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Enhanced drag in pipe turbulent flow by an aqueous electrolyte: An electroviscous effect

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A B S T R A C T

Drag enhancement is reported for turbulent pipe flow of aqueous electrolyte solutions. No electroviscous effect was obtained with laminar flow. Nor was any unusual pressure drop observed for laminar or turbulent flow of non-electrolyte aqueous solutions such as sugar. An electroviscous theory was advanced that predicted the drag enhancement for a 1/1 electrolyte solution. The theory depended on consideration of Debye length.

Keywords:

Drag enhancement
Electroviscosity
Turbulent pipe flow
Aqueous salt solution

1. Introduction

Liquid flow in pipes has been shown to follow the Moody diagram [1] which details the friction in terms of the flow Reynolds number as shown in Fig. 1. Laminar flow occurred at $Re < 2000$ in the streamline region of flow with a single curve representing all data irrespective of pipe roughness δ_T/D . The $2.5 \times 10^3 < Re < 10^5$ flow covers the turbulent regime predicted by the Blasius relation for smooth pipe, i.e., at $\delta_T/D = 0$

$$\phi = 0.0396Re^{-0.25} \quad (1)$$

For rougher pipe the friction factor forms a series of curves in the turbulent region that depend on the roughness ratio δ_T/D . A transitional region lies between laminar and turbulent flow where the friction factor varies over a wide range that depends on the triggering of turbulence.

This general form of pipe flow has been shown by Pigott [2–4] to apply to a wide variety of fluids. One significant deviation from the Moody diagram has been the drag reduction exhibited by dilute aqueous polymer solutions in turbulent flow [5]. Drag reduction also occurred in other systems such as pulp suspensions [6] and in certain gas liquid flows [7–9].

It has always been assumed that aqueous electrolyte solutions follow the Moody diagram. Indeed Gould and Levy [10] and Kratz

et al. [11] reported that aqueous calcium chloride obeyed the expected pipe pressure drop within experimental accuracy. However, the flow was only observed up until the early transition to turbulent flow where abnormal pressure effects would not be expected.

The object of this work was to examine experimentally and theoretically the frictional pressure drop in an aqueous electrolyte in the laminar and turbulent regions of pipe flow.

2. Experimental

The apparatus consisted of four horizontally supported PVC pipes of different diameters 6.2 m long connected one above the other between two large vertical headers. The internal diameters of the pipes were 0.0179, 0.0293, 0.0422 and 0.0524 m determined by the weight of water held. Liquid was circulated by a centrifugal pump which drew from the base of one header. The pump outlet fed to a cooling heat exchanger, a calibrated rotameter bank, a thermometer pocket, into the second header, through one of the connecting horizontal pipes back to the pump inlet. The system was ball valved and fitted with a bypass and relief system. Pressure tapping points along the top of the horizontal pipes were used to ensure that a constant pressure drop existed over a 4 m length for all flow rates. The pressure drop was determined for water and then for aqueous solutions of sugar, glycerol, and sodium chloride at various concentrations. Physical properties of solutions were taken from International Critical Tables for the actual experimental conditions employed.

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Nomenclature

a	flow area [m ²]	z	ion charge number
A, B	integration constants	δ	laminar layer thickness [m]
b	distance [m]	δ_T	pipe roughness [m]
D	diameter [m]	ϵ	dielectric constant [F m ⁻¹], [C V ⁻¹ m ⁻¹], [A ² s ⁴ kg ⁻¹ m ⁻³]
E	streaming potential gradient [V m ⁻¹]	ζ	potential at liquid/wall interface [V]
e	electronic charge [C]	η	viscosity [kg m ⁻¹ s ⁻¹]
F	electric field strength [V m ⁻¹]	η_a	apparent viscosity [kg m ⁻¹ s ⁻¹]
I	electric current [A]	λ	Debye electric double layer thickness [m]
I_c	convective current [A]	ρ	charge density [C m ⁻³]
k	Boltzmann constant [J K ⁻¹], [kg m ² s ⁻² K ⁻¹]	$\bar{\rho}$	density [kg m ⁻³]
l	length [m]	ϕ	moody friction factor
n	electrolyte concentration [m ⁻³]	ϕ_{exp}	experimental friction factors
n_i	ion concentration [m ⁻³]	Ψ	potential [V], [W A ⁻¹], [kg m ² s ⁻³ A ⁻¹]
P	pressure [kg m ⁻¹ s ⁻²]	Ω	average specific conductivity [A V ⁻¹ m ⁻¹]
ΔP	pressure gradient [kg m ⁻² s ⁻²]		
Q	flow rate [m ³ s ⁻¹]		
R	wall shear stress [kg m ⁻¹ s ⁻²]		
R_E	electric resistance [kg m ² s ⁻³ A ⁻²]		
T	absolute temperature [K]		
u	velocity [m s ⁻¹]		
x	distance from wall [m]		
		Subscripts	
		b	at distance b
		i	ion type i
		x	at point x
		δ	at laminar layer

3. Results

Fig. 1 details the Moody diagram obtained for water flow in all four horizontal pipes. The data followed the Hagen-Poiseuille relation for laminar flow within $\pm 1\%$. The transitional region shown was an average with the effect of triggering from laminar to turbulent flow eliminated. The turbulent flow data formed a series of curves depending on the relative roughness δ_T/D . Comparison with published data [12] a value of $\delta_T = 1.56 \times 10^{-5}$ m was obtained. Both calculated and experimentally derived errors were within $\pm 1.0\%$. Much effort was given to obtaining reliable and reproducible water flow data for the pipes in order to achieve accurate estimation of subsequent drag reduction or drag enhancement effects.

The pipe roughness had an effect on the pressure drop in the turbulent region for $Re \geq 7 \times 10^4$ for all the pipes and for the lower diameter tube of 0.01786 m over the whole of the turbulent region. Thus these flow conditions were avoided when investigating aqueous solutions.

All solutions tested gave agreement with the Hagen-Poiseuille relation for laminar flow. The transitional region showed a variation spread similar to that of water flow for all solutions. The aqueous sugar and glycerol solutions followed the same pattern as for

water in the turbulent region. However, the ionic aqueous electrolyte sodium chloride solution exhibited an increase in friction factor above the Moody diagram water only flow in the turbulent region as shown in Figs. 2–4. The calculated error was $\pm 1.5\%$ for the data but the experimentally derived error was $\pm 4\%$. There was no observable effect with the apparatus in the insulated or earthed condition. The reason suggested for the enhancement of drag will now be explored.

4. Electroviscous effect

The electroviscous effect present with solid particles suspended in ionic liquids was shown by Smoluchowski [13] to increase the viscosity over that of the base liquid. Conway and Dobry-Duclanx [14] have identified three aspects of the effect. The primary effect caused by the shear field distorting the electrical double layer surrounding the solid particles in suspension. The secondary effect results from the overlap of the electrical double layers of neighbouring particles. The tertiary effect arises from changes in size and shape of the particles caused by the shear field. The primary electroviscous effect has been the subject of much study

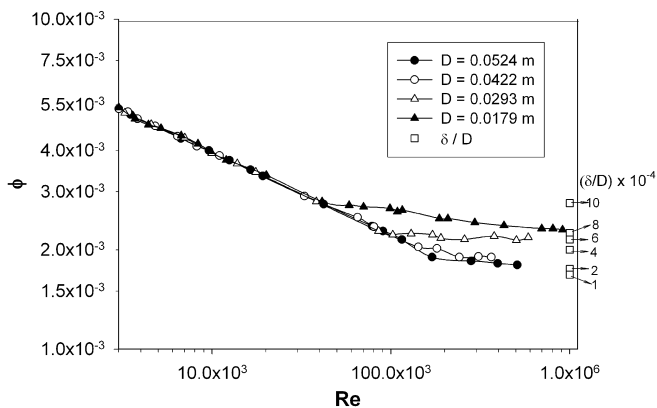


Fig. 1. The Moody diagram of friction factor against Reynolds number and pipe roughness for water flow in the test apparatus.

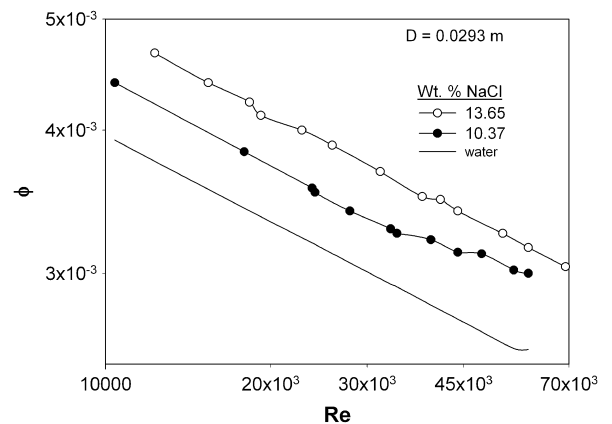


Fig. 2. The friction factor against Reynolds number for NaCl solutions flowing in 0.0293 m diameter pipe.

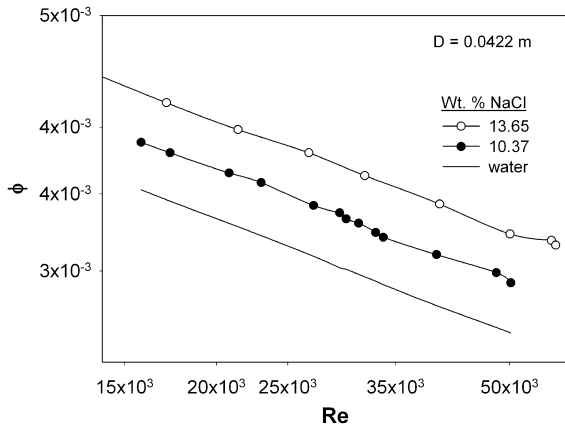


Fig. 3. The friction factor against Reynolds number for NaCl solutions flowing in 0.0422 m diameter pipe.

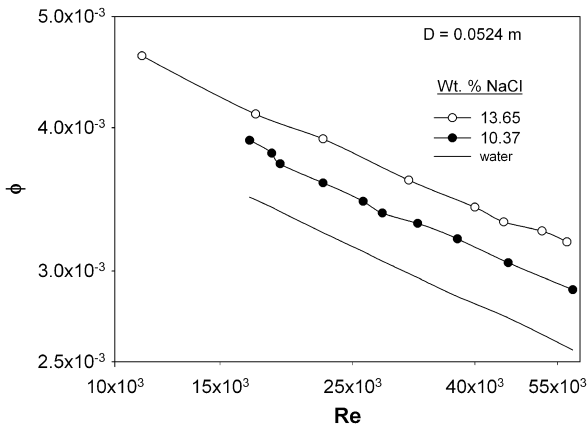


Fig. 4. The friction factor against Reynolds number for NaCl solutions flowing in 0.0524 m diameter pipe.

[15–31] and has been shown to depend on (a) the size of the Debye length of the electrical double layer compared to the size of the suspended particle; (b) the potential at the slipping plane between the particle and the bulk fluid; (c) the Peclet number, i.e., diffusive to hydrodynamic forces; (d) the Hartmann number, i.e., electrical to hydrodynamic forces and (e) variations in the Stern layer around the particle. The secondary and tertiary effects which apply in concentrated suspension and colloids respectively have received attention [32–37] but are of no interest here where solid pipe surfaces are involved.

Reviews of the electroviscous effect for particle suspensions in ionic liquids [14,38–40] and the flow of ionic liquids in porous media [39,40] provide insight for the present study but extension to ionic liquid turbulent flow conditions in pipes is required. Attention has been given to the electroviscous effect of laminar flow in capillary tubes and porous media [41,42], but no corresponding effect has been reported for pipes. On the other hand, Harned and Owen [43] have detailed methods for predicting the increase in viscosity of a base liquid due to the presence of a dissolved electrolyte but viscosity measurements were performed under laminar flow conditions.

General electrokinetic effects have been observed when two phases move tangentially to each other. Kortum [44] has detailed four possible cases. The motion may be due to an external electric field parallel to the phase boundary [44–48] causing (a) electro-osmosis, i.e., liquid motion in a porous body or (b) cataphoresis

or electrophoresis causing suspended particle motion through a liquid. The reverse cases occur when potential differences are established by motion of a liquid through (a) a porous solid which is termed streaming potential or (b) suspended particles are moved through a liquid which is called an electrophoretic potential. This work is concerned with turbulent pipe flow of aqueous ionic solution without an applied electric field in which enhanced drag is observed and an electroviscous effect is observed due to the establishment of a streaming potential.

5. Electroviscous relationship

A relationship will be developed giving the apparent increased viscosity η_a of an ionic liquid in turbulent flow due to the resistance to shear by the electrical double layer on the solid wall attracting ions in the passing liquid. The viscosity increase occurs in the laminar layer on the immediate wall surface, and the geometry can be approximated to a flat plate of unit width (and length) equal to the tube circumference. This is because the actual layer influencing the electroviscosity effect is small compared to the geometry of the overall system. It is necessary to calculate the potential at any point in the fluid as a function of distance x from the plate. The apparent viscosity relation is then obtained by a balance of the forces and the volumetric flow in the laminar layer.

5.1. Potential distribution

Glasstone [49] showed that at a point x up from a plate the potential gradient is related to the excess charge density by the Poisson equation

$$\frac{d^2\psi}{dx^2} = -\frac{\rho_x}{\epsilon} \quad (2)$$

From the Boltzmann equation

$$\rho_x = -zn_i e \sinh\left(\frac{e\psi}{kT}\right) \quad (3)$$

for a uni-univalent electrolyte $z = 2$

$$\text{But } \sinh\left(\frac{e\psi}{kT}\right) = \left(\frac{e\psi}{kT}\right) + \frac{1}{6}\left(\frac{e\psi}{kT}\right)^3 + \frac{1}{120}\left(\frac{e\psi}{kT}\right)^5 \quad (4)$$

If $e\psi/kT$ is small only the first term of Eq. (4) applies.

Thus from Eqs. (2), (3), (4)

$$\frac{d^2\psi}{dx^2} = \frac{2n_i e^2}{\epsilon kT} \psi = \frac{\psi}{\lambda^2} \quad (5)$$

where the Debye double layer thickness is

$$\lambda = \left(\frac{\epsilon kT}{2n_i e^2}\right)^{0.5} \quad (6)$$

Integration of Eq. (5) remembering that

$$\frac{d^2\psi}{dx^2} = \frac{1}{2} \frac{d}{d\psi} \left(\frac{d\psi}{dx}\right)^2 \quad (7)$$

gives

$$\frac{d\psi}{dx} = \left(\frac{\psi^2}{\lambda^2} + A\right)^{0.5} \quad (8)$$

where A is an integration constant and since $d\psi/dx = 0$ at the tube centre line where $\psi = 0$ for a thin electrical double wall layer then $A = 0$

$$\text{Therefore } \frac{d\psi}{dx} = \frac{\psi}{\lambda} \quad (9)$$

Integrating again

$$\frac{x}{\lambda} = \ln \psi + B \quad (10)$$

When $x = 0$ the constant B is

$$B = \pm \ln \psi_0 \quad (11)$$

$$\text{Therefore } \frac{x}{\lambda} = \ln \left(\frac{\psi}{\psi_0} \right) \text{ or } \ln \left(\frac{\psi_0}{\psi} \right) \quad (12)$$

$$\psi = \psi_0 e^{-x/\lambda} \quad (13)$$

as the only viable variable of ψ .

5.2. Pressure force balance

Consider a unit length laminae parallel to the plate at x of thickness δx . A pressure balance gives

$$\text{Viscous retarding force} = \eta \delta x \frac{d^2 u}{dx^2} \quad (14)$$

$$\text{Pressure force} = \Delta P \delta x \quad (15)$$

$$\text{Electrical retarding force} = E \rho_x \delta x \quad (16)$$

At equilibrium the liquid moves without acceleration and the pressure gradient is opposed by the retarding forces.

Simplification and a balance gives

$$-\eta \frac{d}{dx} \left(\frac{du}{dx} \right) - E \rho_x = \Delta P \quad (17)$$

From the Poisson Eq. (2)

$$-\eta \frac{d}{dx} \left(\frac{du}{dx} \right) + \varepsilon E \frac{d}{dx} \left(\frac{d\psi}{dx} \right) = \Delta P \quad (18)$$

Integration gives

$$-\eta \left(\frac{du}{dx} \right) + \varepsilon E \left(\frac{d\psi}{dx} \right) = \Delta P x + A \quad (19)$$

The Newton viscosity law gives

$$R = -\eta \frac{du_x}{dx} \quad (20)$$

Close to the surface in the laminar sub-layer the velocity gradient is linear and from the concept of the universal velocity profile based on mixing length theory [12]

$$\frac{du_x}{dx} = \frac{R}{\eta} = \frac{u_\delta}{\delta} \quad (21)$$

Substitution in Eq. (19) gives

$$-R + \varepsilon E \frac{d\psi}{dx} = \Delta P x + A \quad (22)$$

At $x = 0$, $\psi = \psi_0$, $u = 0$ and $d\psi_0/dx \approx 0$ so $A = -R$.

Integration of Eq. (19) gives

$$-\eta u + \varepsilon E \psi = + \frac{\Delta P x^2}{2} - R x + B \quad (23)$$

At $x = 0$, $\psi = \psi_0$ and $u = 0$ so that

$$B = \varepsilon E \psi_0 \quad (24)$$

So Eq. (23) becomes

$$-\eta u + \varepsilon E (\psi - \psi_0) = \frac{\Delta P x^2}{2} - R x \quad (25)$$

To obtain a useful form for R in Eq. (25), consider flow in the laminar layer between the two flat laminae surfaces of unit width set on the centre line at $x = \delta/2$ and $2b$ apart as shown in Fig. 5.

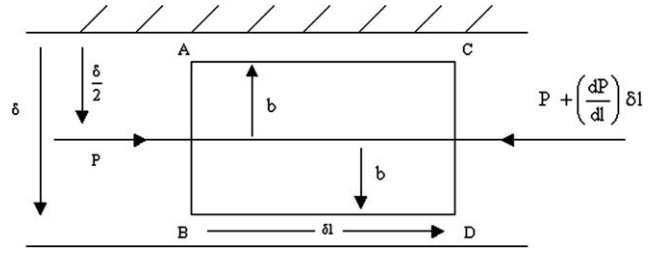


Fig. 5. Laminar sub-layer flow.

By a force balance

$$\text{Force on } AB = 2bP \quad (26)$$

$$\text{Force on } CD = -2b \left(P + \left(\frac{dP}{dl} \right) \delta l \right) \quad (27)$$

$$\text{Force on surfaces } AC \text{ and } BD = 2\eta \delta l \left(\frac{du_b}{db} \right) \quad (28)$$

$$2bP - \left(2bP + 2b \left(\frac{dP}{dl} \right) \delta l \right) + 2\eta \delta l \frac{du_b}{db} = 0 \quad (29)$$

$$-b \left(\frac{dP}{dl} \right) + \eta \left(\frac{du_b}{db} \right) = 0 \quad (30)$$

Since $R = \eta \left(\frac{du_b}{db} \right)$ and $b = \delta/2$

$$R = \Delta P \delta / 2 \quad (31)$$

So Eq. (25) becomes

$$-\eta u + \varepsilon E (\psi - \psi_0) = \Delta P x^2 / 2 - \Delta P x \delta / 2 \quad (32)$$

5.3. Apparent viscosity

To obtain a relationship for the apparent viscosity it is required to determine the liquid volume flowing in the laminar layer in unit time

$$Q = \int_0^\delta u dx = \frac{1}{\eta} \int_0^\delta \left[\frac{\Delta P x}{2} (\delta - x) + \varepsilon E (\psi - \psi_0) \right] dx \quad (33)$$

Substituting Eq. (13)

$$Q = \frac{1}{\eta} \int_0^\delta \left[\frac{\Delta P x}{2} (\delta - x) + \varepsilon E \psi_0 (1 - e^{-x/\lambda}) \right] dx \quad (34)$$

$$= \frac{1}{\eta} \left[\frac{\Delta P}{2} \left(\frac{\delta x^2}{2} - \frac{x^3}{3} \right) - \varepsilon E \psi_0 (x + \lambda e^{-x/\lambda}) \right]_0^\delta \quad (35)$$

$$= \frac{1}{\eta} \left[\frac{\Delta P \delta^3}{12} - \varepsilon E \psi_0 (\delta + \lambda e^{-\delta/\lambda}) \right] \quad (36)$$

The effect of the electrical retarding force of Eq. (16) will be to increase the viscosity to an apparent value η_a and the pressure force balance will be from Eq. (17)

$$-\eta_a \frac{d}{dx} \left(\frac{du}{dx} \right) = \Delta P \quad (37)$$

Following through the development already presented

$$Q = \frac{\Delta P \delta^3}{12 \eta_a} \quad (38)$$

The equivalent streaming potential is found as follows.

Electro osmosis is the fluid flowing through a fixed tube due to the influence of an external electric field. The acting electrical force on a volume is the product of the charge density ρ and the electrical field strength F . The electrical force is balanced by the frictional force between adjacent fluid layers with different velocities. The result is an electrical current I will flow due to ion migration through the flow area a

$$I = \frac{V}{R_E} = \frac{Fl}{R_E} = \Omega a F \quad (39)$$

The reverse effect of liquid forced through the volume will result in the formation of a streaming potential. There is an absorbed ionic layer on the solid surface while the flowing liquid passing carries ions of a similar charge. A convection current I_c is produced and a balance established between ionic migration and the streaming potential

$$I_c = \int u \rho da \quad (40)$$

From Eq. (1)

$$I_c = - \epsilon \int u \frac{d^2 \psi}{dx^2} da \quad (41)$$

Solving by partial integration gives

$$I_c = \frac{\epsilon \zeta a \Delta P}{\eta_a} \quad (42)$$

At equilibrium the connective current I_c must be equal to the opposite conduction current I and with $F = E$

$$I_c = I = \frac{\epsilon \zeta a \Delta P}{\eta_a} = \Omega a E \quad (43)$$

$$E = \frac{\epsilon \zeta a \Delta P}{\eta_a \Omega} \quad (44)$$

Substitution of Eqs. (38) and (44) into Eq. (36) gives

$$\eta_a = \eta + \frac{12 \epsilon^2 \zeta^2}{\Omega \delta^3} (\delta + \lambda e^{-\delta/\lambda}) \quad (45)$$

5.4. Apparent viscosity

Assume the drag enhancement was due to an increase in viscosity from the value for the aqueous solution obtained from International Critical Tables (expected as the full line in Figs. 2–4) to an apparent viscosity for the experimental ionic solutions. From Eq. (1) and experimental data as dashed lines in Figs. 2–4.

$$(\phi)_{\text{exp}} = \left(\frac{\Delta P}{u^2} \right)_{\text{exp}} \frac{D}{4 \bar{\rho}} \quad (46)$$

Eq. (46) is independent of the value of viscosity unlike the Reynolds number. If the experimental friction factor value was moved to the left to coincide with the Blasius full lines in Figs. 2–4 then a value of Reynolds number would result that includes the apparent viscosity.

Thus Eqs. (1) and (46) yield

$$(\phi)_{\text{exp}} = 0.0396 \left(\frac{Du \bar{\rho}}{\eta_a} \right)^{-0.25} \quad (47)$$

$$\eta_a = Du \bar{\rho} \left(\frac{\phi}{0.0396} \right)^4 \quad (48)$$

The resulting apparent viscosity values were checked by back calculation.

6. Discussion

The flow resistance of water or aqueous non-electrolyte solutions followed the usual Moody diagram of friction factor ϕ versus Reynolds number for both laminar and turbulent flows. Elton [42], Hunter [39,40] and others have reported drag enhancement in capillaries and narrow channels for these types of fluids in laminar flow. Such an effect of drag enhancement took place when the diameter was below about 2×10^{-7} m and would not have been observed in this work at much greater diameters or indeed in the normal laminar flow measurement of viscosity in either capillary or rotational viscometers. Electroviscosity was invoked by these authors to explain drag enhancement in capillary flow.

In this work, drag enhancement was observed in turbulent pipe flow of ionic aqueous solutions but not in laminar pipe flow where the normal Hagen-Poiseuille relation was followed. The drag enhancement was directly dependent on the ionic concentration. The average experimental values of the apparent viscosity η_a which was invoked to explain the drag enhancement were calculated from the electroviscous theory using equation (45) assuming that the electrical laminar layer thickness δ was equal to the Debye length λ for each system. These are detailed in Table 1. Agreement between measured and apparent calculated viscosity for the sodium chloride solutions was reasonable.

The actual mechanism for the increased drag in the ionic solutions concerns the electrical double layer on the inner pipe wall. There is an absorbed layer of ions of an ionic dimension or two thicknesses, beyond which the double layer extends out into the bulk liquid. All these phenomena are well within the hydraulic laminar sub-layer which was of the order of $3.5 \times 10^{-4} \pm 100\%$ m. It is the hydrodynamic laminar sub-layer which governs the friction in turbulent flow rather than the bulk of the liquid flow beyond the sub-layer. The actual mechanism of action was that the section of the electrical double layer within the Debye length from the wall attracted ions passing in the bulk within the laminar hydraulic sub-layer. The overall effect was to pull back the passing fluid resulting in an increase in viscosity from η to η_a the apparent viscosity value. This proposed mechanism differs from the viscoelectric effect where an impressed external voltage affects the viscosity in the whole bulk of the fluid. Herein the effect of the surface change is confined to the volume of fluid within the Debye length in from the wall. Hunter [39,40] referring to Davies and Rideal [50] mentioned another viscoelectric effect that was confined to the electrical double layer where the surface voltage affected both the viscosity and dielectric constants. The treatment however was confined to static flow conditions whereas this work was involved with the streaming potential set up by the turbulent fluid flow.

The ionic diameter of the cations and anions were of the order of 3×10^{-12} m which was well within the Debye length of

Table 1
Results of measurements and calculation of apparent aqueous electrolyte solution viscosity

Parameter	NaCl concentration	
	13.65% w/w	10.37% w/w
Measured from pressure drop viscosity [kg m ⁻¹ s ⁻¹]	0.003131 + 10%–12%	0.001973 ± 16%
Calculated viscosity [kg m ⁻¹ s ⁻¹] Eq. (45)	0.003156	0.001930
Specific conductance [A V ⁻¹ m ⁻¹]	17.834	15.020
Solution viscosity [kg m ⁻¹ s ⁻¹]	0.001303	0.001203
Dielectric [F m ⁻¹]	695.24 × 10 ⁻¹²	605.24 × 10 ⁻¹²
Surface interface potential [V]	0.0122	0.0073
λ [m]	1.90 × 10 ⁻¹⁰	2.20 × 10 ⁻¹⁰

2×10^{-10} m. The hydrated ions had a diameter of about 7×10^{-10} m and since the surface interfacial potential falls off rapidly in the electrical double layer it may be assumed that the electroviscous effect was principally concerned with the unhydrated ionic layers close to the inner pipe wall. Eq. (45) only applies to apparent viscosity calculations in turbulent pipe flow. For laminar flow conditions the normal viscosity values apply.

7. Conclusions

Aqueous electrolyte solutions were shown to possess drag enhancement for turbulent pipe flow. The electroviscous effect was advanced to explain the observations and an equation was derived from consideration of electrolyte solution theory. Agreement between theory and experiment was achieved for the 1/1 sodium chloride aqueous solution. The electrostatic attraction by the layer of ions absorbed on the inner pipe wall provides the mechanism for the observed electroviscous effect by holding back the movement of ions passing in the bulk fluid.

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