Active Control of a clamped beam equipped with piezoelectric actuator and sensor using Generalized Predictive Control

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***Abstract*— A predictive control method to perform the active damping of a flexible structure is here presented. The studied structure is a clamped-free beam equipped with collocated piezoelectric actuator/sensor. Piezoelectric transducers advantages lie in theirs compactness and reliability, making them commonly used in aeronautic applications, context in which our study fits. Theirs collocated placement allow the use of well-known control strategies with guaranteed stability. First an analytical model of this equipped beam is given, using the Hamilton's principle and the Rayleigh-Ritz method. After a review of the experimental setup (and notably of the piezoelectric transducers), two control laws are described. The chosen one - Generalized Predictive Control (GPC) - will be compared to a typical control law in the domain of flexible structures, the Positive Position Feedback, one of the control law mentioned above. Majors benefits of GPC lie in its robustness in front of model uncertainties and others disturbances. The results given come from experiments on the structure, performed thanks to a DSP. GPC appears to suit for the considered study's context (i.e. damping of the first vibration mode). Some improvements may be reached. Among them, a more complex structure with more than a single mode to damp, and more uncertainties may be considered.**

***Index Terms*— Active Control - Piezoelectric transducers - Predictive Control - Flexible structures.**

I. Introduction

The reduction of vibrations in flexible structures finds one of its most important applications in avionics. Indeed, considering the vibratory environment in this domain, we can distinguish two kinds on vibration: the transient vibrations which are related to impulses or gusts and the permanent vibrations dues to repetitive efforts or turbulences. Transient vibrations, in particularly, are located around the wings and the fuselage, appears in low frequency and may excite the modes of the structure (See details in [7]).

In this paper, a Generalized Predictive Control (GPC) is used to perform the active damping of a clamped beam equipped with piezoelectric sensor and actuator. The purposes here are on the one hand to perform the damping of the structure i.e. to perform as efficiently as possible the vibratory disturbances reject and on the other hand to evaluate the performances of such a control law, in the domain of flexible structures.

The flexible structure considered has its eigenfunctions in low frequency - around 20 Hz - just like avionics structures. This equipped beam will be described in section II-A, where more details concerning the piezoelectric components and the experimental environment are given. An analytical state model follows this description, taking into account the full control loop i.e. including the piezoelectric actuator and sensor. The section IV deals with the control law. The basis and the main features of the chosen control law are here explained. Finally, the efficiency of such a control law is studied in the section V. These results will allow appreciating the interest of GPC, according to more classical control laws.

II. Experimental Setup

A. The control loop

The whole control loop is represented Fig. 1.

The studied structure in an aluminium clamped-free beam, whose characteristics will be given in Table I. This beam is equipped with piezoelectric sensor and actuator (c.f. next section), bounded in its clamped side. In case of vibrations, the sensor is subject to a bending moment which appears as a charge variation, then as a voltage, thanks to a charge amplifier. In an analog way, the actuator induce into the structure a bending moment proportional to the voltage applied to it.

This flexible structure has its eigenfunctions in low frequency - around 20 Hz - just like avionics structures, domain in which our study fits.

The output and control signals are respectively the stresses measured by the sensor and provided by the actuator at the clamped side of the beam. The full control loop also includes a charge amplifier and a voltage amplifier.

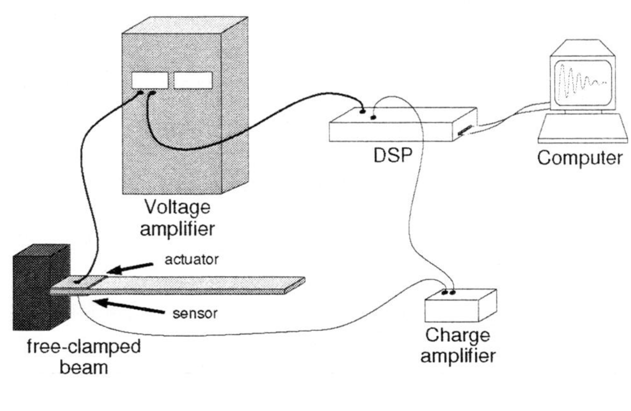

Fig. 1. Control loop

Using such a charge amplifier allow us to measure a charge quantity instead of a voltage and thus to avoid difficulties inherent to the cables impedance. This amplifier set the tension between the sensor's electrodes to zero. The charges are conveyed to a capacity inside the charge amplifier. The quantity of charges is determined by measuring the tension at the capacity's terminals.

The control law is computed by a DSP, a dSpace board.

material	Aluminium
length	$L = 30cm$
width	$l = 2cm$
thickness	$h = 2mm$
Young modulus	$75GPa$
density	$\rho = 2970$

TABLE I
BEAM CHARACTERISTICS

B. *The piezoelectric devices*

The piezoelectric transducers allow the conversion between mechanical and electrical energy. Thanks to the piezoelectric effect, bending moments can be measured and provided (because they are proportional to the delivered and provided voltages).

These patches are bonded on each side of the beam. They are collocated (c.f. the 'Modelling' section). It is shown that the most efficient location for them is on the clamped side of the structure. The chosen actuator material is PZT (*Lead-Zirconate-Titanate*). This actuator will be glue on the beam using an epoxy adhesive. The beam being used as ground, the sensor's electrode has its potential set to zero. A cable is glued on the other electrode.

The sensor will be cut in a PVDF (*Polyvinylidene fluoride*) sheet. It is placed on the other face of the beam, using double-face adhesive, with one cable per electrode.

The main benefits of using piezoelectric transducers lie in the fact that this is nowadays a mature technology (see [1], [2]). Theirs linearity, compactness and reliability should make them particularly efficient for such a structure. Moreover piezoelectric patches are commonly used in aeronautic applications (precisely because of theirs above advantages), context in which our study fits.

Main characteristics of the piezoelectric transducers will be given in Table 7.

III. MODELING

The purpose of this section is to give an analytical state model of the equipped beam.

The first step is to consider the Hamilton's principle (see (1)), thanks to which the relation between the conservative forces's work and the kinetic and potential energies can be expressed.

$$\int_{t_1}^{t_2} \delta(T-U)dt + \int_{t_1}^{t_2} \delta\tau dt = 0 \qquad (1)$$

where:

- τ: conservative force's work

	Actuator/Sensor
material	PZT
length	$L_a = L_c = 2.5cm$
width	$l_a = l_c = 2cm$
thickness	$h_a = 0.5mm$
	$h_c = 25.10^{-6}m$
Young modulus	$E_a = 60GPa$
	$E_c = 60GPa$
piezo. coef	$d_{31a} = 210.10^{-12}mV^{-1}$
	$d_{31c} = 16.10^{-12}mV^{-1}$

TABLE II
PIEZOELECTRIC TRANSDUCERS CHARACTERISTICS

- T: kinetic energy
- U: potential energy

This continuous problem is solved using an approximation method, like the Rayleigh-Ritz (or variational) one, where the considered generalized coordinates q_i will be the nodes displacements. The deflection tip will be approximated by the sum of these generalized displacements weighted by the *shape functions* η, as described in (2)

$$w(x,y,z,t) = \sum_i \eta_i(x,y,z).q_i(t) = \eta^t.q \qquad (2)$$

The Hamilton's principle is rewritten, using (2) and is used jointly with the Lagrange equations, defined in (3).

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} = F \qquad (3)$$

where F represent the *generalized forces*.

This lead to the dynamics equations (4):

$$M\ddot{q} + C\dot{q} + Kq = F \qquad (4)$$

where M and K are respectively the *mass* and *stiffness matrix*. C is the damping matrix. It has been added to improve the model.

Because the aim is to compute a control law, the notions of input and output are added to the above equations.

$$\begin{cases} M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = bu(t) \\ y(t) = c_d q(t) + c_v \dot{q}(t) + c_a \ddot{q}(t) \end{cases} \qquad (5)$$

where u and y are respectively the input and output.

Thus a state-space representation of the structure can be written. However the flexible structures domain deals with the dynamic behavior of these structures. A real modal analysis must be performed so the modal base is chosen to express the state-space representation, where the eigenfunctions will be the mode shapes of the beam.

The state-space representation in such a base will be written as:

$$\begin{cases} \begin{bmatrix} \ddot{\tilde{q}}_i \\ \dot{\tilde{q}}_i \end{bmatrix} = \begin{bmatrix} -2\xi_i\omega_i & -\omega_i^2 \\ 1 & 0 \end{bmatrix} . \begin{bmatrix} \dot{\tilde{q}}_i \\ \tilde{q}_i \end{bmatrix} + \begin{bmatrix} \tilde{b}_i \\ 0 \end{bmatrix} .u \\ \\ y = \begin{bmatrix} 0 & \tilde{c}_i \end{bmatrix} . \begin{bmatrix} \dot{\tilde{q}}_i \\ \tilde{q}_i \end{bmatrix} + 0.u \end{cases} \qquad (6)$$

The sensor and the actuator were previously called 'collocated'. This term is used when a structure is equipped with an actuator applying an generalized effort and with a sensor measuring the corresponding degree of freedom. This case results in a very particular allure of the Bode diagram (see

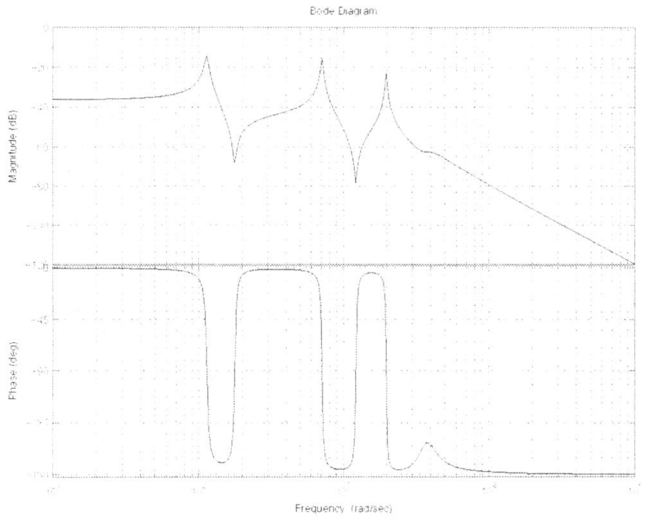

Fig. 2. collocated system (4 first modes)

Fig. 2) and in an alternating pole-zero pattern near the imaginary axis in the root locus. These properties aren't anecdotic, they have an important significance in term of stability. Indeed, such a pattern guarantees the asymptotic stability of some well-known control laws (and allows to avoid the pole-zero flipping phenomenon for example). One of these typical control laws will be reviewed in the next section.

The state-space model must take account into the piezoelectric devices. Bundling the actuator and sensor into the model leads to consider the piezoelectric equations (7).

$$\left\{\begin{array}{l} S = s^{\epsilon}\sigma + d^{t}\epsilon \\ D = d\sigma + \varepsilon^{\sigma}\epsilon \end{array}\right. \tag{7}$$

Those bind electrical (electric field ϵ and displacement D) to mechanical values (stress S and strain σ). *(This notation is only valid for **this** equation set).* Thanks to these equations, the relations between the voltages (applied by the actuator and delivered by the sensor) and the bending moments corresponding can be written. The expressions of $\tilde{b}_i$ and $\tilde{c}_i$ can also be written, in function of piezoelectric variables (see détails in [2]).

Note: Another way lies in adding the energetic contribution of the piezoelectric patches into the Lagrange equations.

The final model include each element quoted above. The last step in the modelling process is the model reduction. Indeed, on the one hand a too large number of states in the model would make the control almost impossible to perform experimentally and on the other hand, the choice has been made to focus the control on the first vibration mode. So the model has been reduced to the second order.

IV. Control law

The two chosen control laws will be here described. As been said above, the control focus on the first flexible mode of the beam. So the disturbance used during the tests will be chosen to suit this aim.

To be able to evaluate the contribution of GPC in the study domain, we need to compare it to a standard. The 'reference' control law selected is one of the typical control laws used for active control of flexible structures, named PPF (Position Positive Feedback). The second one is (of course) the GPC (Generalized Predictive Control).

A. PPF

The interests and benefits of using collocated actuator/sensor couples are already been remarked. A class of control law that guarantee the asymptotic stability has been used and successfully tested.

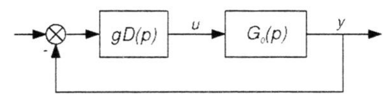

Fig. 3. SISO loop

Such control laws are built on considering independent SISO loops $g.G_0(p).D(p)$ (see Fig.3), where g is a scalar gain, G_0 the sensor/actuator transfer and D the controller. Among these control laws - and because of the natural input and output of our structure i.e. stresses - the Positive Position Feedback (PPF) appears as particularly appropriate (see [3]).

Its starting idea is that some roll-off must exist in $g.G_0(p).D(p)$ to deflect the phase lag due to the actuator dynamic or the digital sampling for example. If the structure (G_0) doesn't include enough roll-off, it must appear in the controller. Let's consider a typical structure to damp. The governing equations are almost similar to a second order filter:

$$\left\{\begin{array}{l} \ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = u \\ \\ \dot{y} = B^T\dot{x} \end{array}\right. \tag{8}$$

The control law u will be defined by :

$$\left\{\begin{array}{l} \ddot{v} + 2\xi_f\omega_f\dot{v} + \omega_f^2 v = \omega_f^2 x \\ u = g\omega_n^2 v \end{array}\right. \tag{9}$$

(ω_n^2 is introduced to make g non-dimensional).

The structure to damp has two more poles than zeros (see (8) and two extra poles are brought by the controller. Thus, as seen in Fig.4, four asymptotes exist in the root locus. An analysis shows that stability of the closed loop system is guaranteed for $0 < g < 1$.

Notes: The aim is to damp the first mode, so these equations have been given in the mono-dimensional case.

B. GPC

Each Predictive control methods involve three steps. First of all comes the *Output prediction*. As Predictive control is a model-based control law, this model allow to predict the future behavior of the system output from the actual data. Then occurs the *Control calculation*, when the control signal is compute to make the predicted output as close as possible to the desired future output. The last step lies in *closing the feedback loop*, during which the current value of the proposed future control signal is applied to the system. Because these three steps happen at each sample instant, predictive control is known as a *receding horizon* strategy.

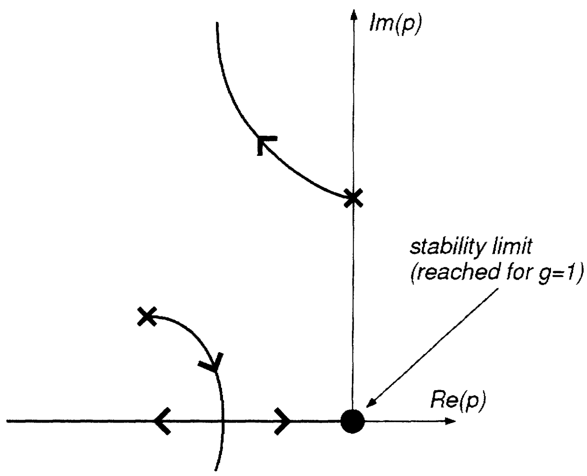

Fig. 4. PPF - root locus

Generalized Predictive Control (GPC) is an interesting and later approach among the differents Predictive Control methods(see [4], [5], [6]).

It uses a CARIMA model of the considered plant (instead of the impulse or step response models previously used, which bring difficulties to handle unstable systems). Consider a plant of the form:

$$A(q^{-1})y(t) = q^{-1}B(q^{-1})u(t) \qquad (10)$$

Where y, u are respectively the output and input of this plant.

The linear CARIMA model will be written as:

$$A(q^{-1})\Delta y(t) = q^{-1}B(q^{-1})\Delta u(t-1) + \xi(t) \qquad (11)$$

Where ξ is the the disturbance.

The control law is computed by minimizing a cost function (see (12)), which involves the predicted output y^* and a reference set r.

$$J = \sum_{j=h_1}^{h_2} [\hat{y}(t+j) - r(t+j)]^2 + \sum_{j=1}^{h_u} \lambda [\Delta u(t+j-1)]^2 \qquad (12)$$

We can distinguish in this cost function the minimum and maximum prediction horizon (h_1 and h_2, respectively), the control horizon (h_u) and the control increment weighting (λ). These four values are the design parameters of GPC.

$\hat{y}(t+j)$ is the predicted output at time $t+j$. This prediction is performed thanks to the available input/output data at time t, as shown in (13).

$$\hat{y}(t+j) = F_j(q^{-1})y(t) + E_j(q^{-1})B(q^{-1})\Delta u(k+j-1) \qquad (13)$$

With the *Diophantine equation* (14):

$$1 = E_j(q^{-1})A(q^{-1}).(1-q^{-1}) + q^{-1}F_j(q^{-1}) \qquad (14)$$

The prediction of the system output y is based on two different components, named *free* and *forced responses*. The free response represents the predicted behavior of the output $y(t+j|t)$ (in the range from $t+1$ to $t+N$), based on previous outputs $y(t-i|t)$ and inputs $u(t-i|t)$, considering no future control. The forced response is the additional component of the output computed from the optimization criterion.

The total prediction is the sum of both components (for linear systems). Together with the known reference values the future errors can be calculated. Caused by these future errors, future control signals are calculated to set the output to the desired reference values.

The following step is to separate the forces and free responses, using (15):

$$\hat{y}(t+j) = \hat{y}(t+j|t) + G(q^{-1})\Delta u(k+j-1) \qquad (15)$$

(15) can also be written :

$$\hat{y} = G.\tilde{u} + f \qquad (16)$$

where:

$$\begin{cases} \hat{y} = [\hat{y}(t+h_1)\ldots\hat{y}(t+h_2)]^t \\ f = [\hat{y}(t+h_1|t)\ldots\hat{y}(t+h_2|t)]^t \\ \tilde{u} = [\Delta u(t)\ldots\Delta u(t+h_u-1)]^t \end{cases} \qquad (17)$$

and

$$G = \begin{pmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ \vdots & & & g_0 \\ g_{h_2-1} & \cdots & \cdots & g_{(h_2-h_u)} \end{pmatrix} \qquad (18)$$

where g_i are the polynomial elements of $G(q^{-1})$.

We obtain:

$$\tilde{u}_{opt} = \left(G^tG + \lambda.Id\right)^{-1}.G^t(r-f) \qquad (19)$$

GPC belongs to the group of "long-range predictive controllers" and generates a set of future control signals in each sampling interval, but only the first element of the control sequence is applied to the system input, as described in (20).

$$u(t) = u(t-1) + \bar{g}^t(r-f) \qquad (20)$$

With $\bar{g}$, being the first row of $\left(G^tG + \lambda.Id\right)^{-1}.G^t$.

In addition to its well-known good control performance the robustness properties makes GPC interesting and realizable for practical control applications. For this purposes GPC offers a compact control strategy in terms of model mismatches, variable dead time and disturbances.

V. Results

As mentioned above, the aim of the control law is to damp the first vibration mode. The tests must reflect this purpose. *Release tests* have thus been performed: the beam's tip is deviated from its equilibrium position and released. This kind of disturbance allows to excite almost exclusively the first mode of the beam. Both of the control laws have been applied to the structure. Two criteria will be used to allow the quantitative comparison between them (see below). The result figures will provide the temporal responses curves using each of the control strategies and a "5% zone", corresponding to 5% of the maximum magnitude of the response.

The first criterion used is a temporal one. Based on the "5% zone", it will indicate the contribution of the considered controller, in terms of velocity for returning to equilibrium state. Concretely, this criterion represent the response times ratio with and without the controller.

The second criterion is an energetic one. It express the corresponding energy of the response signal, i.e. the cumulated sum of the response's square (normalized by the energy for the uncontrolled structure).

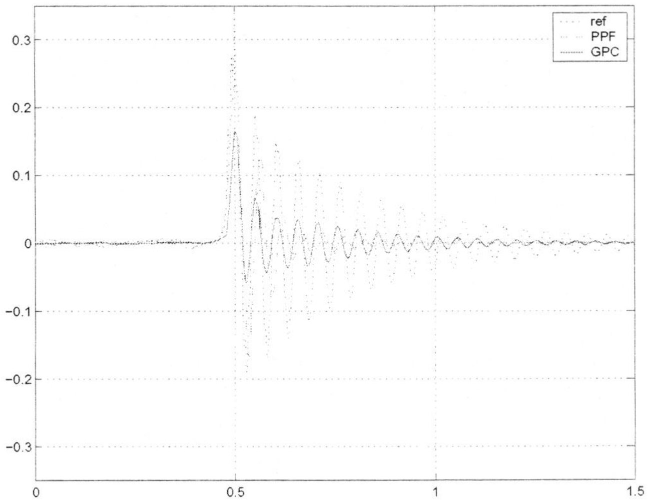

Fig. 5. control laws comparison

First of all, we can make a qualitative remark about the responses curve envelops. The response of the system controlled by PPF has the most "roped" curve: the "5% zone" is reached very quickly. This behavior is noticeable, when observing the experiment, the vibration reduction is particularly impressive. However, the control isn't very efficient at the beginning of the test.

The PPF is noticeably effective according to the temporal criterion. As remarked above, the "5% zone" is more quickly reached than when GPC is used. For that matter, tests using the PPF controller are the most visually impressive.

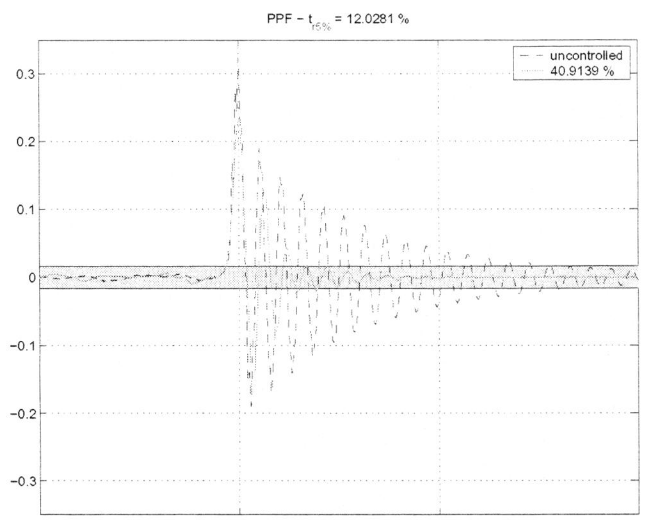

Fig. 6. Positive Position Feedback

However, according to the energetic criterion, GPC is far more efficient. Indeed, as described in Table III the energy corresponding to the response for the system controlled with GPC is near from the value of 16% (versus about 40% for

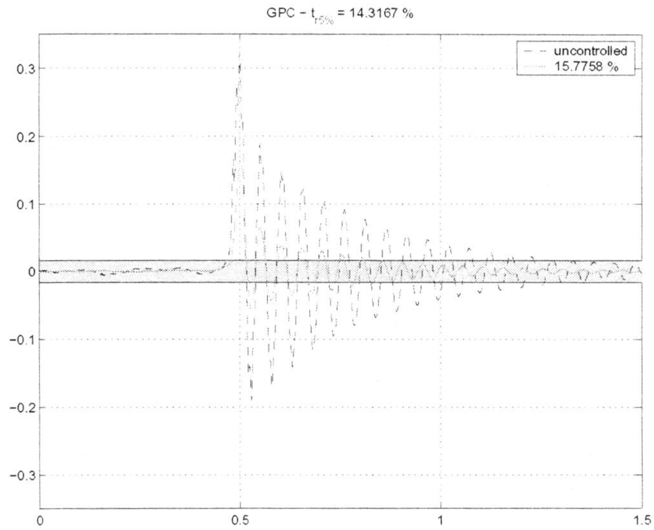

Fig. 7. Generalized Predictive Control

PPF control). We can explain such large differences by looking at the structure behavior during the beginning of tests (see Fig. 5 and 7).

Indeed, the GPC controller sensibly damps vibrations since the first overshoot. In return, as damping is particulary efficient in the beginning of the test, the "5% zone" is reached later than with PPF control. This means less impressive performances according to the temporal criterion.

control law	temporal criterion	energetic criterion
PPF	12.03%	40.91%
CGPC	14.31%	15.78%

TABLE III

COMPARATIVE ANALYSIS

VI. Conclusion and perspectives

This study's purpose was to evaluate how efficient predictive control could be, when applied to the flexible structures domain. For the considered structure and context (i.e. control of the first vibration mode of a clamped-free beam), GPC seems to suit. However, this study represent a first step and fits in a larger context. Indeed, the choice has been made to focus on the first vibration mode and the tests performed have been designed in this perspective.

Some improvements have to be made to complete the present results. Among them, there is the sensor choice. The actual sensor is a PVDF piezoelectric patch. The choice of this material has been motivated by its nature itself: PVDF is a polymer (instead of a ceramic, like PZT). PVDF is noticeably flexible, so may be much thinner and larger than PZT. In return, some problems of noise in the signal measured by the sensor occur during experiments, problems which can be avoided by the use of a PZT patch, thinner than one used for the actuation.

Also concerning the transducers, it would be useful to take a look at the dimensioning. This study is at work, to evaluate the potential performances of the actuator.

At least, the full potential of the predictive control can't be exploited when controlling a unique mode. Indeed, this control law is classified as a *global control law*. It means that such a control law damps not only the mode for which she was designed. In fact, a global control law damps each modes of the structure, and mainly the mode it is designed for.

These improvements refer to a following study, using a more complex structure. This new experimental support calls upon several modes coupled in more than an unique dimension and it needs a particularly robust control law, because of some additional uncertainties due to fluid/structure interactions within it.

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