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**To cite this version**: Gerolymos, G. A. and Chapin, Vincent *Generalized Expression of Chorochronic Periodicity in Turbomachinery Blade-Row Interaction.* (1991) La Recherche Aérospatiale (n° 5). pp. 69-73. ISSN 0034-1223

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SHORT NOTE

## GENERALIZED EXPRESSION OF CHOROCHRONIC PERIODICITY IN TURBOMACHINERY BLADE-ROW INTERACTION, by G. A. GEROLYMOS and V. CHAPIN (*).


### ABSTRACT

The unsteady flow which is generated when 2 turbomachinery blade-rows are in relative angular motion is periodic in time, with a different period in the frame of reference associated with each blade-row, and is characterized by a pitchwise traveling wave chorochronic periodicity. This periodicity is studied for arbitrary angular velocities and pitch-ratio of the 2 blade-row and simple formulae for the corresponding interblade-phase angles are given.


The purpose of this note is to formulate the chorochronic periodicity relations characterizing the behaviour of aeromechanic quantities during the interaction of two adjoining blade-rows, in relative angular motion, with arbitrary pitch-ratio. The formulation includes both rotor/stator interaction and the flow through counter-rotating rotors. Numerical solutions for such flows, which make explicit use of chorochronic periodicity, have been presented in the pioneering work of **Erdos** *et al.* [1], and, later, by **Hodson** [4], **Koya** and **Kotake** [5], **Giles** [2, 3], and **Lewis** *et al.* [7].

Making use of chorochronic periodicity one can compute a single interblade-passage per blade-row, thus acheiving considerable gains in computing-time, without any loss of information concerning the unsteady flowfield. Nevertheless, a generalized expression, allowing straightforward computer implementation of the chorochronic periodicity conditions for arbitrary pitch-ratio and angular velocities of the 2 rows is not found in the literature (*cf.* the complicated relationships used in **Koya** and **Kotake** [5]).

The following analysis concerns the periodicity imposed on the flowfield by kinematic considerations. One cannot overemphasize the fact that other periodicities, independant of the angular velocities, such as separated flow vortex-shedding, may be present in the flowfield (*cf.* **Richardson** [8]).

Consider two adjacent blade-rows, $A$ and $B$, schematically depicted in Figure 1 (the stagger of the cascades and the flow direction have been intentionally omitted since they do not influence periodicity relations, and may be arbitrary), rotating with respect to a frame $\Theta_{ABS}$ with rotational velocities $\Omega_A$ and $\Omega_B$ respectively (there is no need to assume that $\Theta_{ABS}$ is inertial). In the frames connected with each blade-row, the angular position of each blade is then determined by the relations:

$$\Theta_{A_i}=(i-1)\frac{2\pi}{N_A} \qquad \text{and} \qquad \Theta_{B_j}=(j-1)\frac{2\pi}{N_B}$$

Without loss of generality the origine of time and of $\Theta_{ABS}$ may be chosen as the time when the blades $A_1$ and $B_1$ are aligned:

$$t=0 : A_1 \,|\!\vdash B_1$$
$$\Theta_{ABS}=0$$

It follows that at any time $t$, $\Theta_{ABS}$, $\Theta_A$ and $\Theta_B$ are related by:

time $t$:

$$\Theta_A=\Theta_{ABS}-\Omega_A t+2k\pi; \qquad k\in Z$$
$$\Theta_B=\Theta_{ABS}-\Omega_B t+2l\pi; \qquad l\in Z$$

and the kinematic relation between the two frames is obtained:

$$\Theta_A=\Theta_B-(\Omega_A-\Omega_B)t+2k\pi; \qquad k\in Z.$$

It is well established (e.g. **Erdos** *et al.* [1]) that the relative angular motion of the two blade-rows induces a flow that is periodic, with a different period in the


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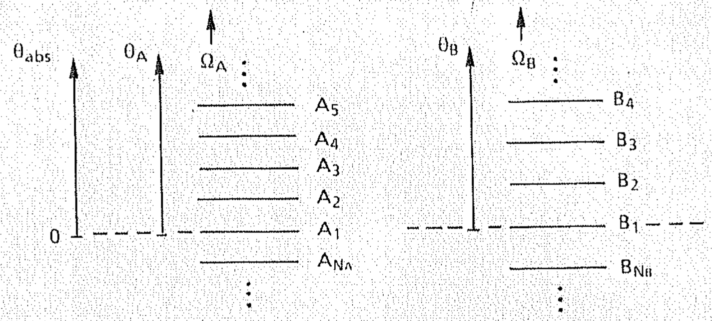


Fig. 1. — Schematic representation of two adjacent blade-rows $A$ and $B$ at time $t=0$.

frame connected with each blade-row, and that the circumferential organization of the flowfield is of a traveling-wave nature (*cf.* **Lane** [6]), so that for any aeromechanical quantity $F_A$, in frame $A$, the following time periodicity holds:
if

$$F_A(x, r, \Theta_A; t) = G_A(2\pi f_A t)$$

then

$$F_A\left(x, r, \Theta_A + k\frac{2\pi}{N_A}; t\right) = G_A(2\pi f_A t + k\delta\Phi_A)\, k \in Z \quad (1)$$

with $G_A$ a periodic function of $2\pi f_A t$ of unit period; a similar relation holds for the $B$-frame.

**PERIODICITY IN FRAME A (rotating at $\Omega_A$)**

The period of unsteady flow phenomena in frame $A$ is the time between the passage of any two consecutive $B$-blades in front of a given $A$-blade, e. g. $A_i$:

For fixed $(i;\ 1 \leqq i \leqq N_A) \exists (j;\ 1 \leqq j \leqq N_B)$:

$$B_j \,|\vdash A_i \Leftrightarrow \Theta_A + \Omega_A t = \Theta_B + \Omega_B t + 2k\pi;\quad k \in Z$$

$$\Leftrightarrow (j-1)\,2\pi/N_B = (\Omega_A - \Omega_B)\,t + (i-1)\,2\pi/N_A + 2k\pi;\quad k \in Z$$

and remarking that $i$ is fixed, the frequency of the phenomenon in frame $A$ is the passing frequency of the $B$-blades:

$$2\pi f_A = N_B\,|\Omega_A - \Omega_B| = N_B\,\Omega \quad (2)$$

with $\Omega \neq 0$ the absolute value of the relative angular velocity

$$\Omega = |\Omega_A - \Omega_B| \quad (3)$$

**PERIODICITY IN FRAME B (rotating at $\Omega_B$)**

In exactly the same way, it may be shown that the period of unsteady flow phenomena in frame $B$ is the passing period of the $A$-blades:

$$2\pi f_B = N_A\,|\Omega_A - \Omega_B| = N_A\,\Omega \quad (4)$$

**PHASE-ANGLE RELATIONS**

Without loss of generality one may consider only the case $N_A \geqq N_B$, since the flow direction, or indeed any flowfield property does not have any effect on periodicity considerations. One may also note that $N_A = N_B$ is a trivial case where classic steady-state space-periodicity (interblade-phase-angle equal to zero) is recovered.

$\underline{N_B \leqq N_A \leqq 2N_B}$

The case $N_B \leqq N_A \leqq 2N_B$ will be examined first (Fig. 2). Since according to the periodicity relations the frequency in each blade-row frame is the passing frequency of one blade of the adjoining row, the phase-angle between two adjacent blades $A_i$ and $A_{i+1}$ (respectively $B_j$ and $B_{j+1}$) is determined by the time

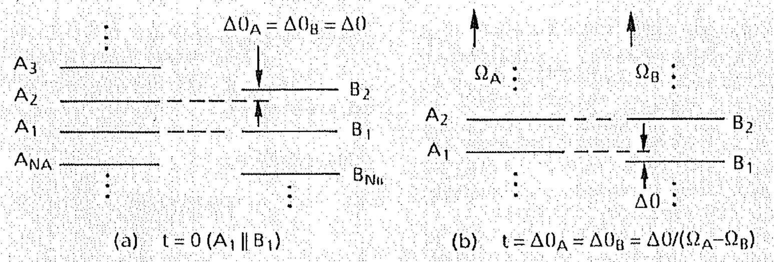


Fig. 2. — Schematic representation of blade-alignment in the case $N_B \leqq N_A \leqq 2N_B$, (a) $t=0$; (b) $t=\delta t_A=\delta t_B$.

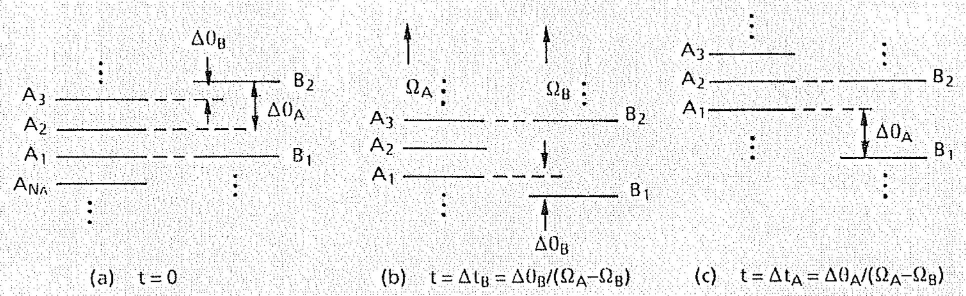


Fig. 3. — Schematic representation of blade-alignment in the case $2N_B \leqq N_A \leqq 3N_B$, (a) $t=0$; (b) $t=\delta t_B$; (c) $t=\delta t_A$.

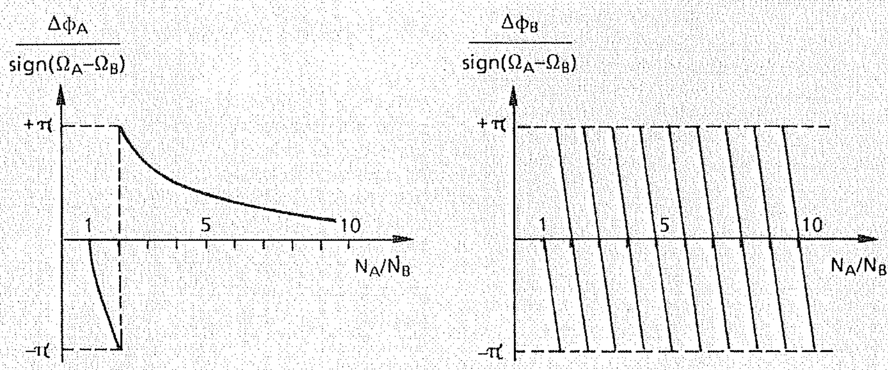


Fig. 4. — Interblade phase-angle $\delta\Phi_A$ and $\delta\Phi_B$ as a function of $N_A$ and $N_B$.

between the alignment of blades $A_i$ and $B_j$ $(A_i \| B_j)$ and the alignment of the blades $A_{i+1}$ and $B_{j+1}$ $(A_{i+1} \| B_{j+1})$, which corresponds to the time needed for a relative rotation of

$$\delta\Theta_A=\delta\Theta_B=2\pi(1/N_B-1/N_A) \quad \text{(Fig. 2).}$$

This may be determined considering, without loss of generality, the periodicity relations for the blades 1 and 2 of each row, *viz.*:

$$F_A\left(x,\ r,\ \Theta_A=\frac{2\pi}{N_A};\ t-\delta t\right)=F_A(x,\ r,\ \Theta_A=0;\ t) \quad (1')$$

$$F_B\left(x,\ r,\ \Theta_B=\frac{2\pi}{N_B};\ t-\delta t\right)=F_B(x,\ r,\ \Theta_B=0;\ t) \quad (1'')$$

for any flow quantity $F$, with $\delta t$ (1):

$$\Theta_{ABS_{A_1}}(t-\delta t)=\Theta_{ABS_{B_2}}(t-\delta t) \quad (^2)$$

$$\frac{2\pi}{N_A}-\Omega_A\,\delta t=\frac{2\pi}{N_B}-\Omega_B\,\delta t-2k\pi;\quad k\in Z$$

$$\delta t(\Omega_A-\Omega_B)=\frac{2\pi}{N_A}-\frac{2\pi}{N_B}+2k\pi;\quad k\in Z$$

and the interblade phase-angle is determined by:

$$N_B \leqq N_A \leqq 2N_B$$

$$\left.\begin{aligned}\delta\Phi_A&=N_B\,\Omega\,\delta t\\&=-2\pi\,\mathrm{sign}(\Omega_A-\Omega_B)\frac{N_A-N_B}{N_A}+2k_A\pi;\\&k_A\in Z\end{aligned}\right\}\quad (5a)$$

$$\left.\begin{aligned}\delta\Phi_B&=N_A\,\Omega\,\delta t\\&=-2\pi\,\mathrm{sign}(\Omega_A-\Omega_B)\left(\frac{N_A}{N_B}-1\right)+2k_B\pi;\\&k_B\in Z\end{aligned}\right\}\quad (5b)$$

$2N_B \leqq N_A \leqq 3N_B$

The case $2N_B \leqq N_A \leqq 3N_B$ (Fig. 3) is slightly more complicated. The frequency of the phenomenon in the frame associated with each blade-row is again the passing frequency of the blades of the adjoining row. The phase-angle between two consecutive blades $A_i$ and $A_{i+1}$ is again determined by the time between the alignment of blades $A_i$ and $B_j$ ($A_i|\vdash B_j$) and the alignment of the blades $A_{i+1}$ and $B_{j+1}$ ($A_{i+1}|\vdash B_{j+1}$), corresponding to the time $\delta t_A$ needed for a relative rotation of $\delta\Theta_A=2\pi(1/N_B-1/N_A)$ (Fig. 3). On the contrary the phase-angle between 2 consecutive blades $B_j$ and $B_{j+1}$ is now determined by the time between the alignment of blades $A_i$ and $B_j$ ($A_i|\vdash B_j$) and the alignment of the blades $A_{i+2}$ and $B_{j+1}$ ($A_{i+2}|\vdash B_{j+1}$), corresponding to the time $\delta\Theta_B$ needed for a relative rotation of $\delta\Theta_B=2\pi(1/N_B-2/N_A)$ (Fig. 3), *viz.*

$$2N_B \leqq N_A \leqq 3N_B$$

$$\left.\begin{aligned}\delta\Phi_A&=N_B\,\Omega\,\delta t_A\\&=-2\pi\,\mathrm{sign}(\Omega_A-\Omega_B)\frac{N_A-N_B}{N_A}+2k_A\pi;\\&k_A\in Z\end{aligned}\right\}\quad (6a)$$

$$\delta\Phi_B=N_A\,\Omega\,\delta t_B=-2\pi\,\mathrm{sign}(\Omega_A-\Omega_B)\left(\frac{N_A}{N_B}-2\right)+2k_B\pi;\quad k_B\in Z$$

$$\left.\begin{aligned}\delta\Phi_B&=N_A\,\Omega\,\delta t_B\\&=-2\pi\,\mathrm{sign}(\Omega_A-\Omega_B)\left(\frac{N_A}{N_B}-1\right)+2k'_B\pi;\\&k'_B\in Z\end{aligned}\right\}\quad (6b)$$

with $k'_B=k_B+\mathrm{sign}(\Omega_A-\Omega_B)$, thus recovering the same relation for the argument as in the $N_B \leqq N_A \leqq 2N_B$ case.

## GENERALIZED RELATIONS

Noting that Eqs. (5) and Eqs. (6) are identical the periodicity relations may be generalized by reccurence for arbitrary $N_A/N_B$ ratio. Chosing the arbitrary integer constants so that $-\pi < \delta\Phi \leqq +\pi$, the phase-angles are computed by:

$$N_B \leqq N_A$$

$$\delta\Phi_A=\arg\{\exp[-2\pi i\,\mathrm{sign}(\Omega_A-\Omega_B)(1-(N_A/N_B)^{-1})]\}\quad (7a)$$

$$\delta\Phi_B=\arg\{\exp[-2\pi i\,\mathrm{sign}(\Omega_A-\Omega_B)((N_A/N_B)-1)]\}\quad (7b)$$

with $i=\sqrt{-1}$ the imaginary unit.

The above phase-angle relations are valid for arbitrary $N_A/N_B$ ratio and rotational velocities $\Omega_A$ and $\Omega_B$ and may be easily programmed on a computer (e.g. using the ATAN2 bivariate function in a FORTRAN program). It is reminded that $A$ is the blade-row with the larger number of blades.

The dependence of $\delta\Phi_A$ and $\delta\Phi_B$ on $N_A/N_B$ is schematically depicted in Figure 4. It is seen that for sign $(\Omega_A-\Omega_B)>0$ the blade $A_{i+1}$ lags the blade $A_i$ when $N_A/N_B<2$. For $N_A/N_B=2$ antiphase variation is observed, and for $N_A/N_B>2$ the blade $A_{i+1}$ leads the blade $A_i$, the phase lead tending to zero when $N_A/N_B\to\infty$. On the contrary $\delta\Phi_B$ follows a periodic variation with $N_A/N_B$.

The practical importance of Eqs. (7) is that they can be implemented, in conjunction with the well known Eqs. (2), (4), in a blade-row-interaction computer program for automatically determining the periodicity relations, for 2 arbitrarily rotating blade-rows and for arbitrary blade-number-ratios. The case of large $N_A/N_B$ is not academic, since it is systematically encountered in rotor/struts interaction, as

(1) If $A_2$ has a phase-lead $\delta\Phi$ with respect to $A_1$ then the alignment $A_2|\vdash B_2$ happens at $\delta t$ time-units before the alignment $A_1|\vdash B_1$.

(2) Taking into account that $\Theta_{ABS_{A_1}}(t)=\Theta_{ABS_{B_1}}(t)$.

well as in volute/impeller interaction where $N_B = 1$ (e.g. **Sideris** and **VandenBraembussche** [9]).

*Manuscript submitted November 7, 1990; accepted March 2, 1991.*

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Achevé d'imprimer à l'ONERA, février 1992 - Dépôt légal 1er trimestre 1992 - n° 920