The impact of local masses and inertias on the dynamic modelling of flexible manipulators

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Abstract

After a brief review of the recent literature dealing with flexible multi-body modelling for control design purpose, the paper first describes three different techniques used to build up the dynamic model of SECAFLEX, a 2 d.o.f. flexible in-plane manipulator driven by geared DC motors : introduction of local fictitious springs, use of a basis of assumed Euler-Bernouilli cantilever-free modes and of 5th order polynomial modes. This last technique allows to take easily into account local masses and inertias, which appear important in real-life experiments. Transformation of the state space models obtained in a common modal basis allows a quantitative comparison of the results obtained, while Bode plots of the various interesting transfer functions relating input torques to output in-joint and tip measurements give rather qualitative results. A parametric study of the effect of angular configuration changes and physical parameter modifications (including the effect of rotor inertia) shows that the three techniques give similar results up to the first flexible modes of each link when concentrated masses and inertias are present. From the control point of view, "pathological" cases are exhibited : uncertainty in the phase of the non-colocated transfer functions, high dependence of the free modes in the rotor inertia value. Robustness of the control to these kinds of uncertainties appears compulsory.

KEYWORDS : FLEXIBLE MULTIBODY SYSTEMS — 2 D.O.F. MANIPULATOR — DY-NAMIC MODELLING — GEAR TRAIN DYNAMICS

1. Introduction

Modelling and control of motor-driven flexible multibody chains has recently received a wide attention from specialists in structural dynamics, mechanical engineering, automatic control, . . . in connection with the various projects of space manipulators, and on a wider scope with the increasing complexity of spacecrafts and the better performances required for independent pointing of payloads. The improvement of the cycle rate of industrial manipulators creates also a concern about the impact of vibrations on performances. Eventually, the flexible manipulator has often been chosen as an emulator of control problems arising from control/structure interaction (CSI). A 2 d.o.f. planar manipulator has been built in this scope at CERT/DERA, and named SECAFLEX (cf. [7]). Its main specificity with respect to other similar set-ups is the drives, which are built with on-the-shelf DC motors and Harmonic Drive gears, in order to get sufficient joint torque to move the experiment and to put energy in the flexible members.

The present paper discusses the modelling issues of such experiments on the ground of the parametric data available on SECAFLEX : we will see that several techniques are available to derive the dynamic model of such an experiment, which are the subject of numerous papers in technical journals even in the case of simple beams, and among which the "assumed modes" approach is the most popular. A major issue is however the

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choice of the shapes depending upon the effective boundary conditions, as the fact that the joints between substructures are actively controlled makes these a function of the stiffness of the effectively implemented control laws.

We compare three of these techniques from the point of view of the data needed to build up the model and the frequency contents and properties of the models obtained : restricting ourselves to the linear case, which seems adequate and sufficient to deal with vibration problems, we define typical transfer functions relating joint torques to measurements on the set-up, and study the variations in the models obtained as functions of the selected distributed element modelling technique and uncertainty on the physical parameters, in the scope of further use of the model for control design and validation in simulation. Basically, we know that the control laws will be grounded on a linear "second order" model including mass, stiffness and damping matrices, and corresponding input injection and output extraction matrices, and we would like to answer the questions : how should we fill in these matrices with correct parameters ? What confidence do we have in these parameters ? What is the influence of local masses and inertias ?

2. Techniques used to derive a dynamic model

2.1. Literature survey

The various journals dealing with dynamics and control (Journal of Guidance, Control and Dynamics, IEEE Transactions on Robotics and Automation, ASME Transactions on Measurement and Control, . . .) reflect the numerous papers published in the conferences on the subject of modelling and control of flexible structures in general and flexible manipulators in particular. The importance of rigid modes make this last case specific in the sense that generally the same actuators are used to control the rigid body motion and to prevent excitation of the flexibility distributed in the links, the interaction between rigid and flexible modes being the lowest frequency expression of the so-called "spillover".

A good part of the literature is dedicated to the establishment of knowledge models in the frame of very general multibody software, oriented towards applications in various fields (space, robotics, automotive industry), and able to take into account arbitrary complicated topologies. On the other side, a lot of papers deal with beams as the influence of distributed flexibility is more apparent in the structural parts which have a large aspect ratio, and a debate is still going on about the convenient assumed modes which should be chosen to build up the dynamic model, and the kinematic parameters liable to describe the largest set of displacements. We will restrict ourselves to the open flexible multibody chains, and to modelling in the scope of control design.

The general purpose multibody software ([4], [19], [9], ...) is generally based on a modelling of the substructures by finite element (FE) methods, and provide the user with a frame allowing to include easily the data coming from FE software and to generate simulations in a reliable manner. These tools seem to appear too powerful in the case of simple beams and a direct approach is more popular among people dealing with control of experiments ([5],[13],[21], ...): this assumed modes approach uses generally a set of analytical mode shapes extracted from Bernouilli-Euler theory of beams with various boundary conditions. The presence of local masses and inertias complicates rapidly the problem as the establishment of shapes taking into account the loaded interfaces implies the solution of transcendental equations. The set of shapes is then used to establish the set of differential equations by a classical Lagrangian or Kane formulation, but the linear part of the model only needs the expression of the kinetic energy or the equivalent simple scalar products. It is worth to be mentioned that several authors point out the relative independence of the results with respect to the precise choice of shapes.

Other approaches can be quoted :

- representation of distributed flexibility by local stiffness, that we proposed for the treatment of singularity in space manipulators ([8]) and included as a basic feature in our space manipulator simulator SMASP; an improvement has been proposed which aims to reduce the degree of approximation through minimization of the difference with a "truth model" obtained by finite elements ([20]);
- modal impedance aims to replace analytical assumed modes by a quadrupole relating forces and displacements at one end to forces and displacements at the other end, and to solve a transcendental equation for the whole chain in a limited frequency range, extracting thus free modes in a more reliable numerical manner ([16], [15]);

• bond-graph techniques have been also proposed to model flexible members ([12], [18]).

These techniques give birth to possibly high order models : the confidence in the prediction of high frequency components (past the first few low frequency flexible modes) is however not very good : it has been pointed out that the modelling of deformation as normal to the equilibrium position, which is a very common hypothesis, leads to inconsistently linearized models, and that a very rough linearization could be preferable to exact derivation of equations taking into account radial deflection ([10][14]). This discussion is related to the substructuring issues ([3]), which look very simple in the case of beams and not representative of the real problem met in the case of large and complicated elements, and we will only keep in mind that it is not sound to take into account too much modes per substructure.

2.2. Selection of three modelling techniques for SECAFLEX

2.2.1. Available geodynamical data

Figure A.1 shows a rough sketch of the geometry of SECAFLEX, including local masses and inertias, to which should be added the inertias due to drive rotors and gear trains, as shown in the more detailed sheet of data:

Component	Index 1	Index 2
Beam length	1.390 m	1.400 m
Beam height	0.1 m	0.1 m
Beam thickness	0.006 m	0.005 m
Inertia of section	$1.8 10^{-9} m^4$	$1.04 \ 10^{-9} \ m^4$
Mass per unit volume	$7800 \ kg/m^3$	2700 kg/m^3
Flexible part mass	6.50 kg	1.89 kg
Young's modulus	20600 Mpa	7360 Mpa
EI product	$371 Nm^2$	$\overline{77 Nm^2}$
Gear stiffness	360 000 Nm/rd	36 000 Nm/rd
Torquemeter stiffness	180 000 Nm/rd	8 000 Nm/rd
Tachometer inertia	$0.15 \ 10^{-3} \ m^2 kg$	$5 \ 10^{-6} \ m^2 kg$
Drive rotor inertia	$0.79 \ 10^{-3} \ m^2 kg$	$2.6 \ 10^{-5} \ m^2 kg$
Wawe generator inertia	$1.50 \ 10^{-3} \ m^2 kg$	$6.0\ 10^{-5}\ m^2 kg$
In-joint total inertia (motor side)	$2.44 \ 10^{-3} \ m^2 kg$	9.1 $10^{-5} m^2 kg$
Gear ratio	80	80
In-joint total inertia (payload side)	$15.6 m^2 kg$	$0.582 m^2 kg$

Table 1 Data sheet for SECAFLEX

2.2.2. Fictitious joint approach

This technique is described in detail in [8]. Due to the planar feature of the experiment, it simply leads to add 1 spring in the middle with stiffness EI/l, 2 springs at 25% and 75% of length with stiffness 2EI/l, ... Inhouse developed SMASP software allows to build up the mass matrix taking into account the concentrated masses and inertias, and inertial effects due to drive rotors having any orientation (a feature not useful here).

2.2.3. Assumed modes approach

We followed the methodology described in [11], with a modification to account for tip payload offset (see fig. A.2 for a sketch of kinematic parameters). The shapes are cantilever-free shapes for an ideal beam : it would

be more exact to account for loads, but this point has been rather developed with the technique described in the following paragraph. With 2 modes per link, the deflection normal to equilibrium position reads :

$$u_i(x,t) = \phi_{i1}(x)q_{i1}(t) + \phi_{i2}(x)q_{i2}(t)$$
(2.1)

The effect of rotor inertia is introduced by modifying the mass matrix obtained.

2.2.4. Assumed polynomial shapes

The sketch of the manipulator shows that geometric offsets and local hub inertias may play an important role in the modelling, and can be accounted for with analytical modes shapes. We chose to derive a more precise model using a set of polynomial shapes, in the line of finite element techniques ; however, the flexible parts are very regular and we felt that superelements describing each link as a whole would perhaps be sufficient (in the line of modal impedance approaches). We developed then an approach grounded on polynomial approximation of the deformation along each link, but we selected 5th order polynomials instead of classical 3rd order ones, to be able to express the interface conditions between the link and the yokes, which are rigidly connected to the beams. Figure A.3 shows the kinematic parameters that can be selected with these polynomials : the bending moments at both ends appear in addition to classical translation and rotation. For instance, the interface condition at the root of the link with a hub of inertia J_h is easily expressed as

$$EIq_3 = J_h \ddot{q_2} \tag{2.2}$$

This finite element technique was first validated on a 1 d.o.f. manipulator (the Stanford experiment described in [17]) and proved efficient to predict the dynamic model : it gives a good accuracy compared to the analytical assumed mode techniques, with less effort than a classical finite elements approach (where several elements par link are required). The transfer functions between tip position and joint torque exhibits the non-minimum phase complex symmetric zeros configuration experimentally found in [17].

Application to SECAFLEX was then made on the basis of the skeleton shown on figure A.4, on which the kinematical parameters are indicated :

$$X = \begin{bmatrix} \theta_1 & \theta_2 & q_{1H} & y_E & \alpha_E & q_{1E} & q_{2E} & y_T & \alpha_T \end{bmatrix}$$
(2.3)

These parameters suppose the mass matrix of each link established in a moving frame of angular and translational rate $\dot{\theta_1}$ and 0 for link 1 and $\dot{\theta_1} + \dot{\alpha_E} + \dot{\theta_2}$ and $\dot{y_E} + l_1\dot{\theta_1}$ for link 2. An alternate set of parameters may be chosen with a reference of kinematic parameters to the equilibrium (undeformed) position. It is easier to use but restricted to small motion around the equilibrium.

The selection of 9 parameters gives a 18th order model that can be reduced to 12th order model by modal truncation or by projection of the kinematic parameters on the first two cantilever free modes of each link (to compare the results with those of the preceding approach).

3. Comparison between dynamic models

3.1. Structure of the differential equations

The three techniques described in the previous section lead to a set of differential equations which linearized form has the classical general structure with mass, damping and stiffness matrices (the "second order" form):

$$M\ddot{q} + D\dot{q} + Kq = B^*\Gamma \quad ; \quad y = C^*q \tag{3.1}$$

The set of equations must be completed by an observation equation which allows to express measurements or adequate signals for performance evaluation. The interesting variables in the case of SECAFLEX are measurements : the motor-driven joints angles and angular velocities θ_i and $\dot{\theta}_i$ ("colocated" optical encoders end tachometer measurements) and the angular velocity ω of the payload w.r.t. an inertial frame ("noncolocated" gyrometer measurement). A state space form is easily derived from this set of equations, at the expense of a loss of the physical insight in the parameters, but is necessary to include the velocities as functions of the state vector. Inside this common structure, the three techniques give birth to quite different models in the second order form, due to the different parametrization choices for the instantaneous field of deformation of the flexible beams. In the following equations, in which the in-joint flexibility has not been considered, the exhibited m_{ij} terms in the mass matrix are the inertia terms corresponding to the rigid body motion and should be the same ; they include the effects of rotor inertia, which is multiplied by the square of the gear ratio on the diagonal and by the gear ratio off-diagonal.

• **fictitious joints approach :** the mass matrix contains on the diagonal the inertias as seen from the successive rotary joints, real or fictitious, and off-diagonal the linear inertial couplings ; the stiffness matrix is diagonal with zeros for real joints ; the joints variables are part of the state vector and the payload angular rate is merely the sum of the angular velocities ; in the case of 2 fictitious joints per link, the sketch of the result is the following :

• analytic assumed modes approach : the configuration parameters selected in [11], that we used initially in this approach, were nor absolute neither relative : the second joint angle θ_2 is defined as the angle between the *undeformed* first link and the root of the second link ; the second member matrix injecting joint torques depends upon the slope of the mode shapes at the tip of the first link ; the mass matrix terms come from integration of the shapes over the space variable, and the stiffness matrix is still diagonal due to orthogonality of the analytical shapes. However a linear transformation to a set of parameter including the relative angle between the tip of the first link and the root of the second one transforms the linear model under the following form (with 2 shapes per link) :

polynomial assumed modes approach : with 9 kinematic parameters, the set of differential equation takes the following form

m_{11}	m_{12}	-	x	x	—	—	x	- 1	$\lceil q_1 \rceil$	٦	Γ0	0	0	0	0	0	0	0	ך 0	$\lceil q_1 \rceil$		Γ1	ך 0	
m_{21}	m_{22}	—	x	x	_	_	x	—	q_2		0	0	0	0	0	0	0	0	0	q_2		0	0	
_	—	_	_	_	_	—	_	_	q_3		0	0	x	x	x	x	0	0	0	q_3		0	0	
x	x	_	x	x	_	—	x	_	q_4		0	0	x	x	x	x	0	0	0	q_4		0	1	
x	x	_	x	x	—	_	x	_	q_5	+	0	0	x	x	x	x	0	0	0	q_5	=	0	0	Γ
—	_	_	—		_	_	—	—	q_6		0	0	x	x	x	x	0	0	0	q_6		0	0	
-	—	_	_	_	_	_	_	_	q_7		0	0	0	0	0	0	x	x	x	q_7		0	0	
x	x	_	x	x	_	_	x	-	q_8		0	0	0	0	0	0	x	x	x	q_8		0	0	
L —	_	_	_	—	_	_	_		$\lfloor q_9 \rfloor$		Lo	0	0	0	0	0	x	x	x _	$\lfloor q_9 \rfloor$		Lo	0	
		$\lceil \theta \rceil$	٦	Γ1	0	0	0	0 0	0	0	0.0	0	0	0	0	0	0	0	0]	-				
		$\hat{\theta}_{0}$	_		1	ñ	ñ	0 0	i n	ñ		n	n N	ñ	n N	ñ	ñ	n	ñ	Q				
		02	_		1	0	0			0	0 1	1	0	0	1	0	0	0	1	Q				
		$\lfloor \omega \rfloor$	1	Lo	0	U	0	0 0	0	U	0 1	T	0	0	T	0	U	U	ŢŢ					(2, 1)
																								(3.4)

The stiffness matrix is no longer diagonal and takes a block-diagonal structure ; the terms in the mass matrix are of varying importance : small terms are represented by -, large ones by x. This set of equation may be projected on a basis of cantilever-free modes to have the same order of the state vector as in the previous ones.

3.2. Adequate transfer functions for further comparison

The interesting transfer functions which we will use for comparisons belong to two classes : colocated ones and non-colocated ones. If we write the transfer matrix from the two drive torques to the three measurements, we isolate the following terms (Δ is the common denominator including two rigid modes and the convenient number of free vibration modes, nj stand for a numerator between joints and ne for a numerator from joint to tip);

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \omega \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} nj_{11} & nj_{12} \\ nj_{12} & nj_{21} \\ ne_1 & ne_2 \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix}$$
(3.5)

The square sub-transfer-matrix between torques and joint angles is symmetric, and transfer functions $\frac{n_{j_{11}}}{\Delta}$ and $\frac{n_{j_{22}}}{\Delta}$ are colocated, while the others are non-colocated.

Other interesting transfer functions can be exhibited : isolation of an element in the preceding transfer matrix implies that the other term in each equation is equal to 0, i.e. that no torque is applied at the other joint ; if we consider now the other joint, say i, as locked, we introduce a mono-input transfer matrix relating each torque to the three measurements with condition $\theta_i = 0$. The fixed torque may be eliminated between the equations to give

$$\frac{\theta_j}{\Gamma_j} = \frac{1}{\Delta n j_{ii}} \left(n j_{ii} n j_{jj} - n j_{ij}^2 \right) = \frac{n j_j^l}{n j_{ii}} \qquad \frac{\omega}{\Gamma_j} = \frac{1}{\Delta n j_{ii}} \left(n e_j n j_{ii} - n e_i n j_{jj} \right) = \frac{n e_j^l}{n j_{ii}}$$
(3.6)

The zeros of the free transfer functions appear in the denominator of the locked ones (cantilever modes), and simplifications appear which eliminate the free modes from the transfer functions : these simplifications raise numerical problems even in simple cases and it is best to establish the locking conditions directly on the second order differential equations. Due to the symmetry of the above expressions in 3.6, the numerators are the same in the locked case and $nj_1^l = nj_2^l$, $ne_1^l = ne_2^l$. The transfer functions are used to build up Bode plots which give rather a qualitative graphical insight into the results ; the gain and phase is reliable only if some natural damping is added (in the modal basis as usual).

3.3. Numerical comparison : the common modal basis

The quite different set of equations described before are not directly adequate for comparison of results: we must first look for representations with a minimum number of parameters; the transfer functions give a possibility (comparison of the denominator and of the 5 different numerators), but we may also look for a linear transformation on the initial differential equations. We may first apply the basic result of modal analysis and find a basis of eigenvectors of the generalized eigenvalue problem on M and K to turn the equations into completely decoupled form, making thus appear the free modes; the norm of the eigenvectors may in particular be chosen to get a new mass matrix equal to identity, which makes the squares of the frequencies appear in the new stiffness matrix.

From the input-output point of view, this state/input normalization is however not sufficient to provide a minimum set of parameters : the expression of the transfer functions as a decomposition in residues shows clearly that the product of input matrix B^* by output matrix C^* is a constant, so that a further step is required : if the first actuator is not located at a shape node for any flexible mode, it is possible to normalize the first column of B matrix to unity (for the flexible modes only) ; moreover the lines of the C matrix corresponding to joint angles will be in the same ratio as the columns of the B matrix, so that finally the really independent parameters (describing the flexible modes) appearing in the set of transformed state space equations are the frequencies, the second column of B^* , the first line of C^* and the full line of C giving the tip angular rate.

The rigid modes raise a problem as the diagonalization has an infinity of solutions, but we can show that the state subspaces describing the flexible modes are identical if they remain decoupled from the rigid modes, so the choice of the basis for these is not critical.

3.4. Numerical example

In order to give some insight in the preceding considerations, we will briefly present the numerical results obtained, summarized on a data sheet for each angular or parametric configuration. In the case of the fictitious joints approach, the transfer function analysis gives the following values, where only the results related to flexible modes are included (frequencies are expressed in rd/s, damping is set to zero) :

- Free modes (non-zero roots of Δ) : $\pm 8.47j \pm 14.76j \pm 42.27j \pm 66.74j$
- Joint 1 zeros (roots of nj_{11}) : $\pm 4.45j \pm 14.57j \pm 41.90j \pm 66.19j$
- Joint 2 zeros (roots of nj_{22}) : $\pm 3.88j \pm 10.01j \pm 38.56j \pm 66.70j$
- Joint 1 to tip zeros (roots of ne_1): $\pm 8.57j \pm 14.21 \pm 72.46 \pm 94.46$
- Joint 2 to tip zeros (roots of ne_2): $\pm 5.50 \pm 8.46j \pm 64.68j \pm 75.52$

Locking of the second joint has the following effects :

- New free modes $\Delta_1 = n j_{22}$
- Joint 1 zeros (roots of nj^i) : $\pm 1.90j \pm 7.22j \pm 38.23j \pm 66.13j$
- Joint 1 to tip zeros (roots of ne^{l}): $\pm 1.84 \pm 11.94 \pm 72.95 \pm 94.00$

Conversely, locking of the first joint has the following effects :

- New free modes $\Delta_2 = n j_{11}$
- Joint 2 zeros : identical to previous roots of nj^l
- Joint 1 to tip zeros : identical to previous roots of ne^{l}

The corresponding common modal basis state space representation takes the following form :

The first two differential equations combined to the first two columns of the observation of joint angles are clearly one decomposition, among the infinity of possibilities, of the rigid motion on a basis of rigid modes. It is implicitly a result of the choice of eigenvectors made by the generalized eigenvalues problem solver.

3.5. Parametric study

3.5.1. Methodology

The parameters considered may be classified in the following manner :

- parameters defining the angular configuration of the setup, which are not "uncertain" but merely liable to changes and are moreover measured : due to symmetry, only the angular variable θ_2 belongs to this class ;
- uncertain parameters, like the natural damping, the various local masses and inertias, the stiffness either located in the joints or spread along the beams ;
- modelling technique driving parameters, that is parameters which may be uncertain in the case of the
 present set-up as it is (and on which we may have a better knowledge), but which also have an influence
 on the validity of the model depending on the modelling technique used, and for which it is interesting
 to consider variations much greater than the a priori confidence that we have for SECAFLEX : these
 are mainly the local masses and inertias, and in particular the local inertias due to rotor inertias "seen"
 through the gear ratios.

The data sheets summarizing the numerical results of the kind exhibited in equation 3.7 have been compared in the following manner : the fictitious joints approach will be the reference case, with 2 springs per link, in the extended angular configuration ($\theta_2 = 0 \text{ deg}$). The other modelling techniques are then compared in that configuration, and then the parameters are varied : angular configuration up to 45 and 90 deg, damping up to 0.01, number of modes taken into account for each link from 1 to 3 (depending on the technique : we have seen that the polynomial technique provides us with a 18th order model which must be reduced to get a reference 12th order model), rotor inertias down to zero, local masses down to 1kg, in-joint stiffnesses to their nominal values instead of infinity.

3.5.2. Reference results

The reference results, given in quantitative form in the preceding section, can be investigated in qualitative form on the Bode plots : an illustration is given on figure A.5 ("colocated" transfer function nj_{22}/Δ which alternates pairs of imaginary zeros and pairs of imaginary poles, a couple of these being called "dipoles" in the sequel) and figure A.6 (non-colocated transfer function ne_2/Δ , which includes non-minimum phase zeros) ; the sketch of these Bode plots differ a bit from the usual results on mono-input flexible beams : the residues of the higher frequency dipoles decreases slower in the first case (and slower that for the case of transfer function nj_{11}/Δ , for which only the first dipole has a significant residue), and combinations of non-minimum phase and imaginary zeros appear in the second case.

The effect of joint locking is a general decrease of the frequencies and an increase of the residues associated to each dipole (if any); the locking of joint 2, which transforms our 2 d.o.f. flexible manipulator in a one d.o.f. manipulator with a length twice as large, gets back a transfer function from joint 1 torque to tip angular rate with no imaginary zeros at all. The locking of joint 1, which emulates a 1 d.o.f. manipulator with a flexible mount, still exhibits more exotic sequences of imaginary and non-minimum-phase zeros.

3.5.3. Influence of modelling techniques

The assumed modes technique with analytical cantilever-free modes leads to a model which resembles the reference model, with however a general decrease in the frequencies obtained. This can be explained by the greater apparent length of the flexible members due to a more rough skeleton of the set-up, the geometry of the yokes being ignored with our implementation of this technique.

The assumed modes with polynomial shapes, once reduced to 12th order, gives a model which differ very slightly from the reference case, the remarkable points being the presence of 4 complex symmetric zeros instead of a double pair of non-minimum-phase zeros, a configuration already published about experimental investigations ([17]). These quantitative differences do not show much on the Bode plots.

3.5.4. Parameter variations

<u>Damping</u> Inclusion of a natural damping in the modal basis does not change much the numerical results, but regularizes the Bode plots, making the phase and amplitude variation less depending upon the resolution in frequency. It shows also that dipoles with small residues disappear almost completely for a 0.01 relative damping.

<u>Number of modes retained per link</u> In the cases of the fictitious joints and assumed analytical mode techniques, to select one mode per link gives almost the same low frequency mode as the truncation of the 12th order model, so that the global modes are clearly related to each individual link. However, in transfer function ne_1 , the "inverted" dipole $\pm 8.57j/\pm 8.47j$ seen before becomes a "regular" one $\pm 8.07j/\pm 8.90j$ if only one fictitious joint per link is selected. This great change in the phase properties of this non-colocated transfer function is due to the near-zero value of the related C coefficient, which expresses a high uncertainty on the slope of the corresponding shape at the tip.

The selection of three modes per link simply adds up higher frequency dipoles without changing much the low frequency properties.

<u>Angular configuration</u> The change of angular configuration is quite important on the coefficients describing the rigid modes (there is no difference with the rigid case from this point of view), but does not change much the free flexible modes : the boundary conditions being pinned, the links vibrate "on themselves", so that the characteristic equation does not depend on this parameter. The situation is different for the numerator roots, which correspond to the cantilever case, and particularly for the numerator of transfer function ne_1 . Figure A.7 shows the variation of the pole and the zero of the lower frequency dipole when the configuration changes from 0 to 90 degrees ; the pair of imaginary zeros become real symmetric.

Local masses and inertias The suppression of rotor inertias have an very strong effect on the free vibration modes, which frequency increases much (multiplication by 3 for the first one, by 2 for the second one) : the parameter variation in the mass matrix is small compared to this effect ($15 m^2 kg$ over 132 in joint 1, $0.6 m^2 kg$ over 17.5 in joint 2). Conversely, the lowest zero of the colocated transfer functions varies very little.

Change in the local masses have less effect on the frequency properties of the model, which increase by a few rd/s, and the change in the elbow masses has a greater effect than the change in the tip mass, which only affects slightly the lowest frequency mode.

If we combine the reduction in mass and inertia, considering a "naked" manipulator in which the local and distributed dynamic parameters are of the same order of magnitude, we can show that the techniques using fictitious joints and polynomial assumed modes give still coherent results, while the analytical assumed modes approach gives quite different results : the cantilever free shapes seem too inadequate in this case where the beams are really pinned on the joints.

4. Conclusion

We would like to highlight the following points, in the scope of further use of the dynamic model to design control laws taking explicitly into account the flexibility :

- flexible modes are more "visible" from the "elbow" than from the "shoulder" (to use anthropomorphic expressions), and the multivariable feature adds to the variety of situations illustrating control/structure interaction problems that can be emulated with a testbed ;
- the three techniques investigated give quite equivalent results for the manipulator including local masses and inertias ; an inadequate choice of the assumed mode shapes has no consequences on the model unless the masses and inertias are regularly distributed along the manipulator : this will not be the case for geared motor drives, in which the concentration of inertia due to gear ratio is much higher than any "natural" inertia ; in other words, these concentrated inertias regularize the dynamic model with respect to flexibility (as they do in fact in the rigid case with respect to payload variation) ;
- as a consequence, the flexible beams of SECAFLEX have quite the behavior of massless springs, which are quite satisfactorily approximated with local fictitious springs : this technique seems sufficient to derive initial models for control design ; supplementary efforts towards a more precise dynamic modelling should rather be directed towards mechanical engineering modelling, including friction, backlash, ..., which importance in the final performances of the control will probably be greater than higher frequency vibration components ;
- 2 modes per link seem quite sufficient to describe the dynamics of the set-up, 1 mode per link giving a reduced order model quite satisfactory for control design ;
- the study allows to exhibit two major "pathological" cases from the point of view of parameter uncertainty, which should be accounted for by the control :
 - for co-located transfer functions relating joint angular measurements to joint torques, the uncertainty in the rotor inertia (including the inertia of the gear train on the motor side) which makes a slight change of one coefficient of the mass matrix, changes the free vibration modes by a great amount ;
 - for non-colocated transfer functions relating the tip angular rate to the joint torques, the angular configuration variation creates a change in the phase properties of the transfer function which could drive easily the control to instability.

The first problem may be academic as the precise identification of these uncertain inertias is possible, even on a simplified model of the joint. The second one is an avatar of the more general problem of the confidence in the numerators of the transfer functions appearing in flexible structure modelling, and raises the challenge of a control law achieving robustness to these kind of uncertainties. A possible approach of the problem is discussed in [2] which addresses the control synthesis on the basis of the models discussed here using recent robust control theory.

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Figure 1: Geodynamic skeleton of 2 d.o.f. manipulator SECAFLEX



Figure 2: Geodynamic skeleton of 2 d.o.f. manipulator for assumed Bernouilli-Euler modes approach



Figure 3: Kinematic parameter of link superelement



Figure 4: Kinematic parameters chosen for assumed polynomial shapes



Figure 5: Bode plot of colocated transfer function nj_{22}/Δ (reference case, 0.001 damping)



Figure 6: Bode plot of non-colocated transfer function ne_2/Δ (reference case, 0.001 damping)



Figure 7: Evolution of low frequency poles (x) and zeros (o) of transfer function during a change in angular configuration from 0 to 90 degrees