PRACTICAL APPLICATION OF ACTIVE NOISE CONTROL IN A DUCT USING PREDICTIVE CONTROL

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Abstract: This paper presents a practical application of Active Noise Control (ANC) using the Generalised Predictive Control (GPC) algorithm. The main objective of this application was to reduce significantly the noise level in a duct using this technique. It is shown that the experimental results obtained using GPC are very close from those obtained with the classical LMS algorithm on the same experimental set-up. On the other side, no adaptation time is needed and a single controller is used over a broad band. The predictive controller is synthetized using a realistic simulation of the process based upon some rough assumptions on the transfer functions.

Keywords: Active Control, Noise Reduction, Predictive Control, Methodology

1. INTRODUCTION

Nowadays the sound level from the human life environment has increased very much due to technological developement reaching the so called *acoustic pollution*. Thus it becomes necessary to control this to take care of human health and to improving the work conditions. Mainly there exist two techniques for this:

- passive control: that uses the absorbtion, reflecting or diffusing properties of some materials,
- active control: that uses a secondary sound field which is in anti-phase with the main noise source.

Active noise control is more efficient for low frecquency (less than 1000 Hz) completing in this way the passive control which is efficient only for high frecquencies (when the thickness of isolation layer is acceptable). In practice active noise control is mainly used for duct-like systems (e.g. blowers, gas turbines and ventilation systems) or enclosures of low model density (aircraft and vehicle cabins, headphones and control rooms).

There are mainly two aproaches for applying the active noise control: the adaptive filter solution which deals with a reference signal, and the fixed filter approach. The well known Least Mean Square (LMS) algorithm, that belongs to the first family, often lacks of robustness and is quite slow in terms of adaptation time.

In this paper we consider the second approach starting from Clarke's work (Clarke, *et al*., 1987) which led to the Generalised Predictive Control (GPC) and the next section presents the theory of GPC.

2. THE GPC ALGORITHM

Predictive control means a large variety of control methods that share certain common concepts:

- a process model, explicitly used to predict the procces output over a fixed number of steps in the future,
- a known future reference trajectory,
- the calculation of a future control sequence minimising a certain quadratic cost function,
- a receding horizon strategy (at each sampling period only de first control signal of the sequence calculated is applied to the process).

The GPC controller, considered in the present application, uses output predictions based upon a CARIMA model (Controlled Autoregressive Integrated Moving Average) which is given as:

$$\Delta A(q^{-1})y(t)=B(q^{-1})\Delta u(t)+C(q^{-1})\xi(t) \qquad (1)$$

where $y(t)$ is the output signal, $u(t)$ is the control signal, $\xi(t)$is the noise signal, A, B, C there are polynomials of the shift operator q^{-1}. The cost function is given as

$$J=\sum_{i=N_1}^{N_2}\left\{P(q^{-1})\hat{y}(k+i)-w(k+i)\right\}^2+\lambda\sum_{i=1}^{N_u}\left\{\Delta u(k+i-1)\right\}^2 \qquad (2)$$

where $\hat{y}(k+i)$ is the *i* steps ahead predicted output, based upon information available at time *t*, $w(k+i)$ is the reference signal, $\Delta u(k+i-1)$ is the sequence of control increments that is to be determined.

The cost function parameters are the followings:

- Horizons N_1, N_2 and N_u called minimum, maximum and control horizon, respectively,
- Reference trajectory is assumed to be known beforehand. The preferred aproach is to use smooth reference trajectory that begins from the actual output value and approaches asymptoticaly via a first order filter the desired setpoint w^{∞}. It is thus given as

$$w(k)=y(k),$$
$$y(k+i)=\alpha w(k+i-1)+(1-\alpha)w^{\infty}. \qquad (3)$$

Then the α parameter determines the smoothness of the trajectory and will be considered as a tuning parameter. In the practical application from this paper it is used a polynomial filter

$$P(q^{-1})=\frac{1-\alpha q^{-1}}{1-\alpha}, \qquad (4)$$

which achieves the same effect.

- The λ parameters gives the weight of command increment.

One of the main advantages of GPC is its ability to stabilize and control non-minimum phase systems, open-loop unstable processes, even in the presence of dead time, this through judicious choice of the tuning parameters N_1, N_2, N_u, λ and α.

The derivation of the predictor starts with the CARIMA model (1) of the form

$$A(q^{-1})y(t)=B(q^{-1})u(t-1)+\frac{C(q^{-1})}{\Delta}\xi(t). \qquad (5)$$

Forward, to obtain the *j* step predictor, two polynomial divisions (or equivalently Diophantine equations) are to be solved:

$$C=E_j\Delta A+q^{-j}F_j\ ; \qquad BE_j=G_jC+q^{-j}\Gamma_j. \qquad (6)$$

Thus the prediction of the output will be:

$$\hat{y}(t+j)=G_j\Delta u(t+j-1)+\frac{\Gamma_j}{C}\Delta u(t-1)+\frac{F_j}{C}y(t) \qquad (7)$$

$$\hat{y}(t+j)=G_j\Delta u(t+j-1)+y_0(t+j) \qquad (8)$$

This predictor can be vectorized as a function of *j*. Hence the predictor, in the vector notation, can be written as

$$\hat{y}=G\cdot\tilde{u}+y_0, \qquad (9)$$

where G is built with the impulse response coefficients

$$\mathbf{G}=\begin{pmatrix} g_0 & 0 & 0 & \cdots & 0\\ g_1 & g_0 & 0 & \cdots & 0\\ g_2 & g_1 & g_0 & \ddots & \vdots\\ \vdots & \vdots & \ddots & \ddots & 0\\ g_{N_2-1} & g_{N_2-2} & \cdots & g_1 & g_0 \end{pmatrix} \qquad (10)$$

and the cost function is expressed as

$$J=c_0+2g^T\tilde{u}+\tilde{u}^TH\tilde{u}, \qquad (11)$$

where the gradient *g* and the Hessian *H* are defined as

$$g^T=G^T(y_0-w), \qquad (12)$$

$$H=G^TG+\lambda I. \qquad (13)$$

Then it is simple to perform the minimization of the cost function (11) as a direct problem of linear algebra, and it finally yields:

$$\tilde{u}=-H^{-1}g. \qquad (14)$$

This equation gives the whole trajectory of the future control increments as such it is a open-loop strategy. For close-loop, only the first element of $\tilde{u}$ (that means $\Delta u(t)$) is applied to the system, and the whole algorithm is recomputed at time t+1 (Receding Horizon Principle strategy).

So until now, in this section, it has been presented the theoretical background used for calculation of the predictor and the future control sequence which caracterize the GPC. More tuning parameters can be included in the design procedure, but in our case, the simplest algorithm is considered, mainly for implementation reasons. In the next section will be presented the experimental set-up for applying the theory.

3. EXPERIMENTAL ENVIRONMENT

The experimental set-up for applying Active Noise Control was designed and made in the laboratory of the Department of Avionics & Systems of ENSICA, Toulouse.

As it can be observed in Figure 1, it consists in a plastic tubular duct with an elbow at one end, where the primary speaker who will produce the noise to be controlled is placed, and an open end. A second speaker is placed in an intermediate place on the tube for generating the antinoise. There are also two microphones, one to measure the reference signal and the other for the error signal.

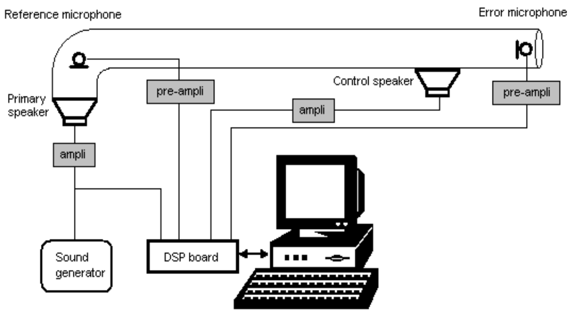

Figure 1 Experimental set-up

For managing the experiment, it was used a high-performance computer which has integrated a Digital Signal Processor (DSP) board DS1102, based on the Texas Instruments TMS 320C31. This will compute the whole process operations with ***dSpace*** software, which allows the user, through its interface ***Control Desk***, to visualize and store the input and output data of the system. The GPC algorithm is implemented with ***Matlab***/***Simulink*** program codes. The experimental set-up is completed by a noise generator and amplifiers for speakers and microphones.

This system having two inputs and two outputs, four transfer function have to be considered and identified:

- H1, the transfer function between primary speaker and error microphone;
- H2, the transfer function between control speaker and error microphone;
- H3, the transfer function between control speaker and reference microphone;
- H4, the transfer function between primary speaker and reference microphone.

Only the first two transfer functions were considered and identified for our experiments. Hence, it has been made the assumption that the H3 transfer function is null (the noise produced by the control speaker receptioned by the reference microphone is very poor) and the H4 transfer function was considered unitary because of the neighbourhood between primary speaker and reference speaker.

The identification of H1 and H2 transfer function was made using sub-space methods from the ***Matlab System Identification toolbox.*** This supposes to send two noise signals via DSP board to the speakers and to receive, in the same way through DSP board, two signal from the microphones.

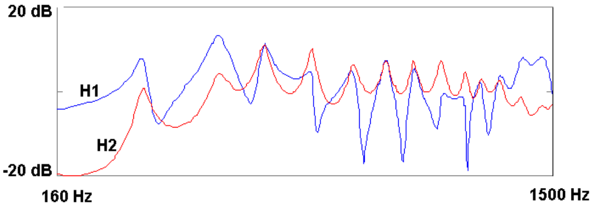

Figure 2. The frequency response of the transfer functions

The sample time t_s was chosen taking into account the Shannon limitations and the operationnal power of the computer processor. So there was made experiments for three values corresponding to t_s : $t_{s1} = 1/2500$ s; $t_{s2} = 1/3000$ s; $t_{s3} = 1/3500$ s.

Having the transfer functions, the system is ready now for the implementation of the GPC controller in order to reduce the noise made by the primary speaker using for this the second speaker.

4. GPC EXPERIMENTAL APPLICATION FOR ANC

First of all, considering the model given in (1), the signals from the experimental set-up will be:

$y(t)$ - the signal from error microphone,

$u(t)$ - the command signal of control speaker,

$\xi(t)$ - the signal from reference microphone.

The ideal objective of the experiment is to peform silence at the error microphone, that means w = 0.

In Figure 2 is shown the block diagram of the GPC control loop.

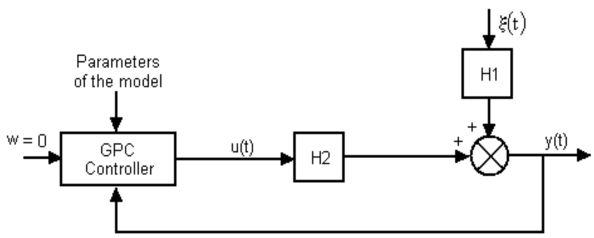

Figure 3. Principle schema for GPC algorithm

In oder to find a simple form of GPC fixed-filter it is necesary to identify the control law equation suitable for this application. For this, it was taken $\Delta u(t)$, the first element of $\tilde{u}$ defined in (14):

$$\Delta u(t) = -h_1 (y_0 - w) \tag{15}$$

with h_1 being the first line of $H^{-1}G^T$ and w = 0.

$$\Delta u(t) = -h_1 \left[\frac{\Gamma_j}{C} \Delta u(t-1) + \frac{F_j}{C} y(t) \right] \tag{16}$$

which can be rewritten as

$$u(t)\left[1+h_1\frac{\Gamma_j}{C}q^{-1}\right]+\frac{h_1}{\Delta}\frac{F_j}{C}y(t)=0\,. \qquad (17)$$

It is now easy to see in (17) the characteristical equation of the RST control law: $Ru(t)+Sy(t)=Tw(t)$, with R, S, T being polynomials of q^{-1}. In the present case, w = 0 so T is not important, then:

$$R=1+h_1\frac{\Gamma_j}{C}q^{-1}\,,\ S=\frac{h_1}{\Delta}\frac{F_j}{C}\,. \qquad (18)$$

Note: In fact, it has experimentally established that stability is better achieved when the R and S polynomials are computed using a CARMA (Controlled Auto-Regressive Moving Average) model:

$$y(t)=\frac{B(q^{-1})}{A(q^{-1})}u(t-1)+\frac{C(q^{-1})}{A(q^{-1})}\xi(t), \qquad (19)$$

which means that $\frac{B(q^{-1})}{A(q^{-1})}=H_2(q^{-1})$ and $\frac{C(q^{-1})}{A(q^{-1})}=H_1(q^{-1})$, the transfer functions who have already been determinated.

But, beside $H_1(q^{-1})$ and $H_2(q^{-1})$, the program which computes the fixed-filter polynomials R and S requires the GPC parameters N_1, N_2, N_u, λ and α. The minumum prediction horizon, N_1, is always taken as a unit ($N_1=1$) because the calculation of the optimal predictor starts at the present moment. The others parameters were chosen or computed taking into account the recomandations from (Fikar, 1999) and the fine tuning was made on the present system for ensuring stability and to improve the noise attenuation, of course.

The simulation of the experiment was achieved on the computer using a ***Simulink*** model which respects all the real conditions of the experiment, as it can be seen in the Figure 3 which shows the simulation scheme.

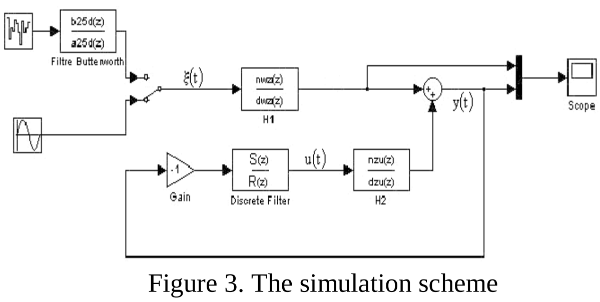

Figure 3. The simulation scheme

The simulations were succesfully done for all three sample times (t_{s1}, t_{s2}, t_{s3}) and the best results were obtained with the next range of values for GPC parameters:

$\alpha\in[0{,}1\ \ldots\ 0{,}15]$, $N_y\in[55\ \ldots\ 70]$,

$\lambda\in[0\ \ldots\ 0{,}03]$ and $N_u=3$.

In the same time the order of the transfer function $H_1(q^{-1})$ and $H_2(q^{-1})$, was increased as long as the sample time t_s was decreased: n = 28 for t_{s1}, n = 32 for t_{s2}, n = 36 for t_{s3}. Once simulations done, the same parameters were used for real-time experiments. In Figure 4 are presented, in graphical form, the results for sample time t_{s2} and sine noise source, both for simulation and experiment. The similitude of simulation results with experimental results, which can be observed in Figure 4, shows a very good modelisation of the acoustical paths in the duct, in spite of some rough assumptions about H3 and H4.

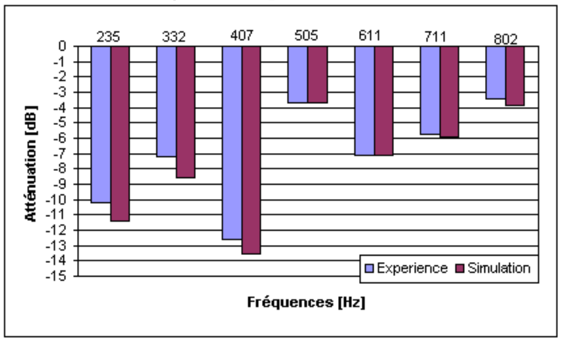

Figure 4. The attenuation results from experience and simulation for t_{s2}

The frequencies for which the calculation of the noise attenuation has been made are the resonance frequencies of the system in the range 200....1000 Hz. As source signal, we used both sine and band limited noise sources. The noise signal was centered on the same frequencies as the sine signal but with a 25 Hz bandwidth.

The noise attenuation was computed by a ***Matlab*** program which has used the data stored by ***Control Desk*** during the experiment. The amount of data corresponded just for 2 seconds action, enough for calculation. The graphical representations of attenuation can be seen in Figures 5 and 6 for a particular case.

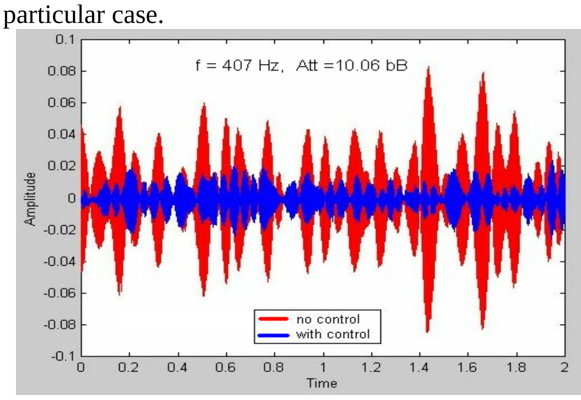

Figure 5 Attenuation for band limited noise

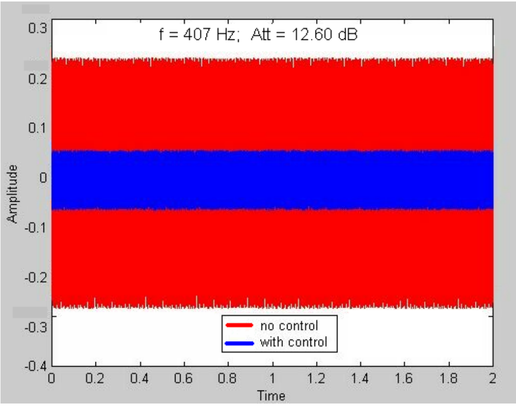

Figure 6 Attenuation for sine noise

All the results of the experiments are shown in Table 1 where can it be noted the variations of noise attenuation (in dB) for each frequency (in Hz) as a function of sample time t_s.

Table 1 The results of the experiences

f / ts	Sine Noise						
ts	*235*	*332*	*407*	*505*	*611*	*711*	*802*
ts_1	**-7.30**	**-5.44**	**-8.38**	**-3.85**	**-10.45**	**-5.91**	**-1.00**
ts_2	**-10.18**	**-7.19**	**-12.60**	**-3.67**	**-7.12**	**-5.77**	**-3.44**
ts_3	**-10.00**	**-8.60**	**-11.94**	**-2.35**	**-3.41**	**-6.06**	**-6.65**
f	Band limited noise						
ts	*235*	*332*	*407*	*505*	*611*	*711*	*802*
ts_1	**-3.11**	**-4.47**	**-5.40**	**-2.41**	**-8.31**	**-4.82**	**-0.20**
ts_2	**-0.89**	**-5.95**	**-10.06**	**-2.25**	**-6.65**	**-4.78**	**-1.86**
ts_3	**-3.95**	**-6.44**	**-9.04**	**-0.93**	**-1.71**	**-4.79**	**-5.35**

With the help of ***Control Desk,*** it can be observed (Figure 7) the quickness of attenuation when the GPC controller is turned on.

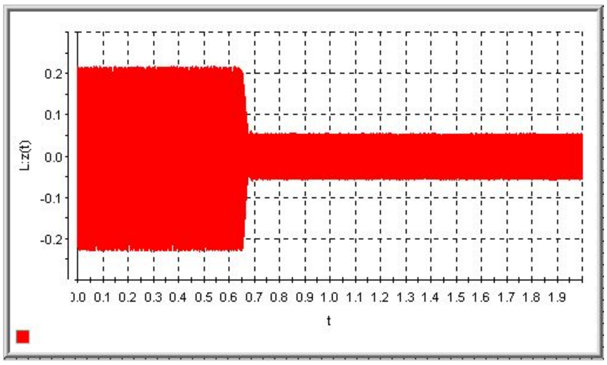

Figure 7. Transient response of the GPC Controller

The transient response is very short and independent of the source signal (frequency, level) unlike the applications using LMS. All the experiments using GPC shown a very good robustness of the controller.

5. CONCLUSION

The experiment shows that the noise attenuation in a duct using GPC algorithm gives good results if the tuning parameters $(N_1, N_2, N_u, \lambda$ and $\alpha)$ are carefully chosen. More, it has been shown that a simple simulation is realistic enough for a fine tuning of the real time controller.

The experimental results in noise reduction obtained using LMS on the same experimental set-up (Rastoul, 2001) led to a better attenuation (about 15%) than the results presented in Table 1. But it is also true that the adaptation time was up to 10 minutes for LMS. Unlike LMS, GPC gives a good attenuation very quickly as it can be seen in Figure 7. In addition of its simplicity, another advantage of GPC is the unicity of the filter for all frequencies and its robustness. For these reasons, GPC could become one of the most suitable predictive technique for practical application in the real life for noise control.

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