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CONTROL OF A LAUNCHER IN ATMOSPHERIC ASCENT WITH GUARDIAN MAPS

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ABSTRACT

This paper describes the synthesis of a SISO scheduled controller for a launcher vehicle. The problem consists in designing a control law which will be valid on the atmospheric ascent trajectory from time $25\,\mathrm{s}$ *to time* $60\,\mathrm{s}$*, while ensuring robustness and performance requirements. Moreover a flexible model with two bending modes is considered, making the problem more challenging. An algorithm based upon guardian maps has been retained in order to find an* a priori *fixed architecture controller. The algorithm yields a sequence of controllers that ensures that pole confinement constraints are fulfilled for any time between* $25\,\mathrm{s}$ *and* $60\,\mathrm{s}$*. The user can then interpolate those controllers to find a scheduled controller with respect to time.*

1. INTRODUCTION

For several years now, H_∞ synthesis has been providing efficient scheduled controllers for launcher vehicle in atmospheric ascent [14]; but depending on the augmented model, the controller order can be very high and has to be reduced before implementation. In [1], the authors use a multi-objective method based on the Youla parameterization to design linear controllers that are scheduled afterwards while guaranteeing stability *a posteriori*. Therefore, our objective is to design a controller with an *a priori* fixed architecture and limited complexity. Moreover, as previous experiments proved it [9], the controller also has to be scheduled with respect to ascent time in order to take into account the dynamic variations along the trajectory.

A guardian map based algorithm has been proposed to design scheduled controllers with fixed architecture and guaranteeing *a priori* robust stability [11] . The requirements are expressed in terms of pole confinement and the algorithm finds a sequence of controllers that ensures stability for the parameter variation domain. We propose to apply this algorithm to a launcher vehicle problem submitted to different constraints.

This paper is structured as follows. Section 2 introduces the launcher vehicle model provided by ASTRIUM-ST and the associated objectives. Section 3 briefly presents the guardian map theory and the algorithm used to synthetise scheduled controllers. Finally Section 4 illustrates the application of this algorithm to our launcher vehicle problem.

2. MODEL AND OBJECTIVES

Launcher vehicle dynamics are generally described by "short-period" equations of motion during the atmospheric flight. Indeed, in this particular flight phase, the main constraint is to minimize the angle of attack α, which generates a lift force acting on the lateral direction of vehicle. Therefore no important maneuvers are commanded. All motion equations are resolved along the launcher vehicle rigidly attached axes with their origin at the center of mass.

A. LAUNCHER MODEL

Relative orientation of the vehicle in the aerodynamic context is defined by the angle of attack α while the launcher vehicle orientation is defined by the attitude θ. Both of them are related by the equation:

$$\alpha = \theta - \gamma - \frac{W}{V_r} \tag{1}$$

with W wind input, V_r launcher vehicle speed and under the hypothesis that path angle $\gamma = 0°$. The launcher model is then described by the LTV model [2, 5]:

$$\left[\begin{array}{c} \ddot{\theta} \\ \dot{\theta} \\ \hline \dot{x}_F \end{array}\right] = \left[\begin{array}{cc|c} 0 & a_{12}(t) & \multirow{2}{*}{$M_F(t)$} \\ 1 & 0 & \\ \hline \multicolumn{2}{c|}{\mathbf{0}} & A_F(t) \end{array}\right] \left[\begin{array}{c} \dot{\theta} \\ \theta \\ \hline x_F \end{array}\right] + \left[\begin{array}{ccc} b_{11}(t) & b_{12}(t) & b_{13}(t) \\ 0 & 0 & 0 \\ \hline \multicolumn{3}{c}{B_F(t)} \end{array}\right] \left[\begin{array}{c} W \\ \beta \\ \ddot{\beta} \end{array}\right] \tag{2}$$

$$y = \left[\begin{array}{c} \theta \\ \alpha \end{array}\right] = \left[\begin{array}{cc|c} 0 & 1 & \multirow{2}{*}{$C_F(t)$} \\ 0 & 1 & \end{array}\right] \left[\begin{array}{c} \dot{\theta} \\ \theta \\ \hline x_F \end{array}\right] + \left[\begin{array}{ccc} 0 & 0 & 0 \\ d_{12}(t) & 0 & 0 \end{array}\right] \left[\begin{array}{c} W \\ \beta \\ \ddot{\beta} \end{array}\right] \tag{3}$$

with β the actual nozzle deflection and x_F the states associated to the 2 bending modes via A_F, M_F, B_F and C_F. The coefficients a_i and b_i are time-varying coefficients and depend on the launcher vehicle characteristics. The inputs β and $\ddot{\beta}$ are generated by a 4^{th}-order actuator:

$$\left[\begin{array}{c} \beta \\ \ddot{\beta} \end{array}\right] = G(s)\beta_c \tag{4}$$

whose dynamic is defined by two pairs of complex poles: one well-damped at $102\,\mathrm{rad/s}$ and one ill-damped at $127\,\mathrm{rad/s}$. A gridding of 18 rigid + flexible models are used for design; we consider three cases (one nominal case denoted `nom` and two worst cases with high and low frequency rigid modes, namely `lf` and `hf`) and for each case, six flight instants equally distributed between 25 s and 60 s are chosen: $t_1 = 25\,\mathrm{s}$, $t_2 = 32\,\mathrm{s}$, $\ldots$, $t_6 = 60\,\mathrm{s}$. Figure 1 illustrates the pole dispersion of the 18 models. Finally the time-varying coefficients are approximated by 5^{th} order polynomials for later synthesis purpose.

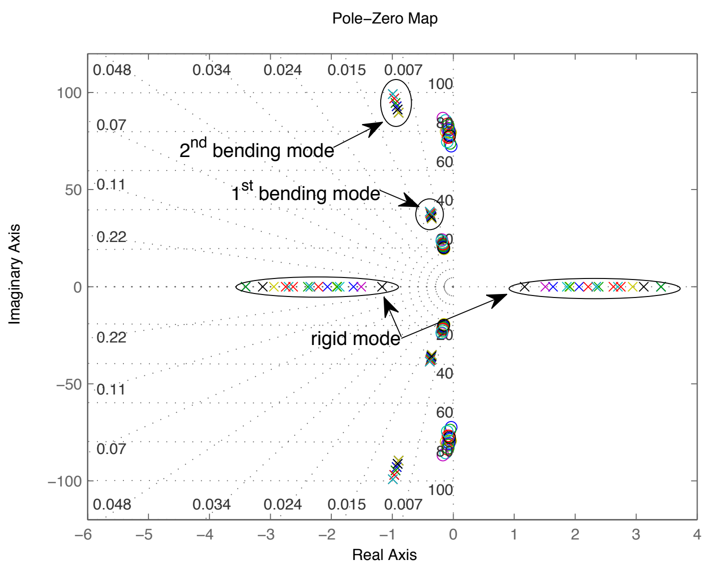


***Fig. 1**: Open loop poles for the 18 models*

B. DESIGN SPECIFICATIONS

Although the main constraint is to minimize the angle of attack α, only the output θ is available for feedback. Moreover the SISO controller structure should be as simple as possible. The objectives are then:

- Closed-loop rigid mode: minimum damping ratio $\zeta > 0.5$ for `nom` case and $\zeta > 0.15$ for `hf` and `lf` cases
- $3\,\mathrm{dB}$ gain margin for `nom` and `hf` cases and $1\,\mathrm{dB}$ gain margin for `lf` case
- The magnitude of α should stay below 3 degrees in response to a worst-case wind input signal W
- Delay margin $> 27\,\mathrm{ms}$ (one control period)
- The bending modes should not be destabilized by the feedback controller

3. GUARDIAN MAPS

Basically, guardian maps are scalar valued maps defined on the set of $n \times n$ real matrices (or n^{th}-order polynomials) that take non-zero values on the set of Ω-stable matrices (or polynomials) and vanish on its boundary. The description below will focus on families of matrices with the understanding that it applies to polynomials as well.

A. DEFINITION

We are here interested in stability sets of the form:

$$S(\Omega) = \{A \in \mathbb{R}^{n\times n} : \sigma(A) \subset \Omega\} \tag{5}$$

were Ω is an open subset of the complex plane of interest, and $\sigma(A)$ denotes the set consisting of the eigenvalues of A. Such sets $S(\Omega)$ will be referred to as *generalized stability sets*, and thus represent the set of all matrices which are stable relative to Ω, i.e. which have all their eigenvalues in Ω.

Definition *Let ν map $\mathbb{R}^{n\times n}$ into $\mathbb{C}$. We say that ν* guards *$S(\Omega)$ if for all $A \in \overline{S}(\Omega)$, the following equivalence holds:*

$$\nu(A) = 0 \Leftrightarrow A \in \overline{S}(\Omega) \tag{6}$$

Here $\overline{S}$ denotes closure of the set S. The map is said to be polynomic *if it is a polynomial function of the entries of its argument.*

Example 1. *Some guardian maps are given for classical regions (Fig. 2).*

- *Hurwitz Stability: for $\Omega = \mathring{\mathbb{C}}_-$, a guardian map is*

$$\nu_H(A) = \det(A \odot I)\det(A) \tag{7}$$

where $\odot$ denotes the bialternate product[13].

- *Stability margin: the open α-shifted half-plane region has a corresponding guardian map*

$$\nu_m(A) = \det(A \odot I - \alpha I \odot I)\det(A - \alpha I) \tag{8}$$

- *The conic sector with inner angle 2θ has a corresponding guardian map given by*

$$\nu_d(A) = \det\left(A^2 \odot I + (1 - 2\zeta^2)A \odot A\right)\det(A) \tag{9}$$

where $\zeta \triangleq \cos\theta$ denotes the limiting damping ratio.

- *Schur stability: for the circle of radius $\omega > 0$, a corresponding guardian map is*

$$\nu_p(A) = \det\left(A \odot A - \omega^2 I \odot I\right)\det\left(A^2 - \omega^2 I\right) \tag{10}$$

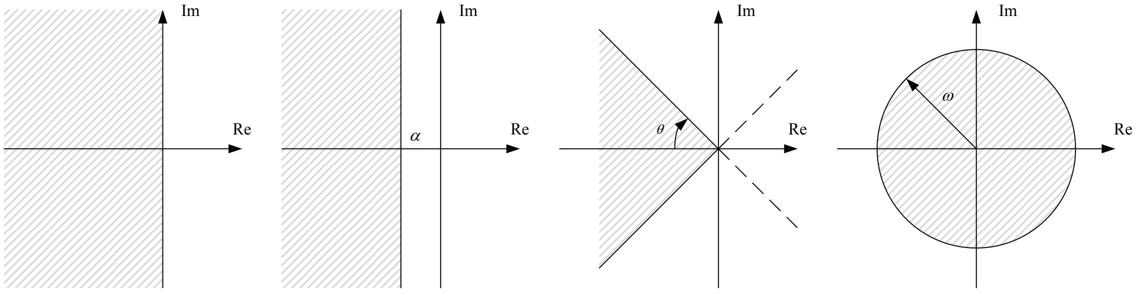


***Fig. 2**: Regions for Example 1*

A systematic way of constructing guardian maps for various Ω regions can be found in Saydy et al. [12].

B. STABILIZING GAIN CHARACTERIZATION

Let $\{A(r) : r \in U \subset \mathbb{R}^k\}$ be a continuous family of $n \times n$ matrices which depend on the (usually) uncertain parameter vector $r := (r_1, \ldots, r_k)$ where each entry lies in a given range for which only the bounds are known, say $r \in U \subset \mathbb{R}^k$.

Theorem 1. *Let $S(\Omega)$ be guarded by the map ν_Ω. The family $\{A(r) : r \in U\}$ is stable relative to Ω if and only if*

(i) it is nominally stable, i.e. $A(r_0) \in S(\Omega)$ for some $r_0 \in U$; and,

(ii) $\forall r \in U$, $\nu_\Omega(A(r^0))\nu_\Omega(A(r)) > 0$. i.e. $\nu_\Omega(A(r))$ does not vanish in U.

Corollary 1. *Let $S(\Omega)$ be guarded by the map ν and consider the family $\{A(r) : r \in U\}$. Then C defined by:*

$$C = \{r \in \mathbb{R}^k : \nu_\Omega(A(r)) = 0\} \tag{11}$$

divides the parameter space $\mathbb{R}^k$ into components C_i that are either stable or unstable relative to Ω. To see which situation prevails for a given component C_i, one only has to test $A(r)$ for any one point in C_i.

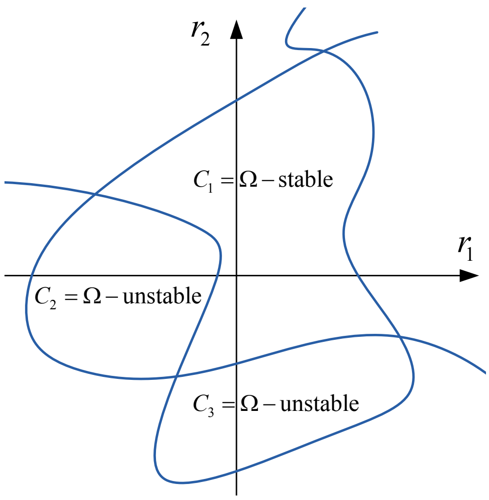


***Fig. 3**: Corollary component illustration*

Example 2. *Suppose that the closed-loop poles of a given system are specified by the polynomial:*

$$p(s) = s^3 + k_1 s^2 + k_2 s + 1 \tag{12}$$

where k_1, k_2 denote some controller gains. If the damping region $\zeta > 0.7$ is the one considered (Fig. 4), then one obtains (e.g. by applying Eq. (9) to the companion matrix corresponding to p):

$$\nu_\Omega(p) = 2k_2^3 - k_1^2 k_2^2 - 4k_1 k_2 + 2k_1^3 + 1 \tag{13}$$

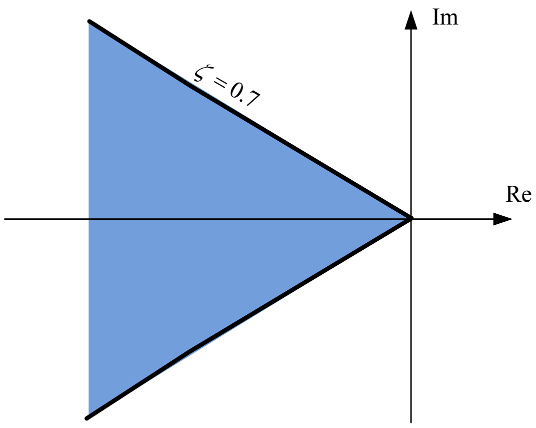


***Fig. 4**: Stability region Ω*

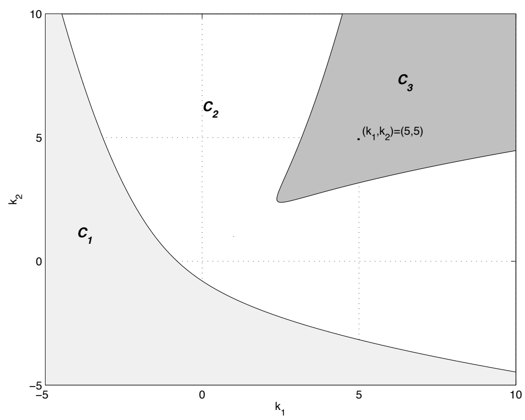


***Fig. 5**: Set C_3 of all gains ensuring $\Omega-$stability*

Setting this quantity to 0 yields the 3 components in the parameter space (k_1, k_2) of Fig. 5. It can be verified that the set of all gains (k_1, k_2) which place all the closed-loop poles within the damping zone above is the component C_3. Any other choice of the gains outside of C_3 yields closed-loop poles outside the damping conic region. We arrive to this conclusion simply by testing Ω-stability of $p(s)$ for any three pairs (k_1, k_2) in C_1, C_2 and C_3 respectively.

C. ROBUST STABILITY

We consider here the stability of one parameter families of real matrices relative to a domain Ω for which $S(\Omega)$ is endowed with a polynomic guardian map ν_Ω. In the following, we consider single-parameter polynomial matrices of the form:

$$A(r) = A_0 + rA_1 + \ldots + r^k A_k \tag{14}$$

with A_i given constant matrices and such that $A(r_0)$ is Ω-stable. The corresponding guardian map $\nu_\Omega(A(r))$ is a polynomial in r. We seek to find the largest open stability interval w.r.t. Ω around r_0. Let

$$r^- \doteq \sup\{r < r_0 : \nu_\Omega(A(r)) = 0\} \text{ (or } -\infty \text{ if none exists)}$$
$$r^+ \doteq \inf\{r > r_0 : \nu_\Omega(A(r)) = 0\} \text{ (or } +\infty \text{ if none exists)}$$

be the maximal perturbation bounds for nonsingularity of matrices around $r = r_0$.

Lemma 1. *Let $A(r) = A_0 + rA_1 + \ldots + r^k A_k$ be a polynomial matrix in the uncertain parameter r real with given constant matrices A_i such that $A(r_0)$ is stable w.r.t. Ω and let $S(\Omega)$ be guarded by a map ν_Ω. Then $A(r)$ is stable relative to Ω for all $r \in (r^-, r^+)$. Furthermore, this interval is the largest one containing r_0.*

D. SINGLE-PARAMETER GAIN-SCHEDULING ALGORITHM

In typical gain-scheduling techniques, LTI controllers have to be designed on different linearized models; controller interpolation is done *a posteriori*, or switching laws are implemented between the various controllers. In the interpolation approach case, even if the controllers designed on each operating point fulfill the requirements locally, there is no guarantee that between the synthesis points stability is retained, especially if the designer did not take enough synthesis points[7, 4]. Moreover, depending on controller complexity, interpolation problems may arise. For example, in the case of H_∞ control or μ-synthesis, an initial order reduction phase is often required on each LTI controller in order to ensure that all the controllers have the same order and structure [6]. In the switching approach case, the designed controllers must cover the entire domain and again the number of synthesis points is crucial. So the three major issues are : the number of synthesis points, the controller structure and the stability and performance satisfaction on the entire domain[3, 8] . Our method proposes to address these issues.

Let an initial fixed-structure controller $\boldsymbol{K}^0$ be designed on a peculiar trim condition. This controller naturally presents some robust performance margins w.r.t. trim variables, i.e. it is still performant for other trim conditions around the initial one. Let then a limit trim condition for which $\boldsymbol{K}^0$ can no longer ensure performance. By adjusting $\boldsymbol{K}^0$ gains (according to a method based on guardian maps), one should obtain a new controller $\boldsymbol{K}^1$ having its own robust performance margins. By taking another worst-case condition where $\boldsymbol{K}^1$ fails, another controller $\boldsymbol{K}^2$ is designed and so on till covering the entire operating domain.

The proposed algorithm is applied in the case of a system with one single parameter r (e.g. t the flight instant in our case). Let Ω be the region of eigenvalue confinement of interest and ν_Ω be a corresponding guardian map. Let $A(r, \boldsymbol{K})$ denote the closed-loop state space matrix with $\boldsymbol{K} = [K_i]$ the gain vector. With a slight abuse in notation, we denote $\nu_\Omega(r, \boldsymbol{K}) := \nu_\Omega(A(r, \boldsymbol{K}))$. If $A(r, \boldsymbol{K})$ depends polynomially on the parameters and the boundary of Ω is also defined polynomially, then $\nu_\Omega(r, \boldsymbol{K})$ is a multivariable polynomial as well. We seek to find $\boldsymbol{K}$ (scheduled w.r.t. r) that stabilizes the system for $r \in [r_{\min}, r_{\max}]$.

For an initial parameter value $r_0 = r_{\min}$, let $\boldsymbol{K}^0$ be a nominal choice of stabilizing gains, that is, such that the eigenvalues of $A(r_0, \boldsymbol{K}^0)$ are inside Ω. With $\boldsymbol{K} = \boldsymbol{K}^0$, we apply Lemma 1 to find the largest stability interval $]\underline{r}_0, \overline{r}_0[$. Thus the vector $\boldsymbol{K}^0$ stabilizes the system for any parameter r in $]\underline{r}_0, \overline{r}_0[$. If $\overline{r}_0 = +\infty$ or $\overline{r}_0 > r_{\max}$, one can stop as $\boldsymbol{K}^0$ ensures stability $\forall r \in [r_{\min}, r_{\max}]$. Moreover, we have $\nu_\Omega(\underline{r}_0, \boldsymbol{K}^0) = \nu_\Omega(\overline{r}_0, \boldsymbol{K}^0) = 0$ if $\underline{r}_0$ and $\overline{r}_0$ happen to be finite.

If $\overline{r}_0 \leq r_{\max}$, we proceed as follows. Fix $r_1 = \overline{r}_0$. The equation $\nu_\Omega(r_1, \boldsymbol{K}) = 0$ defines new components in the space of gain parameters that are either stable or unstable (Corollary 1). By definition of $\overline{r}_0$, $\nu_\Omega(\overline{r}_0, \boldsymbol{K}^0) = 0$ and $\boldsymbol{K}^0$ lies on the boundary of a stable component. In [10], an algorithm was developed to search inside a component in order to find a new vector $\boldsymbol{K}^1$ which places closed-loop poles strictly inside Ω. This new choice leads to a new stability interval $]\underline{r}_1, \overline{r}_1[$ with $\overline{r}_0 \in]\underline{r}_1, \overline{r}_1[$. The same steps are repeated till possibly covering all values of parameter $r \in [r_{min}, r_{max}]$.

If the algorithm succeeds, it yields a sequence of controllers $\left\{\boldsymbol{K}^0, \ldots, \boldsymbol{K}^i, \ldots, \boldsymbol{K}^n\right\}$ satisfying all the criteria on the corresponding intervals $\{[r_{\min}, \overline{r}_0[, \ldots,]\underline{r}_i, \overline{r}_i[, \ldots,]\underline{r}_n, r_{\max}]\}$. Moreover, the entire parameter range $[r_{\min}, r_{\max}]$ is covered since by construction $\underline{r}_{i+1} < \overline{r}_i$. The user is then free to exploit this set of satisfying controllers depending on the way they will be implemented: look-up tables, switching controllers or interpolation of the data to name a few. Algorithm 1 is then proposed.

Remark 1. *Going "rightward" from $r_{\min}$ to $r_{\max}$ is an arbitrary choice and one can adapt the algorithm to make it work "leftward". This leads to different results in general.*

As an example, Fig. 6 deals with the synthesis of a scheduled PI controller with respect to some measurable parameter r. If Ω is a stability region of interest, we suppose that the algorithm starts at $r = r^0$ and find an adequate choice of gains inside the component C^* (blue shape) ensuring the pole confinement in Ω (Fig. 7). After a robustness analysis in r, the upper stability limit of this peculiar controller is reached at $r = r^1$. The procedure is then repeated. The final scheduled affine controller in r (red line) is then chosen.

4. APPLICATION TO THE LAUNCHER VEHICLE PROBLEM

We apply the algorithm presented in the previous section to our launcher vehicle problem. The goal is to find the

Algorithm 1. Single parameter gain-scheduling algorithm

Step 0: Initialization

Let Ω be a region of the complex plane and ν_Ω a corresponding guardian map, $A\,(r, \boldsymbol{K})$ a closed-loop matrix depending polynomially on the single parameter $r \in [r_{\min}, r_{\max}]$ and the gain vector $\boldsymbol{K} = [K_j] \in \mathbb{R}^p$. Obtain a controller $\boldsymbol{K}^0$ designed for the nominal case $r_0 = r_{\min}$ ensures nominal stability relative to Ω. Set $n \leftarrow 0$.

Using Lemma 1 on $A(r, \boldsymbol{K}^0)$, find the largest stability interval $]\underline{r}_0, \overline{r}_0[$ containing r_0.

If $\overline{r}_0 > r_{\max}$ **then Stop else** set the counter $n \leftarrow n + 1$.

Step n.1: Synthesis phase

Find a new gain vector $\boldsymbol{K}^n$ inside a component defined by $\nu_\Omega(\overline{r}_{n-1}, \boldsymbol{K}) = 0$ using search algorithm (see [10]) with initial vector $\boldsymbol{K}^{n-1}$.

Step n.2: Robustness analysis

Using Lemma 1 on $A(r, \boldsymbol{K}^n)$, find the largest stability interval $]\underline{r}_n, \overline{r}_n[$ containing $\overline{r}_{n-1}$.

If $\overline{r}_n > r_{\max}$ **then** go to **Final Step** or **Stop else** set the counter $n \leftarrow n + 1$ and go to **Step n.1**.

Final Step: Interpolation

If an interpolation $\boldsymbol{K} = \boldsymbol{K}(r)$ is sought, use Lemma 1 to check if stability is preserved $\forall r \in [r_{\min}, r_{\max}]$.

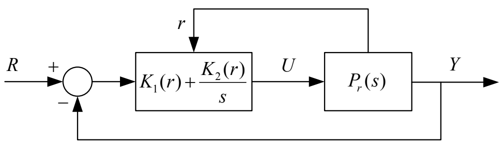


***Fig. 6**: Scheduled PI synthesis*

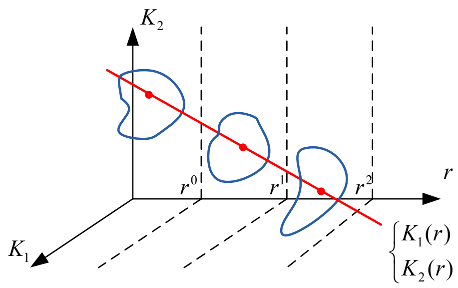


***Fig. 7**: Scheduling function after algorithm application*

simplest controller that will fulfill all the requirements.

A. CONTROLLER ARCHITECTURE

Derived from previous experiments, a controller of the following form is sought:

$$C(s) = \frac{K_1 s^2 + K_2 s + K_3}{s\,(s/35 + 1)\,(s/50 + 1)} \tag{15}$$

The integral effect on θ allows to indirectly minimize the angle of attack α and the roll-off property limits the controller bandwidth. In order not to destabilize the bending modes, a 4^{th} order elliptic is added. The controller poles are deliberately fixed and only the numerator coefficients can be tuned. After state and output augmentation of the model with the controller and filter dynamics, the problem can be cast as a static output feedback:

$$A_{CL}(\Delta) = A(\Delta) - B(\Delta) \begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix} C(\Delta) \tag{16}$$

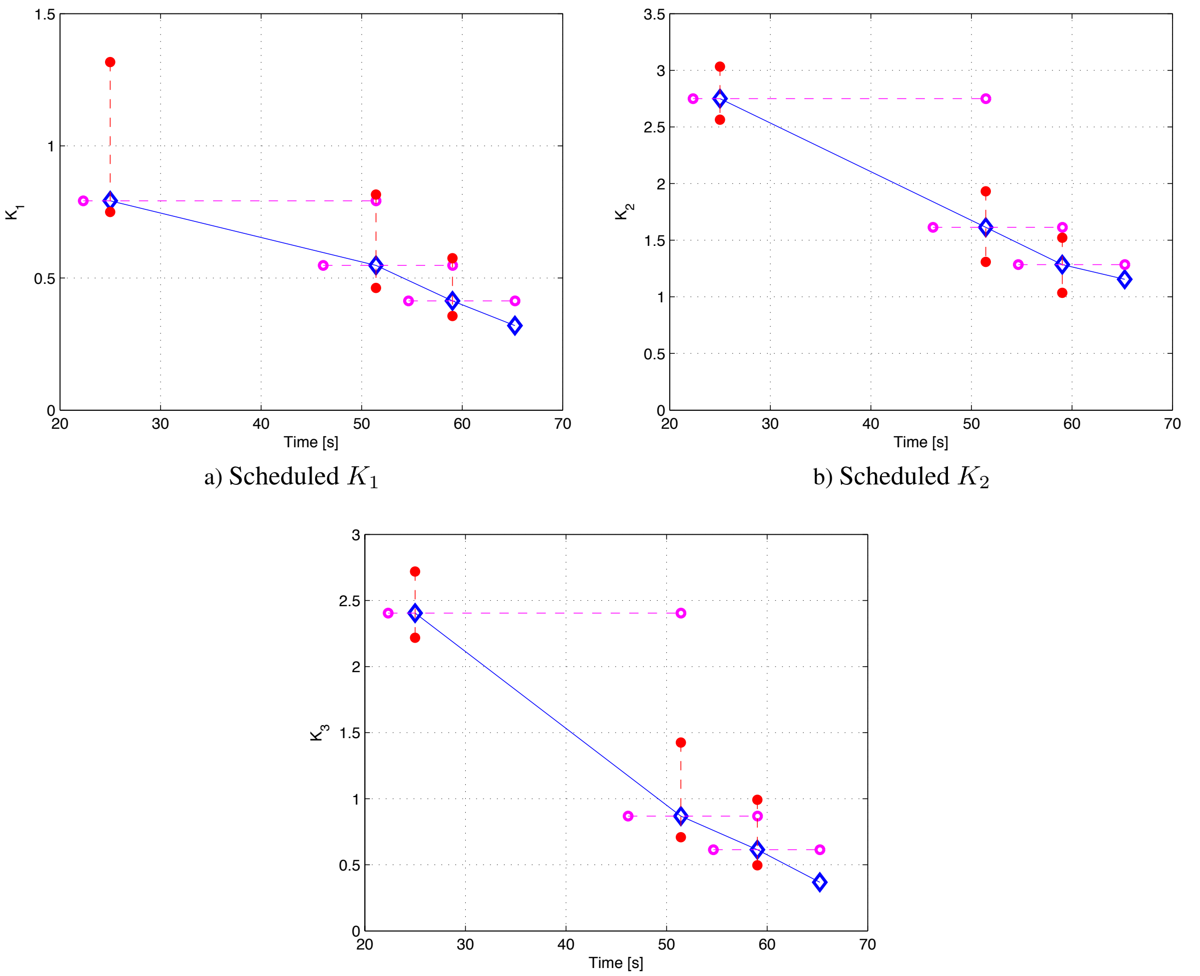
with $\Delta = t$ for LPV design or $\Delta = t_i$ for gridding analysis. It now boils down to adjust the gains K_1, K_2, K_3 in order to satisfy the requirements for any $t \in [25, 60]$.

B. SYNTHESIS

Three pole confinement constraints are considered in the application of the algorithm:

- Hurwitz stability (Eq. 7)
- Real part constraint on the rigid mode (Eq. 8 with $\alpha < -0.4$)
- Damping constraint on the rigid mode (Eq. 9 with $\zeta > 0.5$ for `nom` case, $\zeta > 0.15$ for `lf` and `hf` cases)

After computation, the algorithm delivers the gains of Fig. 8. A sequence of three controllers is sufficient to cover the considered time domain. The diamond signs ($\diamond$) denote the gains (i.e. controller) computed by the algorithm at a specific time t; then the horizontal dashed magenta line indicate the stability interval of the concerned controller. When the other gains and time t are fixed, the vertical red line show the allowable variation of a specific gain before losing general stability. Table 1 gathers the gain values as for their respective robust stability domain (as defined by the pole confinement constraints).



a) Scheduled K_1

b) Scheduled K_2

c) Scheduled K_3

Fig. 8: Scheduled gains found by the algorithm

Controller	K_1	K_2	K_3	Time t	Stability domain in t
#1	0.79	2.75	2.40	25	$t \in]22.33\,,\,51.43[$
#2	0.54	1.61	0.86	51.42	$t \in]46.18\,,\,59.04[$
#3	0.41	1.28	0.61	59.03	$t \in]54.67\,,\,65.25[$

***Table 1**: Sequence of controllers*

Starting from the sequence of controllers found by the algorithm, the following time affinely dependant controller is chosen:

$$C(s,\delta t) = C_0(s) + \delta t C_1(s) \tag{17}$$

$$C_0(s) = \frac{0.98s^2 + 2.95s + 2.3}{s\,(s/35+1)\,(s/50+1)} \tag{18}$$

$$C_1(s) = -\frac{0.605s^2 + 1.85s + 1.7}{s\,(s/35+1)\,(s/50+1)} \tag{19}$$

with $\delta t \in [0\,,1]$ and $\delta t = \dfrac{t-25}{60-25}$.

C. RESULTS

As expected the damping ratio requirement is satisfied for each case and the bending modes are still stable (Fig. 9). The stability margins (Fig. 10) are adequate for delay and phase margins. Concerning the gain margins, at time $t = 25\,\mathrm{s}$, it is less than the $3\,\mathrm{dB}$ requirement for the `hf` case. Nevertheless, we consider this as satisfying. The controller discretization (control period 27 ms) yields satisfying time-responses on the LTV model for all three cases. The maximum α values are $3.71°$ (`nom`), $3.72°$ (`lf`) and $3.78°$ (`hf`).

5. CONCLUSION

In this article, a scheduling algorithm based upon guardian maps is successfully applied to a launcher vehicle control problem. With an *a priori* fixed architecture controller, which is scheduled with respect to time t, robustness and performance objectives are fulfilled for any time between $25\,\mathrm{s}$ and $60\,\mathrm{s}$. The results proved very good even on the flexible models and for worst cases. Moreover the SISO margins satisfy the requirements and the controller discretization with a $27\,\mathrm{ms}$ period does not affect the good results obtained with the continuous time controller. With the addition of the elliptic filter, the controller is a 7^{th} order one, which is very reasonable for such an ambitious problem.

6. ACKNOWLEDGEMENTS

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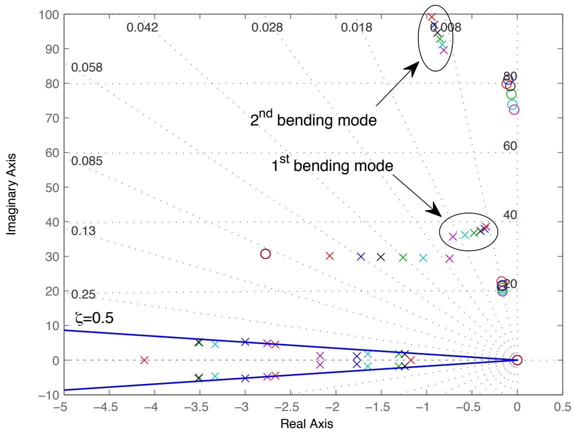
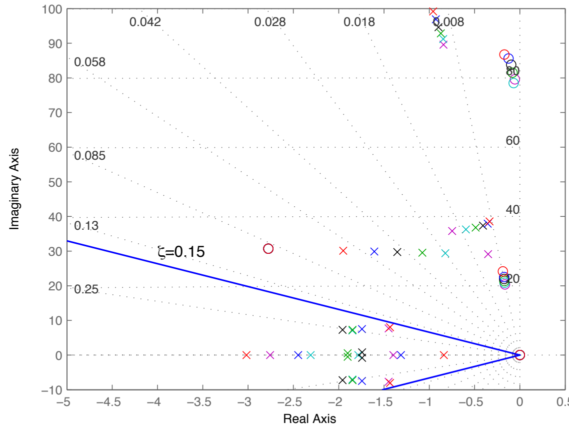
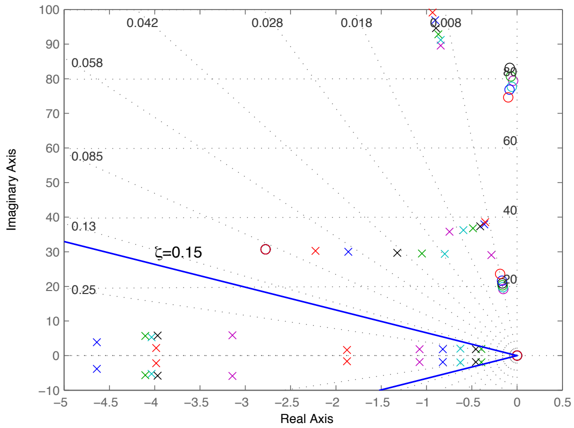


a) `nom` case



b) `hf` case



c) `lf` case

***Fig. 9**: Closed-loop poles*

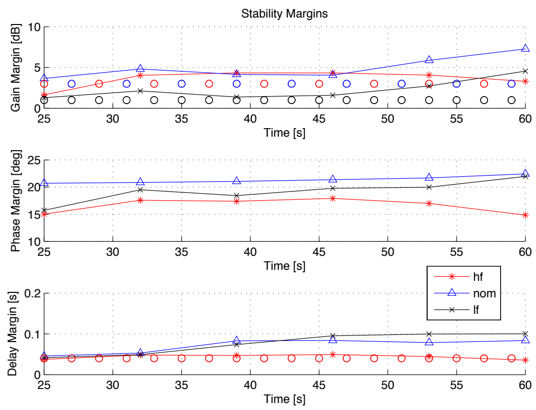


***Fig. 10**: Stability margins*

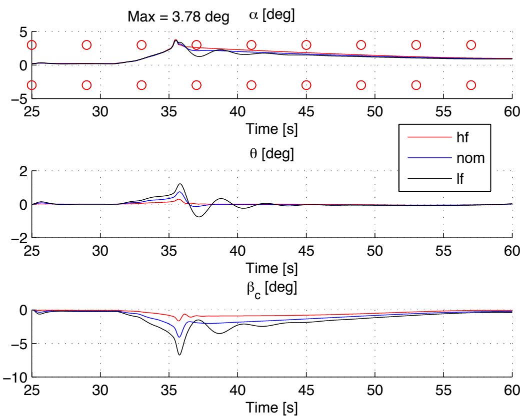


***Fig. 11**: Time-responses to worst-case wind profile*

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