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OPTICAL APPROACH OF A HYPERCATADIOPTRIC SYSTEM DEPTH OF FIELD

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ABSTRACT

A catadioptric system is composed of a mirror and a perspective camera. Since the mirror is curved and the distance between the mirror and the camera is short, some parts of the panoramic image keep blurred. In this article, an optical approach of the panoramic system using a hyperbolic mirror is presented and its depth of field is analyzed. The impact of different parameters of mirror and camera on the quality of the panoramic image is researched and a valid method of choosing camera and mirror is presented. Finally, this article gives some possible perspectives based on these researches.

Index Terms — Panoramic vision, Blur, Depth of Field

1. INTRODUCTION

The omnidirectional camera is more and more used for many applications, e.g. SLAM (Simultaneous Localization and Mapping) for robotics applications. A catadioptric system is an optical system which consists of a reflective element (catoptric) and a refractive element (dioptric) [1]. In this system, the camera observes the reflection of the scene in the mirror and we can obtain a panoramic image with 360-degree field of view.

The type of the mirror has many kinds, for example, spherical mirrors, conic mirrors, parabolic mirrors, hyperbolic mirrors, etc [2]. In this article, a catadioptric system using a hyperbolic mirror (called hypercatadioptric camera) is presented.

This research is especially applied for miniature UAV (Unmanned Aerial Vehicle), therefore, a small and light camera has to be used for mini UAV. While, as the distance between the mirror and the camera is very short, it is difficult to obtain a whole clear panoramic image. Hence, it is necessary to analyze the depth of field of the hypercatadioptric camera.

The Depth Of Field (DOF) of a camera is an intrinsic fundamental property, and models have to be built to know scene areas where objects will seem clear and areas where objects will seem blurred. For a perspective camera, the DOF is defined between two planes. But for a catadioptric camera, finding a model to understand and estimate the DOF is not obvious.

This article deals with DOF of hypercatadioptric cameras. Similar works are almost non-existent. Baker and Nayar [2][3] do an all-sided research about catadioptric system which includes a part of blur, but they did not focus deeply on it. Consequently, a geometric method has been used to establish a complete and detailed analysis about blur of panoramic image.

2. DEFOCUS BLUR OF A HYPERCATADIOPTRIC CAMERA

As already mentioned, it is well known that for a perspective camera (using a monolens model camera), the DOF is defined between two parallel planes, that are perpendicular to the camera axis. Positions of these planes depend on the camera's parameters (focus, diaphragm aperture...) The objects located on the scene between these two planes will be well focused, and others will be seen as blurred. To define quantitatively whether an image is blurred or not is very difficult, because this notion depends on many conditions, including the distance of observer, the size of image, the amount of pixels, etc. To our knowledges, there is no strict definition for blurred image. In this article, the diameter of the circle of confusion is used; its value can be represented by $o = n \times p$, where n is the maximum number of pixels for defining blur; p is the physical size of each pixel which is a property of the camera sensor. Of course, the bigger is the diameter, the more blurred is the image. For $n = 1$, as it is used here, an image is blurred as soon as the circle of confusion is larger than one pixel.

Our panoramic system is constructed by a hyperbolic mirror and a perspective camera. The constraint of Single View Point (SVP) [2] is respected. In fact, the target of the analysis of the DOF is to solve the problem of the blur of panoramic image.

For the panoramic camera, the imaging process includes two steps: 1) each spatial point corresponds to a virtual point in the mirror; 2) each virtual point corresponds to a real point on the image-plane (sensor). Once we know the distribution of all virtual points, we can use the model of perspective camera to analyze the DOF of the hypercatadioptric camera.

For a catadioptric system based on SVP, there is a main ray for each spatial point (the ray towards the focus of the hyperbolic mirror), but we also need an adjacent ray to find the accurate position of the image point. In Fig.1, the point $p(x, y)$ is any spatial point, $p_1(x_1, y_1)$ and $p_2(x_2, y_2)$ are two special points located on the hy-

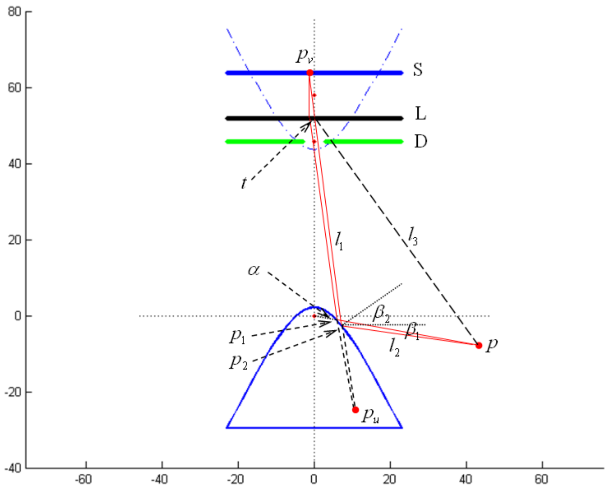


Fig. 1. The geometric analysis of the imaging process of the hypercatadioptric camera.

perbolic mirror. An incident ray from p arrives at p_1, then is reflected by the mirror, passes through the focus of the lens, and finally, arrives at a certain point. Another incident ray from p arrives at p_2, then is reflected by the mirror, passes the center of the lens, finally, arrives at the same point. By these two rays, we can obtain the position of the image-point corresponding to the spatial point.

For the point p_1, according to the formula of hyperbol, we can obtain

$$\frac{(y_1 - c/2)^2}{a^2} - \frac{x_1^2}{b^2} = 1$$

Due to $y_1 = x_1 \cdot tan\alpha$, we can obtain

$$x_1 = \frac{b^2 \cdot c \cdot tan\alpha - a \cdot b \cdot \sqrt{4b^2 \cdot tan^2\alpha + c^2 - 4a^2}}{2(b^2 \cdot tan^2\alpha - a^2)}$$

In the same way, for the point p_2, we can obtain

$$y_2 = \frac{c}{2} - a \cdot \sqrt{\frac{x_2^2}{b^2} + 1}$$

Based on the geometric analysis, we have

$$tan\beta_1 = \frac{y_2 - y}{x - x_2}$$
$$tan\beta_2 = \frac{(c/2 - y_2) \cdot b^2}{a^2 \cdot x_2}$$

We suppose that Θ is the angle between the incident ray and the reflected ray, then we have $\Theta = 2(\beta_1 + \beta_2)$, so

$$\Theta = 2 \cdot \left(arctan(\frac{y_2 - y}{x - x_2}) + arctan(\frac{(c/2 - y_2) \cdot b^2}{a^2 \cdot x_2})\right)$$

At the same time, we know

$$l_1^2 = x_2^2 + (c + f - y_2)^2$$
$$l_2^2 = (x - x_2)^2 + (y - y_2)^2$$
$$l_3^2 = x^2 + (c + f - y)^2$$

Based on the theorem of the trigonometric cosine

$$l_3^2 = l_1^2 + l_2^2 - 2l_1 \cdot l_2 \cdot cos\Theta$$

and

$$y_2 = \frac{c}{2} - a \cdot \sqrt{\frac{x_2^2}{b^2} + 1}$$

As we have $\frac{x_2}{c+f-y_2} = \frac{t}{v}$ and $\frac{x_1}{c-y_1} = \frac{t}{f}$, we can obtain

$$v = \frac{x_1 \cdot f \cdot (c + f - y_2)}{x_2 \cdot (c - y_1)}$$

Then, we substitute x_1, y_1, x_2, y_2 in this formula, by calculating, finally we can obtain the relation between the incident angle and the position of real point. Correspondingly, we can use the imaging principle of lens to obtain the formula of the distribution of the virtual point:

$$u = \frac{f \cdot v}{v - f} = \frac{x_1 \cdot f^2 \cdot (c + f - y_2)}{x_1 \cdot f^2 \cdot (c + f - y_2) - x_2 \cdot f \cdot (c - y_1)}$$

In Fig.2, the blue line is a hyperbolic mirror and the red line shows the distribution of the virtual points. If all the virtual points are located in the DOF of the camera, we can absolutely obtain a clear panoramic image. However, the distance between the mirror and the camera is too short to offer enough DOF for all the virtual points.

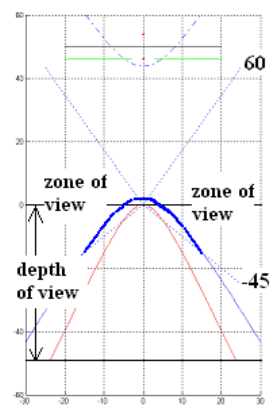


Fig. 2. The distribution of all virtual points.

Normally, the clear zone in a panoramic image is defined between two concentric circles which correspond to two planes of the DOF of the perspective camera (see Fig.3).

Fig.3 shows that a zone (between the two red circles) of the panoramic image is clear and the two other zones are blurred. We should find the best parameters of the camera and the mirror to increase the clear zone.

Fig.4(a) shows the relation between the incident angle and the positions of the virtual points. The red curve corresponds to the spatial points which are 10 meters far away from the mirror and the blue curve corresponds to the spatial points which are 1 meter far away from the mirror. We can find that these two curves are almost coincident, it means that all the points of the same incident ray have a same virtual point.

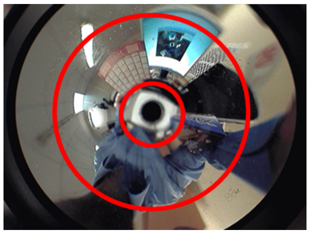
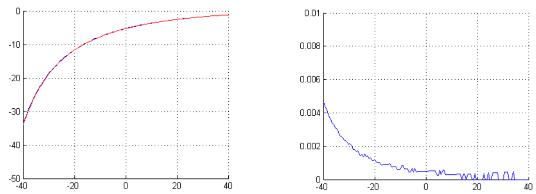


Fig. 3. The clear zone and the blurred zone in a panoramic image.



(a) Position of virtual image (b) Absolute differences

Fig. 4. The differences between two different distances spatial points (1m et 10m from mirror).

In Fig.4(b), we can find that the difference between these two curves is very small (less than 0.5%), so it can be ignored.

3. PRACTICAL TEST

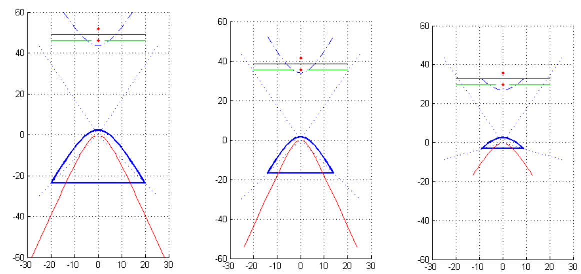
We use three hyperbolic mirrors and one small camera to do some experiments, and the camera can change lens (see Fig.5).

Fig. 5. The three hyperbolic mirrors and one small camera.

The parameters[1] of these mirrors:

- HMN-X50 (left) : $b = 9.892$; $a = 20.764$; $c = 46mm$; $k = 10.812$; the exterior diameter= $40mm$;
- HS-X50 (middle) : $b = 7.434$; $a = 16.173$; $c = 35.6mm$; $k = 11.466$; the exterior diameter= $28mm$;
- HT-N24 (right) : $b = 8.367$; $a = 12.247$; $c = 29.665mm$; $k = 6.285$; the exterior diameter= $18mm$.

To obtain the distribution of virtual points, we also must consider the size of the mirror (the exterior diameter) which can decide directly the maximum lateral angle of vision. Based on these parameters of the mirrors, we do the simulation for every mirror and we get the distribution of virtual points which are shown as red line in Fig.6.

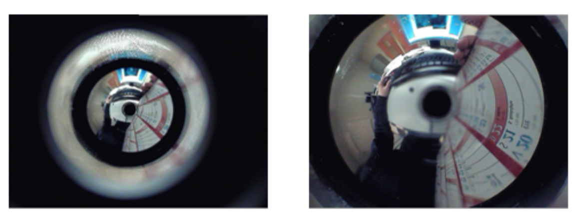


(a) HMN-X50 (b) HS-X50 (c) HT-N24

Fig. 6. The distribution of virtual points of three different mirrors.

In Fig.6, due to the different parameters of the mirrors their lateral angles are different, the angle of the big mirror is from -50^o to 90^o , same angle for the medium mirror, the angle of the small mirror is from -19^o to 90^o. We can find that the small mirror has the smallest distribution of virtual points, so it is easier to obtain a clear image, but it loses some field of view. The choice of the mirror lies on the application.

The focus of lens affects directly the area where the panoramic image occupies on the sensor. The focus is shorter, the image is clearer but the effective area is smaller. Fig.7 shows well this phenomenon.



(a) $f = 3.3mm$ (b) $f = 8mm$

Fig. 7. The panoramic images using different lens.

For the camera, we know that the focus is shorter, the DOF is bigger. In Fig.7, (a) is clearer than (b), but (b) uses more effectively the area of the sensor. In our application, with the used equipment, we can obtain better results from (b) than (a). Moreover, the focus of lens is too small, the deformation of image is very fearful.

[1] In the formula of hyperbol, a and b are the classical parameters, but they are not convenient to analyze, so we use another type parameters of hyperbole: k and c. If we know k and c, we can also know the form of hyperbole, the relation between a, b and k, c are:

$$a = \frac{c}{2}\sqrt{\frac{k-2}{k}} \qquad b = \frac{c}{2}\sqrt{\frac{2}{k}}$$

In fact, k expresses the degree of the convexity of hyperbole, if k is bigger, hyperbole is more convex; c expresses the distance between the two focus of hyperbole. As k and c have very clear physical meaning, we always use them to describe the form of hyperbole.

4. IMPACT OF DIFFERENT PARAMETERS OF OF A HYPERCATADIOPTRIC CAMERA

For a hypercatadioptric camera, the blur of the panoramic image is very hard to avoid completely, so, to decrease the blur, the choice of the parameters of the hyperbolic mirror and the lens is very important. In fact, there are only three parameters that we can regulate: 1) k, the degree of convexity of the hyperbolic mirror; 2) c, the distance between the two focus of the hyperbolic mirror; 3) f, the focus of the lens.

Based on SVP, we can obtain the distribution of the virtual image in the mirror. Then, we can use the virtual image to find the best parameters of the mirror and the lens. We suppose that the size of the sensor is known, and the panoramic image always occupies the biggest possible area of the sensor. As k and c are not independent, we analyze respectively (k, f) and (c, f) (see Fig.8 and Fig.9).

We define: w is the width of the virtual image, h is the height of the virtual image, D is the DOF of the camera. And then, w/h is the rate of the width by the height, D/h is the rate of the clear area of image by the whole image. If w/h is bigger, it will be easy to obtain a clearer image, equally, D/h is bigger, the image will be clearer.

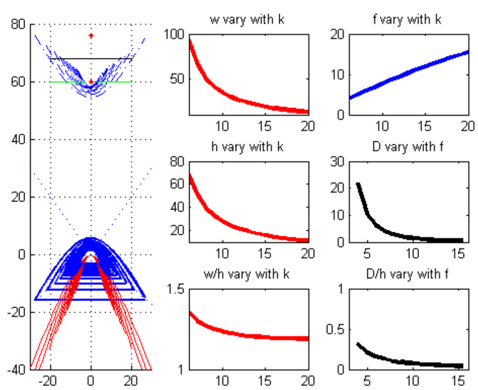


Fig. 8. The impact of the different parameters (k, f).

Fig.8 shows that w/h and D/h decrease when k increases, so we should choose a small value for k. If k decreases, the mirror will be more planar, and we must increase the size of the mirror to keep the same zone of view. However, in our application, the size of the mirror (the weight of the mirror) is an important constraint and must be the smallest possible. This constraint is also for other applications.

Fig.9 shows that w/h and D/h increase when c increases, so we should choose a big value for c. If c increases, the camera will be more far away from the mirror because, based on SVP, the mirror is always located at the second focus of the hyperbolic mirror. However, if the camera is very far away from the mirror, the hypercatadioptric system is not compact, and moreover, this system is difficult to install well and easy to lose coaxial constraint.

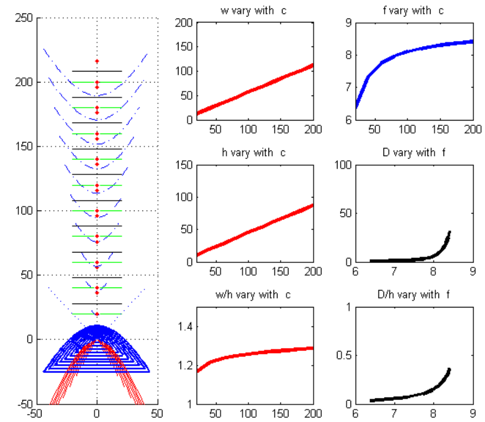


Fig. 9. The impact of the different parameters (c, f).

5. CONCLUSIONS

We have presented the issue of the blur of panoramic image and have analyzed the DOF of the hypercatadioptric camera. The principal reason which causes blur is the short distance between the mirror and the camera, and the small camera can't offer enough DOF for all the virtual points.

To decrease the blur and obtain a clearest possible image, we have done some simulation with different hyperbolic mirrors and different lens. We found that: 1) for large robot (ground robot), we can regulate c to obtain a clear image, normally, when c is bigger than 30cm, the quality will be very good; 2) for small robot (aerial robot), we should increase k to use the small mirror and keep the enough zone of view, at the same time, we should decrease c to use the lens having short focus (attention: if k is too big, the image will be very small; if c is too small, the deformation of image will not be ignored.). Except that, if we have a powerful camera and environment is very brilliant, we can also reduce the diameter of the diaphragm to obtain a clear image. While, each solution has some inconveniences for resolution, weight, stability, intensity, etc.

6. REFERENCES

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