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Direction Finding for an Extended Target With Possibly Non-Symmetric Spatial Spectrum

Andrei Monakov and Olivier Besson

Abstract—We consider the problem of estimating the direction of arrival (DOA) of an extended target in radar array processing. Two algorithms are proposed that do not assume that the power azimuthal distribution of the scatterers is symmetric with respect to the mass center of the target. The first one is based on spectral moments which are easily related to the target's DOA. The second method stems from a previous paper by the present authors and consists of a least-squares fit on the elements of the covariance matrix. Both methods are simple and are shown to provide accurate estimates. Furthermore, they extend the range of unambiguous DOAs that can be estimated, compared with the same previous paper.

Index Terms—Covariance matrix, direction-of-arrival estimation, extended target, spectral moments.

I. INTRODUCTION AND MOTIVATION OF THE WORK

The problem of estimating the direction of arrival (DOA) of an extended target is an important issue in radar array processing. Briefly speaking, a target can be considered as "extended" as soon as its physical dimensions are of the same order as the array beamwidth (although the signal to noise ratio has also to be accounted for in order to define an extended target). In such a case, the energy backscattered by the target seems to no longer emanate from a point source but from multiple, closely spaced scatterers [2]. This in turn implies that the signal received on the array does not result in a rank-one correlation matrix. In fact, the distribution of the eigenvalues correlation matrix (and in particular the value of the second eigenvalue of the correlation matrix compared to the noise floor) serves as an indicator for defining a target as extended; see, e.g., [3] for a related discussion. Interestingly enough, a similar problem has been recently evidenced in the area of wireless communications. Some campaigns of measurements [4] have shown that local scattering in the vicinity of a mobile is a non-negligible phenomenon. Owing to the presence of local scatterers around the mobile, the source appears to be spatially dispersed, as seen from a base station antenna array. This has a potential impact on the performance of any array processing algorithm and, thus, should be taken into account. Finally, note that in underwater acoustics, a nonhomogeneous propagation medium gives rise to coherence loss along the array [5], [6] and, therefore, to a full-rank correlation matrix.

Briefly stated, the signal received on an array of sensors from a spread source can be described by the following model:

$$\mathbf{y}(t) = \mathbf{x}(t) \odot \mathbf{a}(\theta_0)s(t) \quad (1)$$

where $\mathbf{x}(t)$ describes the random multiplicative effect due to local scattering, $s(t)$ is the emitted signal that is independent of $\mathbf{x}(t)$, and $\mathbf{a}(\theta_0)$ is the so-called steering vector. In the previous equation, \odot denotes the Schur–Hadamard (i.e. element-wise) product. In the case of a point

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source, $\mathbf{x}(t) \equiv \mathbf{1}$. The covariance matrix corresponding to (1) is easily obtained as

$$\mathcal{E} \left\{ \mathbf{y}(t) \mathbf{y}^H(t) \right\} = P \mathbf{B} \odot \left[\mathbf{a}(\theta_0) \mathbf{a}^H(\theta_0) \right] \quad (2)$$

where $\mathbf{B} = \mathcal{E} \{ \mathbf{x}(t) \mathbf{x}^H(t) \}$ is the covariance matrix of the multiplicative perturbation, and P stands for the source power. The elements of \mathbf{B} depend on the azimuthal power distribution of the scatterers.

In this correspondence, we address the problem of estimating θ_0 from N snapshots drawn from (1). In order to be insensitive to a possible mismodeling of the azimuthal power distribution, we consider estimators that do not make any assumption on the distribution of the scatterers. Direction finding for scattered sources without any specific assumption on the form of \mathbf{B} has received considerable attention in the last few years. A first approach consists of applying techniques developed originally for point sources. For example, the conventional beamformer was studied in [7] and was shown to produce accurate estimates, provided that the angular spread is not large. Likewise, application of subspace-based techniques such as MUSIC or ESPRIT still enables to recover accurately the mean DOA in the case of small angular spreads [8]. However, these methods are likely to degrade rapidly if source spreading becomes large and is not taken into account [9]. Extension of subspace-based techniques to handle spread sources have been presented, e.g., in [10] and [11]. The proposed methods work rather well; however, selection of the signal subspace dimension still remains a delicate task for which no theoretically sound solution exists. Indeed, the ‘‘optimal’’ number of eigenvectors to be retained depends, among others, on the angular spread, which is unknown, and the performance of these estimators is sensitive to this parameter. Robust and simple methods were recently proposed in [1], [12], and [13]. These methods essentially rely on an unstructured model for the covariance matrix of the multiplicative noise. Hence, they do not assume any specific form for the azimuthal power distribution of the scatterers, which is an appealing feature. Additionally, they provide computationally efficient algorithms. Finally, their performance was shown to approach the Cramér–Rao bound in a wide variety of scenarios. However, these methods are based on, and indeed exploit, the fact that the azimuthal power distribution is *symmetric*, which makes the covariance matrix of the multiplicative noise \mathbf{B} *real-valued*. Therefore, they cannot handle the situation where the distribution of the scatterers is no longer symmetric. Observe that this would be the case with an extended target that is not symmetric with respect to its mass center.

In this correspondence, we relax the assumption of a real-valued covariance matrix of the multiplicative noise and consider the more general situation of a possibly nonsymmetric power distribution. Two simple methods, which do not depend on any assumption on the form or the symmetry of the scatterers distribution, are proposed. The first is based on a general parametrization of the covariance matrix in terms of its spectral moments, as was done in [2]. The second borrows ideas from [1], where a least-squares fit on the elements of the covariance matrix is carried out. In contrast to [1], the assumption of a real-valued covariance matrix of the multiplicative noise is not made. The proposed method exploits the fact that it corresponds to some correlation function. Moreover, compared with [1], it extends the unambiguous range of DOAs that can be estimated.

II. DATA MODEL AND ASSUMPTIONS

Let us consider a uniform linear array (ULA) of m sensors spaced a half-wavelength apart. The snapshot received at time t is assumed to obey the following model:

$$\mathbf{y}(t) = \mathbf{x}(t) \odot \mathbf{a}(\omega_0) s(t) + \mathbf{n}(t) \quad (3)$$

where $\omega_0 = \pi \sin \theta_0$ is the spatial frequency, and $\mathbf{a}(\omega_0) = [1 \ e^{i\omega_0} \ \dots \ e^{i(m-1)\omega_0}]^T$ is the so-called steering vector. $\mathbf{n}(t)$ is assumed to be a zero-mean complex-valued spatially white noise with power σ_n^2 . $s(t)$ denotes the emitted signal with power P , and $\mathbf{x}(t) = [x_1(t) \ \dots \ x_m(t)]^T$ captures the effect of local scattering or that of an extended target in radar array processing. $\mathbf{x}(t)$ is assumed to be stationary both temporally and spatially. With $\mathbf{R}_x = \mathcal{E} \{ \mathbf{x}(t) \mathbf{x}^H(t) \} \mathcal{E} \{ |s(t)|^2 \}$ being the correlation matrix of $\mathbf{x}(t)$ (in which, for the sake of convenience, we have absorbed the signal power), the correlation matrix of $\mathbf{y}(t)$ is readily obtained as

$$\mathbf{R}_y = \mathbf{R}_x \odot \left[\mathbf{a}(\omega_0) \mathbf{a}^H(\omega_0) \right] + \sigma_n^2 \mathbf{I} = \mathbf{R}_s + \sigma_n^2 \mathbf{I} \quad (4)$$

under the additive white noise assumption. For later use, let us define

$$r_x(k) = \mathcal{E} \{ x_{k+\ell}(t) x_\ell^*(t) \} = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_x(\omega) e^{ik\omega} d\omega \quad (5)$$

where $S_x(\omega)$ corresponds to the spectrum of the random process $\mathbf{x}(t)$. Observe that $S_x(\omega)$ is determined by the spatial distribution of the scatterers. In contrast to most studies so far, we do not make the assumption that $r_x(k)$ is real valued, i.e. it can take values in \mathbb{C} . However, we assume that the center frequency of $S_x(\omega)$ is 0, that is

$$\int_{-\pi}^{\pi} \omega S_x(\omega) d\omega = 0. \quad (6)$$

In other words, ω_0 corresponds to the ‘‘mass center’’ of the target. This assumption is important as we can concentrate on the estimation of ω_0 without any additional assumptions concerning the shape of $S_x(\omega)$. Given N snapshots $\{ \mathbf{y}(t) \}_{t=1}^N$ drawn from (3), our goal is to estimate ω_0 . For later use, let

$$\hat{\mathbf{R}}_y = \frac{1}{N} \sum_{t=1}^N \mathbf{y}(t) \mathbf{y}^H(t) \quad (7)$$

denote the sample covariance matrix.

III. ESTIMATION

In this section, two methods for estimating ω_0 are proposed. The first one makes use of spectral moments, whereas the second algorithm consists in a least-squares fit on some statistic build from the sample covariance matrix.

A. Spectral Moments-Based Estimation

Our first method relies on the fact that the covariance matrix \mathbf{R}_s can be written in the following form [2]:

$$\mathbf{R}_s = \sum_{q=0}^{\infty} M_q \mathbf{A}_q \quad (8)$$

$$\mathbf{A}_q(k, \ell) = \frac{1}{q!} [i(k - \ell)]^q e^{i(k - \ell)\bar{\omega}}$$

where M_q are the so-called spectral moments, $\bar{\omega}$ is arbitrary, and $\mathbf{A}_q(k, \ell)$ is the (k, ℓ) element of \mathbf{A}_q . It was shown in [2] that spectral moments have a clear physical meaning. More precisely, the zeroth moment M_0 equals the total scattered power (in our case $P r_x(0)$). The first moment is given by

$$M_1 = M_0(\omega_0 - \bar{\omega})$$

and thus provides information about the sought parameter ω_0 . The previous relation suggests the following very simple method for estimating the center frequency

$$\hat{\omega}_0 = \bar{\omega} + \frac{\widehat{M}_1}{\widehat{M}_0} \quad (9)$$

where \widehat{M}_1 and \widehat{M}_0 correspond to estimates of the moments. The latter can be obtained easily from the sample covariance matrix as the solution of a linear least-squares problem; for the sake of brevity, we omit the details and refer the interested reader to [2] for more details. Note that although an arbitrary $\bar{\omega}$ can be chosen, the closer $\bar{\omega}$ to ω_0 , the better is the accuracy of the estimate. In the case considered herein, the conventional beamformer can provide a rather accurate estimate of the DOA, and therefore, $\bar{\omega}$ can be set to this value. Accordingly, multiple values of $\bar{\omega}$ can be investigated, and the value resulting in the best fitting can be retained.

B. Covariance Matrix Based Estimation

Our second algorithm borrows ideas from [1], which is based on the following observation. For $k = 1, \dots, m-1$, let

$$z_k = \sum_{\ell=1}^{m-k} \mathbf{R}_y(k+\ell, \ell) = (m-k)r_x(k)e^{ik\omega_0} = \zeta_k e^{ik\omega_0} \quad (10)$$

be the statistic obtained by summing along the k th subdiagonal of \mathbf{R}_y , and let

$$\hat{z}_k = \sum_{\ell=1}^{m-k} \widehat{\mathbf{R}}_y(k+\ell, \ell) \quad (11)$$

denote a consistent estimate of z_k . In [1], it was proposed to jointly estimate ω_0 and $\zeta = [\zeta_1 \dots \zeta_{m-1}]^T$ by minimizing

$$Q = \sum_{k=1}^{m-1} |\hat{z}_k - \zeta_k e^{ik\omega}|^2. \quad (12)$$

The method in [1] was shown to produce rather accurate estimates. However, it suffers from two drawbacks. First, it assumes that $r_x(k)$ is real-valued. Next, the unambiguous range of spatial frequencies that can be estimated by this method is restricted to $[-(\pi/2), (\pi/2)]$. Indeed, the frequency estimate of [1] is given by the argument that maximizes

$$V = \text{Re} \left(\sum_{k=1}^{m-1} \hat{z}_k^2 e^{-i2k\omega} \right) \quad (13)$$

However

$$\begin{aligned} \sum_{k=1}^{m-1} \hat{z}_k^2 e^{-i2k(\frac{\pi}{2}+\omega)} &= \sum_{k=1}^{m-1} \hat{z}_k^2 e^{-ik\pi} e^{-i2k\omega} \\ &= \sum_{k=1}^{m-1} \hat{z}_k^2 e^{-i2k\pi} e^{ik\pi} e^{-i2k\omega} \\ &= \sum_{k=1}^{m-1} \hat{z}_k^2 e^{-i2k(-\frac{\pi}{2}+\omega)} \end{aligned}$$

and therefore, ω is uniquely identifiable in $[-(\pi/2), (\pi/2)]$. This corresponds to a $[-30^\circ, 30^\circ]$ DOA sector, which is rather restrictive.

Herein, we relax the assumption of a real-valued $r_x(k)$ and consider a possibly complex-valued $r_x(k)$. At the same time, we try to improve the unambiguous DOA range. Similarly to [1], we propose to minimize Q with respect to ω_0 and ζ . Nevertheless, it is clear that if $r_x(k)$ is arbitrary and complex-valued, then the solution to this problem is not unique. Indeed, for any Δ , $r_x(k)e^{ik\omega_0} = [r_x(k)e^{-ik\Delta}]e^{ik(\omega_0+\Delta)}$,

and therefore, ω_0 and ζ would not be identifiable. However, the latter decomposition would lead to a correlation function $\tilde{r}_x(k) = r_x(k)e^{-ik\Delta}$ (for $k = 1, \dots, m-1$) which (at least for large m) would violate the assumption in (6) as it would correspond to a spectrum $\tilde{S}_x(\omega) = S_x(\omega + \Delta)$ for which

$$\begin{aligned} \int_{-\pi}^{\pi} \omega \tilde{S}_x(\omega) d\omega &= \int_{-\pi}^{\pi} \omega S_x(\omega + \Delta) d\omega \\ &= \int_{-\pi}^{\pi} (\tilde{\omega} - \Delta) S_x(\tilde{\omega}) d\tilde{\omega} \\ &= \int_{-\pi}^{\pi} \tilde{\omega} S_x(\tilde{\omega}) d\tilde{\omega} - \Delta \int_{-\pi}^{\pi} S_x(\tilde{\omega}) d\tilde{\omega} \\ &= -\Delta r_x(0) \neq 0 \end{aligned}$$

which disagrees with (6). From the above discussion, it appears that the physically meaningful assumption (6) prevents ambiguity problems, although it relies on all $r_x(k)$'s, whereas Q depends only on the first $m-1$ correlation lags. In the sequel, we concentrate on the estimation of the spatial frequency only. In contrast to [1], we do not concentrate the criterion with respect to ζ ; rather, we exploit the fact that $r_x(k)$ is a correlation function with spectrum $S_x(\omega)$. Toward this end, we rewrite the minimization problem in terms of $S_x(\omega)$ and then show that asymptotically, the minimizing argument of Q is the true frequency. First, observe that

$$Q = \sum_{k=1}^{m-1} |\hat{z}_k|^2 + |\zeta_k|^2 - 2\text{Re} \left(\hat{z}_k^* \zeta_k e^{ik\omega} \right).$$

Hence, minimizing Q with respect to ω_0 amounts to maximizing

$$\begin{aligned} \tilde{Q} &= \sum_{k=1}^{m-1} (m-k) \text{Re} \left(\hat{z}_k^* r_x(k) e^{ik\omega} \right) \\ &= \frac{1}{2\pi} \sum_{k=1}^{m-1} (m-k) \text{Re} \left(\hat{z}_k^* \int_{-\pi}^{\pi} S_x(u) e^{ik(u+\omega)} du \right) \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} S_x(v-\omega) \left\{ \sum_{k=1}^{m-1} (m-k) \text{Re} \left(\hat{z}_k^* e^{ikv} \right) \right\} dv \end{aligned}$$

where, to obtain the last equality, we used the fact that $S_x(u)$ is real valued. Therefore, \tilde{Q} can be written as

$$\tilde{Q} = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_x(v-\omega) \phi(v) dv \quad (14)$$

with

$$\phi(v) = \sum_{k=1}^{m-1} (m-k) \text{Re} \left(\hat{z}_k^* e^{ikv} \right). \quad (15)$$

Let us now study the asymptotic properties of $\phi(v)$. Using the fact that \hat{z}_k converges to $z_k = (m-k)r_x(k)e^{ik\omega_0}$, it follows that

$$\begin{aligned} \lim_{N \rightarrow \infty} \phi(\omega) &= \sum_{k=1}^{m-1} (m-k)^2 \text{Re} \left(r_k^* e^{ik(\omega-\omega_0)} \right) \\ &= \sum_{k=1}^{m-1} (m-k)^2 \text{Re} \left(\frac{1}{2\pi} \int_{-\pi}^{\pi} S_x(u) e^{ik(\omega-\omega_0-u)} du \right) \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} S_x(\omega-\omega_0-u) \psi(u) du \quad (16) \end{aligned}$$

where

$$\begin{aligned}\psi(u) &= \operatorname{Re} \left(\sum_{k=1}^{m-1} (m-k)^2 e^{iku} \right) \\ &= -\frac{m^2}{2} + \frac{m}{2 \sin^2 \frac{u}{2}} - \frac{\cos^2 \frac{u}{2}}{2 \sin^2 \frac{u}{2}} \frac{\sin mu}{\sin u}.\end{aligned}$$

The function $\psi(u)$ is real and even. It is straightforward to show that it has a unique peak in the vicinity of $u = 0$, whose effective width is equal to $3\pi/(M+1)$. Substituting (16) into (14) yields

$$\begin{aligned}\lim_{N \rightarrow \infty} \tilde{Q} &= \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} S_x(v-\omega) S_x(v-\omega_0-u) \psi(u) du dv \\ &= \int_{-\pi}^{\pi} \psi(u) B(u + \omega_0 - \omega) du\end{aligned}\quad (17)$$

where the function

$$B(u) = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} S_x(v) S_x(v-u) du \quad (18)$$

is the correlation function of the spectrum $S_x(u)$. Clearly, $B(u)$ is positive, symmetric with respect to $u = 0$ and achieves its maximum at $u = 0$. This, along with the properties of $\psi(u)$, implies that the maximum of \tilde{Q} will asymptotically be reached when $\omega = \omega_0$. Now, \tilde{Q} depends on $S_x(\omega)$, which is unknown. However, using (16) and the fact that the spectrum $S_x(u)$ is concentrated in the vicinity of $u = 0$, it follows that $\phi(\omega)$ will asymptotically peak at $\omega = \omega_0$. Therefore, we suggest that we estimate ω_0 as

$$\begin{aligned}\hat{\omega}_0 &= \arg \max_{\omega} \phi(\omega) \\ &= \arg \max_{\omega} \operatorname{Re} \left(\sum_{k=1}^{m-1} (m-k) \hat{z}_k^* e^{ik\omega} \right).\end{aligned}\quad (19)$$

It should be pointed out that the ambiguity described previously—and mainly due to the squaring operation in (13)—is resolved with the new algorithm. In addition, the proposed method can handle complex-valued $r_x(m)$ under the assumption (6); however, the latter condition is mild as it only means that we wish to recover the mass center of the target.

IV. NUMERICAL EXAMPLES

In this section, we provide an illustration of the performances of the proposed methods. The method based on the spectral moments [see (9)] is referred to as SME in the figures, whereas (19) is denoted by “proposed” in the figures. For comparison purposes, the performances of the AML_1 method of [13] and the method of [1] (we refer to it as the “subdiag” method in the figures) are also displayed. Finally, the Cramér–Rao bound for the problem at hand is also given. We consider a linear array of $m = 8$ sensors. Unless otherwise stated, the mass center of the target is located at $\omega_0 = 0.2$, which corresponds to $\theta_0 = 3.65^\circ$. In all simulations, the spatial spectrum of the target is given by Fig. 1, where β will be referred to as the source extent. Observe that β is normalized to the array beamwidth $2\pi/m$, i.e., $\beta = 0.5$ means that the source extent is half the array beamwidth. In addition, note that the mass center of the target is at ω_0 since the condition in (6) is fulfilled. Finally, note that the power spatial density in Fig. 1 is not symmetric with respect to $\omega = 0$, which indicates that $r_x(k)$ is complex valued. The signal-to-noise ratio is defined as $q^2 = 10 \log_{10} P/\sigma_n^2$ in decibels. In all figures, the root mean-square errors of the estimates (normalized to the array beamwidth) are plotted.

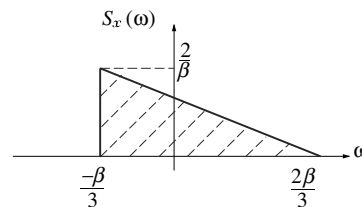


Fig. 1. Power spatial density of the target.

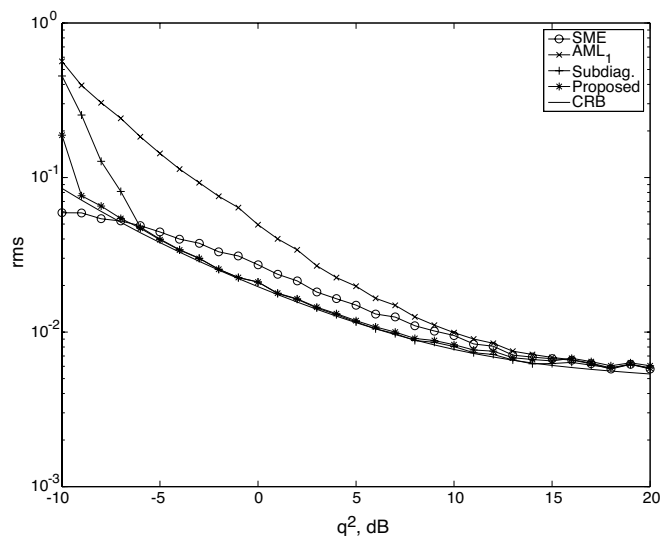


Fig. 2. RMSE of the DOA estimates versus SNR. $m = 8$, $N = 64$, and $\beta = 0.25$.

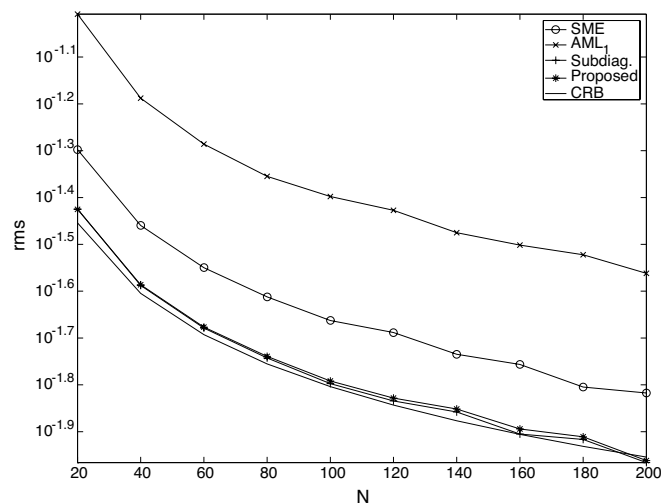


Fig. 3. RMSE of the DOA estimates versus N . $m = 8$, $q^2 = 0$ dB, and $\beta = 0.25$.

In Figs. 2–4, we study the influence of the signal-to-noise ratio, the number of snapshots, and the source extent. From inspection of these figures, it can be seen that the spectral moment estimate (9) performs slightly poorer than the new method (19). The latter has performance similar to the subdiag method of [1] and very close to the CRB over a wide range of scenarios.

Finally, we illustrate the improvement in terms of DOA range ambiguity that is achieved with the new method compared with that of [1]. Toward this end, the target’s position is varied from 0 to 90° , and results are given in Fig. 5. It can be observed that for $\theta_0 > 30^\circ$, the method

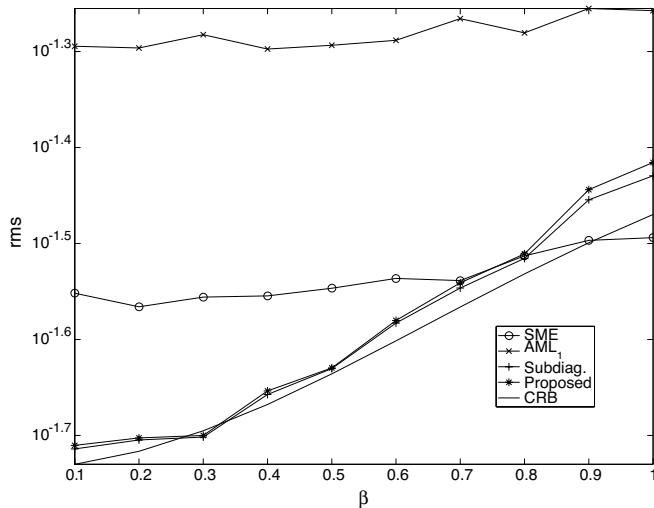


Fig. 4. RMSE of the DOA estimates versus β . $m = 8$, $N = 64$, and $q^2 = 0$ dB.

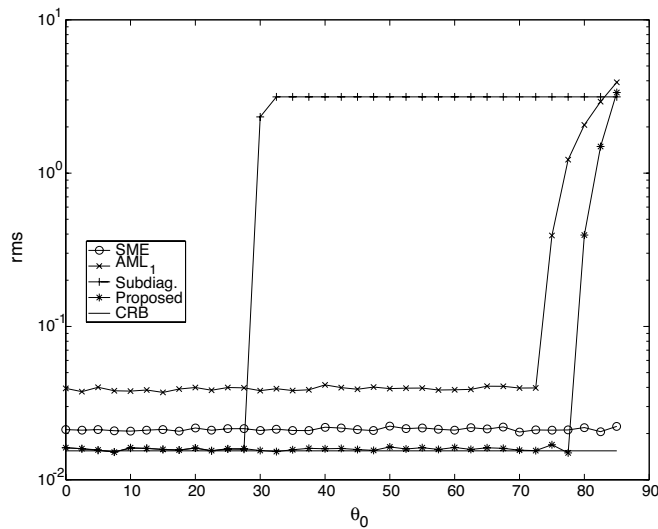


Fig. 5. RMSE of the DOA estimates versus θ_0 . $m = 8$, $N = 64$, $q^2 = 0$ dB, and $\beta = 0.25$.

of [1] provides erroneous estimates, which clearly demonstrates the ambiguity problem. In contrast, our method does not suffer from this problem.

V. CONCLUSIONS

In this correspondence, we consider the direction-finding problem for an extended target whose power spatial density is not necessarily symmetric with respect to its mass center. Two computationally simple algorithms were proposed. One is based on the spectral moments of the target, which are easily related to its DOA. The second borrows ideas from [1] and extends the range of DOAs that can be estimated unambiguously. Both methods provide robust, simple, yet accurate DOA estimates.

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Comments on "A High-Resolution Quadratic Time-Frequency Distribution for Multicomponent Signals Analysis"

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Abstract—It is shown that the time-frequency distribution (TFD) proposed in the above paper is not well defined in the ordinary sense for power signals, including the single-tone sinusoid, and it needs the introduction of generalized functions and transforms. It is also shown that the proposed TFD does not satisfy the conditions cited by the authors of the paper to justify the claim that it has the instantaneous frequency property.

Index Terms—Generalized functions, instantaneous frequency, multicomponent signals, reduced interference distributions, time-frequency analysis.

Recently, a time-frequency distribution (TFD) of Cohen's Class, which is known as the B-distribution (BD), was proposed and claimed

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