Cross Standard Form for generalized inverse problem: application to lateral flight control of a highly flexible aircraft

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Abstract

This paper introduces the Cross Standard Form (CSF) which can be considered as a generalization of the LQ inverse problem to the H_2 and H_∞ inverse problems. The CSF allows to formulate a standard problem on which an initial compensator can be obtained by H_∞ or H_2 synthesis. The definition of the CSF is based on the possibility to construct equivalent observer-based realization of a given compensator. From the robust control application point of view, the general idea is to apply the CSF to a first compensator satisfying nominal performances to initialize a H_∞ procedure in order to handle frequency-domain or parametric robustness specifications. These state observer based tools are then applied to design lateral flight control of a highly flexible aircraft.

1. INTRODUCTION

Many control problem, mainly flexible structures control problem are expressed as multi-objective control problems. Frequency-domain synthesis techniques (H_∞, H_2) are suitable to handle *roll-off* specifications required to prevent *spill-over* on the flexible modes, which are neglected in the synthesis model [1]. Additionally, to prevent pole/zero cancellation [2], it is necessary to take into account parametric uncertainties on the flexible modes to be controlled. μ synthesis [1] or parametric robust linear quadratic gaussian (PRLQG) synthesis [3] are good candidate techniques for parametric robustness specifications. On the other hand, these frequency-domain or quadratic type techniques are less suitable when time domain or modal specifications (eigenvalue assignment, decoupling) on the rigid modes of the system are required. The latter point is particularly significant within the context of flight control laws and more particularly the roll / yaw decoupling required for lateral flight [4]. Sato and Suzuki propose an interesting approach that combines an H_∞ estimator and a state feedback gain designed by eigenstructure assignment to overcome this difficulty[5]. More generally, the combination of eigenstructure assignment and quadratic techniques is often mentioned in aeronautical applications[6, 7, 8]. Implicit model following design is an alternative to mix eigenstructure assignment on a reduced order model and linear quadratic regulation on a full order model [9]. In the same manner, the *cross standard form* we propose in this paper is a competitive technique to combine eigenstructure assignment and optimal (H_2 or H_∞) syntheses.

In the first part of this paper, we recall basic principles to compute the observer-based realization of a given compensator and a given model. An illustration addressing plant state monitoring problem is proposed. The principle of the CSF is then introduced. Next the CSF is applied to the lateral flight control design of a flexible aircraft. This approach proceeds in two steps: the first deals with the synthesis of a dynamic output feedback satisfying modal specifications on the rigid modes, the stability requirements on the structure and performance robustness with respect to the various cases of loading. The second step uses the equivalent observer-based representation of the former compensator to built a two degree of freedom compensator allowing the time domain specifications in response to the pilot's inputs to be met.

2. OBSERVER-BASED STRUCTURES AND CROSS STANDARD FORM

2.1 Observer-based realization of a given compensator

We recall some recent results on the LQG controller structure or more generally on compensators involving a state observer (with an estimation gain K_f), a state feedback (with a gain K_c) and a dynamic Youla's parameter $Q(s)$. This structure allows the parametrization of all stabilizing controllers. In [11] a procedure is proposed to compute the parameters K_c, K_f et Q which characterized this structure, for an arbitrary controller K of order n_K and a system G of ordre n.

We recalls the case of a full order compensator and a continuous-time system $G(s)$ (n states, m inputs, p outputs). The state space realization of $G(s)$ reads :

$$\left[\begin{array}{c}\dot{x}\\ \hline y\end{array}\right]=\left[\begin{array}{c|c}A & B\\ \hline C & D\end{array}\right]\left[\begin{array}{c}x\\ \hline u\end{array}\right] \tag{1}$$

The Youla parametrization of a controller $K(s)$ built on such a structure is depicted in Figure 1 and can be read :

$$\begin{array}{rcl}\dot{\hat{x}} & = & A\hat{x}+Bu+AK_f(y-C\hat{x}-Du)\\ \dot{x_Q} & = & A_Qx_Q+B_Q(y-C\hat{x}-Du)\\ u & = & -K_c\hat{x}+C_Qx_Q+\cdots\\ & & \cdots(D_Q-K_cK_f)(y-C\hat{x}-Du)\end{array} \tag{2}$$

where A_Q, B_Q, C_Q and D_Q are 4 matrices of the state representation of $Q(s)$ associated to the state vector x_Q. $\hat{x}$ is an estimate of the state x_k.

Let consider a controller of order $n_K=n$ defined by the following representation :

$$\left[\begin{array}{c}\dot{x_K}\\ \hline u\end{array}\right]=\left[\begin{array}{c|c}A_K & B_K\\ \hline C_K & D_K\end{array}\right]\left[\begin{array}{c}x\\ \hline y\end{array}\right] \tag{3}$$

The goal is to compute the state feedback gain K_c, the estimate gain K_f, the Youla parameter ($Q(s)=D_Q$) and a transformation matrix T such the controller (3) could be described by the state representation (2) when the following change of variable is performed :

$$x_K=T\hat{x}. \tag{4}$$

Then, the following equations can be derived :

$$[-T\ I]\left[\begin{array}{cc}A+BD_KC & BC_K\\ B_KC & A_K\end{array}\right]\left[\begin{array}{c}I\\ T\end{array}\right]=0 \tag{5}$$

and

$$\begin{array}{rcl}AK_f & = & T^{-1}B_K-BD_K\\ K_c & = & -C_KT-D_KC\\ D_Q & = & D_K+K_cK_f.\end{array} \tag{6}$$

The problem is now to solve the Riccati equation (5) and next to compute K_c, K_f and D_Q using (6).

The Hamiltonian matrix associated with the Riccati equation is nothing else than the closed-loop dynamic matrix constructed on the state vector $[x^T,\ x_K^T]^T$:

$$A_{cl}=\left[\begin{array}{cc}A+BD_KC & BC_K\\ B_KC & A_K\end{array}\right]. \tag{7}$$

The Riccati equation (5) can then be solved in $T\in\mathbb{R}^{n_k\times n}$ by standard subspace decomposition techniques, that is:

- compute an invariant subspace associated with a set of n eigenvalues, spec(Γ_n) spec(A) is the set of eigenvalues of the matrix A), chosen among $2n$ eigenvalues in spec(A_{cl}), that is,

$$\begin{bmatrix} A + BD_K C & BC_k \\ B_K C & A_k \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \Gamma_n, \tag{8}$$

where $U_1 \in \mathbb{R}^{n \times n}$ and $U_2 \in \mathbb{R}^{n_K \times n}$. Such subspaces are easily computed using Schur decompositions of the matrix A_{cl}.
- compute the solution

$$T = U_2 U_1^{-1} \ . \tag{9}$$

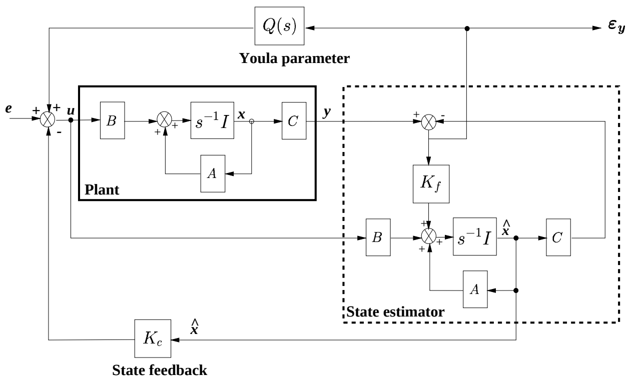

Figure 1: YOULA parameterization with LQG structure.

In the next section, the interest of observer-based realizations of given compensators is highlighted for plant state monitoring.

2.2 Illustration: plant state monitoring

The model of a launcher between the yaw angle ψ and the command β (thruster deflection) can be roughly approximated by the second order transfer function:

$$G(s) = \frac{1}{s^2 - 1}$$

associated with the state-space realization:

$$\begin{bmatrix} \dot{\psi} \\ \ddot{\psi} \\ \hline \psi \end{bmatrix} = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 1 & 0 & 1 \\ \hline 1 & 0 & 0 \end{array}\right] \begin{bmatrix} \psi \\ \dot{\psi} \\ \hline \beta \end{bmatrix} . \tag{10}$$

Let us consider the following stabilizing compensator (positive feedback):

$$K(s) = -\frac{s^2 + 27s + 26}{s^2 + 7s + 18} \ .$$

A state space realization (companion form) of this compensator reads:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \hline \beta \end{bmatrix} = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ -18 & -7 & 1 \\ \hline -8 & -20 & -1 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ \hline \psi \end{bmatrix} . \tag{11}$$

The application of the previous procedure provides the following parameterization:

$$K_c = [3 \quad 3]; \ K_f = [4 \quad 5]^T; \ Q = -1 \ .$$

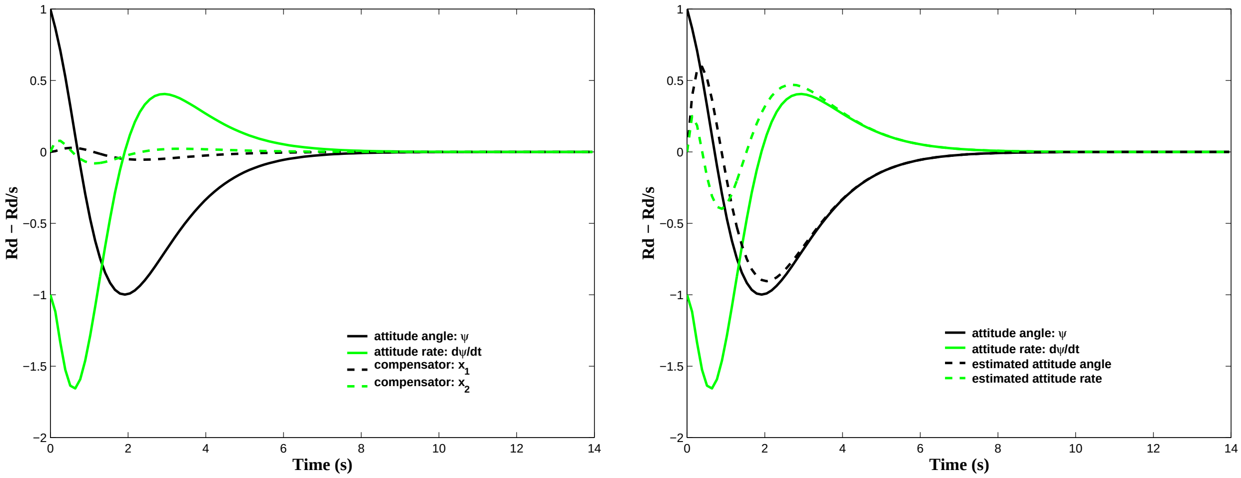

Figure 2: Responses to initial conditions on launcher states - on the left: companion realization of $K(s)$ (equation (11)) - on the right: observer-based realization of $K(s)$ (equation (12)).

Then, the observer-based realization of $K(s)$ reads:

$$\left[\begin{array}{c} \dot{\hat{\psi}} \\ \dot{\hat{\dot{\psi}}} \\ \hline \beta \end{array}\right] = \left[\begin{array}{cc|c} -4 & 1 & 4 \\ -6 & -3 & 4 \\ \hline -2 & -3 & -1 \end{array}\right] \left[\begin{array}{c} \hat{\psi} \\ \hat{\dot{\psi}} \\ \hline \psi \end{array}\right] \tag{12}$$

associated with the estimated state vector $\hat{x} = [\hat{\psi},\ \hat{\dot{\psi}}]^T$.

Figures 2 plot the closed-loop state responses (launcher and compensator) to initial conditions on launcher states. Figure 2.left is obtained when the first compensator realization (equation (11)) is used while Figure 2.right is obtained with the observer-based realization (equation (12)). For both simulations the launcher state responses are the same as the initial conditions are the same and as the input-output behavior of the compensator is independent of its realization. But, one can see in Figure 2.left that there is no straightforward relation between compensator states and launcher states (ψ and $\dot{\psi}$) while Figure 2.right highlights the compensator states of the observer-based realization are good estimated of launcher states and can be used to monitor launcher state for off-line or in-line analysis (failure diagnosis for instance). As the plant state are meaningful variables (ψ (rd) and $\dot{\psi}$ (rd/s)), one can also conclude that the state feedback gain K_c has a physical dimension: $K_c = [3.\,rd/rd \quad 3.\,s]$; while the dimension of the various components of realization (11) obscure.

2.3 Cross Standard Form

The CSF we detail in this section is based on the previous observer based realization of a given compensator.

Let us consider a system $G(s)$ defined by equation (1) and the standard problem $P_p(s)$ called *Cross Standard Form* (CSF), associated with this system :

$$P_p(s) \quad : \quad \left[\begin{array}{c} \dot{x} \\ \hline z \\ \hline y \end{array}\right] = \left[\begin{array}{c|c|c} A & K_f & B \\ \hline K_c & 0 & I_m \\ \hline C & I_p & D \end{array}\right] \left[\begin{array}{c} x \\ \hline w \\ \hline u \end{array}\right] \tag{13}$$

where K_c et K_f are respectively a state feedback gain and a state estimation gain synthesized to meet some specifications by a suitable technique (modal, LQG, ...). We will also assume that $A - BK_c$ et $A - K_fC$ are stable.

Theorem 0.1 *H_2 and H_∞ (optimal or sub-optimal) syntheses carried out on the standard problem $P_p(s)$ defined by (13) provide the same solution. This solution coincides with the LQG controller (noted : $K_{LQG}(s)$) build on K_c and K_f and reads :*

$$\left[\begin{array}{c}\dot{\hat{x}}\\ \hline u\end{array}\right] = \left[\begin{array}{c|c} A - BK_C - K_fC + K_fDK_c & K_f \\ \hline -K_c & 0\end{array}\right]\left[\begin{array}{c}\hat{x}\\ \hline y\end{array}\right]. \tag{14}$$

Proof: The state-space representation of the standard form $P_p(s)$ fed-back by the controller $K_{LQG}(s)$, *i.e.* $F_l(P_p(s), K_{LQG}(s))$ [1], reads :

$$\left[\begin{array}{c}\dot{x}\\ \dot{\hat{x}}\\ \hline z\end{array}\right] = \left[\begin{array}{cc|c} A & -BK_c & K_f \\ K_fC & A - BK_c - K_fC & K_f \\ \hline K_c & -K_c & 0\end{array}\right]\left[\begin{array}{c}x\\ \hat{x}\\ \hline w\end{array}\right]. \tag{15}$$

Let us carry out the internal state mapping using the estimation error $\varepsilon = x - \hat{x}$ associated with the KALMAN'S filter :

$$\left[\begin{array}{c}x\\ \hat{x}\end{array}\right] = \mathcal{M}\left[\begin{array}{c}x\\ \varepsilon\end{array}\right] \text{ with } \mathcal{M} = \left[\begin{array}{cc} I_n & 0 \\ I_n & -I_n\end{array}\right]. \tag{16}$$

Then, the new representation of $F_l(P_p(s), K_{LQG}(s))$ reads :

$$\left[\begin{array}{c}\dot{x}\\ \dot{\varepsilon}\\ \hline z\end{array}\right] = \left[\begin{array}{cc|c} A - BK_c & BK_c & K_f \\ 0 & A - K_fC & 0 \\ \hline 0 & K_c & 0\end{array}\right]\left[\begin{array}{c}x\\ \varepsilon\\ \hline w\end{array}\right]. \tag{17}$$

We can notice that the states ε associated with the estimation error are uncontrollable by w and that the states x of the plant are unobservable by z. The transfer between w and z is therefore null :

$$F_l(P_p(s), K_{LQG}(s)) = 0 \tag{18}$$

and, consequently :

$$\|F_l(P_p(s), K_{LQG}(s))\|_2 = \|F_l(P_p(s), K_{LQG}(s))\|_\infty = 0 \tag{19}$$

This controller is thus the optimal solution to this standard problem ($P_p(s)$) in H_2 and H_∞ norm sense.

□

This result can be considered as a generalization, for H_2 and H_∞ criteria, of the solution to the problem known as *LQ inverse* raised in the Sixties and Seventies and which consisted in finding the LQ cost whose minimization restore a given state feedback. This CSF used as such is not of interest since it is necessary to know the gains K_c and K_f to format the standard problem $P_p(s)$ and to find finally the initial LQG compensator. On the other hand, from a state feedback K_c satisfying some time domain specifications relating to particular states of the system, the CSF can be very useful to initialize a standard set-up which will be completed by dynamic weightings to take into account some complementary frequency domain specifications.

3. FLEXIBLE AIRCRAFT: MODELS AND SPECIFICATIONS

The models used in this study are linearized models of the lateral motion of a flexible aircraft around equilibrium points. The system is a large carrier aircraft in which flexibility was intentionally degraded in order to evaluate the relevance of control law synthesis techniques in highly critical cases. The models[2] are 60^{th}-order state-space representations with 2 control inputs (aileron deflection δ_l and rudder deflection δ_n) and 6 measurements: lateral acceleration n_y, roll rate p, roll angle ϕ and yaw rate r (n_y and r are measured in two different locations on the fuselage; see [10] for more details). The state vector x contains:

[1] $F_l(P, K)$ means the *lower* Linear Fractional Transformation of P and K.

[2] The reader will find the MATLAB™ source files required to load these models online at `http://www.cert.fr/fr/dcsd/CDIN/CDINPUB/alazard/AIAA_JGCD/`

- 4 rigid states (yaw angle β, roll rate p, yaw rate r, roll angle ϕ),
- 36 states corresponding to 18 flexible modes modeled between 8 and $80\, rad/s$ (generalized coordinates q_j and $\dot{q}_j$, $(j = 1, ..., 18)$),
- 20 secondary states that represent the dynamics of the servo-control surfaces and aerodynamic lags.

The models are available for three different flight conditions and six different loading cases (corresponding to six distributions of the mass inside the plane). Only one model, noted G_f, is considered in this paper. We will also refer to the corresponding rigid models G_r in which only the 4 rigid states ($x_r = [\beta,\ p,\ r,\ \phi]^T$) are considered.

The following list summarizes the various specifications:

- **S1**: Dutch roll damping ratio $>\ 0.5$,
- **S2**: templates on the step responses w.r.t. β and p,
- **S3**: roll/yaw channel decoupling,
- **S4**: no degradation of the damping ratios of flexible modes, or furthermore, an increase of the damping ratios of low frequency flexible modes in order to improve comfort during turbulence,

Modal and time-domain specifications (**S1**) to (**S3**) involve the rigid dynamics of the aircraft. If the system is assumed to be rigid, eigenstructure assignment techniques are particularly effective at handling these specifications, especially since we have a sufficient number of outputs (≥ 4) to implement an ideal rigid state feedback by a static output feedback. These techniques will not be detailed in this paper [12]. A "rigid" state feedback K_{x_r} on the 4 rigid states (x_r) was thus calculated on model G^r in order to meet the following modal specifications (see chapter 8, part 3 in[4]):

- the Dutch roll mode is damped and assigned to $-1 + 1.3\, i$ and is decoupled from ϕ,
- the pure roll mode is assigned to -1.1 and is decoupled from β (that is, the natural dynamics is preserved),
- the spiral mode is assigned to -1 and is decoupled from β.

Note that eigenstructure assignment involves the rigid dynamics, not the flexible dynamics. Although flexible mode damping could be easily handled by eigenstructure design, such a design might lead to undesirable high efforts on control surfaces. Thus, we have chosen to handle flexible mode control by an H_2 design.

Time responses obtained on the rigid model G_r are presented in Figure 4 (black curve). These responses satisfy (**S1**) to (**S3**) and will be used as a reference to appreciate the solutions proposed on the full-order models. This simulation plots the rigid state responses ($\beta(t)$, $\phi(t)$ $p(t)$ and $r(t)$) to β_{ref} and p_{ref} step inputs. In fact, the actual command that is applied is:

$$\begin{bmatrix} \delta_l \\ \delta_n \end{bmatrix} = H \begin{bmatrix} \beta_{ref} \\ \frac{1}{s}\, p_{ref} \end{bmatrix} - K_{x_r}\, x_r \tag{20}$$

where H is a feed-forward static matrix computed to ensure the steady state constraint:

$$\lim_{t\to\infty} \begin{bmatrix} \beta(t) \\ \phi(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \beta_{ref} \\ \frac{1}{s}\, p_{ref} \end{bmatrix}. \tag{21}$$

The problem can now be restated in the following manner:

- synthesize a control law satisfying the frequency domain and modal specifications (**S4**) on the flexible modes,
- preserve as much as possible the "rigid" performances obtained with the modal gain K_{x_r}.

4. OUTPUT FEEDBACK SYNTHESIS BY MULTI-OBJECTIVE H_2 DESIGN

The multi-objectives synthesis setup is described in Figure 3. The synthesis model (A, B, C, D) corresponds to the 30^{th}-order reduced model $G(s)$ in which 12 relevent flexible modes are retained after a balanced reduction of the full order model $G_f(s)$ (see [10]). This setup can be clarified as follows:

- a controlled output $z_1 = u + K\,x$ to inflect the synthesis towards the nominal rigid solution (Cross Standard Form property),
- a controlled output z_2 to specify a r^{th}-order roll-off beyond the pulsation ω ($r = 3$ and $\omega = 2\pi$ on both control channel),
- very low measurement noises w_1 ($\varepsilon = 0.001$) since the number of measurements is high enough to directly and statically estimate the rigid states. This tuning leads to very fast dynamics in the compensator which can be reduced after the synthesis,
- initial conditions w_2 on the 4 rigid states ($V = [I_{4\times 4},\, 0,\, \cdots,\, 0]$),
- the transfer between w_3 and z_3 takes into account variations on flexible mode damping ratios: the 12 inputs w_3 (through the matrix M) and the 12 outputs z_3 (through the matrix N) are in fact given by the linear fractional transformation (LFT) modeling of these parametric variations, completed by static weightings ρ_e and ρ_s, respectively.

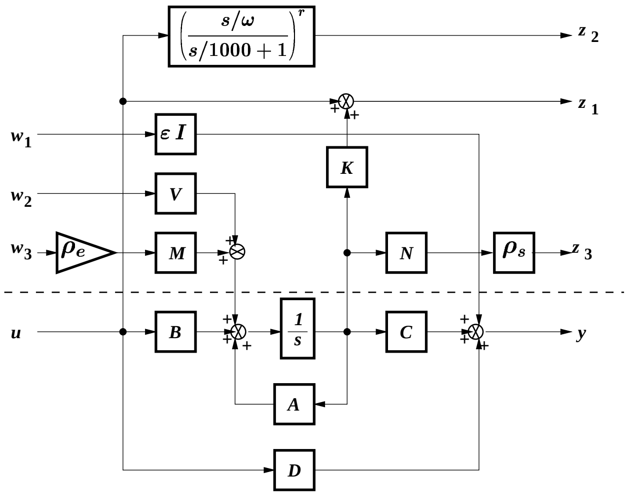

Figure 3: Setup for active synthesis.

The H_∞ synthesis thus performed provides a 36 order compensator $K(s)$. The analysis of this solution is presented in Figure 4. On can notice in Figure 4.left that all the rigid dynamics is correctly assigned and that the low frequency flexible modes damping ratios are increased in closed-loop. This behavior is robust to the various cases of loading. Figure 4.right allows a comparison between the pure rigid state feedback K_{x_r} applied on the rigid model and the nominal solution applied on the full order model.

5. TWO D.O.F COMPENSATOR SYNTHESIS BY EQUIVALENT LQG STRUCTURE

The previous solution is interesting for its robust performance properties, particularly the robustness of the damping introduced on the flexible modes. On the other hand, the time responses presented in Figure 4 are not satisfactory due to the overshoot of β, a non-minimum phase response on ϕ for a step in β_{ref} and a too slow a rise time for p. It is thus necessary to synthesize a dynamic feed-forward to overcome these defects. This point is a very significant aspect in flight control design.

The results presented in the first section are used here to compute the observer-based realization of this compensator involving a judicious *on-board* model (model A, B and C in Figure 1). We chose to use the rigid model here as the *on-board* model. The uncontrollability from e of the rigid states estimation modes leads to a dynamic feed-forward effect [13]. Thus, we need not introduce additional dynamics such as with classical structures where the feed-forward path and the feedback path are uncoupled. The structure we obtain is said to be a 2 degree of freedom compensator.

Note that there are several solutions, depending on the choice of the distribution of the closed-loop eigenvalues between the state-feedback dynamics (spec$(A - BK_c)$), the state-estimation dynamics (spec$(A -$

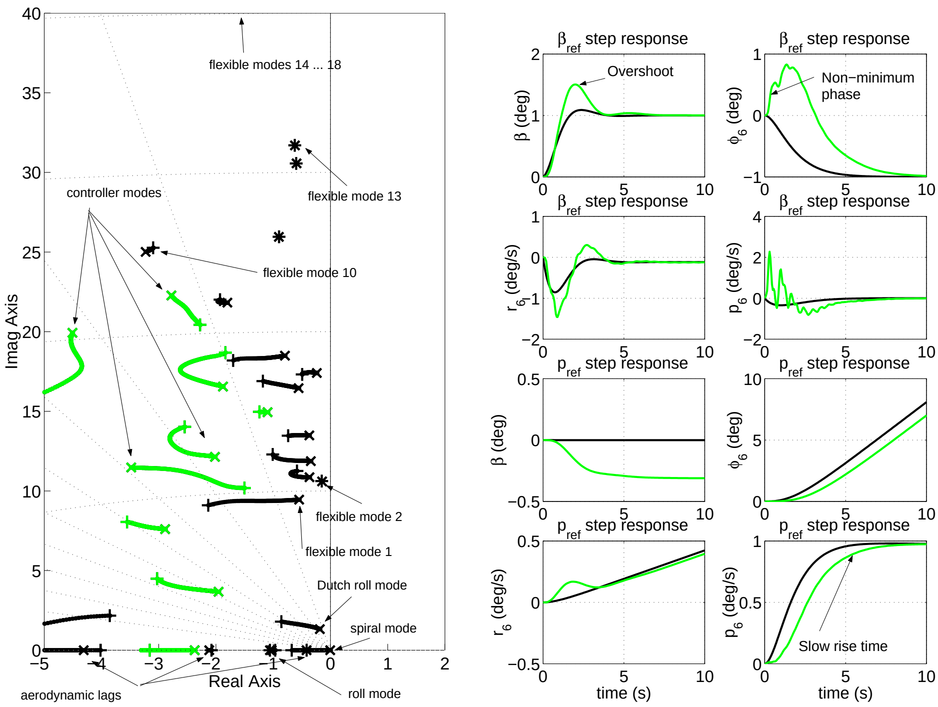

Figure 4: On the left: Root locus $K(s)G_f(S)$ - On the right: time domain responses (G_r, K_{x_r}) (black), (G_f, K) (green)

$K_fC)$) and the Youla parameter dynamics (spec(A_Q)). The transfer between e and u, that is, the feed-forward path, depends directly on this choice. The range of solutions can be reduced according to the following considerations:

- a set of self-conjugated eigenvalues must be chosen in order to find a real parameterization,
- an uncontrollable (resp. unobservable) eigenvalue in the system must be selected in the state-feedback dynamics (resp. state-estimation dynamics),
- thirdly, one usually chooses the state-estimation dynamics (spec$(A - K_fC)$) faster than the state-feedback dynamics (spec$(A - BK_c)$).

The nominal compensator $K(s)$ was first reduced to 20^{th} order (truncation in the balanced realization), and then transformed into LQG form on the 4^{th} order rigid model (then, we obtain a 16^{th} order Youla parameter). The closed-loop eigenvalue distribution (in the vicinity of the rigid dynamics) we have chosen is depicted in Figure 5. The reference signal e really applied is:

$$e = H \begin{bmatrix} \beta_{ref} \\ \frac{1}{s} p_{ref} \end{bmatrix} \tag{22}$$

where H is a static feed-forward matrix computed to ensure the prescribed steady state (equation 21). The simulation thus obtained is presented in Figure 6: we can see that the overshoot on β is reduced, the non-minimum phase response on ϕ has disappeared, the roll/yaw decoupling is restored and the settling time on p was halved. However, this improvement in response has the undesirable effect of exciting the flexible modes. Nevertheless, the transient responses now satisfy the flying quality requirements.

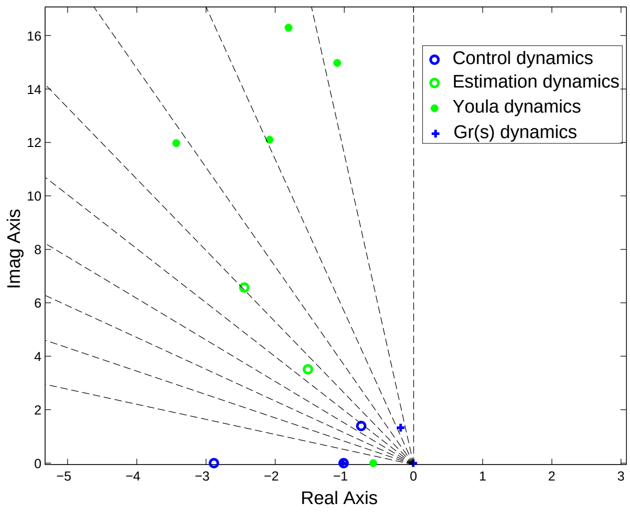

Figure 5: Eigenvalue distribution (around rigid modes).

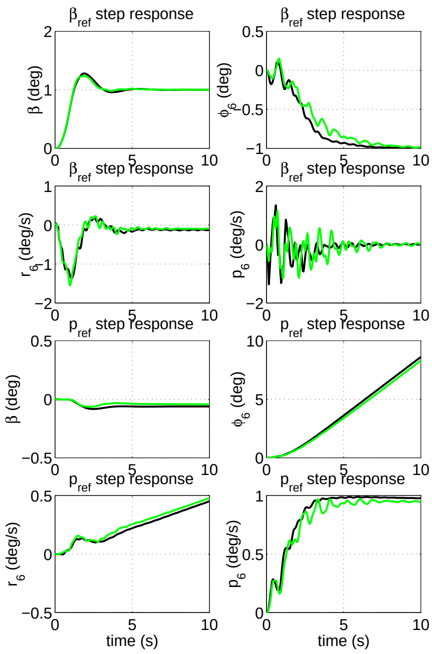

Figure 6: Simulations with the 2 d.o.f. equivalent LQG compensator.

6 CONCLUSIONS

Two observer based tools, the computation of the observer-based realization of a given compensator and the Cross Standard Form, was presented in this paper. From a practical point of view, they are interesting and complementary tools to master multi-objectives control problem. They have been applied to the lateral flight control of a highly flexible aircraft. Firstly a dynamic output feedback was performed using the cross standard form to take into account a previously define state feedback, a roll-off specification and parametric robustness requirements. Then, a two degrees-of freedom has been computed using the observer based realization to fulfill specifications on transient responses to pilot input references.

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