Robust Multivariable Predictive Control

How Can It Be Applied to Industrial Test Stands?

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One of the fundamental difficulties encountered throughout process control is the presence of time delay (often referred to as "dead time"). This time delay is often a result of the flow rate of material through a process. Consideration of this problem led to the development of predictive control strategies in the 1980s. Such algorithms are now widely used in industrial environments, and many successful applications have been reported in the literature (for instance, by [1]-[3]). As a result of theoretical work dedicated to advanced predictive control that has been conducted at ENSICA (see [4] and [5]), a multivariable predictive controller (MPC) has been developed to regulate a general M-input, N-output system in a stochastic framework. This MPC algorithm has been successfully applied [6] to an industrial test stand for air conditioning systems (Fig. 1).

The air conditioning system of an aircraft is used to regulate the cockpit temperature and pressure during flight and usually generates its airflow from the compressor turbine of the jet engine. Testing an air conditioning system requires simulation of the running conditions at ground level. More particularly, carrying out these experiments without running the jet engine requires the ability to simulate the thermodynamic conditions at the high-pressure stages. The certification of modern systems, whose operating conditions are subjected to wide-scale and very rapid fluctuations depending on the engine rating, requires advanced technologies. For this purpose, the French Aeronautical Test Center (Centre d'Essais Aéronautique de Toulouse, or CEAT) has recently developed a new test stand for air conditioning systems. An installation has been created to generate an airflow at a given temperature and pressure, controlling two high-pressure sources connected by two servovalves. The control law primarily installed uses two proportional-integral-differential (PID) controllers, implemented on an April 5000 automaton. These controllers are single-input, single-output (SISO) type (i.e., the multivariable nature of the system is not taken into account explicitly). The first results are not entirely satisfactory due to important coupling effects and insufficient dynamic performances relative to a very strict specification sheet: pressure gradients of 10 bar/s and temperature gradients up to 100 °C/s. Finally the last constraint is related to the real time implementation: the controller has to be implemented on the existing hardware system (i.e., the automaton), which can accommodate very limited memory and computation load.

The controller we propose in this article, denoted α-MPC, is a robust extension of the initial multivariable predictive control law that improves the disturbance-rejection properties of the closed-loop system, reducing the H_∞-norm of the multivariable sensitivity function with an extra parameter. This augmented algorithm has been chosen to carry out the new tests on the industrial process. Experimental recordings reported here have confirmed significant performance improvement with this new approach relative to the former PID regulation. The paper is organized as follows. First we introduce the original MPC. Next we describe the extended α-MPC algorithm and analyze the robustness of the closed-loop system through the H_∞ approach. Then we discuss the methodology of the control design task and describe the experimental test stand, focusing on the software and hardware implementation. Finally, we report the results of the α-MPC control law on the actual test stand. Special attention is given to the comparison with the former control system.

The Multivariable Predictive Controller

System Model

Ever since the original generalized predictive controller (GPC) was introduced by Clarke et al. in [7], studies have been done to extend such algorithms to the multivariable case, first in a deterministic framework [8], [9] and more recently in an entirely stochastic context [10], [11]. Our control law is mostly based on these last two approaches. The algorithm is developed for a general M-input, N-output system described by the following controlled auto-regressive integrated moving average (CARIMA) model:

$$A(q^{-1})\Delta(q^{-1})y(t) = B(q^{-1})\Delta u(t-1) + C(q^{-1})e(t) \qquad (1)$$

where $y(t)$, $u(t-1)$, and $e(t)$ are the output, the input, and the disturbance vectors of respective dimensions N, M, and N. The $\{e(t)\}$ sequence is assumed to satisfy

$$E\{e(t) \mid t-1\} = 0 \quad E\left\{e(t)\, e(t)^T\right\} = \sigma_0$$

where σ_0 is a positive definite matrix. $A(q^{-1})$, $B(q^{-1})$, and $C(q^{-1})$ are matrices of respective dimensions $N \times N$, $N \times M$, and $N \times N$ whose elements are polynomials in the unit delay operator q^{-1}, and $\Delta(q^{-1})$ is the diagonal polynomial matrix

$$\Delta\left(q^{-1}\right) = \mathrm{diag}_{i=1,\ldots,N}\left\{1-q^{-1}\right\}.$$

Finally, $C(q^{-1})$ is such that $C(0) = I$ and $\det C(q^{-1})$ has all its roots strictly outside the unit circle (in the q^{-1} plane). Notice that in most industrial applications a successful identification of this matrix is unlikely, and it would be preferable to consider it as a design parameter extending the robustness results introduced recently in the monovariable case [12].

Synthesis Filtered Predictions

In the original version of the GPC, Clarke et al. [7] introduced an auxiliary quantity $\Psi(t)$. It represents the output of a synthesis filter applied to the system response $y(t)$:

$$\Psi(t) = P_N\left(q^{-1}\right)\left\{P_D\left(q^{-1}\right)\right\}^{-1} y(t)$$

where $P_N(q^{-1})$ and $P_D(q^{-1})$ are $N \times N$-dimensional matrices of polynomials. This synthesis filter is used to tune the servo behavior of the closed-loop system.

In the same way, as proposed by Gu et al. [11] in the multivariable case, we have introduced a second intermediate variable that acts upon the frequency spectrum:

$$\Phi(t) = Q_N\left(q^{-1}\right)\left\{Q_D\left(q^{-1}\right)\right\}^{-1} \Delta u(t) \tag{2}$$

with $Q_N(q^{-1})$ and $Q_D(q^{-1})$ being $M \times M$-dimensional matrices of polynomials.

These additional variables have to be predicted over the prediction horizon H_p. Denote the j-step-ahead optimal predictor of the auxiliary output as

$$\hat{\Psi}(t+j) = E\left\{\Psi(t+j)\middle| t\right\}.$$

The two optimal predictors are respectively given by Kinnaert [10] and Gu et al. [11]:

$$\begin{aligned}\tilde{\Psi}(t) &= G\,\Delta\tilde{u}(t) + \tilde{\Psi}_0(t)\\ \tilde{\Phi}(t) &= T\,\Delta\tilde{u}(t) + \tilde{\Phi}_0(t)\end{aligned} \tag{3}$$

with

$$\begin{aligned}\tilde{\Psi}(t) &= \left[\hat{\Psi}(t+1)^T \ldots \hat{\Psi}(t+H_p)^T\right]^T\\ \tilde{\Phi}(t) &= \left[\Phi(t)^T \ldots \Phi(t+H_p-1)^T\right]^T\\ \Delta\tilde{u}(t) &= \left[\Delta u(t)^T \ldots \Delta u(t+H_p-1)^T\right]^T\end{aligned}$$

where G and T are lower triangular matrices of respective dimensions $(N \times H_p) \times (M \times H_p)$, and $(M \times H_p) \times (M \times H_p)$, resulting from Diophantine equations [13].

Notice that the global predictive model depends on future controls $\Delta\tilde{u}(t)$, and on what has been measured until time t through the right-hand terms $\tilde{\Psi}_0$ and $\tilde{\Phi}_0$.

Then we can express:

$$\begin{aligned}\tilde{\Psi}_0(t) = {} & H_{\mathrm{MPC}}\begin{pmatrix}\Delta u(t-1)\\ \vdots\\ \Delta u(t-\mathrm{deg}B)\end{pmatrix} + F_{\mathrm{MPC}}\begin{pmatrix}y_F(t)\\ \vdots\\ y_F(t-\mathrm{deg}F)\end{pmatrix}\\ & + V_{\mathrm{MPC}}\begin{pmatrix}e(t)\\ \vdots\\ e(t-\mathrm{deg}C+1)\end{pmatrix}\\ \tilde{\Phi}_0(t) = {} & S_{\mathrm{MPC}}\begin{pmatrix}\Delta u_F(t-1)\\ \vdots\\ \Delta u_F(t-\mathrm{deg}S-1)\end{pmatrix}\end{aligned}$$

where

$$\begin{aligned}y_F(t) &= \{P_D\}^{-1} y(t)\\ \Delta u_F(t) &= \{Q_D\}^{-1}\Delta u(t)\end{aligned}$$

with H_{MPC}, F_{MPC}, V_{MPC}, and S_{MPC} being matrices of respective dimensions $(N \times H_p) \times (M \times \mathrm{deg}B)$, $(N \times H_p) \times (M \times (\mathrm{deg}F+1))$, $(N \times H_p) \times (M \times \mathrm{deg}C)$, and $(M \times H_p) \times (M \times (\mathrm{deg}S+1))$, where

$$\begin{aligned}\mathrm{deg}F &= \max\left\{\mathrm{deg}P_N\ 1, \mathrm{deg}A + \mathrm{deg}P_D\right\}\\ \mathrm{deg}S &= \max\left\{\mathrm{deg}Q_N - 1, \mathrm{deg}Q_D - 1\right\}.\end{aligned}$$

Note: When X is a matrix of polynomials, $\mathrm{deg}X$ is the maximum degree of all the polynomials.

Control Law

With these equations, we can predict the behavior of the system and thus determine the best inputs to be applied. For this, we form the vector containing the H_p desired process outputs

$$\tilde{w}(t) = \left[w(t+1)^T \ \ldots \ w(t+H_p)^T\right]^T$$

and the corresponding predictions

$$\begin{aligned}\overline{\Psi}(t) &= \left[\Psi(t+1)^T \ \ldots \ \Psi(t+H_p)^T\right]^T\\ \overline{\Phi}(t) &= \left[\Phi(t)^T \ \ldots \ \Phi(t+H_p-1)^T\right]^T.\end{aligned}$$

Then the following criterion will be minimized:

$$J_1 = E\left\{\left\|\overline{\Psi}(t) - \tilde{w}\right\|_\Lambda^2 + \left\|\overline{\Phi}(t)\right\|_\Omega^2 \middle| t\right\} \tag{4}$$

where $\|x\|_R = x^T Rx$ and Λ and Ω are weighting diagonal matrices with respective dimensions $(N \times H_p) \times (N \times H_p)$ and $(M \times H_p) \times (M \times H_p)$. Later, we will consider $\lambda(j)$ and $r(j)$, the submatrices of Λ and Ω, with respective dimensions $N \times N$ and $M \times M$:

$$\Lambda = \begin{bmatrix} \lambda(1) & 0 & \cdots & 0 \\ 0 & \lambda(2) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \lambda(H_p) \end{bmatrix} \qquad \Omega = \begin{bmatrix} r(1) & 0 & \cdots & 0 \\ 0 & r(2) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & r(H_p) \end{bmatrix}$$

with

$$\lambda(j) = \begin{bmatrix} \lambda_1^j & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \lambda_N^j \end{bmatrix} \qquad r(j) = \begin{bmatrix} r_1^j & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & r_M^j \end{bmatrix}.$$

According to the two optimal predictors presented earlier, these two vectors satisfy

$$E\left\{\overline{\Psi}(t)|t\right\} = \tilde{\Psi}(t) \qquad E\left\{\overline{\Phi}(t)|t\right\} = \tilde{\Phi}(t).$$

On the assumption that the control increments are all taken to be zero after the control horizon N_u, the optimal control law is given by Vaucoret et al. [6]:

$$\Delta\tilde{u}(t) = \left(G_{N_u}^T \Lambda G_{N_u} + T_{N_u}^T \Omega T_{N_u}\right)^{-1} \left[G_{N_u}^T \Lambda \left(\tilde{w} - \tilde{\Psi}_0\right) - T_{N_u}^T \Omega\, \tilde{\Phi}_0\right] \tag{5}$$

where G_{N_u} (respectively, T_{N_u}) is the submatrix built from the $(M \times N_u)$ first rows of G (respectively, T). In a classic way, this control law will be implemented in the receding horizon sense. Thus, only the M first lines of the matrix relation (5) (thus forming matrices L_{MPC} and M_{MPC}) are needed to determine the control increment to be applied as the calculation is repeated at each sampling time

$$\Delta u(t) = L_{\mathrm{MPC}}\left(\tilde{w} - \tilde{\Psi}_0\right) - M_{\mathrm{MPC}}\tilde{\Phi}_0.$$

Dimensions of the L_{MPC} and M_{MPC} matrices are $(M \times NH_p)$ and $(M \times MH_p)$, respectively.

Robust Extension: The α-MPC Algorithm

Modified Criterion

Though there exist studies on the robustness of multivariable predictive control (see [14]), the user's constraints and specifications prevent any complex algorithm to be used. This is why we have tried to improve the above control law using few tuning parameters.

The second synthesis filter $Q_N(q^{-1})\{Q_D(q^{-1})\}^{-1}$ introduced through (2) improves the robustness of the final control law, as shown by Soeterboek [2] in the monovariable case. Indeed, when one frequency f_0 (e.g., a resonance frequency) in the controller output is to be attenuated, one can design the filter $Q(q^{-1}) = Q_N(q^{-1})\{Q_D(q^{-1})\}^{-1}$ such that this frequency f_0 is considerably more weighted than others. Then, minimizing the criterion (4), this frequency f_0 will be strongly attenuated in the controller outputs. Thus in the case of an open-loop stable process, a high-pass Q-filter will be a useful means of improving the robustness of the control law by attenuating high frequencies in the controller outputs (see [2]). At this point, the robustness of the closed-loop system can still be improved. Indeed, if the Q-filter amplifies high frequencies of the future increments $\Delta\tilde{u}(t)$ in (3) (which tend to smooth the controller behavior), it also increases the level of high-frequency disturbances in past events $\tilde{\Phi}_0$ (Fig. 2). All things considered, this tends to degrade the signal-to-noise ratio of past measurements. To counter this drawback, a correction term $\overline{\Phi}_1$ has been included in the criterion function

$$J_2 = E\left\{\left\|\overline{\Psi}(t) - \tilde{w}\right\|_\Lambda^2 + \left\|\overline{\Phi}(t) + \overline{\Phi}_1(t)\right\|_\Omega^2 |t\right\}.$$

A simple way to correct the effects of the high-pass Q-filter is to build this correction term from the past measurements

$$\overline{\Phi}_1(t) = -\alpha\ \tilde{\Phi}_0(t) \qquad \text{with} \quad 0 \le \alpha$$

such that past gauge measurements of the algorithm become

$$\tilde{\Phi}_0(t) + \overline{\Phi}_1(t) = (1-\alpha)\,\tilde{\Phi}_0(t).$$

The relative part of high frequencies in the frequency spectrum of this term is then attenuated for $0 \le \alpha$, leveled for $\alpha = 1$, as illustrated in Fig. 2, or even inverted in favor of low frequencies for $\alpha > 1$.

The α-MPC control law with the extra parameter is then directly derived from (5)

$$\begin{aligned}\Delta\tilde{u}(t) = {} & \left(G_{N_u}^T \Lambda G_{N_u} + T_{N_u}^T \Omega T_{N_u}\right)^{-1} \\ & \times\left[G_{N_u}^T \Lambda \left(\tilde{w} - \tilde{\Psi}_0\right) + (\alpha - 1)\, T_{N_u}^T \Omega\, \tilde{\Phi}_0\right].\end{aligned} \tag{6}$$

The effects obtained by filtering the errors and control moves can also possibly be obtained by replacing the weighting matrices Λ and Ω in (4) by two full nondiagonal positive definite matrices. But in the latter case, tuning is more difficult and the corresponding physical sense is very weak compared to the proposed approach. In an industrial context where the users of the control system are not specialized, this point is crucial.

Robustness Analysis and Design

From here on, the α-MPC algorithm is a robust extension of the initial control law that corresponds to the special case $\alpha = 0$. The H_∞-norm of the sensitivity function S_{yp} will be considered in analyzing the robustness of a given closed-loop system. Indeed, the literature [15] shows that the modulus margin ΔM is equal to the inverse of the maximum of the modulus of this function

$$\Delta M = \left(\max_{\omega} \left|S_{yp}\left(e^{-j\omega}\right)\right|\right)^{-1} = \left(\left\|S_{yp}\right\|_\infty\right)^{-1}.$$

As a consequence, the reduction of $\left\|S_{yp}\right\|_\infty$ will imply the increase of ΔM. Denote $\overline{\sigma}$ (respectively, $\underline{\sigma}$) the largest (respectively, smallest) singular value of the multivariable sensitivity function S_{yp} [16]. Then the H_∞-norm is defined as

$$\left\|S_{yp}\right\|_\infty = \sup_{\omega}\ \overline{\sigma}\left(S_{yp}\left(e^{-j\omega}\right)\right).$$

Thus far, the robust synthesis has been based upon the determination of an *optimal* value for α, denoted α_{opt} using the H_∞-norms of the sensitivity function S_{yp} and the complementary sensitivity function T_{yb}. But as a theoretical analysis of the robustness is not yet available, the design has been performed numerically. So, given a nominal model, an α-MPC controller is first tuned for $\alpha = 0$. Then, varying α, we look for the minimum of the H_∞-norm of the sensitivity functions S_{yp} and T_{yb}. The observed results from the simulations are as follows.

- Starting from $\alpha = 0$, as α increases, the H_∞-norms decrease smoothly or do not change (i.e., the robustness either improves or is constant but never deteriorates). In particular, one of the two considered norms can reach a minimum, as will be presented in the next case study. But the theoretical conditions leading to a minimum are not guaranteed yet, so only a numerical computation is performed. Once the minimum of one of the two norms is found, the corresponding α_{opt} is considered for the control law (6), and the plots of the singular values of $S_{yp}(e^{-j\omega})$ and $T_{yb}(e^{-j\omega})$ are used to verify *a posteriori* the gain over the robustness.
- In the case where either of the two norms reach as a minimum (i.e., when the norms continuously decrease), there exists a bound, denoted α_{bound}, beyond which the H_∞-norm rapidly increases. This bound has to be numerically detected. In addition, as we are considering the control of a real process, we apply a security factor of 20%. The magnitude of this margin is chosen as a function of the validity of the plant model (the more precise the model, the smaller the factor). Thus, in this case, we have

$$\alpha_{\text{opt}} = 0.8\ \alpha_{\text{bound}}.$$

As an illustration, we can consider the example of an SISO system, with H_∞-norms of the sensitivity functions as depicted in Figs. 5 and 6. In this case, only $\left\|S_{yp}\right\|_\infty$ is influenced by variations of α, with $\left\|T_{yb}\right\|_\infty$ remaining constant. In addition, the break on the H_∞-norms evolution is clearly shown, leading to about $\alpha_{\text{bound}} \approx 2.5$. Then, applying the security factor, the control law (6) will be implemented using $\alpha_{\text{opt}} = 0.8\ \alpha_{\text{bound}} = 2.0$, leading to an $\left\|S_{yp}\right\|_\infty$ gain of about 4 dB (Fig. 5).

Case Study

As a complete illustration of the robust control law design, let us consider the following two-input, two-output CARIMA model, which results from the on-line identification of the industrial plant, for a given nominal operating point of $P = 10$ bar and $T = 120$ °C:

$$\begin{bmatrix} A_{11}\left(q^{-1}\right) & A_{12}\left(q^{-1}\right) \\ A_{21}\left(q^{-1}\right) & A_{22}\left(q^{-1}\right) \end{bmatrix} \Delta\left(q^{-1}\right) \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{bmatrix} q^{-d_1} B_{11}\left(q^{-1}\right) & q^{-d_1} B_{12}\left(q^{-1}\right) \\ q^{-d_2} B_{21}\left(q^{-1}\right) & q^{-d_2} B_{22}\left(q^{-1}\right) \end{bmatrix} \begin{pmatrix} \Delta u_1 \\ \Delta u_2 \end{pmatrix} + C\left(q^{-1}\right) e(t)$$

with

$$\begin{aligned}
A_{11} &= 1 - 1.4067\, q^{-1} + 0.4348\, q^{-2} \\
A_{12} &= 0.0086\, q^{-1} - 0.0082\, q^{-2} \\
A_{21} &= -1.5319\, q^{-1} + 1.4497\, q^{-2} \\
A_{22} &= 1 - 1.5939\, q^{-1} + 0.6151\, q^{-2} \\
q^{-d_1} B_{11} &= 0.0056\, q^{-1} + 0.0894\, q^{-2} + 0.8134\, q^{-3} + 2.1714\, q^{-4} \\
q^{-d_1} B_{12} &= -0.0104\, q^{-1} + 0.0336\, q^{-2} + 0.0287\, q^{-3} + 0.3387\, q^{-4} \\
q^{-d_2} B_{21} &= -2.2561\, q^{-5} + 5.8590\, q^{-6} + 4.6272\, q^{-7} - 0.5204\, q^{-8} \\
q^{-d_2} B_{22} &= 3.5972\, q^{-5} + 3.4753\, q^{-6} + 3.2996\, q^{-7} + 1.5961\, q^{-8}
\end{aligned}$$

where y_1 stands for the pressure P, and where y_2 stands for the temperature T.

Note that time delay of five sampling periods for the last two polynomials in (29) is one of the motivations for the use of predictive control methods. This time delay, relative to the temperature control, includes the physical phenomena due to material propagation and the thermocouple dynamic.

The controller has been tuned with the following parameters:

$$H_p = 15 \quad N_u = 2 \quad \lambda(j) = \begin{bmatrix} 1.0 & 0 \\ 0 & 0.003 \end{bmatrix} \quad r(j) = \begin{bmatrix} 10.0 & 0 \\ 0 & 10.0 \end{bmatrix}$$

and with the following synthesis filters:

$$P_N(q^{-1}) = \begin{bmatrix} 1-0.9\,q^{-1} & 0 \\ 0 & 1-0.9\,q^{-1} \end{bmatrix} \quad P_D(q^{-1}) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$Q_N(q^{-1}) = \begin{bmatrix} 1-0.96\,q^{-1} & 0 \\ 0 & 1-0.96\,q^{-1} \end{bmatrix} \quad Q_D(q^{-1}) = \begin{bmatrix} 0.04 & 0 \\ 0 & 0.04 \end{bmatrix}.$$

The result of the controller design is clearly in accordance with the theory, in the sense that the synthesis filters are high-pass types, which is a reasonable way to improve robustness for open-loop stable processes [2].

Then, considering the evolution of $\|S_{yp}\|_\infty$ and $\|T_{yb}\|_\infty$ as a function of α (Figs. 7 and 8), it is found that $\|S_{yp}\|_\infty$ reaches a minimum for $\alpha = 1.28$, which will be chosen as α_{opt}. Finally, the control law (6) is calculated with $\alpha_{\text{opt}} = 1.28$, and the maximum and minimum singular values of the sensitivity functions are then plotted (Figs. 9 and 10) to confirm the obtained gain: about 1 dB for $\|S_{yp}\|_\infty$ and $8\,dB$ for $\|T_{yb}\|_\infty$.

Looking more precisely at Figs. 7 and 8, it appears that a greater value could have been chosen for α. Indeed, it can be observed on Fig. 7 that $\|S_{yp}\|_\infty$ is increasing very little from $\alpha = 1.28$ to $\alpha = 2.8$, while $\|T_{yb}\|_\infty$ is still decreasing significantly. In this case, the choice for α may be made considering which of the two sensitivity functions must be improved as a priority.

Finally, it is interesting to perform a trade-off between the closed-loop behavior of both controllers, with $\alpha = 0$ and $\alpha = 1.28$. The corresponding plots (Figs. 11 and 12) show that the global performance is not modified; the time responses are identical, while a minor deterioration of decoupling is observed.

Control Design for the Experimental Test Stand

The simulation of onboard air conditioning led the CEAT to build a test stand, called REBECA, which is depicted in Fig. 1. To run under variable pressure conditions, the tests are performed inside a large vacuum chamber, whose diameter is about 6 m. The different supplies (hot and cold high-pressure sources) are located outside the chamber. The test stand is controlled by means of two valves u_{hot} and u_{cold}, and the variables to be controlled are the airflow pressure P and temperature T. The main components of the overall system are depicted in Fig. 2. Inside the vacuum chamber, the two airflows mix in a tube (77 mm in diameter and 3 m in length). The expected performances are: pressure gradients of 10 bar/s and temperature gradients up to 100 °C/s without offset. Finally, to simulate extended flight domains, pressure and temperature should vary widely (P from 5 to 25 bar, T from 50 to 400 °C).

Numerical Simulation of the Process

The first step of the control design was to develop a software simulation of the physical phenomena, the most difficult point being the transition between subsonic and supersonic airflows. Indeed, an analytical study of the thermodynamics led to the following expression for the temperature and pressure derivatives.

- Subsonic conditions

$$\frac{dP}{dt} = (Au_{\text{hot}} + Bu_{\text{cold}})T\sqrt{P_1^2 - P^2} - \alpha QT$$
$$\frac{dT}{dt} = (Cu_{\text{hot}} + Du_{\text{cold}})\frac{T}{P}\sqrt{P_1^2 - P^2} - \beta Q\frac{T}{P}$$

- Supersonic conditions

$$\frac{dP}{dt} = \left(\bar{A}u_{\text{hot}} + \bar{B}u_{\text{cold}}\right)T - \alpha QT$$
$$\frac{dT}{dt} = \left(\bar{C}u_{\text{hot}} + \bar{D}u_{\text{cold}}\right)\frac{T}{P} - \beta Q\frac{T}{P}$$

where $A, B, C, D, \bar{A}, \bar{B}, \bar{C}, \bar{D}$, and α, β are constant, and Q is the airflow. The transition between both flow conditions is determined by the difference between the air supplies upper pressure P_1 and the airflow pressure P; the flow is subsonic (respectively, supersonic) when this difference is lower (respectively, higher) than the following limit value:

$$\Delta P_{\text{lim}} = P_1\left(1 - \sqrt{\frac{10.013}{11.56}C_f^2}\right)$$

with C_f being a dimensionless coefficient characterizing the servovalves. In our case, $P_1 = 25$ bar and $C_f = 0.8$, so that $\Delta P_{\text{lim}} = 6.4$ bar. Thus, the transition between both flow types will occur around 18 bar.

The nonlinear behavior is obvious from the above equations, and in the same way coupling effects will be inherent. It is also interesting to note that the dynamics are a function of the airflow, which is neither controlled nor measured during normal operating conditions; this is simply because the airflow is subject to unpredictable evolution during flight.

The simulator has been validated by comparing real-time and simulated recordings. As the plant was initially running in a closed loop with two PID controllers, we have simulated the entire control loop, including controllers. The results were very good; comparing both outputs, errors between simulations and real data remained very small, even during transients due to setpoint changes. This simulator will be the basis for the next step of identification, and later for roughly tuning the MPC parameters. Indeed, running periods are very hard to negotiate (and, of course, very expensive), because one fundamental constraint prescribed by the CEAT was that the control design tests should not disturb the normal (and commercial) use of the test stand.

Preliminary Identification

The test stand has a wide operating range, with temperatures from 150 °C up to 650 °C and pressures from 2 up to 40 bar. Thus, the global behavior is nonlinear, and it is clear that a unique finite dimensional linear model cannot render the entire input-output behavior for any operating condition. On the other hand, the constraint of a unique controller with fixed parameters completely prevents consideration of any adaptive method. So the first step has been an open-loop identification of the two-input, two-output model, considering various operating zones. The two input signals were an uncorrelated pseudorandom binary sequence (PRBS) with fixed amplitude (10% of the maximum amplitude). The sampling period is $T_s = 40$ ms. The operating ranges have been chosen by the test stand users. Fig. 13 illustrates the resulting zones in terms of standard deviations around the nominal points for both temperature and pressure.

The standard deviations of the 11 tests clearly reveal the two kinds of behavior: the first four tests with pressures lower than 18 bar cover the supersonic operating range, whereas the others tests cover the subsonic operating range. In addition, Fig. 7 clearly shows that the open-loop gain drops as pressure increases, and that this gain is less dependent on temperature.

The model structure, directly from (1), is as follows:

$$\begin{bmatrix} A_{11}(q^{-1}) & A_{12}(q^{-1}) \\ A_{21}(q^{-1}) & A_{22}(q^{-1}) \end{bmatrix} \Delta(q^{-1}) \begin{pmatrix} P \\ T \end{pmatrix} = \begin{bmatrix} q^{-d_1} B_{11}(q^{-1}) & q^{-d_1} B_{12}(q^{-1}) \\ q^{-d_2} B_{21}(q^{-1}) & q^{-d_2} B_{22}(q^{-1}) \end{bmatrix} \begin{pmatrix} \Delta u_1 \\ \Delta u_2 \end{pmatrix} + C(q^{-1}) e(t)$$

where u_1 and u_2 now stand for u_{hot} and u_{cold}.

The first step was to determine the two time delays d_1 and d_2. This was done by minimizing Akaike's Final Prediction Error (FPE) criterion [17]:

$$\text{FPE} = \frac{1 + \frac{\dim\Theta}{N}}{1 - \frac{\dim\Theta}{N}} V_N(\Theta)$$

where Θ is the model parameter vector, N is the number of data points, and $V_N(\Theta)$ is the determinant of the estimated covariance matrix of the innovations. The minimal value has been obtained for

$$d_1 = 1, \qquad d_2 = 5.$$

These results, in accordance with reality (pressure sensors are quite instantaneous, thermocouples much slower), are another argument for the use of long-range predictive control techniques: Indeed, the presence of this (not negligible) pure time delay renders classical PID regulation insufficient, as will be shown next.

The second step was the choice of a *nominal model* as a basis for the MPC design. This model has been computed minimizing the sum of squared prediction errors using least-squares methods [18]. It describes the process behavior over the entire supersonic operating range

$$\begin{aligned}
A_{11}(q^{-1}) &= 1 - 0.5359q^{-1} - 0.4031q^{-2} \\
A_{12}(q^{-1}) &= 0.0012q^{-1} - 0.0005q^{-2} \\
A_{21}(q^{-1}) &= -0.4039q^{-1} + 0.3128q^{-2} \\
A_{22}(q^{-1}) &= 1 - 0.5040q^{-1} - 0.4122q^{-2} \\
q^{-d_1} B_{11}(q^{-1}) &= 0.0762q^{-1} + 0.1927q^{-2} + 1.0249q^{-3} + 1.5084q^{-4} \\
q^{-d_1} B_{12}(q^{-1}) &= 0.0248q^{-1} + 0.0411q^{-2} + 0.2994q^{-3} + 0.4109q^{-4} \\
q^{-d_2} B_{21}(q^{-1}) &= 4.1547q^{-5} + 6.2671q^{-6} + 7.1529q^{-7} + 11.642q^{-8} \\
q^{-d_2} B_{22}(q^{-1}) &= 9.4131q^{-5} + 10.081q^{-6} + 8.7914q^{-7} + 17.224q^{-8}.
\end{aligned}$$

To validate this model, we have expressed the correlation rates ρ_{PP_s} and ρ_{TT_s} between the measured outputs (P and T) and the simulated ones (P_s and T_s) obtained with the same input PRBS. This has been realized with the first four tests, which cover the supersonic operating range. Thus, with correlation rates always greater than 96%, this mean model has been validated over the entire supersonic operating range.

Controller Implementation

As previously mentioned, the system is controlled by an April 5000 automaton. As this automaton also covers other tasks (mainly monitoring and supervision of the whole test stand), it was not possible to replace it with another hardware system more adapted to control tasks. This constraint led to a drastic optimization of the computation burden to make it compatible with the sampling period. Indeed, this industrial automaton is not especially dedicated to the implementation of sophisticated amenable controllers.

In the monovariable case, the algorithm is usually rewritten in polynomial form:

$$R(q^{-1})u(t) = T(q^{-1})w(t) - S(q^{-1})y(t)$$

where $R(q^{-1})$, $S(q^{-1})$, and $T(q^{-1})$ are polynomials easily calculated from the model and from the controller parameters. From there, implementation of the controller in the C language is very easy and not time consuming.

In the multivariable case, however, the polynomial form is not obvious, and of course $R(q^{-1})$, $S(q^{-1})$, and $T(q^{-1})$ become polynomial matrices. To render the algorithm as simple as possible, we first made an assumption about the disturbance polynomial (i.e., $C(q^{-1}) = I_d$), and the synthesis filters were chosen so that

$$P_D(q^{-1}) = \begin{bmatrix} p_{D_1} & 0 \\ 0 & p_{D_2} \end{bmatrix} \qquad Q_D(q^{-1}) = \begin{bmatrix} q_{D_1} & 0 \\ 0 & q_{D_2} \end{bmatrix}$$

(i.e., matrices with constant parameters). Then the expressions of the $R(q^{-1})$, $T(q^{-1})$, and $S(q^{-1})$ polynomial matrices are made simpler [13]. Finally, the calculation time has been measured to be less than 20 ms, which is compatible with the 40 ms sample time constraint.

Results

Two-Controller Structure

To date, the industrial process was controlled with two independent PIDs, one for controlling pressure and the other for temperature. For the treated example, set points are chosen to be square signals with large amplitudes (10 – 20 bar and 200 – 450 °C to explore wide operating zones (Fig. 14). The performance is summarized in Table 1, where both 95% rise times and the maximum deviations, as measured on the real test stand, are listed.

Both the rise times and deviations are greater than expected. In addition, coupling between the two control loops is greatly affecting the global behavior, leading to critical oscillations when the operating zone is quite different from the zone considered to tune the PID controllers. Clearly, in this case, the PID controllers have been tuned for large pressure and temperature levels, but not for $T = 200$ °C and $P = 20$ bar (Fig. 14).

A good way to compare the capabilities of predictive control with the existing control structure is to keep the two SISO loops and to replace the PID controllers with SISO α-MPCs. Using the same criteria as before, the results are reported in Table 2. The controllers have been tuned using the parameters of Table 3.

Note: These results are not sufficient to render the benefits of predictive control and to compare with the former method. Indeed, taking into account the user specifications, the performance can be shaped in different ways using the controller parameters H_p, N_u, the weighting matrices Λ and Ω, and the synthesis filters.

One-Controller Structure

The α-MPC was designed and tuned using numerical simulations; this first step resulted in a rough "pretuning" of the controller parameters and especially in the calculation of the optimal value for α. The controller was then implemented on the April 5000 in the polynomial form described earlier. The corresponding plots, shown in Fig. 15, illustrate a large improvement of both transients and coupling rejection. With maximum deviations kept smaller than 1.5 bar for pressure and 40 °C for temperature, the α-MPC also resulted in substantial reduction in rise times, as reported in Table 4.

The controller was tuned using the following parameters:

$$H_p = 15 \quad N_u = 2 \quad \alpha = 2$$

$$\lambda(j) = \begin{bmatrix} 1 & 0 \\ 0 & 0.003 \end{bmatrix} \qquad r(j) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$P_N = \begin{bmatrix} 1-0.9q^{-1} & 0 \\ 0 & 1-0.9q^{-1} \end{bmatrix} \qquad P_D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$Q_N = \begin{bmatrix} 1-0.96q^{-1} & 0 \\ 0 & 1-0.96q^{-1} \end{bmatrix} \qquad Q_D = \begin{bmatrix} 0.04 & 0 \\ 0 & 0.04 \end{bmatrix}.$$

The weighting parameters are the same as in the two-controller case, so only the prediction and control horizons have been changed. In particular, H_p has been doubled, thus improving stability robustness, as is well known [2], without any deterioration of performance. Changing H_p from 6 to 15 has not affected the transient performance.

It should be underscored that the performance improvement, in terms of time responses and coupling rejection, did not lead to excessive variance of both output signals (from the user's point of view). These parameters remain inside reasonable limits, as do the input signals. This particular point is certainly the main advantage of the full multivariable algorithm with respect to the previous one. Indeed, as the model explicitly accounts for the coupling effects, tuning the controller parameters to get high performance *and* sufficiently smooth signals is feasible. Figs. 16 and 17 report other tests realized with this new controller.

Concerning the robustness analysis, Table 5 reports the H_∞-norms of the sensitivity functions for the classical multivariable controller ($\alpha = 0$) and the proposed one ($\alpha = 2$). In this case, the robustness gain is significant mainly for the complementary sensitivity function $\|T_{yb}\|_\infty$, so the robustness of the closed loop with respect to output disturbances is greatly improved. This point is crucial in considering such an industrial application.

Conclusion

To cope with recent technological evolutions of air conditioning systems for aircraft, the French Aeronautical Test Center built a new test stand for certification at ground level. The constraints specified by the industrial users of the process seemed antagonistic for many reasons. First, the controller had to be implemented on an industrial automaton, not adaptable to modern algorithms. Then the specified dynamic performances were very demanding, especially taking into account the wide operating ranges of the process. Finally, the proposed controller had to be easy for nonspecialist users to handle.

Thus, the control design and implementation steps had to be conducted considering both theoretical and technical aspects. This finally led to the development of a new MPC, called α-MPC, whose main characteristic is the introduction of an extra tuning parameter α that has enhanced the overall control robustness. In particular, the H_∞-norm of the sensitivity functions can be significantly reduced by tuning this single new parameter. This turns out to be a simple but efficient way to improve the robustness of the initial algorithm. The other classical tuning parameters are still physically meaningful, as is usual with predictive techniques. The initial results are very promising and this controller has already been adopted by the industrial users as the basis of the control part for future developments of the test stand.

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Figure 1. *Global view of the industrial test stand.*

Figure 2. *Schematic diagram of the test stand.*

Figure 3. *Example of* $\tilde{\Phi}(t)$ *frequency spectrum for a classical MPC design.*

Figure 4. *Example of* $\tilde{\Phi}(t)$ *frequency spectra for several* α*-MPC designs.*

Figure 5. H_∞*-norm of* S_{yp} *as a function of* α*: SISO example. Sensitivity function* S_{yp}.

Figure 6. H_∞*-norm of* T_{yb} *as a function of* α*: SISO example. Complementary sensitivity function* T_{yb}.

Figure 7. H_∞*-norm of* S_{yp} *as a function of* α*. Sensitivity function* S_{yp}

Figure 8. H_∞*-norm of* T_{yb} *as a function of* α*. Complementary sensitivity function* T_{yb}

Figure 9. *Singular values of the sensitivity function* S_{yp} *for* $\alpha = 0$ *and* $\alpha = 1.28$.

Figure 10. *Singular values of the complementary sensitivity function* T_{yb} *for* $\alpha = 0$ *and* $\alpha = 1.28$.

Figure 11. *Closed-loop response for* $\alpha = 0$.

Figure 12. *Closed-loop response for* $\alpha = 1.28$.

Figure 13. *Operating zones for the open-loop identification.*

Figure 14. *Experimental results using PID control.*

Figure 15. *Experimental results using* α*-MPC control.*

Figure 16. *Example of closed-loop response at constant temperature.*

Figure 17. *Example of closed-loop response at constant pressure.*

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Callouts:

The first step of the control design was to develop a software simulation of the physical phenomena..

The test stand has a wide operating range, and thus the global behavior is nonlinear.

These results (instantaneous pressure sensors, slower

thermocouples) are another argument for the use of long-range predictive control techniques.

Table 1. Closed-loop performance with two PID controllers.			
Pressure		Temperature	
$t_{95\%}$	Max. deviation	$t_{95\%}$	Max. deviation
3.7 s	2 bar	8.4 s	50 °C

Table 2. Closed-loop performances with two α-MPC controllers.			
Pressure		Temperature	
$t_{95\%}$	Max. deviation	$t_{95\%}$	Max. deviation
2.4 s	1.9 bar	5 s	40 °C

Table 3. Parameter tuning for the two α-MPC controllers.

	H_p	N_u	Λ	Ω	α
$\alpha - \text{MPC}_1$	6	3	I_6	$10I_6$	2
$\alpha - \text{MPC}_2$	8	3	$0.003I_6$	$10I_6$	2

Table 4. Closed-loop performances with one α-MPC controller.			
Pressure		Temperature	
$t_{95\%}$	Max. deviation	$t_{95\%}$	max. deviation
2.4 s	1.5 bar	2.2 s	40 °C

Table 5. H_∞-norm of the sensitivity functions.

	$\alpha = 0$	$\alpha = 2$
$\lVert S_{yp} \rVert_\infty$	16.9 dB	15.7 dB
$\lVert T_{yb} \rVert_\infty$	22.5 dB	13.6 dB