear equilibrium and noisy signals with heterogeneous beliefs, there exists an optimal level of noise in the signals which maximizes each individual profit for $N \ge 4$.

model as it is already the case in Admati and Pfleiderer (1988a,b), Subrahmanyam (1991), Holden and Subrahmanyam (1992) and Foster and Viswanathan (1996). ¹⁷ As an unexpected result, Proposition 3.1 shows that in this non-linear framework, we obtain that the profit is proportional as in Kyle (1985) to the product $\sigma_u \sigma_v$. When the number of insiders N increases competition quickly destroys individual profits - whereas in the Kyle model the individual profit tends towards 0 more slowly. For the sake of comparison, in the monopoly case (N = 1), the expected profit in the bullish/bearish model is equal to 0.36 times the Kyle one. Moreover, Figure 3 shows that the individual expected profit goes to zero even faster in the case of the bullish/bearish oligopoly when the number of competitors in the market is increasing. The rate of convergence is indeed an exponential one $\frac{1}{N} \exp\left(-\frac{N}{2}\right)$. The rate of convergence in the models of Admati and Pfleiderer (1988b), Holden and Subrahmanyam (1992), Foster and Viswanathan (1996), and Dridi and Germain (1999) is of the order of $\frac{1}{N\sqrt{N}}$.



Figure 4: Aggregate Expected Profit: a Comparison.

Figure 4 shows that the expected aggregate profit¹⁸ in the bullish-bearish setup (dash line) is a decreasing function of the number of insiders. Moreover, in this case, it goes to zero even faster

¹⁸Again expressed as a fraction of the standard Kyle (1985) expected profits $\frac{1}{2}\sigma_u\sigma_v$.

¹⁷However, Dridi and Germain (1999) show that this is not always the case when traders collect noisy information. In effect, for certain regions of noise when the number of competitors is greater than four, one insider could achieve greater profits with noisy information than with perfect information.

when the number of informed traders is large in comparison with the noisy informed framework as developed by Admati and Pfleiderer (1988b), see the AP curve.¹⁹ This can be interpreted, all other things being equal, as quicker revelation of information. In the model of Dridi and Germain (1999) (ON) the aggregate profit does not go to zero for a certain region of noise as long as there are more than four insiders competing in the market.

3.2 Liquidity

Proposition 3.2 : We define the liquidity parameter as the price pressure, $\lambda^*(w, N) = \frac{\partial p}{\partial w}$. Then:

$$\lambda^*(w,N) = \frac{8\sigma_v}{\sqrt{2\pi}\sigma_u} \frac{\sqrt{N}\exp\left(\frac{2w\sqrt{N}}{\sigma_u}\right)}{\left[1 + \exp\left(\frac{2w}{\sigma_u}\sqrt{N}\right)\right]^2}.$$
(3.3)

Moreover when N is large, and for any given level of noise trading u, liquidity and the pricing schedule converge to the following limits:

$$\lim_{N \to +\infty} \lambda^* (w, N) = 0, \quad if \ S = 1 \quad or \ S = -1,$$
(3.4)

$$\lim_{N \to +\infty} \tilde{p} = p^* \quad if \ S = 1, \tag{3.5}$$

$$\lim_{N \to +\infty} \tilde{p} = -p^* \quad if \ S = -1.$$
(3.6)

Proof : See Appendix B.

¹⁹Note that in the Admati and Pfleiderer model the aggregate profit is first increasing and then decreasing. This is due to the fact that the signals are heterogeneous and as it has been shown in Dridi and Germain (1999) in this case competition does not always destroys the aggregate profit. On the other hand, with homogenous signals the aggregate profit function is monotone and converges to zero. In the proposition we defined naturally the price pressure or the liquidity parameter λ as the derivative of the price function with respect to the order flow i.e. price sensitivity to a small change in the order imbalance. Market depth is the inverse of the price pressure ; this is in accordance with what has been previously introduced in the microstructure literature.

Figure 5 represents the liquidity parameter λ as a function of the order flow for the monopoly (N = 1), duopoly (N = 2) and oligopoly (N = 6) cases, and respectively for large and small order flow. Unlike in the Kyle (1985) linear framework, both functions are non-constant in order flow. We see that price pressure is bell-shaped. ²⁰ Price pressure is high for small order flow and it then decreases with order imbalance. In other words, the price function is less sensitive for large aggregate order flow. This is because order flow becomes marginally less informative as its magnitude increases.

We observe that, while for small order flow, the liquidity parameter is lower for the monopoly case (N=1), for larger order flow the liquidity parameter is lower in the duopoly case. This is not the case in Admati and Pfleiferer (1988), or in Holden and Subrahmanyam (1992) where the liquidity parameter is lower for the duopoly case whatever the amount of the order imbalance. In our model, there is no straightforward interpretation of this feature, however, because order flow itself depends on N.

²⁰The market depth function not plotted here is U-shaped.



Figure 5: Liquidity parameter λ .

Market depth in the bullish-bearish case is increasing faster with the number of traders than in the case of Admati and Pfleiderer (1988b), Holden and Subrahmanyam (1992), and Foster and Viswanathan (1996). Indeed, in those models the liquidity parameters are the following ²¹:

- 1. computing the liquidity parameter in Holden and Subrahmanyam (1992) i.e when the signals collected by the informed traders are perfect (S = v), the liquidity parameter is $\lambda = \frac{\sqrt{N}}{N+1} \frac{\sigma_u}{\sigma_u}$,
- 2. computing the liquidity parameter in Admati and Pfleiderer (1988b) in the case where the signals collected are homogeneous and noisy $(S = v + \varepsilon)$, the liquidity parameter is $\lambda = \frac{\sqrt{N}}{N+1} \frac{\sigma_v}{\sigma_v} \frac{1}{\sqrt{1-\varepsilon}}$,

$$\lambda = \frac{\sqrt{N}}{N+1} \frac{\sigma_v}{\sigma_u} \frac{1}{\sqrt{1 + \frac{\sigma_e^2}{\sigma_v^2}}},$$

²¹In those models N is the number of informed traders, σ_v^2 , σ_u^2 and σ_ε^2 are the variances of u the noise trading, v the liquidation value of the asset, and ε_i or ε the noise in the signal of the informed traders ; u, v and ε or ε_i are mutually independent.

3. computing the liquidity parameter in Foster and Viswanathan (1996) or in Admati and Pfleiderer (1988b) i.e when the signals collected are heterogeneous $(S_i = v + \varepsilon_i)$, the

liquidity parameter is expressed by $\lambda = \frac{\sigma_v}{\sigma_u} \frac{\sqrt{N}\sqrt{1+\frac{\sigma_{\varepsilon}^2}{\sigma_v^2}}}{N+1+2\frac{\sigma_{\varepsilon}^2}{\sigma_v^2}}.$

The liquidity parameters in those models decreases with the number of informed traders at a rate of $\frac{1}{\sqrt{N}}$ whereas in the bullish-bearish case it decreases at an exponential rate (for non-zero order flow).²²

4 Concluding Remarks

We have proposed, in this paper, to study a financial market where risk neutral traders are endowed with information of the bullish/bearish type. At the equilibrium, the optimal strategy consists of buying (respectively selling) when bullish (respectively bearish) a fixed quantity. The pricing schedule is a non-linear function of the volumes. The main results of the paper are the following i) the price function is Sigmoid-shaped. ii) Liquidity parameter λ is a bell-shaped curve of the order flow. This is consistent with the empirical evidence. iii) A monopolistic bullish/bearish trader makes nearly thirty six percent of the profits she would have made with a perfect signal in a linear model à la Kyle (1985). iv) In the presence of competition, the market reveals private information quicker than in a perfect informed strategic oligopoly.

Our results suggest several directions for future research. A key testable implication of our model is a link between coarseness of the information structure and concavity of the price function. This is an interesting prediction to test experimentally. On the theoretical side, it would be interesting to generalize this model to a multi-level bounded information structure where the traders would be endowed of the probability that the value of the risky asset belongs in different ranges. An example of this type of generalization is a signal the structure of which would be "strong buy", "buy", "hold", "sell", "strong sell". Another interesting study would generalize this model in a multi-periodic setting.

²²This result is also different from the results in Subrahmanyam (1991) as in his model, due to the risk aversion of the market makers and of the informed traders, the liquidity parameter λ is first increasing and then decreasing with the number of traders. Nevertheless, in Subrahmanyam (1991) the liquidity parameter is greater than in the standard risk neutral case of Admati and Pfleiderer (1988b).

References

- Admati, A., and P. Pfleiderer (1988a), "Selling and Trading on Information in Financial Markets", American Economic Review, 78, 1165-1183.
- Admati, A., and P. Pfleiderer (1988b), "A Theory of Intraday Patterns: Volume and Price Variability", *The Review of Financial Studies*, 1, 3-40.
- Admati, A., and P. Pfleiderer (1990), "Direct and Indirect Sale of Information", Econometrica, 58, 901-928.
- Algert, P. M. (1990), "Estimates of Non-linearity in the Response of Stock Prices to Order Imbalances", mimeo, University of California, Berkeley.
- Back, K. (1992), "Insider Trading in Continuous Time", *Review of Financial Studies*, 5, 387-402.
- Bhattacharya, U., and M. Spiegel (1991), "Insiders, Outsiders, and Market Breakdowns", The Review of Financial Studies, 4, 255-282.
- Biais, B., P. Hillion., and C. Spatt (1995), "An Empirical Analysis of the Order Book and the Order Flow in the Paris Bourse", *Journal of Finance*, 50, 1655-1689.
- Brav, A., and R. Lehavy (2003), "An Empirical Analysis of Analysts' Target Prices: Short Term Informativeness and Long-Term Dynamics", *forthcoming Journal of Finance*.
- Cho, K-H., and N. El Karoui (1997), "Insider Trading and Non-linear Equilibria: Single Auction Case", *mimeo HEC Lausanne*.
- De Jong, F., T. Nijman., and A. Röel (1998), "A Comparison of the Cost of Trading French Shares on the Paris Bourse and on the SEAQ International", *European Economic Review*, 39, 1277-1301.
- Dridi, R., and L. Germain (1999), "Noise and Competition in Strategic Oligopoly", mimeo London Business School, London School of Economics.
- Easley, D., and M. O'Hara (1987), "Price, Trade Size, and Information in Securities Markets", Journal of Financial Economics, 19, 69-90.
- Easley, D., and M. O'Hara (1992), "Time and the Process of Security Price Adjustment", Journal of Finance, 47, 577-605.

- Foster, F. D., and S. Viswanathan (1990), "A Theory of the Inter-day Variations in Volume, Variance, and Trading Costs in Securities Markets", *The Review of Financial Studies*, 3, 593-624.
- Foster, F. D., and S. Viswanathan (1993a), "Variations in Trading Volume, Return Volatility, and Trading Costs: Evidence on Recent Price Formation Models", *Journal* of Finance, 48, 187-211.
- Foster, F. D., and S. Viswanathan (1993b), "The Effect of Public Information and Competition on Trading Volume and Price Volatility", *The Review of Financial Studies*, 6, 23-56.
- Foster, F. D., and S. Viswanathan (1994), "Strategic Trading with Asymmetrically Informed Traders and Long-lived Information", *Journal of Financial and Quantitative Analy*sis, 29, 499-518.
- Foster, F. D., and S. Viswanathan (1996), "Strategic Trading When Agents Forecast the Forecasts of Others", *Journal of Finance*, 51, 1437-78.
- Gennotte, G., and H. Leland (1990), "Market Liquidity, Hedging and Crashes", American Economic Review, 80, 999-1021.
- **Glosten, L., and P.Milgrom** (1985), "Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders", *Journal of Financial Economics* 14, 71-100.
- **Grossman, S. J.** (1976), "On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information", *Journal of Finance*, 31, 573-585.
- Grossman, S. J., and J. E. Stiglitz (1980), "On the Impossibility of Informationally Efficient Markets", American Economic Review, 70, 393-408.
- Hasbrouck, J. (1988), "Trades, Quotes, Inventories and Information", Journal of Financial Economics, 22, 229-252.
- Hasbrouck, J. (1991), "Measuring the Information Content of Stock Trades., Journal of Finance, 46, 179-207.
- Hausman, J. A., Lo, A. W., and MacKinlay, A. C. (1992), "An Ordered Probit Analysis of Transaction Stock Prices", *Journal of Financial Economics*, 31, 319-379.
- Holden, C.W., and A. Subrahmanyam (1992) "Long-Lived Private Information and Imperfect Competition", *Journal of Finance*, 47, 247-270.

- Kempf, A., and O. Korn (1999), "Market Depth and Order Size", Journal of Financial Markets, 2, 29-48.
- Kim, T., Lin J-C., and M. Slovin (1997), "Market Structure, Informed Trading, and Analysts' Recommendations", Journal of Financial and Quantitative Analysis, 32, 507-524.
- Kyle, A. (1984), "Market Structure, Information, Futures Markets and Price Formation in International Agricultural Trade", Advanced Reading in Price Formation, Market Structure, and Price Instability ed. Story, Schmitz and Sarris (Westview Press, Boulder and London), 45-64.
- Kyle, A. (1985), "Continuous Auctions and Insider Trading", Econometrica, 53, 1315-1335.
- Kyle, A. (1989), "Informed Speculation with Imperfect Competition", Review of Economic Studies, 56, 317-356.
- Nöldeke, G., and T. Tröger (1998), "On Linear Equilibria in the Kyle Model", mimeo, University of Basel.
- O'Hara, M. (1995), "Market Microstructure Theory", Blackwell Publishers, Cambridge (MA).
- Rochet, J-C., and J-L. Vila (1994), "Insider Trading without Normality", *Review of Economic Studies*, 61, 131-152.
- Saar, G., (2001), "Price Impact Asymmetry of Block Trades: An Institutional Trading Explanation" Review of Financial Studies, 14, 1153-1181.
- Seppi, D., (1990) "Equilibrium Block Trading and Asymmetric Information", Journal of Finance, 45, 73-94.
- Stickel, S. (1995), "The Anatomy of the Performance of Buy and Sell Recommendations", *Financial Analysts Journal*, 51, 25-39.
- Subrahmanyam, A., (1991), "Risk Aversion, Market Liquidity and Price Efficiency", Review of Financial Studies, 4, 417-441.
- Vives, X. (1995), "The Speed of Information Revelation in a Financial Market Mechanism", Journal of Economic Theory, 67, 178-204.
- Womack, K. (1996), "Do Brokerage Analysts' Recommendations Have Investment Value", Journal of Finance, 51, 137-167.

Appendices

A Proof of propositions 2.1

The informed agent *i* maximizes his conditional expected profit conditionally on the observed signal *S*. If the informed agent *i* observes the value S = 0 (which occurs with probability 0), it is straightforward to show that he optimally submits 0. Given the structure of his information and the symmetry of the problem, his optimal strategy is of the form:

$$\widetilde{x}_i = \gamma_i, \quad if \ S = 1,$$

 $\widetilde{x}_i = -\gamma_i, \ if \ S = -1.$

Given their prior, the market makers conjecture that the informed agent i submits the following order: $\tilde{x} = \gamma^c, \quad if \ S = 1.$

$$\begin{aligned} x &= \gamma_i, \quad if \ S = 1, \\ \tilde{x}_i &= -\gamma_i^c, \ if \ S = -1, \end{aligned}$$

The market makers set prices according to (4.1):

$$\widetilde{p} = E[\widetilde{v}/w] = \frac{\sigma_v}{\sqrt{2\pi}} [K_2(w, N) - K_1(w, N)],$$
where :

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}x^2\right),$$

$$K_1(w, N) = \frac{2\varphi\left(\frac{w + \sum_{i=1}^N \gamma_i^c}{\sigma_u}\right)}{\varphi\left(\frac{w - \sum_{i=1}^N \gamma_i^c}{\sigma_u}\right) + \varphi\left(\frac{w + \sum_{i=1}^N \gamma_i^c}{\sigma_u}\right)},$$
(4.1)

 $K_2(w,N) = 2 - K_1(w,N) \cdot$

The proof proceeds in two steps. We first compute the joint density of $(\tilde{v}, \tilde{w}) : f_{\tilde{v}, \tilde{w}}$ and therefore

the conditional p.d.f. of \tilde{v} given w. We thus deduce the conditional expectation $E[\tilde{v}/w]$.

$$\begin{split} F_{\widetilde{v},\widetilde{w}}(v,w) &= \operatorname{Prob}[\widetilde{v} \leq v, \widetilde{w} \leq w], \\ &= \operatorname{Prob}[\widetilde{v} \leq v, \widetilde{w} \leq w/\widetilde{v} < 0]\operatorname{Prob}[\widetilde{v} < 0] + \operatorname{Prob}[\widetilde{v} \leq v, \widetilde{w} \leq w/\widetilde{v} > 0]\operatorname{Prob}[\widetilde{v} > 0], \\ &= \frac{1}{2} \left\{ \operatorname{Prob}[\widetilde{v} \leq v, -\sum_{i=1}^{N} \gamma_{i}^{c} + \widetilde{u} \leq w/\widetilde{v} < 0] + \operatorname{Prob}[\widetilde{v} \leq v, \sum_{i=1}^{N} \gamma_{i}^{c} + \widetilde{u} \leq w/\widetilde{v} > 0] \right\}, \\ &= \frac{1}{2} \left\{ \operatorname{Prob}\left[\widetilde{v} \leq v, -\sum_{i=1}^{N} \gamma_{i}^{c} + \widetilde{u} \leq w/\widetilde{v} < 0 \right] + \operatorname{Prob}\left[\widetilde{v} \leq v, \sum_{i=1}^{N} \gamma_{i}^{c} + \widetilde{u} \leq w/\widetilde{v} > 0 \right] \right\}, \\ &= \operatorname{Prob}\left[\widetilde{v} \leq \min(0, v), -\sum_{i=1}^{N} \gamma_{i}^{c} + \widetilde{u} \leq w \right] + \operatorname{Prob}\left[0 < \widetilde{v} \leq v, \sum_{i=1}^{N} \gamma_{i}^{c} + \widetilde{u} \leq w \right], \\ &= \Phi\left(\frac{\min(0, v)}{\sigma_{v}} \right) \Phi\left(\frac{w + \sum_{i=1}^{N} \gamma_{i}^{c}}{\sigma_{u}} \right) + \left[\Phi\left(\frac{v}{\sigma_{v}} \right) - \Phi(0) \right] \operatorname{I}_{v > 0} \Phi\left(\frac{w - \sum_{i=1}^{N} \gamma_{i}^{c}}{\sigma_{u}} \right), \end{split}$$

$$\begin{split} F_{\widetilde{v},\widetilde{w}}(v,w) &= \\ \Phi\left(\frac{v}{\sigma_{v}}\right) \Phi\left(\frac{w+\sum\limits_{i=1}^{N}\gamma_{i}^{c}}{\sigma_{u}}\right) \, \mathrm{I}_{v\leq0} + \, \mathrm{I}_{v>0}\frac{1}{2}\Phi\left(\frac{w+\sum\limits_{i=1}^{N}\gamma_{i}^{c}}{\sigma_{u}}\right) + \Phi\left(\frac{v}{\sigma_{v}}\right) \Phi\left(\frac{w-\sum\limits_{i=1}^{N}\gamma_{i}^{c}}{\sigma_{u}}\right) \, \mathrm{I}_{v>0} - \frac{1}{2} \, \mathrm{I}_{v>0}\Phi\left(\frac{w-\sum\limits_{i=1}^{N}\gamma_{i}^{c}}{\sigma_{u}}\right), \\ &= \Phi\left(\frac{v}{\sigma_{v}}\right) \left[\Phi\left(\frac{w+\sum\limits_{i=1}^{N}\gamma_{i}^{c}}{\sigma_{u}}\right) \, \mathrm{I}_{v\leq0} + \Phi\left(\frac{w-\sum\limits_{i=1}^{N}\gamma_{i}^{c}}{\sigma_{u}}\right) \, \mathrm{I}_{v>0}\right] + \frac{1}{2} \, \mathrm{I}_{v>0}\left[\Phi\left(\frac{w+\sum\limits_{i=1}^{N}\gamma_{i}^{c}}{\sigma_{u}}\right) - \Phi\left(\frac{w-\sum\limits_{i=1}^{N}\gamma_{i}^{c}}{\sigma_{u}}\right)\right]. \end{split}$$
 If $v < 0$:

$$F_{\widetilde{v},\widetilde{w}}(v,w) = \Phi\left(\frac{v}{\sigma_v}\right) \Phi\left(\frac{w + \sum_{i=1}^N \gamma_i^c}{\sigma_u}\right),$$
$$f_{\widetilde{v},\widetilde{w}}(v,w) = \frac{1}{\sigma_u \sigma_v} \varphi\left(\frac{v}{\sigma_v}\right) \varphi\left(\frac{w + \sum_{i=1}^N \gamma_i^c}{\sigma_u}\right).$$

If v > 0:

$$\begin{split} F_{\widetilde{v},\widetilde{w}}(v,w) &= \Phi\left(\frac{v}{\sigma_v}\right) \Phi\left(\frac{w-\sum\limits_{i=1}^N\gamma_i^c}{\sigma_u}\right) + \frac{1}{2} \left[\Phi\left(\frac{w+\sum\limits_{i=1}^N\gamma_i^c}{\sigma_u}\right) - \Phi\left(\frac{w-\sum\limits_{i=1}^N\gamma_i^c}{\sigma_u}\right)\right],\\ f_{\widetilde{v},\widetilde{w}}(v,w) &= \frac{1}{\sigma_u\sigma_v}\varphi\left(\frac{v}{\sigma_v}\right)\varphi\left(\frac{w-\sum\limits_{i=1}^N\gamma_i^c}{\sigma_u}\right). \end{split}$$
 Moreover $F_{\widetilde{w}}(w) &= \frac{1}{2} \left[\Phi\left(\frac{w+\sum\limits_{i=1}^N\gamma_i^c}{\sigma_u}\right) + \Phi\left(\frac{w-\sum\limits_{i=1}^N\gamma_i^c}{\sigma_u}\right)\right],$ therefore:
 $f_{\widetilde{w}}(w) &= \frac{1}{2\sigma_u} \left[\varphi\left(\frac{w+\sum\limits_{i=1}^N\gamma_i^c}{\sigma_u}\right) + \varphi\left(\frac{w-\sum\limits_{i=1}^N\gamma_i^c}{\sigma_u}\right)\right] \cdot$ We also have, applying Bayes rule, that:
 $f_{\widetilde{v}/\widetilde{w}}(v,w) &= \frac{f_{\widetilde{v},\widetilde{w}}(v,w)}{f_{\widetilde{w}}(w)}, \end{split}$

and thus:

•
$$if \ v < 0$$
, $f_{\widetilde{v}/\widetilde{w}}(v,w) = \frac{2}{\sigma_v} \frac{\varphi\left(\frac{w}{\sigma_v}\right)\varphi\left(\frac{w+\sum\limits_{i=1}^N \gamma_i^c}{\sigma_u}\right)}{\varphi\left(\frac{w-\sum\limits_{i=1}^N \gamma_i^c}{\sigma_u}\right) + \varphi\left(\frac{w+\sum\limits_{i=1}^N \gamma_i^c}{\sigma_u}\right)}$,
• $if \ v > 0$, $f_{\widetilde{v}/\widetilde{w}}(v,w) = \frac{2}{\sigma_v} \frac{\varphi\left(\frac{w}{\sigma_v}\right)\varphi\left(\frac{w-\sum\limits_{i=1}^N \gamma_i^c}{\sigma_u}\right)}{\varphi\left(\frac{w-\sum\limits_{i=1}^N \gamma_i^c}{\sigma_u}\right) + \varphi\left(\frac{w+\sum\limits_{i=1}^N \gamma_i^c}{\sigma_u}\right)}$.

We define:

$$K_{1}(w,N) = \frac{2\varphi\left(\frac{w+\sum_{i=1}^{N}\gamma_{i}^{c}}{\sigma_{u}}\right)}{\varphi\left(\frac{w-\sum_{i=1}^{N}\gamma_{i}^{c}}{\sigma_{u}}\right)+\varphi\left(\frac{w+\sum_{i=1}^{N}\gamma_{i}^{c}}{\sigma_{u}}\right)},$$
$$K_{2}(w,N) = \frac{2\varphi\left(\frac{w-\sum_{i=1}^{N}\gamma_{i}^{c}}{\sigma_{u}}\right)}{\varphi\left(\frac{w-\sum_{i=1}^{N}\gamma_{i}^{c}}{\sigma_{u}}\right)+\varphi\left(\frac{w+\sum_{i=1}^{N}\gamma_{i}^{c}}{\sigma_{u}}\right)}.$$

Of course $K_1(w, N) + K_2(w, N) = 2$.

$$\begin{split} f_{\widetilde{v}/\widetilde{w}}(v,w) &= \frac{1}{\sigma_v} \varphi\left(\frac{v}{\sigma_v}\right) [K_1(w,N) \, \mathrm{I}_{v<0} + K_2(w,N) \, \mathrm{I}_{v>0}], \\ E[\widetilde{v}/w] &= \int_{-\infty}^{0^-} \frac{v}{\sigma_v} \varphi\left(\frac{v}{\sigma_v}\right) K_1(w,N) dv + \int_{0^+}^{+\infty} \frac{v}{\sigma_v} \varphi\left(\frac{v}{\sigma_v}\right) K_2(w,N) dv, \\ E[\widetilde{v}/w] &= \frac{\sigma_v}{\sqrt{2\pi}} [K_2(w,N) - K_1(w,N)]. \end{split}$$

We define $\gamma = (\gamma_1, \dots, \gamma_N)'$ and $\gamma_c = (\gamma_1^c, \dots, \gamma_N^c)'$. The expected profit $\pi_i(\gamma, \gamma_c)$ of agent *i* is: $\pi_i(\gamma, \gamma_c) = E[\gamma_i(\tilde{v} - \tilde{p})/\tilde{v} > 0]Prob[\tilde{v} > 0] + E[-\gamma_i(\tilde{v} - \tilde{p})/\tilde{v} < 0]Prob[\tilde{v} \le 0],$

$$\pi_{i}(\gamma,\gamma_{c}) = E[\gamma_{i}(v-p)/v > 0]Prob[v > 0] + E[-\gamma_{i}(v-p)/v < 0]Prob[v \le 0],$$

$$= \frac{1}{2} \left\{ E[\gamma_{i}(\tilde{v}-\tilde{p})/\tilde{v} > 0] + E[-\gamma_{i}(\tilde{v}-\tilde{p})/\tilde{v} < 0] \right\},$$

$$= \frac{1}{2} \left\{ \int_{0}^{+\infty} \int_{-\infty}^{+\infty} \gamma_{i}(v-\tilde{p}) \frac{2}{\sigma_{u}\sigma_{v}} \varphi\left(\frac{v}{\sigma_{v}}\right) \varphi\left(\frac{w-\sum_{i=1}^{N} \gamma_{i}}{\sigma_{u}}\right) dv dw$$

$$+ \int_{-\infty}^{0} \int_{-\infty}^{+\infty} -\gamma_{i}(v-\tilde{p}) \frac{2}{\sigma_{u}\sigma_{v}} \varphi\left(\frac{v}{\sigma_{v}}\right) \varphi\left(\frac{w+\sum_{i=1}^{N} \gamma_{i}}{\sigma_{u}}\right) dv dw \right\}.$$

We make the following change in variables for the second integral $\begin{pmatrix} v \\ w \end{pmatrix} \rightarrow \begin{pmatrix} -v \\ -w \end{pmatrix}$ while taking into account that $\varphi(x) = \varphi(-x), \ P(-w) = -P(w)$. Moreover, for simplification we

define the change in variables $t = \frac{w - \sum_{i=1}^{N} (\gamma_i^c)}{\sigma_u}$ with $dt = \frac{dw}{\sigma_u}$.

$$\begin{split} \pi_{i}\left(\gamma,\gamma_{c}\right) &= \frac{1}{2} \left\{ \int_{0}^{+\infty} \int_{-\infty}^{+\infty} \gamma_{i}(v-\tilde{p}) \frac{2}{\sigma_{u}\sigma_{v}} \varphi\left(\frac{v}{\sigma_{v}}\right) \varphi\left(\frac{w-\sum\limits_{i=1}^{N} \gamma_{i}}{\sigma_{u}}\right) dv dw \right. \\ &+ \int_{0}^{+\infty} \int_{-\infty}^{+\infty} \gamma_{i}(v-\tilde{p}) \frac{2}{\sigma_{u}\sigma_{v}} \varphi\left(\frac{v}{\sigma_{v}}\right) \varphi\left(\frac{w-\sum\limits_{i=1}^{N} \gamma_{i}}{\sigma_{u}}\right) dv dw \right\}, \\ &= \frac{2\gamma_{i}}{\sigma_{u}\sigma_{v}} \int_{0}^{+\infty} \int_{-\infty}^{+\infty} (v-\tilde{p}) \varphi\left(\frac{v}{\sigma_{v}}\right) \varphi\left(\frac{w-\sum\limits_{i=1}^{N} \gamma_{i}}{\sigma_{u}}\right) dv dw, \\ &= \frac{2\gamma_{i}}{\sigma_{v}} \int_{0}^{+\infty} \left[v - \int_{-\infty}^{+\infty} 2\frac{\sigma_{v}}{\sqrt{2\pi}} (K_{2}(w,N)-1) \frac{\varphi\left(\frac{w-\sum\limits_{i=1}^{N} \gamma_{i}}{\sigma_{u}}\right)}{\sigma_{u}} dw \right] \varphi\left(\frac{v}{\sigma_{v}}\right) dv, \\ &= \frac{2\gamma_{i}}{\sigma_{v}} \left[\frac{\sigma_{v}^{2}}{\sqrt{2\pi}} - \frac{\sigma_{v}^{2}}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} (K_{2}(w,N)-1) \frac{\varphi\left(\frac{w-\sum\limits_{i=1}^{N} \gamma_{i}}{\sigma_{u}}\right)}{\sigma_{u}} dw \right], \\ &= \frac{2\gamma_{i}\sigma_{v}}{\sqrt{2\pi}} \left[1 - \int_{-\infty}^{+\infty} [K_{2}(t\sigma_{u} + \sum\limits_{i=1}^{N} \gamma_{i}, N) - 1]\varphi(t) dt \right], \\ &= \frac{4\gamma_{i}\sigma_{v}}{\sqrt{2\pi}} \left[1 - \frac{1}{2} \int_{-\infty}^{+\infty} K_{2}(t\sigma_{u} + \sum\limits_{i=1}^{N} \gamma_{i}, N)\varphi(t) dt \right], \end{split}$$

$$=\frac{4\gamma_i\sigma_v}{\sqrt{2\pi}}\left\{1-\int_{-\infty}^{+\infty}\frac{\varphi\left(\sum_{t+\frac{i=1}{\sigma_u}}^N\gamma_i-\gamma_i^c\right)}{\varphi\left(\sum_{t+\frac{i=1}{\sigma_u}}^N\gamma_i-\gamma_i^c\right)}+\varphi\left(\sum_{t+\frac{i=1}{\sigma_u}}^N\gamma_i+\gamma_i^c\right)}\varphi(t)dt\right\}\cdot$$

To simplify the remaining computations we define

$$\mu_i = \frac{\gamma_i}{\sigma_u}$$

$$\mu_i^c = \frac{\gamma_i^c}{\sigma_u}$$

and according $\mu = (\mu_1, ..., \mu_N)', \ \mu^c = (\mu_1^c, ..., \mu_N^c)':$

,

$$\pi_i(\mu,\mu_c) = \frac{4\mu_i \sigma_u \sigma_v}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{\varphi\left(t + \sum_{i=1}^N (\mu_i + \mu_i^c)\right)}{\varphi\left(t + \sum_{i=1}^N (\mu_i - \mu_i^c)\right) + \varphi\left(t + \sum_{i=1}^N (\mu_i + \mu_i^c)\right)} \varphi(t) dt,$$

$$\pi_{i}(\mu,\mu_{c}) = \frac{2\sigma_{u}\sigma_{v}}{\pi}\mu_{i}\int_{-\infty}^{+\infty} \frac{dt}{\exp\left(\frac{1}{2}t^{2}\right) + \exp\left(\frac{1}{2}\left(t + 2\sum_{i=1}^{N}\mu_{i}^{c}\right)^{2}\right)\exp\left(2\sum_{i=1}^{N}\mu_{i}^{c}\sum_{i=1}^{N}(\mu_{i} - \mu_{i}^{c})\right)},$$

At the value $\mu_i = 0$ and when μ_i tends towards infinity the value of the expected profit is zero, therefore there exists an interior solution $\mu_i^*(\mu_{-i}, \mu_c)$ which maximises the expected profit. This solution must verify the first order condition.

$$\begin{aligned} \frac{\partial \pi_i}{\partial \mu_i}(\mu,\mu_c) &= \frac{2\sigma_u \sigma_v}{\pi} \left\{ \int_{-\infty}^{+\infty} \frac{dt}{\exp\left(\frac{1}{2}t^2\right) + \exp\left(\frac{1}{2}\left(t + 2\sum_{i=1}^N \mu_i^c\right)^2\right) \exp\left(2\sum_{i=1}^N \mu_i^c\sum_{i=1}^N (\mu_i - \mu_i^c)\right) - \frac{1}{2} \left(\frac{1}{2}\left(t + 2\sum_{i=1}^N \mu_i^c\right)^2\right) 2\sum_{i=1}^N \mu_i^c \exp\left(2\sum_{i=1}^N \mu_i^c\sum_{i=1}^N (\mu_i - \mu_i^c)\right) dt \right) \right\} \\ \mu_i \int_{-\infty}^{+\infty} \frac{\exp\left(\frac{1}{2}\left(t + 2\sum_{i=1}^N \mu_i^c\right)^2\right) 2\sum_{i=1}^N \mu_i^c \exp\left(2\sum_{i=1}^N \mu_i^c\sum_{i=1}^N (\mu_i - \mu_i^c)\right) dt}{\left[\exp\left(\frac{1}{2}t^2\right) + \exp\left(\frac{1}{2}\left(t + 2\sum_{i=1}^N \mu_i^c\right)^2\right) \exp\left(2\sum_{i=1}^N \mu_i^c\sum_{i=1}^N (\mu_i - \mu_i^c)\right)\right]^2}\right\}. \end{aligned}$$

Each agent i chooses μ_i to maximise his expected profit, and therefore at equilibrium we have:

This implies that:

$$\int_{-\infty}^{+\infty} \frac{dt}{\exp\left(\frac{1}{2}t^{2}\right) + \exp\left(\frac{1}{2}\left(t + 2\sum_{i=1}^{N}\mu_{i}\right)^{2}\right)} - \mu_{i}\int_{-\infty}^{+\infty} \frac{2\sum_{i=1}^{N}\mu_{i}\exp\left(\frac{1}{2}\left(t + 2\sum_{i=1}^{N}\mu_{i}\right)^{2}\right)dt}{\left[\exp\left(\frac{1}{2}t^{2}\right) + \exp\left(\frac{1}{2}\left(t + 2\sum_{i=1}^{N}\mu_{i}\right)^{2}\right)\right]^{2}} = 0,$$

Thus, for all $i = 1, \ldots, N$ we have

$$\mu_{i} = \frac{\int_{-\infty}^{+\infty} \frac{dt}{\exp\left(\frac{1}{2}t^{2}\right) + \exp\left(\frac{1}{2}\left(t + 2\sum_{j=1}^{N}\mu_{j}\right)^{2}\right)}}{\int_{-\infty}^{+\infty} \frac{2\sum_{j=1}^{N}\mu_{j}\exp\left(\frac{1}{2}\left(t + 2\sum_{j=1}^{N}\mu_{j}\right)^{2}\right)dt}{\left[\exp\left(\frac{1}{2}t^{2}\right) + \exp\left(\frac{1}{2}\left(t + 2\sum_{j=1}^{N}\mu_{j}\right)^{2}\right)\right]^{2}}$$

We have proved, at the equilibrium, that the first order conditions implie that all agents have the same strategy, in other words, there exists $\mu^*(N)$ independent of i such as for all i = 1, ..., N we have

$$\mu_i = \mu^*(N). \tag{4.3}$$

Let $H(\cdot)$ and $K(\cdot)$ be two functions defined by :

$$H(x) = \int_{-\infty}^{+\infty} \frac{dt}{\exp\left(\frac{1}{2}t^2\right) + \exp\left(\frac{1}{2}(t+2x)^2\right)}$$

and

$$K(x) = \int_{-\infty}^{+\infty} \frac{2x^2 \exp\left(\frac{1}{2}(t+2x)^2\right) dt}{\left[\exp(\frac{1}{2}t^2) + \exp\left(\frac{1}{2}(t+2x)^2\right)\right]^2}$$

For all x, it can be shown that

$$K(x) = x^2 H(x) \tag{4.4}$$

Using the definitions of $H(\cdot)$ and $K(\cdot)$ and (4.2) and (4.3) implie that

$$H(N\mu^{*}(N)) - \frac{1}{N}K(N\mu^{*}(N)) = 0,$$

Reorganising terms using (4.4)

$$\iff H(N\mu^*(N))\left(1 - N\mu^{*^2}(N)\right) = 0,$$
$$\iff \mu^*(N) = \frac{1}{\sqrt{N}},$$

Concerning the second order conditions, we need to prove that $\frac{\partial^2 \pi}{\partial \mu_i^2} \left(\frac{1}{\sqrt{N}}, \dots, \frac{1}{\sqrt{N}}, \frac{1}{\sqrt{N}}, \dots, \frac{1}{\sqrt{N}}\right) < 0.$

$$\begin{split} &\frac{\partial^2 \pi}{\partial \mu_i^2}(\mu,\mu_e) = \\ &\frac{2\sigma_u \sigma_v}{\pi} \begin{cases} -4\sum_{i=1}^N \mu_i^c \int_{-\infty}^{+\infty} \frac{\exp\left(\frac{1}{2}\left(t+2\sum_{i=1}^N \mu_i^c\right)^2\right) \exp\left(2\sum_{i=1}^N \mu_i^c\sum_{i=1}^N (\mu_i - \mu_i^c)\right) dt \\ &\left[\exp\left(\frac{1}{2}t^2\right) + \exp\left(\frac{1}{2}\left(t+2\sum_{i=1}^N \mu_i^c\right)^2\right) \exp\left(2\sum_{i=1}^N \mu_i^c\sum_{i=1}^N (\mu_i - \mu_i^c)\right)\right]^2 - \\ &4\mu_i \left(\sum_{i=1}^N \mu_i^c\right)^2 \int_{-\infty}^{+\infty} \frac{\exp\left(\frac{1}{2}\left(t+2\sum_{i=1}^N \mu_i^c\right)^2\right) \exp\left(2\sum_{i=1}^N \mu_i^c\sum_{i=1}^N (\mu_i - \mu_i^c)\right) dt \\ &\left[\exp\left(\frac{1}{2}t^2\right) + \exp\left(\frac{1}{2}\left(t+2\sum_{i=1}^N \mu_i^c\right)^2\right) \exp\left(2\sum_{i=1}^N \mu_i^c\sum_{i=1}^N (\mu_i - \mu_i^c)\right)\right]^2 + \\ &8\mu_i \left(\sum_{i=1}^N \mu_i^c\right)^2 \int_{-\infty}^{+\infty} \frac{\exp\left(\left(t+2\sum_{i=1}^N \mu_i^c\right)^2\right) \exp\left(2\sum_{i=1}^N \mu_i^c\sum_{i=1}^N (\mu_i - \mu_i^c)\right) dt \\ &\left[\exp\left(\frac{1}{2}t^2\right) + \exp\left(\frac{1}{2}\left(t+2\sum_{i=1}^N \mu_i^c\right)^2\right) \exp\left(2\sum_{i=1}^N \mu_i^c\sum_{i=1}^N (\mu_i - \mu_i^c)\right)\right]^3 \end{cases}, \\ &\frac{\partial^2 \pi}{\partial^2 \mu_i} \left(\mu^*(N), \mu^*(N)\right) = \frac{2\sigma_u \sigma_v}{\pi} \left\{ -8\sqrt{N} \int_{-\infty}^{+\infty} \frac{\exp\left(\frac{1}{2}\left(t+2\sqrt{N}\right)^2\right) dt \\ &\left[\exp\left(\frac{1}{2}t^2\right) + \exp\left(\frac{1}{2}\left(t+2\sqrt{N}\right)^2\right)\right]^3 \right\}, \\ &\frac{\partial^2 \pi}{\partial^2 \mu_i} \left(\mu^*(N), \mu^*(N)\right) = -\frac{16\sigma_u \sigma_v \sqrt{N}}{\pi} \int_{-\infty}^{+\infty} \frac{\exp\left(\frac{1}{2}\left(t+2\sqrt{N}\right)^2\right) \exp\left(\frac{1}{2}\left(t+2\sqrt{N}\right)^2\right)}{\left[\exp\left(\frac{1}{2}t^2\right) + \exp\left(\frac{1}{2}\left(t+2\sqrt{N}\right)^2\right)\right]^3} < 0. \end{split}$$

This ends both proofs of existence and uniqueness.

$$\pi_i\left(\mu^*(N),\mu^*(N)\right) = \frac{2\sigma_u\sigma_v}{\pi\sqrt{N}} \int_{-\infty}^{+\infty} \frac{dt}{\exp\left(\frac{1}{2}t^2\right) + \exp\left(\frac{1}{2}\left(t + 2\sqrt{N}\right)^2\right)}.$$

B Proof of propositions 3.1 - 3.2:

Profits:

The individual expected profit is given by:

$$\begin{aligned} \pi^*(\mu^*(N),\mu^*(N)) &= \frac{2\sigma_u \sigma_v}{\pi\sqrt{N}} \int_{-\infty}^{+\infty} \frac{dt}{\exp\left(\frac{1}{2}t^2\right) + \exp\left(\frac{1}{2}\left(t + 2\sqrt{N}\right)^2\right)}, \\ &= \frac{2\sigma_u \sigma_v}{\pi\sqrt{N}} \int_{-\infty}^{+\infty} \frac{dt}{\exp\left(\frac{1}{2}\left(t - \sqrt{N}\right)^2\right) + \exp\left(\frac{1}{2}\left(t + \sqrt{N}\right)^2\right)}, \\ &= \frac{4\sigma_u \sigma_v}{\pi\sqrt{N}} \int_{0}^{+\infty} \frac{dt}{\exp\left(\frac{1}{2}\left(t - \sqrt{N}\right)^2\right) + \exp\left(\frac{1}{2}\left(t + \sqrt{N}\right)^2\right)}, \\ &= \frac{4\sigma_u \sigma_v}{\pi\sqrt{N}} \int_{0}^{+\infty} \frac{\exp\left(-\frac{1}{2}\left(t + \sqrt{N}\right)^2\right) dt}{1 + \exp\left(-2t\sqrt{N}\right)}, \\ &= \frac{4\sigma_u \sigma_v}{\pi\sqrt{N}} \theta\left(\sqrt{N}\right), \end{aligned}$$

with $\theta(x) = \int_0^{+\infty} \frac{\exp\left(-\frac{1}{2}(t+x)^2\right)}{1+\exp(-2tx)} dt$, For x > 0, we perform the following changes in variables: u = tx. $\theta(x) = \frac{1}{x} \int_0^{+\infty} \frac{\exp\left[-\frac{1}{2}\left(\frac{u}{x}+x\right)^2\right]}{1+\exp(-2u)} du$,

$$\theta(x) = \frac{1}{x} \int_0^{+\infty} \frac{\exp\left(-\frac{1}{2}x^2\right) \exp(-u) \exp\left(-\frac{1}{2}\frac{u^2}{x^2}\right)}{1 + \exp(-2u)} du,$$

$$\theta(x) = \frac{1}{x} \exp\left(-\frac{1}{2}x^2\right) \int_0^{+\infty} \frac{\exp(-u) \exp\left(-\frac{1}{2}\frac{u^2}{x^2}\right)}{1 + \exp(-2u)} du \cdot \\ \lim_{N \to +\infty} \theta\left(\sqrt{N}\right) = \frac{1}{\sqrt{N}} \exp\left(-\frac{1}{2}N\right) \int_0^{+\infty} \frac{\exp(-u)}{1 + \exp(-2u)} du = \frac{1}{\sqrt{N}} \exp\left(-\frac{1}{2}N\right) \int_0^1 \frac{dv}{1 + v^2}, \\ \lim_{N \to +\infty} \theta\left(\sqrt{N}\right) = \frac{1}{\sqrt{N}} \exp\left(-\frac{1}{2}N\right) \frac{\pi}{4},$$

therefore:

$$\lim_{N \to +\infty} \pi_i^* \left(\mu^*(N), \mu^*(N) \right) = \frac{\sigma_u \sigma_v}{N} \exp\left(-\frac{1}{2}N\right),$$
$$\lim_{N \to +\infty} N \pi_i^* \left(\mu^*(N), \mu^*(N) \right) = \sigma_u \sigma_v \exp\left(-\frac{1}{2}N\right).$$

In other words, at the limit we have :

$$\lim_{N \to +\infty} \pi_i^* \left(\mu^*(N), \mu^*(N) \right) = \lim_{N \to +\infty} \frac{\sigma_u \sigma_v}{N} \exp\left(-\frac{1}{2}N\right) = 0, \tag{4.5}$$

$$\lim_{N \to +\infty} N\pi_i^*\left(\mu^*(N), \mu^*(N)\right) = \lim_{N \to +\infty} \sigma_u \sigma_v \exp\left(-\frac{1}{2}N\right) = 0.$$
(4.6)

Pricing Schedule:

$$P(w) = \frac{2\sigma_v}{\sqrt{2\pi}} [K_2^*(w, N) - 1],$$

$$K_2^*(w, N) = \frac{2\varphi\left(\frac{w}{\sigma_u} - \sqrt{N}\right)}{\varphi\left(\frac{w}{\sigma_u} + \sqrt{N}\right) + \varphi\left(\frac{w}{\sigma_u} - \sqrt{N}\right)},$$

$$P(w) = \frac{2\sigma_v}{\sqrt{2\pi}} \frac{\varphi\left(\frac{w}{\sigma_u} - \sqrt{N}\right) - \varphi\left(\frac{w}{\sigma_u} + \sqrt{N}\right)}{\varphi\left(\frac{w}{\sigma_u} + \sqrt{N}\right) + \varphi\left(\frac{w}{\sigma_u} - \sqrt{N}\right)},$$

$$P(w) = \frac{2\sigma_v}{\sqrt{2\pi}} \frac{\exp\left(-\frac{1}{2}\left(\frac{w}{\sigma_u} - \sqrt{N}\right)^2\right) - \exp\left(-\frac{1}{2}\left(\frac{w}{\sigma_u} + \sqrt{N}\right)^2\right)}{\exp\left(-\frac{1}{2}\left(\frac{w}{\sigma_u} - \sqrt{N}\right)^2\right) + \exp\left(-\frac{1}{2}\left(\frac{w}{\sigma_u} + \sqrt{N}\right)^2\right)},$$
in the result:

thus we obtain the result:

$$\lim_{N \to +\infty} P(w, N) = \frac{2\sigma_v}{\sqrt{2\pi}},\tag{4.7}$$

if S=1,

$$\lim_{N \to +\infty} P(w, N) = -\frac{2\sigma_v}{\sqrt{2\pi}},\tag{4.8}$$

if S=-1

Liquidity:

 $\lambda(w, N) = \frac{\partial p}{\partial w}(w)$ is the price pressure and corresponds to the inverse of the market depth:

$$\lambda(w,N) = \frac{8\sigma_v}{\sqrt{2\pi}\sigma_u} \frac{\sqrt{N}\exp\left(\frac{2w\sqrt{N}}{\sigma_u}\right)}{\left[1 + \exp\left(\frac{2w\sqrt{N}}{\sigma_u}\right)\right]^2},$$

if S=-1,

$$\lim_{N \to +\infty} \lambda\left(w, N\right) = \frac{8\sigma_v}{\sqrt{2\pi}\sigma_u} \sqrt{N} \exp\left(\frac{2w\sqrt{N}}{\sigma_u}\right),$$

if S=1,

$$\lim_{N \to +\infty} \lambda\left(w, N\right) = \frac{8\sigma_v}{\sqrt{2\pi}\sigma_u} \sqrt{N} \exp\left(-\frac{2w\sqrt{N}}{\sigma_u}\right),$$

$$\lim_{N \to +\infty} \lambda\left(w, N\right) = \lim_{N \to +\infty} \frac{8\sigma_v}{\sqrt{2\pi}\sigma_u} \sqrt{N} \exp\left(-2N\right) \exp\left(\frac{2u\sqrt{N}}{\sigma_u}\right) = 0, \tag{4.9}$$

if S=-1,

$$\lim_{N \to +\infty} \lambda\left(w, N\right) = \lim_{N \to +\infty} \frac{8\sigma_v}{\sqrt{2\pi}\sigma_u} \sqrt{N} \exp\left(-2N\right) \exp\left(-\frac{2u\sqrt{N}}{\sigma_u}\right) = 0, \tag{4.10}$$

if S=1.