Enhanced genetic algorithm-based fuzzy multiobjective strategy to multiproduct batch plant design

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A B S T R A C T

This paper addresses the problem of the optimal design of batch plants with imprecise demands in product amounts. The design of such plants necessary involves how equipment may be utilized, which means that plant scheduling and production must constitute a basic part of the design problem. Rather than resorting to a traditional probabilistic approach for modeling the imprecision on product demands, this work proposes an alternative treatment by using fuzzy concepts. The design problem is tackled by introducing a new approach based on a multiobjective genetic algorithm, combined wit the fuzzy set theory for computing the objectives as fuzzy quantities. The problem takes into account simultaneous maximization of the fuzzy net present value $N\tilde{P}V$ and of two other performance criteria, i.e. the production delay/advance and a flexibility index. The delay/advance objective is computed by comparing the fuzzy production time for the products to a given fuzzy time horizon, and the flexibility index represents the additional fuzzy production that the plant would be able to produce. The multiobjective optimization provides the Pareto's front which is a set of scenarios that are helpful for guiding the decision's maker in its final choices. About the solution procedure, a genetic algorithm was implemented since it is particularly well-suited to take into account the arithmetic of fuzzy numbers. Furthermore because a genetic algorithm is working on populations of potential solutions, this type of procedure is well adapted for multiobjective optimization.

1. Introduction

In recent years, there has been an increasing interest in the design of batch plants due to the growth of specialty chemical, biochemical, pharmaceutical and food industries. Batch processes have thus emerged as the preferred mode of operation for the low-volume synthesis of many high-value added products, mainly due to their flexibility in a market-driven environment.

More precisely, the problem of optimal design of a multiproduct batch chemical plant is defined by Papageorgaki and Reklaitis [1] in the following terms: determine a structure of workshop which makes it possible to ensure the production (capacity and a number of the equipment and storage tanks) to optimize some performance criteria, being given:

- The set of products, the specifications on their production and a horizon of time.
- The set of available equipment.
- Recipes for manufacturing each product including the relations of anteriority between operations and corresponding operating times.
- The availability of storages.

According to this definition the optimal design of multiproduct batch plants was formulated in the years 90s as a single-objective mixed-integer nonlinear programming (MINLP) problem (see Patel et al. [2], Montagna et al. [3]), where a techno-economic objective was optimized. However, in real industrial applications, engineers often need to make decisions when faced with competing objectives, related for example to environment, security, flexibility, etc. Indeed, the optimal design problem becomes today essentially a multiobjective one. Another supplementary difficulty is that at the design stage the problem data are not exactly known, as for example some costs and the future demand for products. From a practical point of view, the design occurring at a preliminary stage where the historical data on uncertainties is not yet made up, a probabilistic approach of the problem seems unrealistic, and an efficient way to tackle the problem is to resort to the fuzzy set theory. So this article deals with multiobjective design of batch plants according to three objective functions related to economics

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Nomenclature

A_p	depreciation ($)
AV	annual volume
a	tax rate
B_j	batch j
$Cost$	investment cost ($)
$\tilde{D}_p$	operation cost ($)
d_j	term used for computing the processing time for the batch j
f	working capital ($)
g_{ij}	term used for computing the processing time of product i in the batch j
$\tilde{H}$	time horizon (h)
$\tilde{H}_i$	production time for product i (h)
I	number of products
J	number of batch steps
K	number of semi-continuous steps
$MINLP$	mixed-integer nonlinear programming
$MOOP$	multiobjective optimization problem
m_j	number of parallel items for batch j
$N\tilde{P}V$	net present value ($)
n	number of periods
n_k	number of parallel items of semi-continuous in step k
p_{ij}	constant term used in the computation of the processing time of product i in the batch j (h)
$\tilde{Q}_i$	demand in product i (kg)
$\tilde{Q}_{new}$	new production (kg)
$\tilde{Q}_{initial}$	initial production (kg)
r	discount rate
S	number of sub-processes
SC_i	semi-continuous operation i
SQP	successive quadratic programming
s_{ij}	size factor (l/kg)
$TrFN$	trapezoidal fuzzy number
t_{ij}	processing time of product i in the batch j (h)
V	volume (l)
V_j	volume of batch j (l)
$\tilde{V}_p$	revenue ($)
ω	penalty factor

(the net present value), the respect of due dates (advances/delays) and a flexibility criterion. The innovative approach presented in this paper combines a multiobjective genetic algorithm with fuzzy arithmetic for computing the three objective functions above mentioned. The fuzzy net present value $N\tilde{P}V$ is calculated from fuzzy revenue $\tilde{V}_p$ and fuzzy depreciation $\tilde{D}_p$, the advance/delay criterion is given by the common surface between the time horizon $\tilde{H}$ represented by a rectangular fuzzy number and the fuzzy trapezoidal production time $\tilde{H}_i$ for product i, and the flexibility, in the case of an advance (respectively a delay), represents the additional production (the demand not satisfied) that the batch plant is able to produce.

The article is organized as follows. Section 2 presents the literature analysis; Section 3 is devoted to process description and problem formulation. Section 4 presents a brief overview of fuzzy set theory; the multiobjective genetic algorithm is briefly described in the following part. The paper is then illustrated by some typical results presented in Section 6. Finally, the general conclusions on this work are drawn in the last part.

2. Previous works

2.1. Uncertainties

The most common form of batch plant design formulation considered in the literature is a deterministic one, in which fixed production requirements of each product must be fulfilled. However, because of the constant evolution of the environment and complexity of the needs, the specifications of the production system are often imperfectly known (Shah and Pantelides [4]). The initial data of design present a vague, dubious character and even vagueness. Even if an optimal process design is performed, it cannot give satisfactory results if the data available for the design phase are erroneous or too vague. Thus, it is often impossible to obtain precise information on the future demand for products, at the step of design of a discontinuous workshop. Nevertheless, decisions must be taken on the plant capacity. This capacity should be able to balance the product demand satisfaction and extra plant capacity in order to reduce the loss on the excessive investment cost or on market share due to the varying product demands (Cao and Yuan [5]). Consequently, the mission of the designer, assisted by traditional tools, may prove to be hazardous and makes essential the resort to a more robust approach.

The literature presents three basic approaches to the problem of design under uncertainty (Vajda [6]). They can be classified within the wait-and-see approach, the two-stage formulation and the probabilistic model.

In the wait-and-see formulation, a separate optimal design is found for each realization of the set of uncertain parameters. The cost of the plant is then calculated as the expected value of the separate designs. The practical difficulties with this model are that it is generally difficult to identify the design which yields a value of the plant cost which is equal to the expected value, and even if such a design can be identified, there is usually no direct way establishing to what extent this design will accommodate other values of the uncertain parameters.

In the two-stage formulation, also called the "here-and-now" model, the design variables are selected "here-and-now" so as to accommodate any future uncertain parameter realizations or perhaps those which fall within some specified confidence limits. The equipment sizes are determined at the first step or design stage and the effect of the uncertain parameters on system performances is established in the second or operating stage. The second stage is, of course, the most important part of model since this is the stage at which the flexibility of the design is checked, by including considerations of variations of the operating variables to accommodate the uncertain parameter realizations. Since this part of the model is also the most computationally demanding, researchers have sought to reduce the computational burden by proposing various alternatives fully solving the second-stage problem (Wellons and Reklaitais [7]). Two-stage stochastic programming approaches has also been applied in several works (see Ierapetritou and Pistikopolous [8]; Harding and Floudas [9]; Petkov and Maranas [10]; Cao and Yuan [5]).

The more traditional way would consist in using probabilistic approaches representing the imprecision of demand by probability distributions, considered as independent. In this approach, also called the chance constrained model, a probability of constraint satisfaction must be specified by the designer. As shown by Charnes and Cooper [11], if normal distributions are assumed, if the uncertain parameters linearly occur in the constraints and if the constraints can be considered independent, then the probabilistic constraints can be reduced to a deterministic form and thus the model converts to an ordinary deterministic optimization problem. But, these simplifying assumptions do not represent reality since many parameters are, in practice, dependant from/to

each other and cannot follow laws of symmetrical distributions. Moreover, the design of workshop occurs at a preliminary stage where the historical data base on the demand is not yet made up, leading to practical difficulties for identifying probabilistic laws.

The techniques based on the theory of probability make it possible to quantify only uncertainty (the probability of guaranteeing an annual volume of *AV* units for a given demand in product *A* being equal to *P*). Actually information is also vague (the annual volume of demand in product *A* lies in the range [*N*1, *N*2]). So in the problem formulation, many data are vague, and cannot be quantified by classical arithmetic. The well fitted formalism, to handle this type of vague and imprecise data, is the theory of the fuzzy subsets. In particular, the theory of the possibilities (Zadeh [12], Dubois and Prade [13]) is particularly well adapted to the treatment of subjective information. Thus, in the design phase, the requests can be characterized by functions of membership expressing the designer perception of the imprecision (Jacqmart and Gien [14]).

2.2. Multobjective optimization

In conventional optimal design of a multiproduct batch chemical plant, the production requirements for each product and total production time for all products must be specified. The number, required volume and size of parallel equipment units in each stage are to be determined in order to minimize the investment. Such an approach formulates the optimal design problem as a single-objective mixed-integer nonlinear programming (MINLP) problem (Grossmann and Sargent [15]; Patel et al. [2], Montagna et al. [3]). However, in real world applications, the chemical engineers often need to make decisions when faced with competing objectives (Yao and Yuan [16], Collette and Siarry [17]). Using the formulation of multiobjective constrained problems of Fonseca and Fleming [18], a general multiobjective problem consists of a set f of n criteria f_k, $k = 1, \ldots, n$ to be minimized or maximized. Each f_k may be nonlinear, but also discontinuous with respect to some components of the general decision variable x in an m-dimensional universe U:

$$f(x) = (f_1(x), \ldots, f_n(x)) \tag{1}$$

This kind of problem has not a unique solution in general, but presents a set of non-dominated solutions named Pareto-optimal set or Pareto-optimal front. The Pareto-domination concept lies on the basic rule: in the universe U a given vector $u = (u_1, \ldots, u_n)$ dominates another vector $v = (v_1, \ldots, v_n)$, if and only if,

$$\forall i \in \{1, \ldots, n\} : u_i \leq v_i \wedge \exists i \in \{1, \ldots, n\} : u_i < v_i \tag{2}$$

For a concrete mathematical problem, Eq. (2) gives the following definition of the Pareto front: for a set of n criteria: a solution $f(x)$, related to a decision variable vector $x = (x_1, \ldots, x_m)$, dominates an another solution $f(y)$, related to $y = (y, \ldots, y_m)$ when the following condition is checked (for a minimization problem):

$$\forall i \in \{1, \ldots, n\} : f_i(x) \leq f_i(y) \wedge \exists i \in \{1, \ldots, n\} : f_i(x) < f_i(y) \tag{3}$$

The last definition concerns the Pareto optimality: a solution $x_u \in U$ is called Pareto-optimal if and only if there is no $x_v \in U$ for which $v = f(x_v) = (v_1, \ldots, v_n)$ dominates $u = f(u_v) = (u_1, \ldots, u_n)$. These Pareto-optimal non-dominated individuals represent the solutions of the multiobjective problem. In practice, the decision maker has to select a single solution by searching among the whole Pareto front, and it may be difficult to pick one "best" solution out of a large set of alternatives. Branke et al. [19], and Taboada and Coit [20] suggest to pick the knees in the Pareto front, that is to say, solutions where a small improvement in one objective function would lead to a large deterioration in at least one other objectives.

2.3. Metaheuristic multiobjective optimization

A metaheuristic is a heuristic method for solving a large class of combinatorial problems by combining user-given black-box procedures whose derivatives are not available, with heuristics in the hope of obtaining a good solution for the problem. Some metaheuristics maintain at any instant a single current state, and replace that state by a new one (state transition or move). Some metaheuristics work on pool of states containing several candidate states. The new states are generated by combination or crossover of two or more states of the pool. Since 1975, many metaheuristics appear: genetic algorithms (Holland [21]), simulated annealing (Kirkpatrick et al. [22]), artificial immune systems (Farmer et al. [23]), ant colonies (Dorigo [24]), particle swarm (Kennedy and Eberhart [25]), artificial bee colonies (Nakrani and Tovey [26]).

All the above algorithms can be adapted to the multiobjective case, but the two most popular in the chemical engineering field are MOGA (multiobjective genetic algorithm, see Konac et al. [27]) and MOSA (multiobjective simulated annealing, see Shu et al. [28], Smith et al. [29], Bandyopadhyay et al. [30]). None of these two methods is perfect and selecting one depends on the requirements of the particular design situation under consideration. From the literature survey (Van Veldhuizen and Lamont [31], Branke et al. [19], Turinsky et al. [32], Mansouri et al. [33]) it appears that MOGA is generally preferred to MOSA. Indeed, the main advantage of genetic algorithms over other methods is that a GA manipulates a population of individuals. It is therefore tempting to develop a strategy in which the population captures the whole Pareto front in one single optimization run.

2.4. Basic principles of genetic algorithms (GA)

The choice of a GA as the solving procedure for multiobjective optimization problems is all the more interesting as it provides a set of compromise solutions (not dominated solutions, i.e. Pareto front), by opposition to classical optimization techniques (deterministic like SQP – successive quadratic programming – or stochastic like simulated annealing) which give only one solution. This property is a paramount advantage for using a genetic algorithm. Genetic algorithms are mathematical optimization techniques that simulate a natural evolution process. They are based on Darwinian Theory, in which the fittest species survive and propagate while the less adapted tend to disappear. The search procedure consists in maintaining a population of potential solutions while conducting a parallel investigation for non-dominated solutions. Three main steps exist in a genetic algorithm: crossover, mutation, and selection. Many variants for crossover operator are proposed in the literature, but the common principle is to combine two chromosomes to generate next-generation chromosomes, by a simple gene exchange with, or not, small variations. Mutation randomly changes the gene's values to generate a new combination of genes for the next generation. Mathematically, the main interest of mutation consists in jumping out of local optimal solutions. Selection is the last step where the best chromosome solutions are copied in the next generation.

2.5. Genetic algorithms and fuzzy multiobjective optimization

Among the most widespread multiobjective optimization methods based on genetic algorithms, the following ones can be mentioned: VEGA (genetic vector evaluated algorithm, Schaffer [34]), NPGA (niched Pareto genetic algorithm, Horn et al. [35]), MOGA (multiple objectives genetic algorithm, Fonseca and Fleming [36]) and SPEA (strength Pareto evolutionary algorithm, Zitzler and Thiele [37]). Comparative studies on multiobjective genetic algorithms can be found in Coello Coello [38,39].

In the last decade, only a little number of papers dealing with the fuzzy set theory combined with multiobjective GA for solving engineering problems was published. See for example the books of Sakawa [40,41] concerning process scheduling and operation planning, the paper of Fayad and Petrovic [42] related to scheduling in a printing company, and the articles of Yang and Sun [43] dealing with water management in a river basin and Huang and Wang [44] devoted to the design of multipurpose batch plants.

Amongst the above references, only the paper of Huang and Wang [44] deals with a subject nearby close by the one presented in this article—the fuzzy decision-making design of a chemical plant, but a brief comparison shows that the problems are different. The authors perform a multiobjective optimization (maximization of the revenue and minimization of the investment cost, operation cost and total production time). The fuzzy aggregation functions presented by Sakawa [40] are implemented. A membership function is used to define the degree of satisfaction of each objective function, so that the problem is converted to a highly nonlinear MINLP one. The problem is solved by using a mixed-integer hybrid differential evolution procedure, which belongs to the class of genetic algorithms. In the article presented here, the uncertainties lie on demand and fuzzy sets are implemented for maximizing the net present value together two other performance objectives related to delay/advance and flexibility.

Instead of adapting one of these above procedures, a specific algorithm, based on the previous works of Dietz [45] and Aguilar et al. [46] is used in this study to simultaneously maximize the net present value $N\tilde{P}V$ and two other performance indexes, i.e. the production delay/advance and a flexibility criterion.

3. Problem formulation

3.1. Problem statement

In real world applications, the chemical engineers often need to make decisions when faced with competing objectives. The designers must not only satisfy various techno-economic criteria, but also respect some due dates. In this framework, this study introduces a new design approach to maximize the net present value and two other performance criteria, i.e. the production delay/advance and a flexibility criterion detailed below. Such the optimal design problem falls into the class of multiobjective optimization problems (MOOP).

A significant level of difficulty lies in the fact that in order to specify the production requirements for each product and total production time for all the products, it is almost impossible to obtain some precise information. Indeed, the ability of batch plants to deal with irregular product demand patterns reflecting market uncertainties or seasonal fluctuations is one of the main reasons for the recently renewed interest in batch operations. So the problem to solve is of MOOP type, where some part of the objectives and constraints are imperfectly known.

3.2. Assumptions

The model formulation for batch plant design problems adopted in this paper is based on the Modi's approach (Modi and Karimi [47]). It considers not only treatment in batch stages, which usually appears in all types of formulation, but also represents semi-continuous units that are part of the whole process (pumps, heat exchangers, etc.). A semi-continuous unit is defined as a continuous unit alternating idle times and normal activity periods.

Besides, this formulation takes into account mid-term intermediate storage tanks. They are just used to divide the whole process into sub-processes, in order to store an amount of materials corresponding to the difference between sub-process productivity. This representation mode confers to the plant a better flexibility: it prevents the whole process production from being paralysed by one limiting stage. So, a batch plant is finally represented by series of batch stages, semi-continuous stages and storage tanks.

The modeling process is based on the following assumptions:

(i) The devices used in a same production line cannot be used twice by one same batch.
(ii) The production is achieved through a series of single product campaigns.
(iii) The units of the same batch or semi-continuous stage have the same type and size.
(iv) There is no limitation for utilities.
(v) The cleaning time of the batch items is included into the processing time.
(vi) The item sizes are continuous bounded variables.

3.3. Formulation of objectives

The model considers the synthesis of I products treated in J batch stages and K semi-continuous stages. Each batch stage consists of m_j out-of-phase parallel items with same size V_j. Each semi-continuous stage consists of n_k out-of-phase parallel items with same processing rate R_k (i.e. treatment capacity, measured in volume unit per time unit). The item sizes (continuous variables) and equipment numbers per stage (discrete variables) are bounded. The $S-1$ storage tanks, divide the whole process into S sub-processes.

3.3.1. Economic objective function

In the following of the paper symbols surmounted by a tilde ($\sim$) represent fuzzy terms. The net present value method ($N\tilde{P}V$) of evaluating a major project allows to consider the time value of money. Essentially, it helps to find the present value in "today's value money" of the future net cash flow of a project. Then, this amount can be compared with the amount of money needed to implement the project. When using the formula below, the values of the number of periods (n), discount rate (r) and tax rate (a) take respectively the following classical values 5, 10(and 0 (computation before tax). In order to calculate investment cost ($Cost$), the working capital (f), revenue ($\tilde{V}_P$), operation cost ($\tilde{D}_P$) and depreciation (A_P) are introduced.

$$Max(N\tilde{P}V) = -Cost - f + \sum_{p=1}^{n} \frac{(\tilde{V}_p - \tilde{D}_p - A_p)(1-a) + A_p}{(1+r)^n} \tag{4}$$

$$Cost = \sum_{j=1}^{J}(m_j a_j V^{\alpha_j}) + \sum_{k=1}^{K}(n_k b_k R_k^{\beta_k}) + \sum_{s=1}^{S}(c_s V_s^{\gamma_s}) \tag{5}$$

3.3.2. Advance/delay objective function

This criterion translates the delays and advances for the production time necessary for the synthesis of all the products: for this purpose, the time horizon $\tilde{H}$ represented by a fuzzy quantity has to be compared with the production time $\tilde{H}_i$ (see below). For the comparison of fuzzy numbers, the Liou and Wang's method (Liou and Wang [48]) was adopted.

The criterion relative to the advances or to the delays is calculated by the formulas 6 and 7, respectively. The corresponding mathematical expressions of the objective functions are proposed as follows, where the term "common surface" noted x,

is defined below:

$$Max\,(Criterion\,o\,f\,ad) = x \times \varpi \tag{6}$$

$$Max\,(Criterion\,o\,f\,delays) = \frac{x}{\varpi} \tag{7}$$

The penalty term ω is defined in order to penalize more delays than advances. A sensitivity analysis leads to adopt a value of 3 for ω.

3.3.3. Flexibility index objective function

Finally, an additional criterion was computed in the case of an advance (respectively a delay), representing the additional production (the demand not satisfied) that the batch plant is able to produce. Without going further in the detailed presentation of the computation procedure, it can be simply said that this flexibility index is computed by dividing the potential capacity of the plant by its actual value.

3.3.4. Constraint formulation

The problem statement involves three forms of different constraints as reported in the literature (Modi and Karimi [47]):

(i) Dimension constraints: every unit has to be restricted into its allowable range.
(ii) Time constraint: the summation of available production time for all the products is less than to the total production time.
(iii) Productivity constraint: the global productivity for product i (on the whole process) is equal to the lowest local productivity (of each sub-process).

4. Fuzzy computations

4.1. Representation of fuzzy numbers

The proposed approach involves arithmetic operations on fuzzy numbers and quantifies the imprecision of the demand by means of fuzzy sets (trapezoidal). In this case, the flat line over the interval (q_2, q_3) represents the precise demand with an interval of confidence at level $\alpha = 1$, while the intervals (q_1, q_2) and (q_3, q_4) represent the "more or less possible values" of the demand, where $0 < \alpha < 1$ (see Fig. 1). For example, the net present value ($N\tilde{P}V$) is shown in Fig. 2. For each product i, the production time $\tilde{H}_i$ is also represented by a TrFN. In order to carry out fuzzy number comparisons by using the Liou and Wang's method (Liou and Wang [48]), the given time horizon is represented by a rectangular fuzzy number (see Fig. 3).

4.2. Computation of advance/delay objective function

The production time necessary to satisfy each product demand must be less than the given time horizon, but due to the nature of the fuzzy numbers, eight different cases for determination of the advance/delay criterion may occur. These different cases are reported in Fig. 4.

The advance/delay objective function is computed according to the "common surface", representing the intersection between the sum of the production times (trapezoid) and the horizon of time to respect (rectangle). The calculation of the criterion depends on each case: for example, case 1 illustrates the solutions which arrive just in time. The criterion relative to the advances (2, 4, 6 and 8) or to the delays (3, 5 and 7) is calculated by the formulas 6 and 7 given before.

4.3. Computation of the flexibility index objective function

This flexibility index represents the gain (respectively the loss) in production (during the time interval Δt shown in Fig. 5, respectively Fig. 6) in the case of an advance (respectively delay). The new production $\tilde{Q}_{new}$ is computed according to Δt by summing for each product its production computed on this new time interval and compared with the initial production $\tilde{Q}_{initial}$. In fact, the flexibility index objective function is given by the ratio ($\geq$1 in the

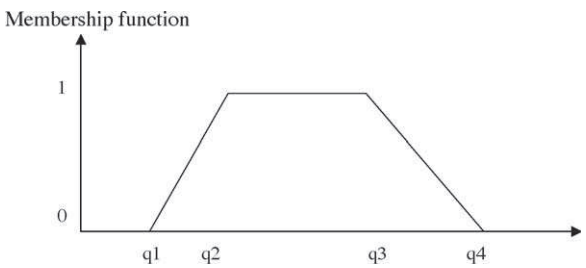

Fig. 1. Trapezoidal fuzzy number TrFN Q $\tilde{Q} = (q1, q2, q3, q4)$.

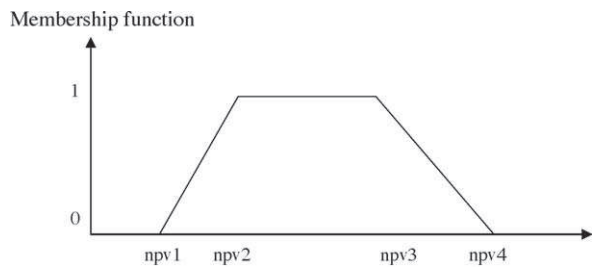

Fig. 2. Fuzzy NPV $N\tilde{P}V = (npv1, npv2, npv3, npv4)$.

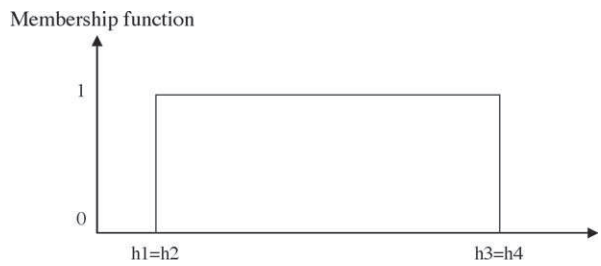

Fig. 3. Rectangular time horizon.

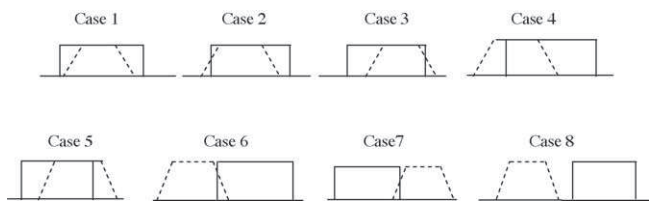

Fig. 4. Eight cases for the advances/delays.

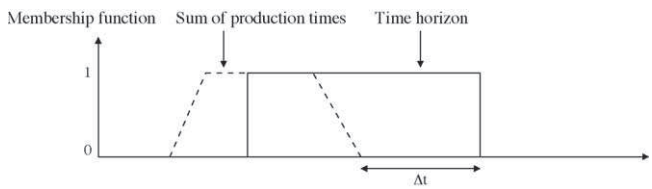

Fig. 5. Computation of the flexibility index for an advance case.

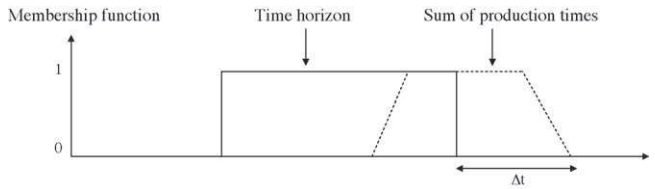

Fig. 6. Computation of the flexibility index for a delay case.

case of an advance and ≤1 in the case of a delay):

$$Flexibilityindex = \frac{Q_{new}}{Q_{initial}} \tag{8}$$

where the terms Q_{new} and $Q_{initial}$ represent the defuzzified values of $\tilde{Q}_{new}$ and $\tilde{Q}_{initial}$ (computed according to the method of the centre of gravity).

5. Solving the MOOP

5.1. Encoding of solutions

The encoding of potential solutions in the form of a numerical chromosome is a fundament al step for using a GA, insofar as it guides the scanning of the solution set. The solution encoding was carried out by dividing the chromosome, i.e. the complete set of code, into two parts. The first one deals with the item volumes, which are continuous in the initial formulation. Nevertheless, on the market the equipment volumes must fall in standard discrete values, and consequently they were discretized here with a 50 unit step within the range defined by their upper and lower bounds. The second part of the chromosome corresponds to the number of equipment items per stage.

5.2. Generation of the initial population

The procedure for creating the initial population corresponds to a random sampling of each decision variable within its specific range of variation. This strategy guarantees a population various enough to cover large zones of the search space.

5.3. Fitness evaluation

The optimization criterion considered for fitness evaluation involves the net present value $N\tilde{P}V$ and two other performance criteria, i.e. the production delay/advance and the flexibility criterion. Traditionally, a GA uses a fitness function, which must be maximized. The fitness for these criteria is equal to their calculated values (the fuzzy $N\tilde{P}V$ is defuzzified).

5.4. Selection of survivals

The multiobjective aspects are taken into account during the selection procedure, inspired of the work of Dietz [45]. On the current population a first selection is performed by implementing the classical Goldberg's wheel for each criterion. The method of Liou and Wang [48] is used to compare the objectives. Then a hybrid selection based on Pareto rank-tournament was proposed and showed a better performance than the classical Goldberg's wheel, systematically leading to a higher number of not dominated solutions. The procedure is detailed in Aguilar [49].

Table 1
Parameters of the GA.

Population size	200
Number of generations (stopping criterion)	400
Crossover probability	0.40
Mutation probability	0.30
Elitism	The best individuals of each generation

5.5. Crossover

Two randomly selected parents are submitted to the crossover operator to produce two children. The crossover is carried out with an assigned probability, which is generally rather high. If a randomly generated number is superior to the probability, the crossover is performed. Otherwise, the children are copies of the parents. The crossover operator is a classical one-point crossover.

5.6. Mutation

The genetic mutation introduces diversity in the population by an occasional random replacement of some individuals. Like for the crossover, the mutation is performed on the basis of an assigned probability, generally less than the probability of crossover. A random number is used to determine if a new individual will be produced to substitute the one generated by crossover. The mutation procedure consists in replacing one of the decision variable values of the chosen individual, while keeping the remaining variables unchanged. The replaced variable is randomly chosen, and its new value is calculated by randomly sampling within its specific range.

5.7. Elitism

In order to preserve the best individuals of the current generation, a single elitism procedure is carried out by systematically copying the best individual according to each objective function in the next generation.

5.8. Generation of the Pareto's front

A Pareto's sort procedure is carried out at the end of the algorithm over all the evaluated solutions during the procedure, so the whole set of the not dominated Pareto's optimal solutions, is obtained.

5.9. Parameters of the procedure

The parameters of the GA are summarized in Table 1 and Fig. 7 shows the main steps of the procedure.

6. Illustrative example

6.1. Example definition

The example chosen to illustrate the approach fuzzy-multi-objective optimization was initially presented by Ponsich et al. [50]: the plant, divided into two sub-processes, consists of six batch stages B_i and eight semi-continuous processes SC_j to manufacture three products A, B and C. So it comes $I = 3$, $J = 6$, $K = 8$ and $S = 2$. The storage tank is assumed large enough to be considered as infinite, it will not be studied here. The first sub-process is composed by the sequence [SC_1, B_1, SC_2, B_2, SC_3] and the second one by the sequence [SC_4, B_3, SC_5, B_4, SC_6, B_5, SC_7, B_6, SC_8]. In the table of results (see Table 4) equipments are numbered from 1 (1 corresponds to SC_1) to 14 (14 corresponds to SC_8).

Table 2
Data for the products.

Product	Term	B_1	B_2	B_3	B_4	B_5	B_6
1	s_{ij}	8.28	6.92	9.70	2.95	6.57	10.60
2		5.58	8.03	8.09	3.27	6.17	6.57
3		2.34	9.19	10.30	5.70	5.98	3.14
1	p_{ij}	1.15	3.98	9.86	5.28	1.20	3.57
2		5.95	7.52	7.01	7.00	1.08	5.78
3		3.96	5.07	6.01	5.13	0.66	4.37
1	g_{ij}	0.20	0.36	0.24	0.40	0.50	0.40
2		0.15	0.50	0.35	0.70	0.42	0.38
3		0.34	0.64	0.50	0.85	0.30	0.22
	d_j	0.40	0.29	0.33	0.30	0.20	0.35

Table 3
Economic data.

Product	Unit selling price ($/kg)	Unit operating cost ($/kg)
1	0.70	0.08
2	0.74	0.10
3	0.84	0.07

From Table 2, the quantity of product in a batch is calculated thanks to the size factor s_{ij} [l kg^{-1}] representing the volume of batch j occupied per unit of mass of product i. For the storage tank, a size factor of 1 is assumed for all the products. The terms p_{ij}, g_{ij} and d_i are used to compute the processing time t_{ij} (h) of product i in batch j according to the following equation:

$$t_{ij} = p_{ij} + g_{ij} s_{ij}^{d_j} \tag{9}$$

The economic data are reported in Table 3. For all the semi-continuous processes it is assumed that the cost ($) is given by $250V^{0.6}$ (where the volume V is expressed in liters). All the other computations are given in Aguilar [49]. For all the equipments (batch or semi-continuous) the minimum size is 250 l and the maximum one is 10 000 l.

6.2. Constructing fuzzy data

Starting from the crisp values used by Ponsich et al. [50] for the demands on products (in kg) $Q_1 = 437\,000$, $Q_2 = 324\,000$, $Q_3 = 258\,000$ and for the time horizon $H = 6000$ h, the data were arbitrarily fuzzified into TrFN for the demands and rectangular form for the time horizon as indicated in Fig. 8. The following fuzzy numbers are deduced:

$$\tilde{Q}_1 = [419\,520, 428\,260, 441\,370, 454\,480]$$

$$\tilde{Q}_2 = [311\,040, 319\,140, 330\,480, 336\,960]$$

$$\tilde{Q}_3 = [247\,680, 258\,000, 263\,160, 268\,320]$$

$$\tilde{H} = [5760, 5760, 6240, 6240]$$

6.3. Optimization results

A mono-objective optimization of the fuzzy net present value is first performed, in order to study the dissipation of the (defuzzified) $N\tilde{P}V$ when other objectives like advance/delay or efficiency index are also simultaneously optimized.

6.3.1. Mono-objective optimization of $N\tilde{P}V$

In this mono-optimization case, at each generation the best individuals, that are the surviving ones, are chosen according to their fitness which is directly the $N\tilde{P}V$. Since these fitness values are represented by fuzzy numbers, they were defuzzified before performing the selection.

GA typical results are presented in Table 4 (taking into account the stochastic nature of the procedure, 10 runs of the GA were performed). In each run, the value of the best individual of each generation and the average value of the objective function computed on each generation take a traditional form of regular increase, to stabilize itself at the end of the research. In Table 4,

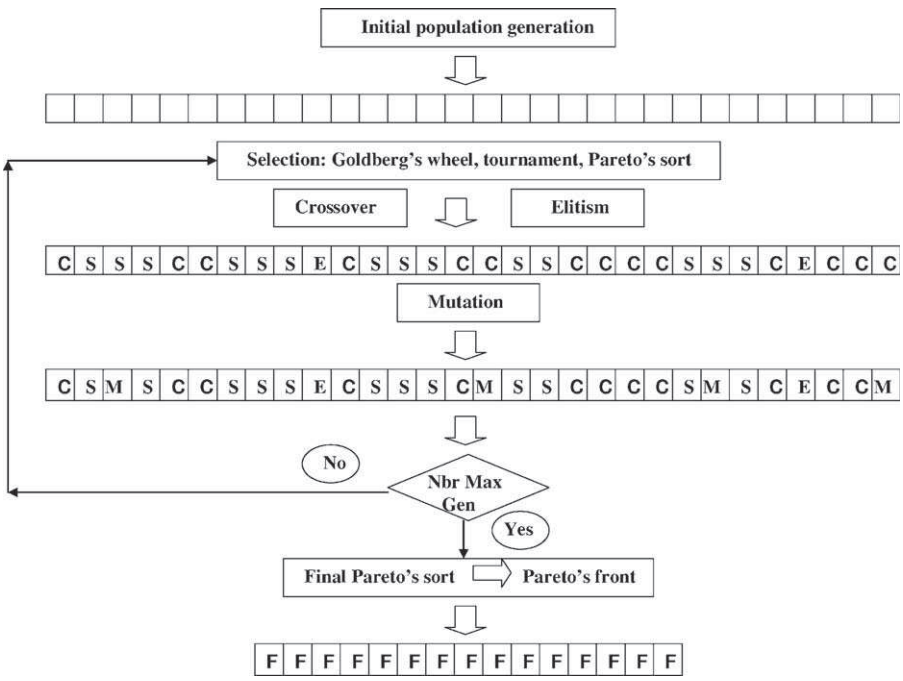

Fig. 7. Multiobjective genetic algorithm.

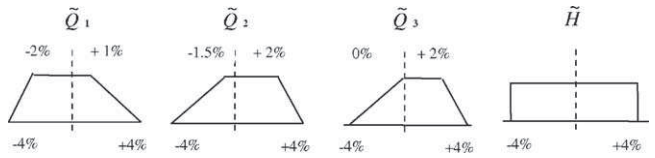

Fig. 8. Fuzzy representation of product demands and of time horizon.

Table 4
Optimal values of volumes and number of parallel units for each operation.

V_1	V_2	V_3	V_4	V_5	V_6	V_7
8042.2	9787.5	9267.8	5128.9	7068.2	9999.0	301.0
n_1	n_2	n_3	n_4	n_5	n_6	n_7
2	2	3	2	1	2	1
V_8	V_9	V_{10}	V_{11}	V_{12}	V_{13}	V_{14}
3210.0	427.1	495.0	1592.0	4181.0	886.8	1271.0
n_8	n_9	n_{10}	n_{11}	n_{12}	n_{13}	n_{14}
1	1	1	1	1	1	1

Table 5
Number and percentage of not dominated solutions obtained for each case.

	Case							
	1	2	3	4	5	6	7	8
Not dominated solutions	0	12	145	0	69	0	12	0
%	0	5.0	60.9	0	29.0	0	5.0	0

only the results (volume of each operation and number of parallel items constituting the operation) corresponding to the best defuzzified value of $N\tilde{P}V$ are reported. For example, V_1 corresponds to the total volume of the semi-continuous operation SC_1, which involves two parallel and identical units. The optimal value of the $N\tilde{P}V$ (\$) is the TrFN [740 641, 804 244, 921 524, 989 552] and its average (defuzzified) value is 863 990. The sum of production times for the three products is given by $\sum_{i=1}^{3} \tilde{H}_i = [5760, 5925, 6097, 6240]$, and its average value is 6005. Let us note that when optimization is performed with the crisp values of demands and horizon time, the solutions obtained are near these average values.

6.3.2. Bicriteria optimization $N\tilde{P}V$ – advances/delays

This first bicriteria analysis concerns the simultaneous optimization of $N\tilde{P}V$ and the objective which represents the advances or delays of the time horizon. Three tests were made with the GA and the algorithm did not find any solution belonging to case 1, because the rectangle representing the horizon of time to respect is smaller than the trapezoids obtained by the sum of times of production for the three products. In Table 5, the 238 not dominated individuals obtained and the results for the various cases are presented.

The analysis is only performed on solutions having on the one hand, the larger common surface (corresponding to an advance of case 2—see Fig. 4) and, on the other hand, the best $N\tilde{P}V$. Table 6 shows the results of this bicriteria optimization, where it can be

Table 6
Bicriteria optimization $N\tilde{P}V$ – advances/delays for case 2 (advance).

	$N\tilde{P}V$ (mean value)	Common surface	$N\tilde{P}V$ (\$) and production time $\sum \tilde{H}_i$ (h)
Case 2	860 358	653	$N\tilde{P}V = [736\,120, 817\,367, 906\,144, 981\,801]$ $\sum \tilde{H}_i = [5758, 5916, 6089, 6238]$

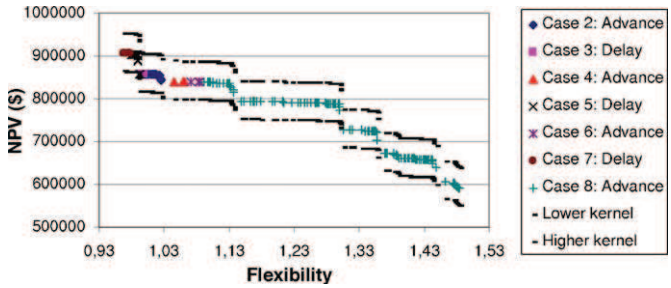

Fig. 9. Bicriteria optimization $N\tilde{P}V$ – flexibility index.

Table 7
Number and percentage of not dominated solutions obtained for each case.

	Case							
	1	2	3	4	5	6	7	8
Not dominated solutions	0	28	18	4	29	5	9	184
%	0	10	6.5	1.4	10.5	1.8	3.2	66.4

noted that the mean value of $N\tilde{P}V$ decreases compared with the mono-objective case.

6.3.3. Bicriteria optimization $N\tilde{P}V$ – flexibility index

The second bicriteria analysis takes into account $N\tilde{P}V$ and the criterion which represents the flexibility index of the configuration chosen to produce a possible additional demand. Fig. 9 and Table 7 Exhibit 277 not dominated solutions of the advance cases (2, 4, 6 and 8) and of the delay cases (3, 5 and 7).

Two solutions of case 2 (the first one is the solution with the best $N\tilde{P}V$ and the second configuration has the best index of flexibility) are presented in Table 8. As in the previous case, the mean value of $N\tilde{P}V$ decreases compared with the mono-objective case. From Table 8, it can also be observed that the mean value of $N\tilde{P}V$ decreases when the flexibility index increases. However, for the second value of flexibility index (1.020), if the value of extra NPV (12 383) is added to the mean value of $N\tilde{P}V$ (854 711), this new mean value (867 094) is greater than the mean value (863 990) obtained in the mono-objective study. This result shows the relevance of carrying out the bicriteria optimization of $N\tilde{P}V$ – flexibility index.

6.3.4. Tricriteria optimization $N\tilde{P}V$ – advances/delays – flexibility index

Finally, the fuzzy optimal design of batch plant takes simultaneously into account the three criteria, i.e. $N\tilde{P}V$, advances or delays (common surface) and index of flexibility. The method proposes a sufficiently large range of compromise solutions making it possible to the decision's maker to tackle the problem of the final choice, with relevant information. Fig. 10 displays the results (5881 not dominated solutions) obtained after three runs of the GA on a three-dimensional curve. In Table 9 it can be observed

Table 8
Bicriteria optimization $N\tilde{P}V$ – flexibility index for case 2 (advance).

	$N\tilde{P}V$ (mean value)	Flexibility index	$N\tilde{P}V$ (\$) and production time $\sum \tilde{H}_i$ (h)
Case 2	857 085	1.005	$N\tilde{P}V = [732\,860, 814\,104, 902\,868, 978\,510]$ $\sum \tilde{H}_i = [5728, 5886, 6058, 6206]$ Extra NPV = 3110 \$
Case 2	854 711	1.020	$N\tilde{P}V = [730\,352, 811\,691, 900\,550, 976\,256]$ $\sum \tilde{H}_i = [5637, 5792, 5961, 6107]$ Extra NPV = 12 383 \$

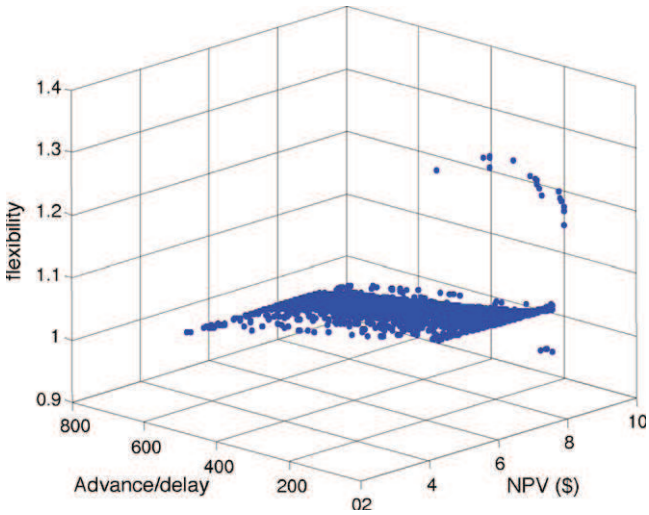

Fig. 10. Tricriteria optimization.

Table 9
Number and percentage of not dominated solutions obtained for each case for the tricriteria optimization.

	Case							
	1	2	3	4	5	6	7	8
Not dominated solutions	0	2467	0	2527	0	868	0	19
%	0	41.9		43.0	0	14.7	0	0.3

that no solution corresponding to a delay case (cases 3, 5 and 7) was obtained by the three GA's.

To analyze the results obtained from the tricriteria optimization, six not dominated solutions are selected: three of case 2, two of case 4 and one of case 6 (see Table 10). These solutions were selected by taking into account the values of the net present value and the index of flexibility, giving a possibility of obtaining an additional benefit. Like in the bicriteria study, for the higher value of the flexibility index (1.066 for case 6) if the value of extra *NPV* (34 400) is added to the mean value of $N\tilde{P}V$ (830 164), this new mean value (864 564) is greater than the mean value (863 990) obtained in the mono-objective study. Compared with the bicriteria case ($N\tilde{P}V$ – flexibility index), the gain in the mean $N\tilde{P}V$ value is lower, but the advance/delay objective is more satisfied.

7. Conclusions

In conventional design of multiproduct batch chemical plants, the designers have to specify the production requirement of each product and the total production time. However, at the step of preliminary design no precise product demand predictions and total horizon time are generally known. In most cases, these data are imprecisely defined. For this reason, an efficient treatment of the imprecision by using fuzzy concepts is introduced in this paper.

In real world applications, designers not only search for minimizing the investment cost, but have also to perform an economic study based on the computation of the net present value (*NPV*) of a project (considering investment operating cost and revenue). In addition, other objectives like the production delays/advances and flexibility measurement of the future plant have to be considered together with the *NPV*.

This multiobjective optimization problem with imprecise data is tackled in this study, by defining a multiobjective genetic algorithm able to handle imprecise values represented in the form of fuzzy numbers (trapezoidal or rectangular). The study is illustrated by an example coming from the literature.

First a mono-objective of the fuzzy *NPV* ($N\tilde{P}V$) is performed for defining a basis of comparison. Then bicriteria optimizations of $N\tilde{P}V$ and advances/delays of products and $N\tilde{P}V$ and a flexibility index of the plant representing the possible additional production are carried out. Finally a tricriteria optimization including the three objectives brings still further information than in the bicriteria case, insofar as a sufficiently broad set of compromise solutions are proposed. This set of relevant solutions will be helpful for guiding the decision-maker in its final choices.

The main advances of the paper can be summarized as follows.

- Fuzzy concepts allow to model imprecision particularly in cases where historical data are not readily available for using a probabilistic representation.
- Heuristic search algorithms can be easily extended to the fuzzy case, insofar as they do not resort to complex calculations such as computations of derivatives, matrix manipulations needed in deterministic optimization.

Table 10
Results of the tricriteria optimization.

	$N\tilde{P}V$ (mean value)	Common surface	Flexibility index	$N\tilde{P}V$ ($) and production time $\sum \tilde{H}_i$ (h)
Case 2	826 932	561	1.020	$\sum \tilde{H}_i = [5647, 5810, 5979, 6118]$ $N\tilde{P}V = [701\,315, 783\,386, 873\,188, 949\,839]$ Extra *NPV* = 10 371 $
Case 2a	825 821	643	1.005	$\sum \tilde{H}_i = [5731, 5897, 6068, 6209]$ $N\tilde{P}V = [700\,577, 782\,396, 871\,917, 948\,395]$ Extra *NPV* = 2585 $
Case 2b	816 533	622	1.010	$\sum \tilde{H}_i = [5699, 5864, 6034, 6174]$ $N\tilde{P}V = [691\,187, 773\,067, 862\,663, 939\,217]$ Extra *NPV* = 5536 $
Case 4	827 359	377	1.038	$\sum \tilde{H}_i = [5554, 5713, 5880, 6017]$ $N\tilde{P}V = [702\,231, 783\,983, 873\,490, 949\,733]$ Extra *NPV* = 19 522 $
Case 4a	826 564	438	1.032	$\sum \tilde{H}_i = [5582, 5742, 5910, 6047]$ $N\tilde{P}V = [701\,377, 783\,160, 872\,720, 949\,002]$ Extra *NPV* = 16 763 $
Case 6	830 164	75	1.066	$\sum \tilde{H}_i = [5409, 5564, 5726, 5860]$ $N\tilde{P}V = [705\,422, 786\,993, 876\,139, 952\,102]$ Extra *NPV* = 34 400 $

- Because it is working on populations of potential solutions, a genetic algorithm is well-suited for multiobjective optimization.

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