Direct numerical simulation of inertia-sensitive turbulence

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Abstract

Direct numerical simulations were performed of the turbulent mixing of a contrasted density field by a statistically homogeneous velocity field. The density ratio between the two initially unmixed fluids is 3, thus well beyond the passive-scalar situation. This low Mach number variable density turbulence is relevant to very large Froude-number turbulent mixing processes. In this paper the flow configuration is a forced homogeneous turbulence mixing an initially sharp density-gradient layer. Depending on the characteristic length scales of the velocity and density fields, the order of magnitude of the vortex-stretching and baroclinic enstrophy sources is compared. In the situation where turbulence is inertia affected and where strong baroclinic events are encountered, the deviations from the passive-scalar case are analysed both from the spectral and the statistical points of view.

Introduction

In any statistical approach of variable density turbulence, the question arises to know if density variations significantly alter the features of homogeneous turbulence. This answer, beyond challenging our understanding of the physics of turbulent flows, is also of crucial interest to those designing closure schemes for averaged Navier-Stokes solvers or for large eddy simulations of variable-density flows. In this respect much more has been done upon compressible turbulence due to the evidence that compressibility alters the turbulent momentum fluxes with direct consequences on global parameters such as the expansion rate of the mixing layer. Chassaing, Harran & Joly (1994) have shown that, in low-speed variable-density mixing, the correlations with the density fluctuation play a central role in the flow prediction and are not correctly represented by gradient diffusion schemes.

The focus of the present contribution is on baroclinic effects beyond the Boussinesq approximation but uncorrelated to compressibility. The baroclinic torque results from the inertial component of the pressure gradient only. The vorticity evolves within a quasi-solenoidal velocity. In this intermediate case, turbulence may be affected by the density fluctuation and the first point of the paper is to propose an effective scaling for the inertia effect in the turbulent context. A transposal to the compressible situation is out of the scope of the present study.

The baroclinic torque and numerical procedure

The baroclinic effect is the additionnal torque that is felt when an inhomogeneous mass field is submitted to a pressure gradient normal to the local density gradient. In the limit of inviscid two-dimensionnal incompressible flows, the baroclinic torque is the only source of vorticity variation along the particle path, and the pressure gradient is connected to the material acceleration $\mathbf{a} = \mathrm{d}\mathbf{u}/\mathrm{d}t$ such that the baroclinic torque reads : $\mathbf{b} = \mathbf{a} \times \boldsymbol{\nabla}(\ln \rho)$. A combination of acceleration and inertia differences thus results in vorticity sources and sinks. It is the only added mechanism on the "standard" incompressible picture.

The complete vorticity budget includes the vortex stretching term and the diffusion one :

$$\frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t} = (\boldsymbol{\omega} \cdot \boldsymbol{\nabla})\mathbf{u} - \frac{1}{\rho^2}\boldsymbol{\nabla} P \times \boldsymbol{\nabla}\rho + \nu\Delta\boldsymbol{\omega} \qquad (1)$$

In the zero Mach number limit, the normalised velocity divergence is :

$$d^* = -\frac{1}{ReSc}\boldsymbol{\nabla}(\frac{1}{\rho}\boldsymbol{\nabla}\rho)^* \qquad (2)$$

which means that a high Reynolds/Schmidt evolution is quasi-solenoidal. This situation falls under the acoustically filtered regime as reviewed in Chassaing (2001).

Neglecting the dilatation contribution to the viscous dissipation, the momentum equation can be recasted in the following form :

$$\partial_t \mathbf{u} = \mathbf{u} \times \boldsymbol{\omega} - \boldsymbol{\nabla}\pi + p\boldsymbol{\nabla}\left(\frac{1}{\rho}\right) + \nu\Delta\mathbf{u} \qquad (3)$$

where is $\pi = p/\rho + u^2/2$ the specific pressure-head.

These equations are solved under periodic conditions using a classical pseudo-spectral approach on a 220^3 mesh yielding a turbulence Reynolds numbers around 50 (a targetted 512^3 simulation will be available by the spring). The random forcing scheme is the one proposed by Alvelius (1999). The density being non-periodic in the z-direction, a standard use of cosine and sine transforms allows for the projection of even and odd functions with respect to z. The time-stepping scheme is a low-storage RK3 combined with an implicit treatment of viscous terms. This allows for high CFL numbers and an associated fast time marching.

The flow configuration

At $t = 0$, a pure turbulent velocity (no mean flow), resulting from a statistically-steady homogeneous isotropic turbulence, mixes an initially smooth density profile $\rho^0(z) = \rho_1 + 0.5\Delta\rho \tanh[(z - z_0)/\sigma^0_\rho]$ between two pure species which densities are ρ_1 and $\rho_2 = \rho_1 + \Delta\rho$. Here the overall density ratio $s = \rho_2/\rho_1$ is set to 3. Due to the homogeneity in xy-planes normal to the density gradient, instantaneous fields ϕ are decomposed into an xy-averaged mean field $\langle\phi\rangle$ and a fluctuation ϕ' relative to the z-local mean value :

$$\phi(\mathbf{x}, t) = \langle\phi\rangle(z, t) + \phi'(\mathbf{x}, t) \qquad (4)$$

The thickness of the density gradient layer spreads as a result of the turbulent diffusion and its thickness is measured by : $\sigma_\rho(t) = \Delta\rho/\partial_z\langle\rho\rangle_{|\max}$. The isosurface at mean density, $\rho_m = (\rho_1 + \rho_2)/2$, is represented on figure 1 after one half of the large eddy turnover time. The simulation is halted when the mean density thickness σ_ρ reaches two thirds of the domain height. The local density contrast is given by the ratio of the density root mean square ϱ' with $\varrho'^2 = \langle\rho'^2\rangle$ to the local mean density $\langle\rho\rangle$: $C_\rho = \varrho'/\langle\rho\rangle$.

A Scaling for the inertia effects

Let u/λ be an estimate the strain rates when energetic structures of velocities u spread over length scales ℓ. Density gradients will be significative over distances λ_ρ and C_ρ is retained for relative density differences. Initially λ_ρ is well approximated by σ^0_ρ. The ratio of the time scale associated to the baroclinic torque τ_b to one associated with the vortex stretching τ_vs can thus be estimated as :

$$\boxed{\frac{\tau_\mathrm{b}}{\tau_\mathrm{vs}} \sim \frac{1}{C_\rho} \cdot \frac{\lambda_\rho}{\lambda} \cdot \frac{\ell}{\lambda}} \qquad (5)$$

The high Reynolds number scaling $\ell/\lambda \sim Re_\lambda$ yields that the baroclinic torque is a rather slow mechanism in high Reynolds number turbulence.

Besides, the density contrast is bounded by the constraint that the density is in the range $[\rho_1, \rho_2]$ such that $\langle\rho\rangle - c\varrho' > \rho_1$. With $c \sim \mathcal{O}(1)$, it gives $C_\rho < 1$. Only the ratio between the length scales λ_ρ and λ may give the baroclinic torque a significative role in the vorticity budget. This is the case here where $\lambda/\sigma^0_\rho \sim 50$. The ratio λ/λ_ρ decreases in time to reach unity due to the mixing of the density field by turbulence. This situation is likely to occur downstream a splitter plate between two turbulent flows mixing pure species at different densities.

On figure 2 is presented the order of magnitude of the vortex stretching and the baroclinic torque from the enstrophy budget. The previous analysis thus allows to carry out numerical simulations relevant to inertia-affected turbulence, in a more efficient way than in the previous simulations by Joly, Chassaing & Castaldi (1997).

Conclusion and perspectives

The frame for direct numerical simulation of variable density mixing has been described and an original flow configuration proposed. A efficient scaling of inertia effects points toward real situations where low-speed turbulence is baroclinically affected. The statistics of the variable density mixing in kinematically homogeneous turbulence is under analysis. This and other aspects will be investigated from finer resolution and more contrasted density fields.

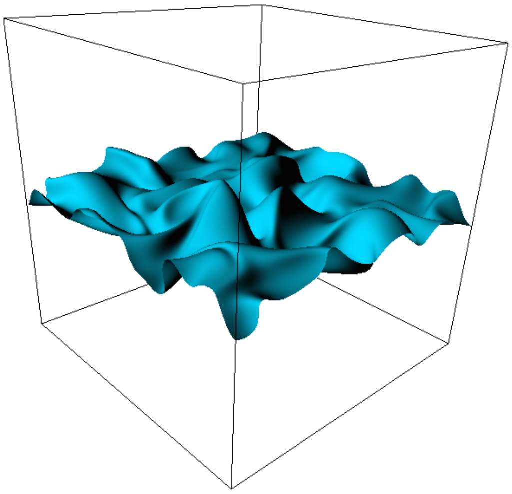

Figure 1: Mean density isosurface after one-half of a large eddy turnover time $\tau = \ell/u$.

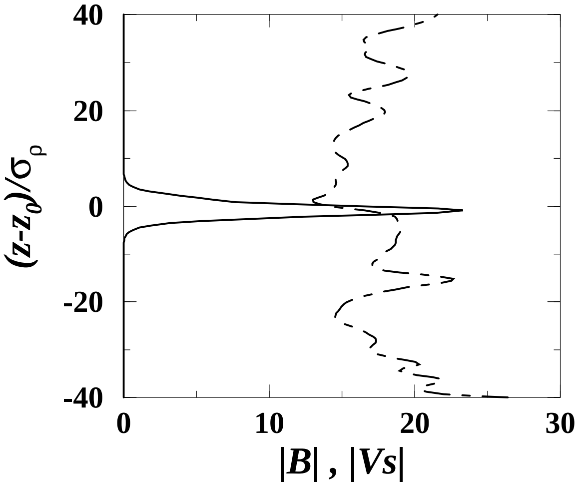

Figure 2: Normalised intensity of the baroclinic enstrophy source $|\mathcal{B}|$ (solid line) compared to the vortex stretching one $|\mathcal{V}_\mathrm{s}|$ (dot-dashed line) versus the normalised coordinate in the inhomogeneous direction.

References

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