A RAO-BLACKWELLIZED PARTICLE FILTER FOR INS/GPS INTEGRATION

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ABSTRACT

The localization performance of a navigation system can be improved by coupling different types of sensors. This paper focuses on INS-GPS integration. INS and GPS measurements allow to define a non-linear state space model, which is appropriate to particle filtering. This model being conditionally linear Gaussian, a Rao-Blackwellization procedure can be applied to reduce the variance of the estimates.

1. INTRODUCTION

The science of positioning consists of using measurements of a cluster of sensors to estimate the trajectory of a vehicle. Global Positioning System (GPS) and Inertial navigation systems (INS) are among the most popular and reliable forms of navigation. However, both suffer from different deficiencies. The overall performance (in terms of accuracy, cost, security, self-containment and availability) can be improved by combining GPS and INS measurements. Indeed, the long term accuracy of GPS can compensate for the drift affecting INS fixes. Moreover, the INS is not subjected to reception limitations and interferences as for GPS.

Section 2 briefly reviews the techniques of navigation considered in this paper, including their limitations. The state space model associated to the GPS/INS integration architecture is also detailed. Section 4 studies a Rao-Blackwellized Particle Filter, which estimates the trajectory of a vehicule by combining GPS and INS measurements. This method is validated by simulations presented in section 5. Conclusions are reported in section 6.

2. GPS/INS INTEGRATION

2.1. The Global Positioning System

The GPS is a satellite-based navigation system which provides precise positioning and timing information to any user equipped with a proper receiver. GPS is widely used in both military and civil areas, due to its accuracy, global availability and low cost.

The user's location is computed from the distances to satellites with known positions. These distances called ranges are obtained as the product of the light velocity and the transit time of satellite-to-receiver signals. A minimum of four measurements is necessary to determine the user's position (in three dimensions) since the receiver's clock offset with respect to the GPS time is an additional unknown.

The GPS signal carries useful information to determine the receiver position. It is composed of a navigation message (which provides the current state of the constellation) and a pseudo-random code. Spread-spectrum techniques allow the receiver to compute the propagation delay as the amount of shift between the transmitted code and a local replica. It is interesting to note that the GPS system has a global availability, due to a constellation of 24 satellites in six 12-hour orbit planes.

The GPS measurements are called pseudo-ranges (instead of ranges) since the estimated time of transmission is corrupted by different biases. The positioning equation for n_s satellites in sight can be defined as:

$$\rho_i = r_i + c\tau_r + w_i, \qquad 1 \leq i \leq n_s \tag{1}$$

where ρ_i is the pseudo-range between the user and the ith satellite, w_i is measurement error, r_i is the geometric range from the user to the satellite, τ_r is the user's clock offset relative to GPS time and c is the light velocity. This paper assumes that the atmospheric errors as well as the satellite clock offset can be corrected by using data from the navigation message. This allows to incorporate the residual delays in the additive noise w_i.

Note that the signals coming from the satellites can experience interferences and jamming, which is the main drawback of GPS.

2.2. Inertial navigation systems (INS)

Inertial navigation is self-relying and autonomous, contrary to satellite-based systems. Based on measured forces and torques, it maintains an estimate of the kinematics of a vehicule by applying the physical laws of motion.

The INS consists of two parts: an inertial measurement unit with inertial sensors (accelerometers and gyrometers) and a computer that provides the mobile with its position, velocity and attitude angles. Two classical types of INS systems are available (strapdown and gimbaled systems). This paper focuses on strapdown systems which are characterized by a sensor platform rigidly attached to the vehicule. The accelerometers deliver a non gravitational acceleration (also referred to as specific force $\boldsymbol{f}^p$) and the gyrometers measure the rotation rate of the sensor cluster $\boldsymbol{\Omega}_{ip}^p$ in order to keep track of the vehicule orientation. For convenience, we use the following notations:

R_{a2b} : rotation matrix from frame a to frame b,

$\boldsymbol{p}_b$: location of the vehicule in the frame b,

$\boldsymbol{\Omega}_{ab}^b$: rotation rate from frame a to frame b, resolved in frame b,

$\boldsymbol{v}_a^b$: velocity relative to frame a, resolved in frame b,

$S(\boldsymbol{u})$: skew-symmetric matrix such that $S(\boldsymbol{u})y = u \wedge y$.

The subscripts and superscripts refer to the different coordinate frames, i.e. i: inertial frame, e: earth centered earth fixed frame, n: local geographic frame, p: platform frame. The differential equations relating the measured quantities to the dynamics are defined as follows:

$$\dot{\boldsymbol{v}}_e^n = R_{p2n} \boldsymbol{f}^p + \boldsymbol{g}^n - (\boldsymbol{\Omega}_{en}^n + 2\boldsymbol{\Omega}_{ie}^n) \wedge \boldsymbol{v}_e^n - \boldsymbol{\Omega}_{ie}^n \wedge \boldsymbol{\Omega}_{ie}^n \wedge \boldsymbol{p}^n, \quad (2)$$

$$\dot{\mathbf{p}} = \begin{pmatrix} \dot{\lambda} \\ \dot{\phi} \\ \dot{h} \end{pmatrix} = \begin{pmatrix} \frac{1}{R_\lambda + h} & 0 & 0 \\ 0 & \frac{1}{(R_\phi + h)\cos\lambda} & 0 \\ 0 & 0 & -1 \end{pmatrix} \boldsymbol{v}_e^n, \quad (3)$$

where λ, ϕ and h are the latitude, longitude and height of the mobile. These equations are integrated to obtain the velocity and the position. However, this operation assumes that the rotation matrix R_{p2n} (from the platform frame to the local geographic frame) is known. In practical situations, R_{p2n} is determined as the solution of the differential equation:

$$\dot{R_{p2n}} = S(\boldsymbol{\Omega}_{ie}^n) + S(\boldsymbol{\Omega}_{en}^n) - R_{p2n} S(\boldsymbol{\Omega}_{ip}^p), \quad (4)$$

It is important to note that the vector of attitude angles (roll, pitch, and heading), denoted as $\boldsymbol{\rho} = (\Phi, \theta, \psi)^T$, completely defines the three rotations from the platform frame to the locally level frame. Consequently, they can be computed directly from the elements of $\dot{R_{p2n}}$ via nonlinear relations (see [1, p. 37]).

Equations (2), (3) and (4) are known as the *navigation equations*. These equations take the form $\dot{\boldsymbol{X}} = f(\boldsymbol{X}, \boldsymbol{U})$, where X stands for the unknown kinematics (such as $\boldsymbol{v_e^n}$, $\boldsymbol{\rho}$, λ, ϕ and h) and U for the INS sensor outputs (such as f^p and $\boldsymbol{\Omega}_{ip}^p$). The double integration entails a drift in the stand-alone INS estimates due to a bias affecting the INS measurements. The bias terms $\boldsymbol{b_a}$ and $\boldsymbol{b_g}$ satisfy the standard equations:

$$\begin{aligned} \boldsymbol{f}_{\text{INS}}^p &= \boldsymbol{f}^p + \boldsymbol{b_a} + \boldsymbol{w_{b_a}}, && (5) \\ \boldsymbol{\Omega}_{ip_{\text{INS}}}^p &= \boldsymbol{\Omega}_{ip}^p + \boldsymbol{b_g} + \boldsymbol{w_{b_g}}, && (6) \end{aligned}$$

where the subscript INS stands for the quantities measured by the gyrometers and accelerometers and (w_{b_a}, w_{b_g}) are additive random errors. By replacing the specific force $\boldsymbol{f}^p$ and the angular rate $\boldsymbol{\Omega}_{ip}^p$ in the navigation equations by the values effectively measured by the sensors, we obtain the so-called *mechanization equations:* $\dot{\boldsymbol{X}}_{\text{INS}} = f(\boldsymbol{X}_{\text{INS}}, \boldsymbol{U}_{\text{INS}})$.

The drift in the INS outputs can be reduced by coupling INS and GPS systems, which is described in the next section.

2.3. Models for INS / GPS coupled units

Integrated INS-GPS is studied through tightly coupled architectures, which are expected to yield better estimations than loosely coupled units. They are more accurate in the sense that raw GPS pseudo-ranges are used as inputs for the navigation filter instead of GPS pre-processed data (position, velocity and time) associated with artificial noises.

2.3.1. State model

The filters classically used in navigation applications directly provide an estimate of the dynamical quantities of interest (position, velocity, attitude). However, the errors of the INS system form a more relevant choice for GPS/INS integration. Indeed, they require a lower update rate (compared to filters constructed from position, velocity and attitude), hence less computations. Besides, if GPS measurements go wrong, the integrated system can still rely on INS outputs.

The errors of the INS system are defined as the difference between the actual and the INS computed values $\delta \boldsymbol{X} = \boldsymbol{X} - \boldsymbol{X}_{\text{INS}}$. Their dynamic behavior is obtained by substracting the *mechanization equations* from the *navigation equations* (and neglecting the second-order terms), which produces:

$$\delta \dot{\boldsymbol{X}} = f(\boldsymbol{X}, \boldsymbol{U}) - f(\boldsymbol{X}_{\text{INS}}, \boldsymbol{U}_{\text{INS}}). \quad (7)$$

These equations can be linearized about the inertial quantities, provided INS errors are small enough. In this case, straightforward computations lead to (see [1] or [2] for more details):

$$\delta \dot{\boldsymbol{\rho}} = -\delta \boldsymbol{\Omega}_{in}^n - S(\boldsymbol{\Omega}_{in}^n) \delta \boldsymbol{\rho} + R_{p2n} \delta \boldsymbol{\Omega}_{ip}^p \quad (8)$$

$$\begin{aligned} \delta \dot{\boldsymbol{v}}_e^n = R_{p2n} \boldsymbol{b_a} + S(\boldsymbol{\rho}) \boldsymbol{f}^n - S(\boldsymbol{\Omega}_{en}^n + 2\boldsymbol{\Omega}_{ie}^n) \delta \boldsymbol{v}_e^n + \\ \delta \boldsymbol{g}^n - S(\delta \boldsymbol{\Omega}_{en}^n + 2\delta \boldsymbol{\Omega}_{ie}^n) \boldsymbol{v}_e^n, \quad (9) \end{aligned}$$

$$\delta \dot{\boldsymbol{p}}^n = S(\boldsymbol{\Omega}_{en}^n) \delta \boldsymbol{p}^n + \delta \dot{\boldsymbol{v}}_e^n, \quad (10)$$

where $\rho = (\Phi, \theta, \psi)^T$ is the vector of attitude angles linked to R_{p2n} (see (2)) and $p = (\lambda, \phi, h)^T$ the geodetic position (see (3)). In addition to these quantities, GPS errors such as the satellite clock offset and its derivative are usually considered. This leads to the following state vector:

$$\boldsymbol{\delta X} = (\delta \boldsymbol{v}_e^n, \delta \boldsymbol{\rho}, \boldsymbol{b_a}, \boldsymbol{b_g}, \delta\lambda, \delta\phi, \delta h, b, d), \quad (11)$$

where $b = c\tau_r$ denotes the GPS receiver clock offset in meters and d its derivative. The dynamic model used in this paper for (b, d) is defined by $\dot{b} = d + w_b$ and $\dot{d} = w_d$, where w_b and w_d are white Gaussian noises (see [1, p. 153]).

The state vector will be denoted $\boldsymbol{X}$ instead of $\boldsymbol{\delta X}$ for brevity. The discretization of the previous continuous-time state model takes the form:

$$X_{n+1} = A_n X_n + v_n, \quad (12)$$

where the additive noise is referred to as v_n and A_n is the following block diagonal matrix:

$$A_n = \begin{pmatrix} A_{vv} & A_{v\gamma} & A_{vba} & A_{vbg} & [0] & [0] \\ A_{\gamma v} & A_{\gamma\gamma} & [0] & A_{\gamma bg} & A_{\gamma p} & [0] \\ [0] & [0] & [0] & [0] & [0] & [0] \\ [0] & [0] & [0] & [0] & [0] & [0] \\ A_{pv} & [0] & [0] & [0] & A_{pp} & [0] \\ [0] & [0] & [0] & [0] & [0] & A_{bb} \end{pmatrix}.$$

The different elements of the matrix A_n are detailed in many textbooks such as [1, p. 204].

2.3.2. Measurement model

The observations are the GPS pseudo-ranges, which depend on the GPS clock offset b and the current position of the vehicule $(x, y, z)^T$ (in the rectangular coordinate system WGS-84 [1]). The observation equation associated to the *ith* satellite is given by:

$$\rho_i = \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} + b + w_i, \quad (13)$$

where $(X_i, Y_i, Z_i)^T$ is the position of the ith satellite. However, these observations have to be expressed as functions of the state vector components to make the filtering problem tractable. The transformation from the geodetic to the rectangular coordinate system is:

$$\begin{aligned} x &= (N + h + \delta h) \cos(\lambda + \delta\lambda) \cos(\phi + \delta\phi) && (14) \\ y &= (N + h + \delta h) \cos(\lambda + \delta\lambda) \sin(\phi + \delta\phi) && (15) \\ z &= (N + h + \delta h) \sin(\lambda + \delta\lambda), && (16) \end{aligned}$$

where $N = \frac{a}{\sqrt{1-e^2 \sin^2 \lambda}}$. The parameters a and e denote the semi-major axis length and the eccentricity of the earth's ellipsoid. These expressions have to be substituted in (13) to obtain the highly non-linear measurement equations.

3. PARTICLE FILTER

Sequential Monte Carlo methods SMC (also known as particle filtering methods) have recently received much attention in the signal processing litterature (see [3]). These methods allow to estimate the state of non linear possibly non-Gaussian dynamical systems. They have proved interesting alternatives to the extended Kalman filter in the case of highly non-linear systems. Indeed, the linearization around the latest estimate can lead to coarse approximations. On the contrary, SMC algorithms are expected to perform more efficiently since they take into account the salient structure of the model. Besides, these methods estimate the whole posterior probability density function (pdf) of the state given the observations, instead simple point estimates such as the minimum mean square error estimate. All inference on the unknown parameters can then be derived from the estimated posteriors within the Bayesian setting.

Particle filtering methods construct a point mass representation of a distribution, from a set of random samples (called particles) that explore the state-space. The principle of these methods is briefly recalled below. For convenience, the state space model is presented in a probabilistic form. The unobserved process $\{\boldsymbol{X}_n; n \in \mathbb{N}\}$ is completely described by its initial distribution $p(\boldsymbol{X}_0)$ and the transition pdfs $p(\boldsymbol{X}_n|\boldsymbol{X}_{0:n-1})$. The observations $\{\boldsymbol{Y}_n; n \in \mathbb{N}^*\}$ are assumed to be conditionally independent given the process $\{\boldsymbol{X}_n; n \in \mathbb{N}\}$ with distribution $g(\boldsymbol{Y}_n|\boldsymbol{X}_{0:n}, \boldsymbol{Y}_{1:n-1})$. The standard notations $\boldsymbol{X}_{0:n} = \{\boldsymbol{X}_0, \ldots, \boldsymbol{X}_n\}$ and $\boldsymbol{Y}_{1:n} = \{\boldsymbol{Y}_1, \ldots, \boldsymbol{Y}_n\}$ are used for the state and observation sequences (up to time n).

If we were able to draw N i.i.d. samples $\{\boldsymbol{X}^i_{0:n}, i = 1, \ldots, N\}$ from the unknown posterior distribution, the following empirical approximation:

$$p(\boldsymbol{X}_{0:n}|\boldsymbol{Y}_{1:n}) \simeq \frac{1}{N}\sum_{i=1}^{N} \delta(\boldsymbol{X}_{0:n} - \boldsymbol{X}^i_{0:n}) \qquad (17)$$

would have good convergence properties because of the strong law of large numbers. Unfortunately, it is usually impossible to sample from the sequence of distribution and alternative methods need to be used. The key idea of Importance Sampling (IS) is to represent the posterior distribution as a non-uniform discrete distribution defined by weighted particles through:

$$p(\boldsymbol{X}_{0:n}|\boldsymbol{Y}_{1:n}) \simeq \sum_{i=1}^{N} w(\boldsymbol{X}^i_{0:n})\delta(\boldsymbol{X}_{0:n} - \boldsymbol{X}^i_{0:n}) \qquad (18)$$

. The weights quantify the relevance of the particles with respect to the distribution of interest. Based on these comments, IS simulates samples according to an arbitrary proposal distribution $\pi(\boldsymbol{X}_{0:n}|\boldsymbol{Y}_{1:n})$ (called *importance distribution*) whose support includes the support of $p(\boldsymbol{X}_{0:n}|\boldsymbol{Y}_{1:n})$. The weights are computed as follows:

$$w(\boldsymbol{X}^i_{0:n}) = \frac{p(\boldsymbol{X}_{0:n}|\boldsymbol{Y}_{1:n})}{\pi(\boldsymbol{X}_{0:n}|\boldsymbol{Y}_{1:n})}, \qquad i = 1, \ldots, N \qquad (19)$$

An appropriate importance function (avoiding to store the past trajectories) allows to evaluate recursively the weights according to the following procedure:

$$\frac{w(\boldsymbol{X}^i_{0:n})}{w(\boldsymbol{X}^i_{0:n-1})} \propto \frac{g(\boldsymbol{Y}_n|\boldsymbol{X}^i_{0:n}, \boldsymbol{Y}_{1:n-1})p(\boldsymbol{X}^i_n|\boldsymbol{X}^i_{0:n-1})}{\pi(\boldsymbol{X}^i_n|\boldsymbol{X}^i_{0:n-1}\boldsymbol{Y}_{1:n})} \qquad (20)$$

More details regarding this classical operation can be found in [3]. A classical choice for the proposal distribution is $p(\boldsymbol{X}_n|\boldsymbol{X}_{0:n-1})$. The inefficiency and degeneracy of IS has been observed in many applications. Indeed, after a few time steps, all but one particle have weights close to 0. Thus, the last step of SMC filtering consists of *resampling* the particles, i.e. multiplying (resp. discarding) particles with high (resp. low) weights. In this paper, we use the stratified resampling procedure proposed by Kitagawa [4].

4. RAO BLACKWELLIZATION

Rao-Blackwellized filters (RBFs) have been proposed to reduce the variance of the state estimates [5]. The main idea of these filters is to integrate out some components of the state vector, thus reducing the dimension of the state-space to explore. The algorithm usually increases the efficiency of the estimation by making the most of the analytical structure of the model.

The INS-GPS state-space model is well suited to Rao Blackwellization. Indeed, the state vector can be partitioned so that some components can be marginalized analytically. Let us rewrite the state space equations to emphasize this interesting structure:

$$\boldsymbol{X}_{n+1} = \begin{pmatrix} A^1_n & C^2_n & [0] \\ C^1_n & A^2_n & [0] \\ [0] & [0] & A^3_n \end{pmatrix} \boldsymbol{X}_n + \begin{pmatrix} B^1_n & [0] \\ D^1_n & [0] \\ [0] & B^3_n \end{pmatrix} \begin{pmatrix} v^1_n \\ v^3_n \end{pmatrix} \qquad (21)$$

$$\boldsymbol{Y}_n = h_n(\boldsymbol{X}^2_n) + C^3_n \boldsymbol{X}^3_n + D^3_n \boldsymbol{W}_n \qquad (22)$$

where $\boldsymbol{X_n} = (\boldsymbol{X}^1_n, \boldsymbol{X}^2_n, \boldsymbol{X}^3_n)^T$ and $\boldsymbol{X}^1_n = (\delta \boldsymbol{v}^n_e, \delta\boldsymbol{\rho}, \boldsymbol{b_a}, \boldsymbol{b_g},)^T$, $\boldsymbol{X}^2_n = (\delta\lambda, \delta\phi, \delta h)^T$, $\boldsymbol{X}^3_n = (b, d)^T$. Note that the measurement equation does not depend on $\boldsymbol{X}^1_n$. Moreover, the observations depend non-linearly on $\boldsymbol{X}^2_n$ and linearly on $\boldsymbol{X}^3_n$. Consequently, $\boldsymbol{X}^1_{0:n}$ and $\boldsymbol{X}^3_{0:n}$ satisfy a linear Gaussian model conditionally to $\boldsymbol{X}^2_{0:n}$ and $\boldsymbol{Y}_{0:n}$. In other words, the posterior distribution of the state can be factorised as follows:

$$p(\boldsymbol{X}^1_{0:n}, \boldsymbol{X}^2_{0:n}, \boldsymbol{X}^3_{0:n}|\boldsymbol{Y}_{0:n}) = p(\boldsymbol{X}^1_{0:n}|\boldsymbol{X}^2_{0:n}) \times p(\boldsymbol{X}^3_{0:n}|\boldsymbol{Y}_{0:n}, \boldsymbol{X}^2_{0:n})p(\boldsymbol{X}^2_{0:n}|\boldsymbol{Y}_{0:n}) \qquad (23)$$

The distributions $p(\boldsymbol{X}^1_{0:n}|\boldsymbol{X}^2_{0:n})$ and $p(\boldsymbol{X}^3_{0:n})|\boldsymbol{Y}_{0:n}, \boldsymbol{X}^2_{0:n})$ are clearly Gaussian. Thus, their means and variances, denoted respectively $\boldsymbol{m}_{n|n,1}, \boldsymbol{m}_{n|n,3}, P_{n|n,1}$ and $P_{n|n,3}$, can be computed by standard recursions associated to the following Kalman state-space models:

$$\left\{ \begin{array}{lcl} \boldsymbol{X}^1_n & = & A^1_n \boldsymbol{X}^1_{n-1} + C^2_n \boldsymbol{X}^2_{n-1} + B^1_n v^1_n, \\ \boldsymbol{X}^2_n & = & C^1_n \boldsymbol{X}^1_{n-1} + A^2_n \boldsymbol{X}^2_{n-1} + D^1_n v^1_n, \end{array} \right. \qquad (24)$$

$$\left\{ \begin{array}{lcl} \boldsymbol{X}^3_n & = & A^3_n \boldsymbol{X}^3_{n-1} + B^3_n v^3_n, \\ \boldsymbol{Y}_n & = & h_n(\boldsymbol{X}^2_n) + C^3_n \boldsymbol{X}^3_n + D^3_n w_n, \end{array} \right. \qquad (25)$$

The posterior distribution of the state $\boldsymbol{X}^2_n$ (non linearly related to the observations) can be estimated by a SMC method:

$$p(\boldsymbol{X}^2_n|Y_n, \boldsymbol{X}^2_{0:n-1}) \simeq \sum_{i=1}^{N} w^{(i)}_n \delta(\boldsymbol{X}^2_n - \boldsymbol{X}^{2,i}_n), \qquad (26)$$

It should be noticed that the Kalman filters provide both the importance distribution $p(\boldsymbol{X}_n^2|\boldsymbol{X}_{0:n-1}^2)$ and the likelihood $p(Y_n|\boldsymbol{X}_{0:n}^2)$. The pdfs $p(\boldsymbol{X}_n^1|Y_{1:n}, \boldsymbol{X}_{0:n-1}^1)$ and $p(\boldsymbol{X}_n^3|Y_n, \boldsymbol{X}_{0:n-1}^3)$ are finally approximated by a mixture of Gaussian distributions:

$$p(\boldsymbol{X}_n^1|Y_n, \boldsymbol{X}_{0:n-1}^1) \simeq \sum_{i=1}^{N} w_n^{(i)} \mathcal{N}(\boldsymbol{X}_n^1, \boldsymbol{m}_{n|n,1}^i, P_{n|n,1}^i)$$

$$p(\boldsymbol{X}_n^1|Y_n, \boldsymbol{X}_{0:n-1}^3) \simeq \sum_{i=1}^{N} w_n^{(i)} \mathcal{N}(\boldsymbol{X}_n^3, \boldsymbol{m}_{n|n,3}^i, P_{n|n,3}^i).$$

5. SIMULATION RESULTS

Many simulations have been conducted to illustrate the performance of the RBF. A GPS-INS simulation can be divided into three parts:

- **Trajectory**: the vehicule dynamics is simulated according to *position-velocity-acceleration model*, which assumes that the acceleration is a random-walk.
- **INS data**: the sensor outputs are calculated from the trajectory via the *navigation equations*. They are corrupted by bias and additive noise according to (5) and (7). The efficiency of the algorithm is investigated for low cost inertial sensors whose biases have been modeled as random walks with initial values $b_a^0 = 500\mu g$ (accelerometers) and $b_g^0 = 1dgr/hr$ (gyrometers). The *mechanization equations* allow to compute the corresponding INS estimates (in position, velocity and attitude),
- **GPS data**: the pseudo-ranges corresponding to the satellites visible from the vehicule are evaluated (the variance of the measurement noise is $E[w_i^2] = 100$ meters). Beforehand, the satellite positions in their orbits have been calculated all along the trajectory.

This paper assumes that the drifts in altitude and vertical velocity have already been estimated thanks to an external source (a barometric altimeter for instance)as in [2]. Consequently, they are removed from the state vector.

The performance of the RBF (section 4) is studied for the estimation of latitude and longitude drifts. We have noted that the particles tend to explore regions of low posterior probability because the prior distribution does not overlap significantly with the likelihood. This is due to the small variance of the additive noise v_n appearing in the state model (12). This degeneracy increases with the resampling procedure that leads to *sample impoverishment*. Indeed, the particles with high weights are selected many times which precludes sample diversity. Regularization techniques are classically used to overcome this problem. One possible strategy consists of adding artificial noise to maintain enough samples with a high likelihood [6]. An alternative referred to as *auxiliary particle filter* [7] (APF) takes into account the value of the actual measurement instead of a blind exploration of the state-space. Both methods have provided similar results when applied to INS-GPS integration.

Figs 1 and 2 show the estimated INS drifts in latitude and longitude (obtained with APF regularization), computed from 100 Monte Carlo runs. Note that the INS drifts reach about a couple of kilometers for a simulation duration of $600s$. These figures (obtained with a reasonable number of samples $N = 1000$) show the good tracking performance of the RBF: the average error between the actual and estimated trajectories drops below 5 meters.

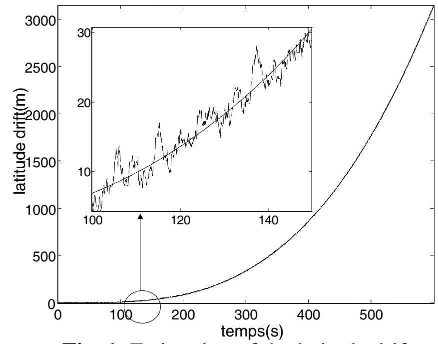

Fig. 1. Estimation of the latitude drift.

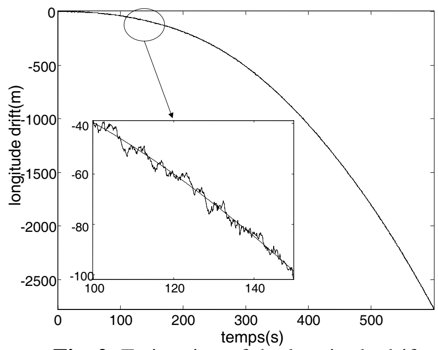

Fig. 2. Estimation of the longitude drift.

6. CONCLUSIONS

This paper has addressed the problem of INS-GPS integration by using a Rao-Blackwellized filter. This filter has shown interesting results for the proposed application. An importance function making use of the measurement might improve the results. A comparison with other estimation strategies (such as the extended Kalman filter) is currently under investigation. Different scenarios will be considered including loss of GPS measurements and/or multipath effects on GPS pseudo-ranges.

7. REFERENCES

[1] J. A. Farrell and M. Barth, *The Global Positioning System and Inertial Navigation*. New York: McGraw–Hill, 1999.

[2] P. J. Nordlund, *Sequential Monte Carlo filters and integrated navigation*. PhD thesis, Linkping University, Linkping, Sweden, 2002.

[3] A. Doucet, N. de Freitas, and N. Gordon, "An introduction to sequential Monte Carlo methods," in *Sequential Monte Carlo methods in practice* (A. Doucet, N. de Freitas, and N. Gordon, eds.), pp. 3–14, New York: Springer Verlag, 2001.

[4] G. Kitagawa, "Monte-Carlo filter and smoother for non-Gaussian nonlinear state space model," *Journal of Computational and Graphical Statistics*, vol. 5, no. 1, pp. 1–25, 1996.

[5] A. Doucet, S. J. Godsill, and C. Andrieu, "On sequential Monte Carlo sampling methods for Bayesian filtering," *Statistics and Computing*, vol. 10, no. 3, pp. 197–208, 2000.

[6] N. J. Gordon, D. J. Salmond, and A. F. M. Smith, "Novel approach to nonlinear/non-Gaussian Bayesian state estimation," *Proc. IEE-F*, vol. 140, pp. 107–113, April 1993.

[7] M. K. Pitt and N. Shephard, "Auxiliary variable based particle filters," in *Sequential Monte Carlo methods in practice* (A. Doucet, N. de Freitas, and N. Gordon, eds.), pp. 273–293, New York: Springer Verlag, 2001.