

Turbulence et vortex à densité variable

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1. Mélange inhomogène : exemples et modèle de description
2. Méthodes de projection et code pseudo-spectral,
3. Dynamique rotationnelle en milieu inhomogène,
4. Vortex lourd versus vortex léger (2D),
5. Ségrégation de masse en turbulence bidimensionnelle,

I *Ecoulements géophysiques*
Ecoulements inhomogènes accélérés

Milieu inhomogène dans un champ de gravité

Milieu inhomogène dans un champ d'accélération externe

II *Mélange à grand nombre de Froude*

Milieu inhomogène et accélération interne

Quelques exemples ...

Expériences de réservoir incliné : « A method of producing a shear flow in a stratified fluid »
S.A. Thorpe JFM32 1968

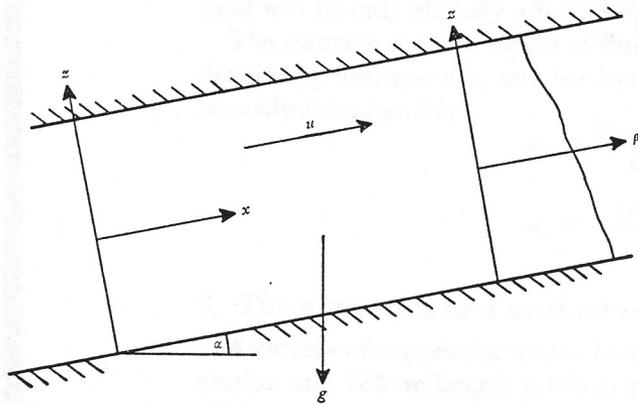


FIGURE 1. Notation.

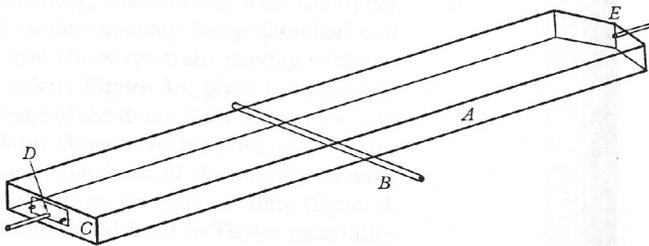
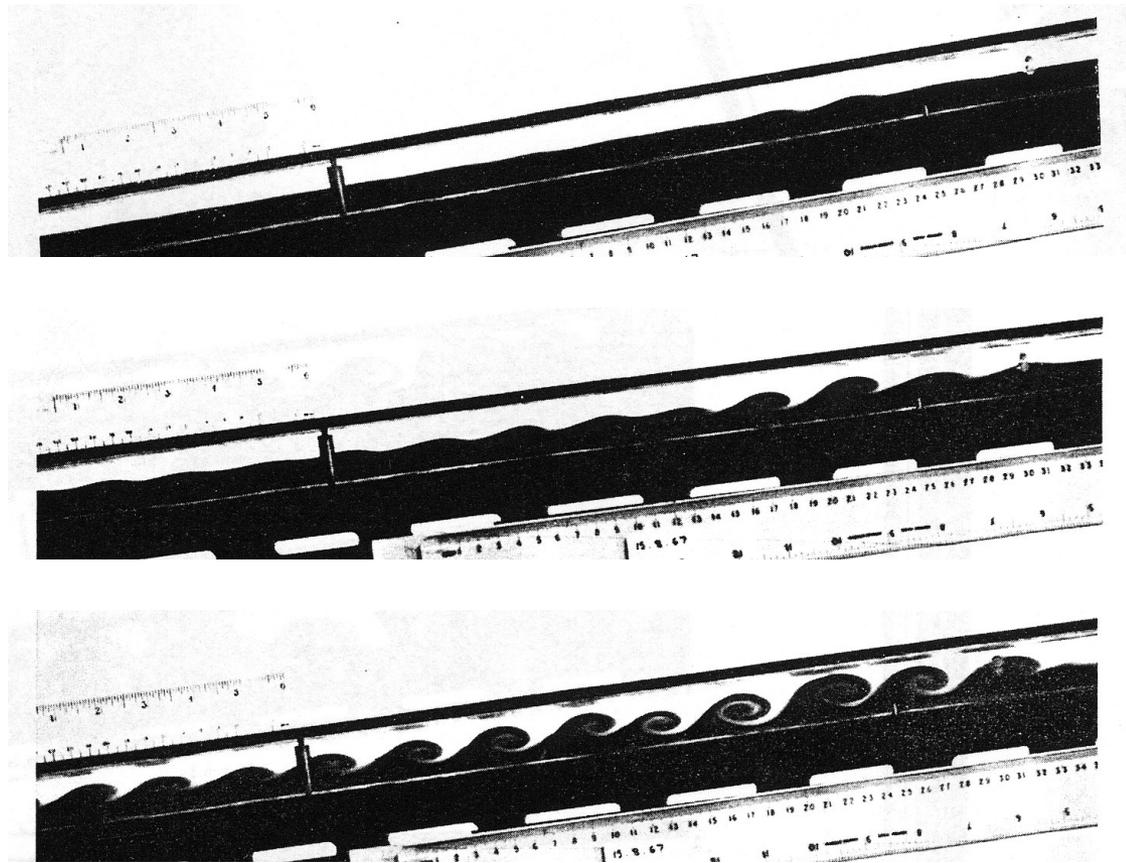


FIGURE 2. The apparatus.



Two-dimensionnal Stratified Mixing Layer (vertical shear)

Klaassen and Peltier JFM227 (1991)

Re=300

$$Ri = g \frac{\Delta\rho}{\bar{\rho}} \frac{\delta\omega}{\Delta U^2} = \frac{1}{Fr^2}$$

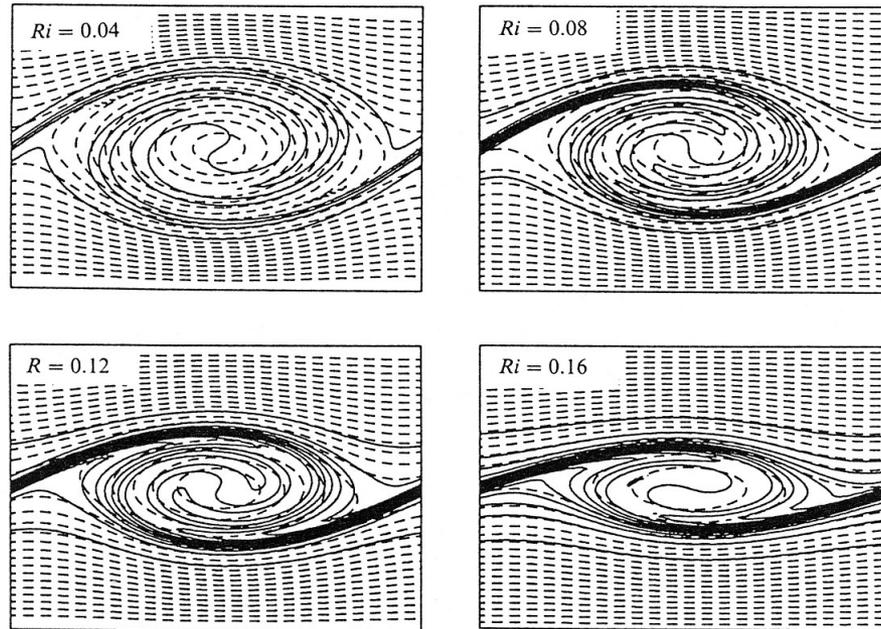
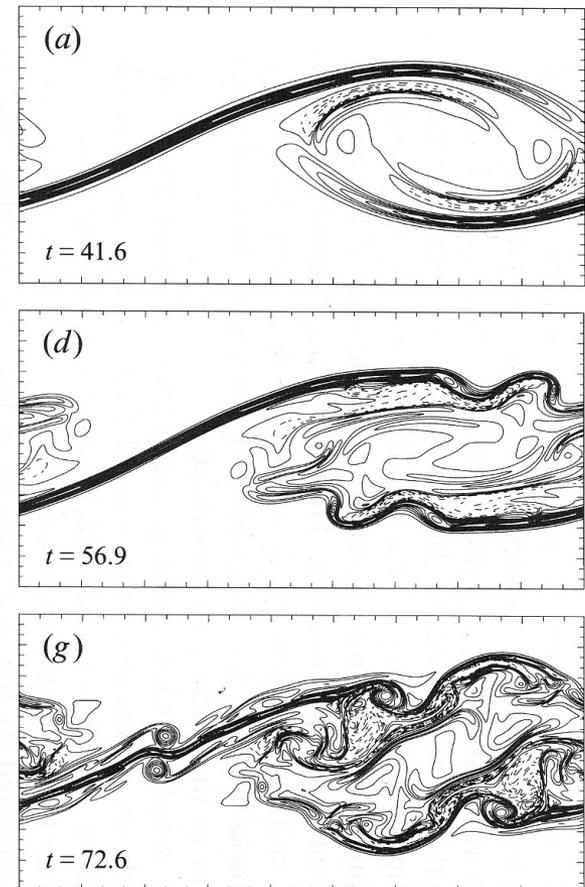


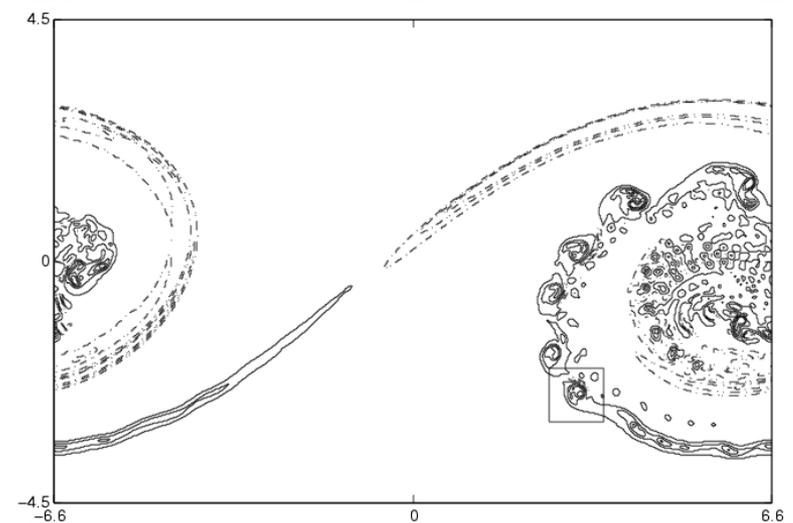
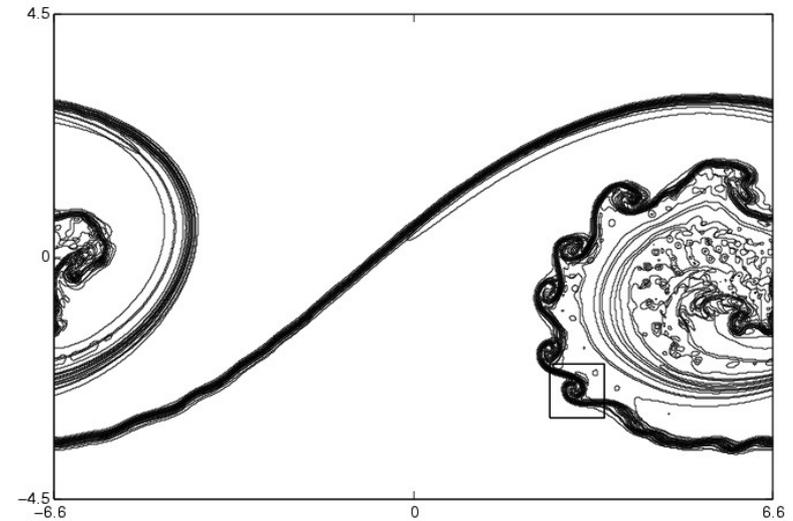
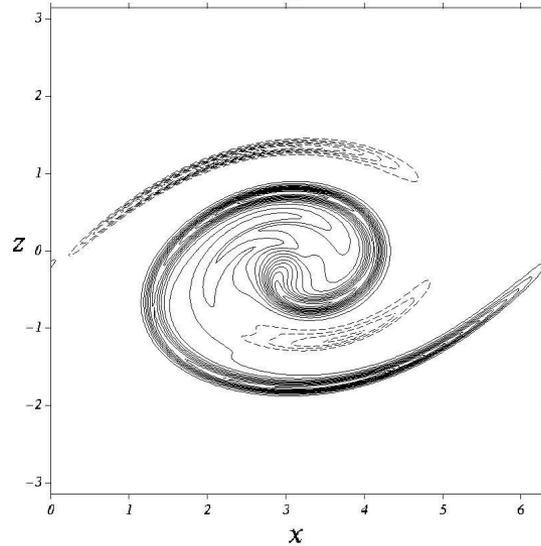
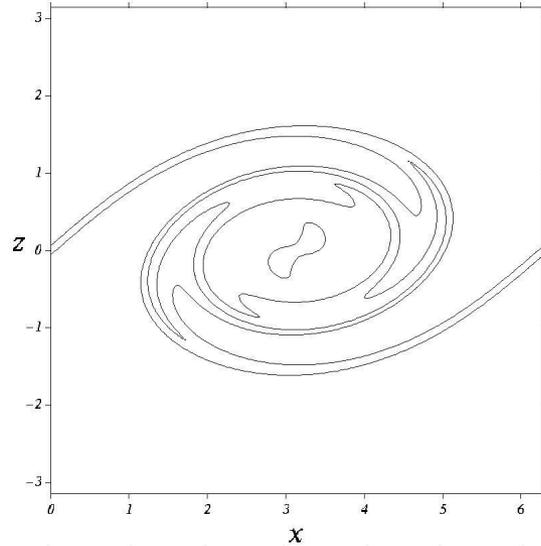
FIGURE 2. Stream function (dashed contours) and potential temperature field (solid contours) for stratified Kelvin-Helmholtz billows at various kinetic Richardson numbers Ri and $Re = 300$, $Pr = 1$. The waves are shown at the times of maximum kinetic energy, which are $t = 26$ ($Ri = 0$), 30 ($Ri = 0.04$), 34 ($Ri = 0.08$), 42 ($Ri = 0.12$) and 52 ($Ri = 0.16$). Contour intervals are the same for each wave. The horizontal period is $14h$ and the domain height is $10h$.

Staquet JFM296 (1995)

Re=2000 - Ri = 0.167



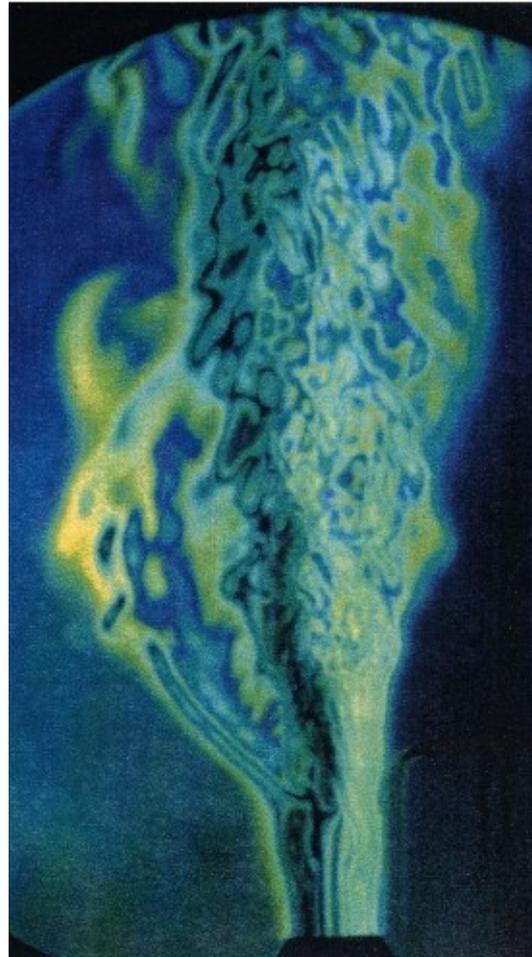
« Two-dimensional secondary baroclinic instability » Reinaud, Joly and Chassaing,
PoF vol 12(10), pp 2489-2505, 2000



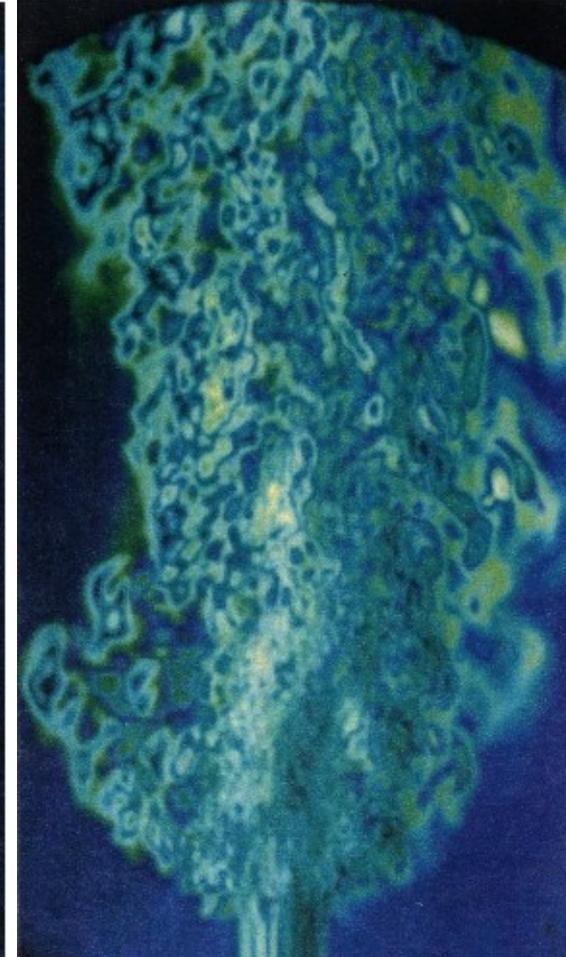
Side ejections in a round laminar helium-jet $S=0,14$: Hermouche, Thèse IMFT 1996



Re = 750



Re = 1000



Re = 2600

Un modèle pour le ...

- II** *Mélange à grand nombre de Froude et faible nombre de Mach*

Kleinermann et Majda (1982), Paolucci (1985), Cook et Riley (1996)

$$\frac{1}{\tau_d} d^* = \frac{M^2}{\gamma \tau_s} \left(-\frac{1}{p} (d_t p + (\gamma - 1)\phi) \right)^* - \frac{C_\theta}{Re Pr} \frac{1}{\tau_s} \left(\frac{1}{\rho c_p T} \nabla \cdot \mathbf{q} \right)^*$$

Mach asymptotiquement nul :

$$\left| \begin{aligned} d &= -\frac{1}{\rho c_p T} \nabla \cdot \mathbf{q} \\ \tau_d &\sim \frac{Re Pr}{C_\theta} \tau_s \end{aligned} \right.$$

$$d = -\nabla \cdot \left(\frac{a}{\rho} \nabla \rho \right)$$

1. Concentration

$$\rho = \alpha \rho C + \beta$$

2. Transport de scalaire passif

$$\rho \, d_t C = \nabla \cdot (\rho a \nabla C)$$

3. Continuité

$$d = \nabla \cdot \mathbf{u} = -\frac{1}{\rho} d_t \rho$$

$$d = -\nabla \cdot \left(\frac{a}{\rho} \nabla \rho \right)$$

$$\frac{1}{\rho} d_t \rho = -d = \nabla \cdot \left(\frac{a}{\rho} \nabla \rho \right)$$

$$\varrho = \ln \rho$$

Sandoval (1995)

$$d_t \varrho = \nabla \cdot (a \nabla \varrho)$$

Deux types de contraintes divergentielles

$$\nabla \cdot \mathbf{u} = -\nabla \cdot (a \nabla \varrho)$$

$$\nabla \cdot (\rho \mathbf{u}) = -\partial_t \rho$$

Dynamique sous forme conservative :

$$\partial_t(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot (2\mu \mathcal{S} - \frac{2}{3}\mu d\mathcal{I})$$

Dynamique sous forme transport :

$$d_t \mathbf{u} = -\frac{1}{\rho} \nabla p + \frac{1}{\rho} \nabla \cdot \mathcal{T} \quad \pi = p/\rho$$

$$d_t \mathbf{u} = -\nabla \pi - \pi \nabla \varrho + \nu_\rho \nabla \cdot (2\mathcal{S} - \frac{2}{3}d\mathcal{I})$$

Non-linéarité

Contraste de densité :

$$C_\rho = \Delta\rho/\bar{\rho}$$

$$d_t \varrho = \frac{1}{ReSc} \Delta \varrho = -\frac{1}{C_\rho} d$$

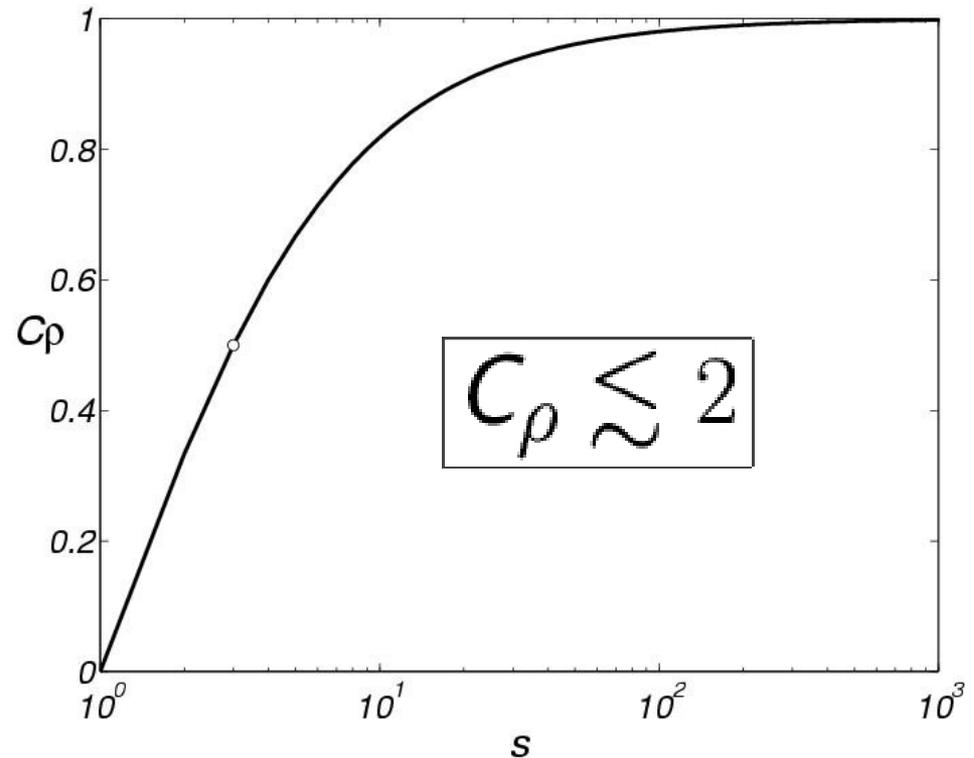
$$d_t \mathbf{u} = -\nabla \pi - C_\rho \pi \nabla \varrho + \frac{1}{Re} \Delta \mathbf{u}$$

$$s = \rho_2 / \rho_1$$

$$C_\rho = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

$$C_\rho = (s - 1) / (1 + s)$$

Condition : $2\rho_m - \Delta\rho > 0$



$$d_t \varrho = \frac{1}{Re Sc} \Delta \varrho = -\frac{1}{C_\rho} d$$

$$d_t \mathbf{u} = -\nabla \pi - \underline{C_\rho \pi \nabla \varrho} + \frac{1}{Re} \Delta \mathbf{u}$$

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Chorin (1969) et Bell, Colella, Glaz (1989)

Décomposition de hodge :

$$\mathbf{u} = \mathbf{u}_d + \nabla \phi, \quad P(\mathbf{u}) = \mathbf{u}_d$$

Incompressible homogène : $\mathbf{u} = \mathbf{u}_d$

$$\partial_t \mathbf{u} = -\nabla p + \underbrace{\nu \Delta \mathbf{u} - (\mathbf{u} \cdot \nabla) \mathbf{u}}_{AD} \longrightarrow \partial_t \mathbf{u} = P(AD)$$

$$1. \quad \mathbf{u}^* = \mathbf{u}^n + \int AD dt$$

$$3. \quad \mathbf{u}^{n+1} = \mathbf{u}^* - \int \nabla p dt$$

Equation de Poisson

$$2. \quad \Delta \bar{p} = \frac{1}{\Delta t} \nabla \cdot \mathbf{u}^*$$

Sandoval (1995), Cook et Riley (1996) et Xavier Coré (2002)

$$\partial_t(\rho \mathbf{u}) = -\nabla p + \underbrace{\nabla \cdot (\mathcal{T} - \rho \mathbf{u} \mathbf{u})}_{AD}$$

Fractionnement :

1. $(\rho \mathbf{u}^*)^{n+1} = (\rho \mathbf{u})^n + \int AD dt$
2. $\Delta \bar{p} = \frac{1}{\Delta t} [\nabla \cdot (\rho \mathbf{u}^*)^{n+1} + \partial_t \rho^{n+1}]$
3. $(\rho \mathbf{u})^{n+1} = (\rho \mathbf{u}^*)^{n+1} - \Delta t \nabla \bar{p}$

Nicoud (1998) : ne converge pas vers $\text{Div } \mathbf{v} = 0$ par passage au non-visqueux

Cook et Riley (1996) : l'évaluation de $\partial_t \rho^{n+1}$ est délicate (instabilité + cout mémoire)

Bell et Marcus (1992), Colella et Pao (1999) mais ici en diffusif

$$\partial_t \mathbf{u} = -\nabla \pi_h + \underbrace{\nabla \cdot \mathcal{T} + \omega \times \mathbf{u} - \pi \nabla \varrho}_{AD}$$

Fractionnement :

$$1. \quad \mathbf{u}^* = \mathbf{u}^n + \int AD dt$$

$$2. \quad \Delta \bar{\pi}_h = \frac{1}{\Delta t} [\nabla \cdot \mathbf{u}^* + \nabla (a \nabla \varrho^{n+1})]$$

$$3. \quad \mathbf{u}^{n+1} = \mathbf{u}^* - \Delta t \nabla \bar{\pi}_h$$

Converge bien vers $\text{Div } \mathbf{v} = 0$ par passage au non-diffusif ($a=0$)

Cout mémoire nul

Précision de l'ordre de celui pour le laplacien de la densité

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Rotationnel de $d_t \mathbf{u} = -\nabla \pi - \pi \nabla \rho + \nu \Delta \mathbf{u}$

$$d_t \boldsymbol{\omega} = (\boldsymbol{\omega} \cdot \nabla) \mathbf{u} - d \boldsymbol{\omega} - \nabla \pi \times \nabla \rho + \nu \Delta \boldsymbol{\omega}$$

$$\frac{d\boldsymbol{\omega}}{dt} = \underbrace{(\boldsymbol{\omega} \cdot \nabla) \mathbf{u}}_{\text{green}} - \underbrace{\frac{1}{\rho^2} \nabla P \times \nabla \rho}_{\text{red}} - \underbrace{d \boldsymbol{\omega}}_{\text{blue}} + \underbrace{\nu \Delta \boldsymbol{\omega}}_{\text{yellow}}$$

Couple barocline

Écoulement de fluide parfait inhomogène

$$d_t \rho = 0$$

$$d_t \mathbf{u} = -g' \mathbf{i}_z - \dot{\mathbf{u}}_r - \frac{1}{\rho} \nabla p$$

$$g' = \frac{g}{\rho_0} (\rho - \rho_{\text{hydro}}) = g \frac{\rho'}{\rho_0} \sim \mathcal{O}(g C_\rho)$$

$$\mathbf{a} = \underbrace{d_t \mathbf{u}}_{\text{II}} + \underbrace{g' \mathbf{i}_z + \dot{\mathbf{u}}_r}_{\text{I}} = -\frac{1}{\rho} \nabla p$$

$$\text{Ba : } -\frac{1}{\rho} \nabla p \times \nabla \varrho \equiv \mathbf{a} \times \nabla \varrho$$

$$d_t \boldsymbol{\omega} = \underbrace{(\boldsymbol{\omega} \cdot \nabla) \mathbf{u}}_{\text{blue}} - d \boldsymbol{\omega} - \underbrace{\nabla \pi \times \nabla \varrho}_{\text{red}} + \nu \Delta \boldsymbol{\omega}$$

$$V_s \quad (\boldsymbol{\omega} \cdot \nabla) \mathbf{u} \sim \mathcal{O}\left(\frac{u^2}{\lambda^2}\right)$$

$$Ba \quad \nabla \pi \times \nabla \varrho \sim \mathcal{O}\left(\frac{u^2}{\ell} \frac{C_\rho}{\lambda_\rho}\right)$$

$$\frac{Ba}{V_s} \sim C_\rho \cdot \frac{\lambda}{\lambda_\rho} \cdot \frac{\lambda}{\ell}$$

$$\frac{Ba}{Vs} \sim C_\rho \cdot \frac{\lambda}{\lambda_\rho} \cdot \frac{\lambda}{\ell}$$

- Turbulence pleinement développée (3D) $\frac{Ba}{Vs} \sim \frac{\lambda}{\lambda_\rho} \cdot \frac{C_\rho}{Re_\lambda}$

Turbulence haut Reynolds peu sensible aux variations de densité

- Ecoulements 2D : pas d'étirement tourbillonnaire $\boldsymbol{\omega} \perp \nabla \mathbf{u}$

Le couple barocline est la seule source/puit de vorticité

- Ecoulements de transition $\frac{\lambda}{\ell} \sim \mathcal{O}(1)$

Le couple barocline très sensiblement la transition

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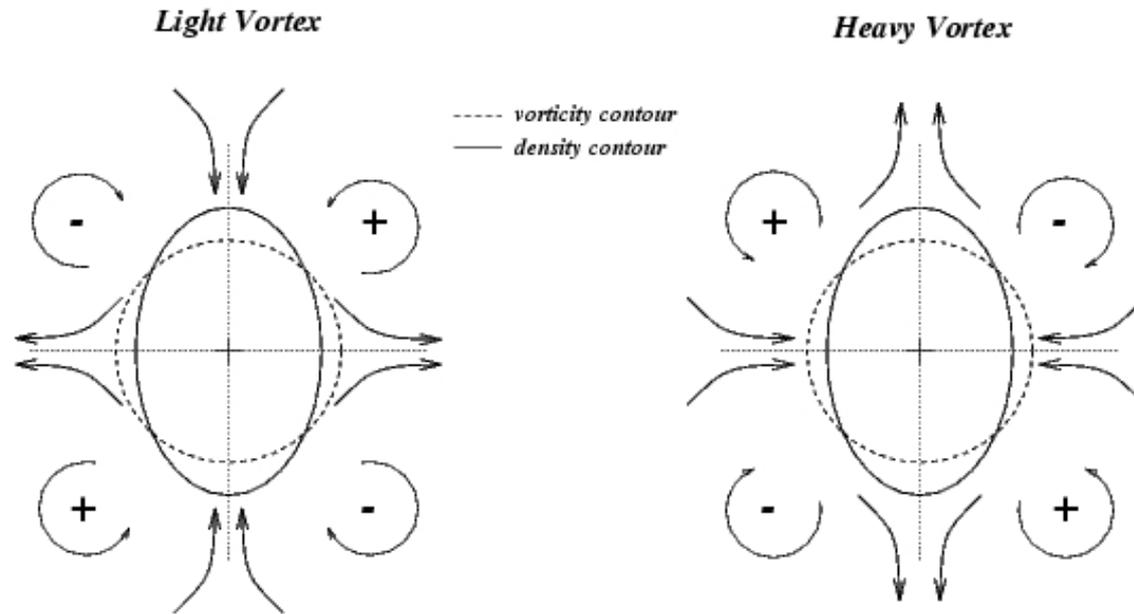


Figure 4.4: Sketch of the baroclinic effects on heavy and light vortices.

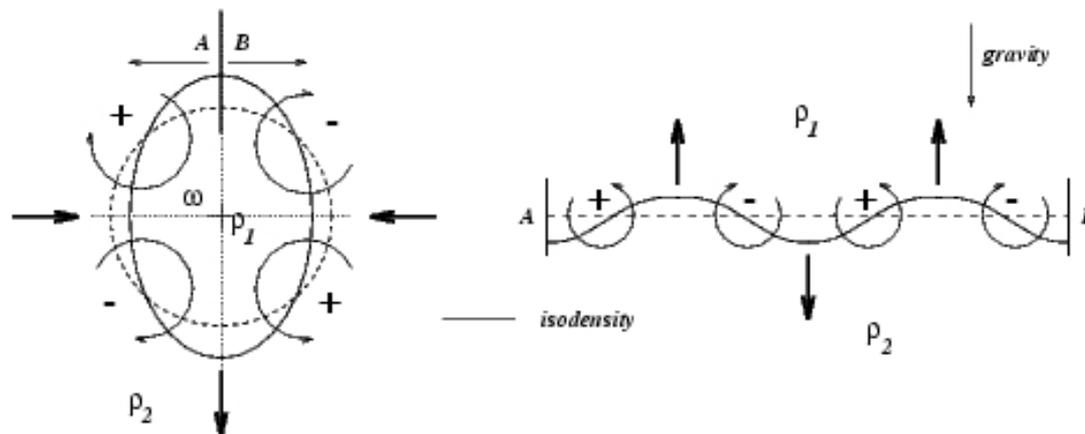
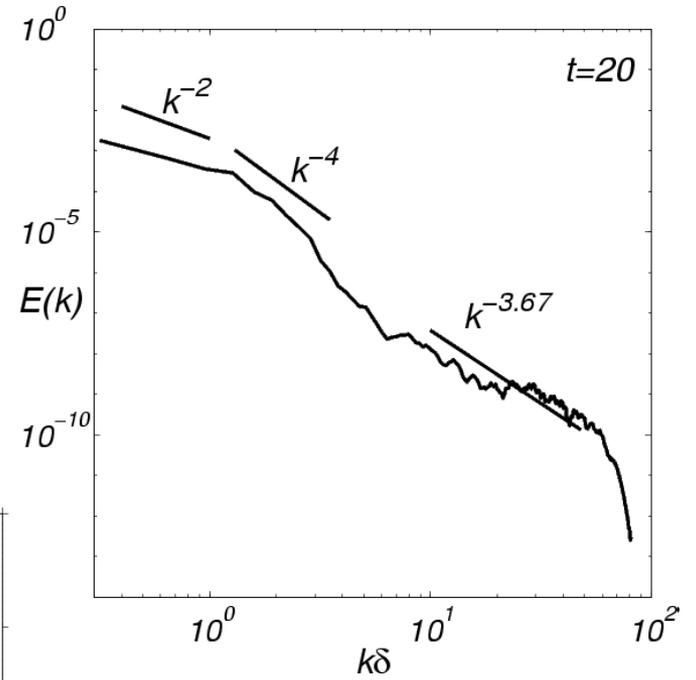
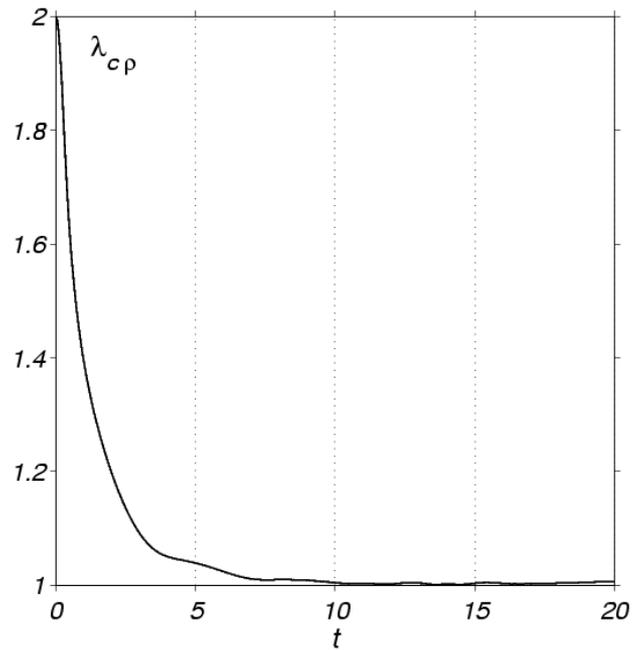
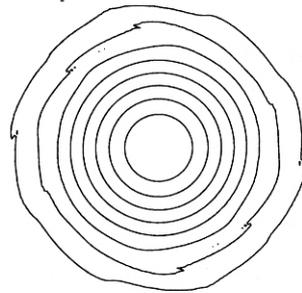
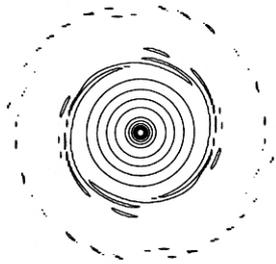


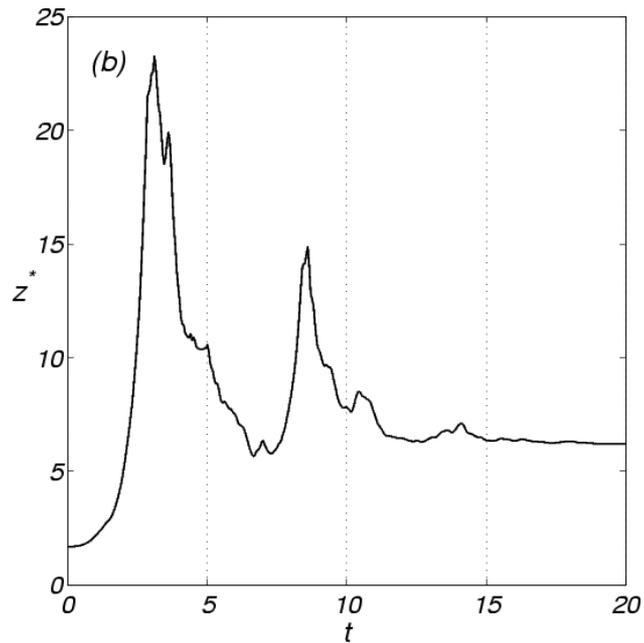
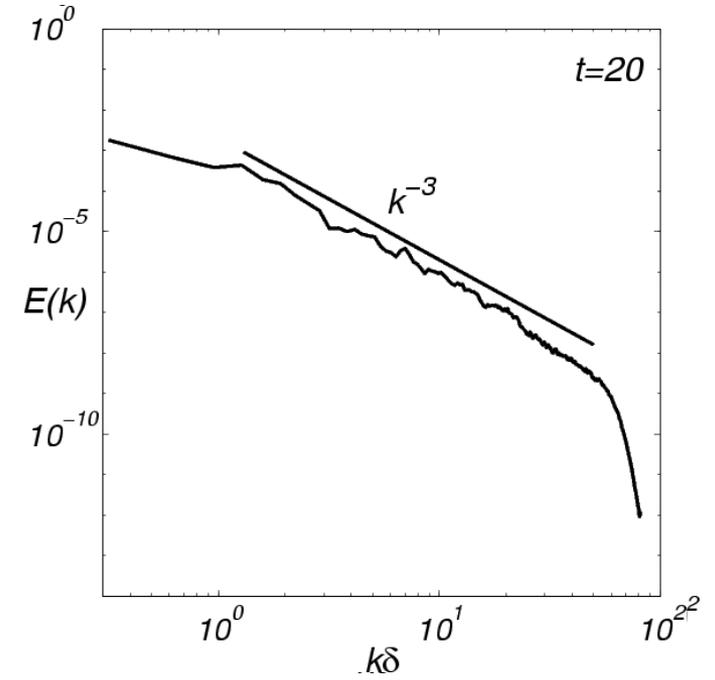
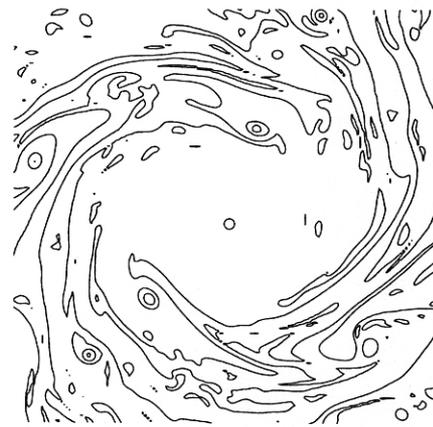
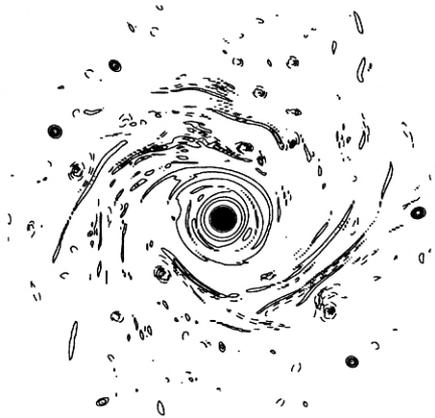
Figure 4.5: Analogy between the heavy vortex and the equivalent unrolled density stratification submitted to a downward gravity field ($\rho_1 > \rho_2$).

- *Vorticité :*
$$\omega(r, \theta) = \frac{\Gamma}{\pi\delta^2} \exp(-r^2/\delta^2)$$
- *Vitesse azimuthale et acceleration centripète :*
$$u_\theta = [1 - \exp(-\frac{r^2}{\delta^2})] \frac{\Gamma}{2\pi r}$$

$$a_r = -u_\theta^2/r$$
- *Champ de densité elliptique :*
$$r_\rho^2(\theta) = r^2[1 + (e^{-2} - 1) \sin^2 \theta]$$
- *Gradient de densité :*
$$\rho(r, \theta) = \rho_e + (\rho_i - \rho_e) \exp(-\frac{r_\rho^2(\theta)}{\delta^2})$$
- *Couple barocline :*
$$\mathbf{g} = \nabla(\ln \rho) = \begin{cases} g_r = -(1 - \frac{\rho_e}{\rho}) \frac{2r}{\delta^2} [1 + (e^{-2} - 1) \sin^2 \theta] \\ g_\theta = (1 - \frac{\rho_e}{\rho}) \frac{r}{\delta^2} (1 - e^{-2}) \sin(2\theta) \end{cases}$$

$$b = a_r g_\theta = [1 - \exp(-\frac{r^2}{\delta^2})]^2 \frac{\Gamma^2}{4\pi^2 r^2 \delta^2} (1 - e^{-2}) \frac{\rho_e - \rho}{\rho} \sin(2\theta)$$





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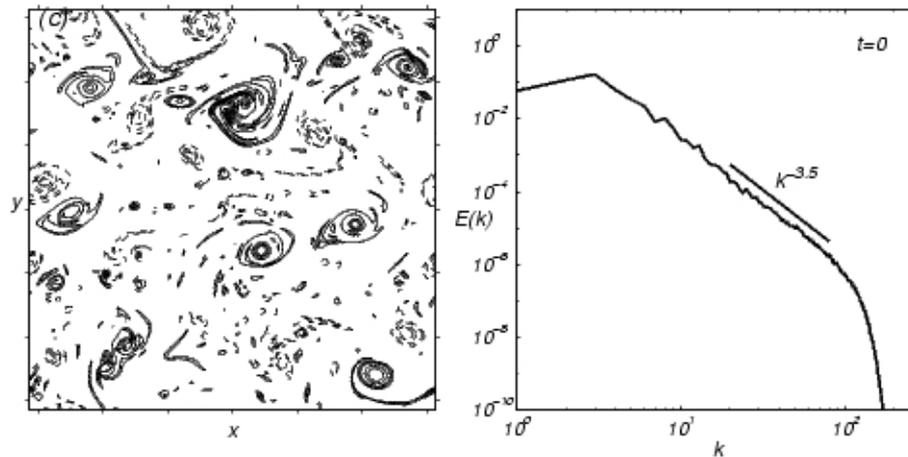


Figure 4.13: Initial vorticity field with solid positive contours and dashed negative ones, increment between contour is $\max(\omega)/2\pi$ and the zero contour is omitted (left); the corresponding energy spectrum (right).

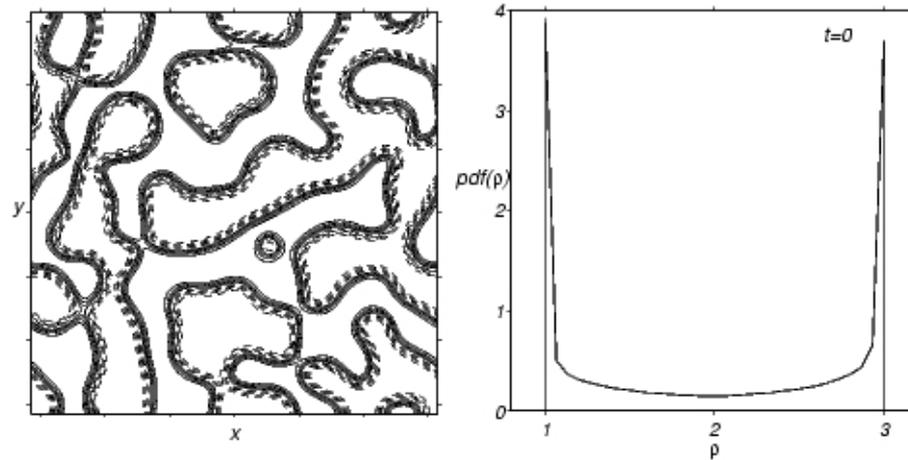
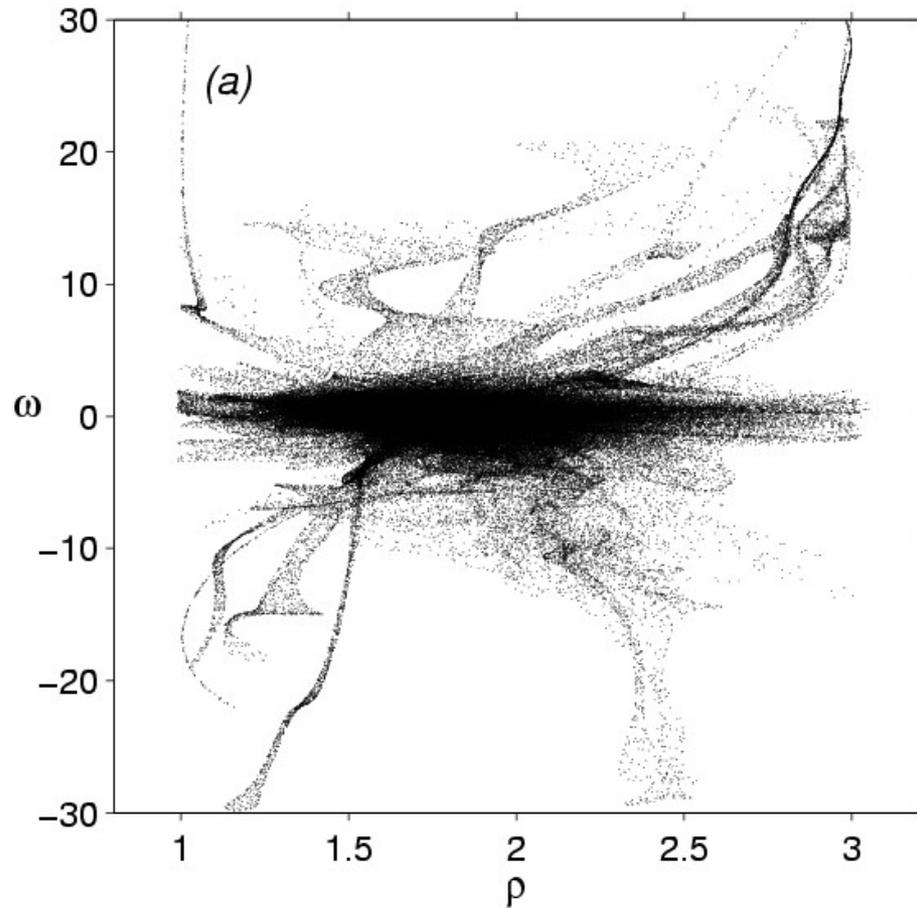
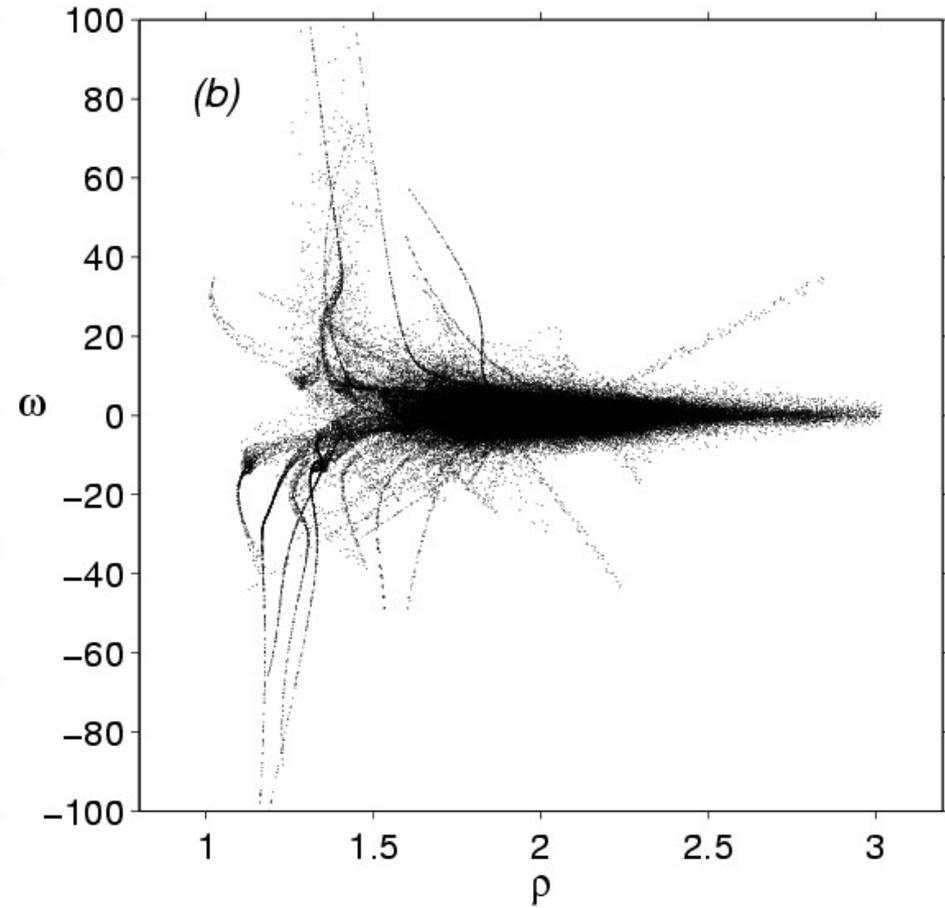


Figure 4.14: Initial density field with solid contours above mean and dashed contours beneath, increment between contour is $\Delta\rho/10$ (left); the corresponding probability density function (right).

Passive scalar 2D turbulence



Inhomogeneous 2D turbulence



1. Deux « lieux » de couplage (non-linéarité et divergence)
2. Différents choix de la contrainte divergentielle
3. Contraintes de résolution spatiale et temporelle (adaptatif)
4. Mécanismes d'instabilité pilotés par la redistribution de vorticité
5. Ségrégation de masse en turbulence bidimensionnelle,
6. Loi d'échelle : la turbulence développée peu sensible à l'inhomogénéité de masse