Launcher attitude control: some additional design and optimization tools

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Abstract

This paper deals with the launcher attitude control during atmospheric flight. A two step approach combining an H_∞ control design and an optimization procedure is proposed. The first step is multi-objective stationary H_∞ design based on the Cross Standard Form. It provides easily a first rough solution from a few physical tuning parameters. The second step is a fine tuning using an multi-constraint satisfaction algorithm. This algorithm enables the certification criteria computed on the validation model to be met and is also used to propagate the nominal tuning to the full flight envelope.

Keywords : multi-objective synthesis, performance, robustness, Cross Standard Form, launcher

1 Introduction

In this paper, the low-level control loop of a non-stationary launcher during atmospheric flight is considered. Only the yaw attitude is explored: the problem is formulated in terms of angle of attack regulation in face of a typical wind profile (disturbance rejection problem, see Figure 4) and consumption reduction. Robustness specifications are expressed in the frequency domain for a set of operating instants regularly spaced along the flight path: the open loop transfer ($L(z) = K(z)G(z)$) must satisfy templates on the Nichols chart for various critical configurations sampled in the uncertain parameter space (see Figure 5). Uncertain parameters are the main dynamic parameters on the rigid mode (aerodynamic coefficient, thruster efficiency,...) and on the bending modes (natural frequencies, modal participation factors).

With respect to the pure stationary synthesis problem at one flight instant, there is no methods, to our knowledge, that can handle such a set of specifications (time-domain performance, open-loop frequency-domain specifications and parametric robustness specifications) in a streamlined manner. Then, a two step approach combining a control design, which can provide easily a first rough solution from a few physical tuning parameters, and

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a fine optimization initialized from this solution seems a good alternative between a tedious trials and errors procedure or a blind optimization from an arbitrary initialization.

Although the control design we proposed is an indirect approach, its capability to take advantage of know-how is particularly highlighted in this application: the time-domain performance specification (angle-of-attack peak amplitude in response to typical wind profiles) is handled by a non-conventional LQG (Linear Quadratic Gaussian) synthesis based on physical considerations. Then, this synthesis is incorporated into a standard H_∞ problem in order to meet frequency-domain templates. The final H_∞ synthesis meets all the specifications and produces a low-order compensator in regard to alternative approaches applied on the same problem.

This design is then refined by an optimization procedure. From a practical point of view, the expression of a scalar cost function combining the various objectives, that is heterogeneous terms for instance, the incidence response peak (degree) and the gain margin (dB). Furthermore, when these various objectives correspond to physical specifications, it is more interesting to appreciate the sharpness of the trade-offs between these specifications rather than to perform a pure optimization. For these reasons, it seems more tractable to solve a multi-constraint satisfaction problem than an optimization problem. For the non-stationary launcher application, such a procedure enables to propagate the tuning at one flight instant to the full flight envelope.

In the first part of this paper, the launcher model and the specifications are described. In the second part, the stationary H_∞ design is presented and applied at the flight instant with maximal aerodynamic pressure. The third part is devoted to the multi-constraint satisfaction algorithm. In the last part of the paper, this algorithm is used to propagate the nominal tuning to the full flight envelope and the gain scheduling of the various compensators is presented.

2 Launcher control problem

2.1 Description

This application considers the launcher inner control loop. Referring to Figure 1, the following notation is used:

- G : the center of gravity,
- i : the launcher angle of attack,
- ψ : the deviation angle around axis w.r.t. the guidance attitude reference,
- V_a and V_r : respectively, the absolute and the relative velocity,
- w : the wind velocity,
- β : the thruster angle of deflection,
- $\dot{z}$: the lateral drift rate.

The rigid behavior is modeled by a third-order system with state vector :$x^r = [\ \psi \quad \dot{\psi} \quad \dot{z}\ ]^T$. This rigid model strongly depends on the 2 uncertain dynamic parameters A_6 (aerodynamic efficiency) and K_1 (thruster efficiency).

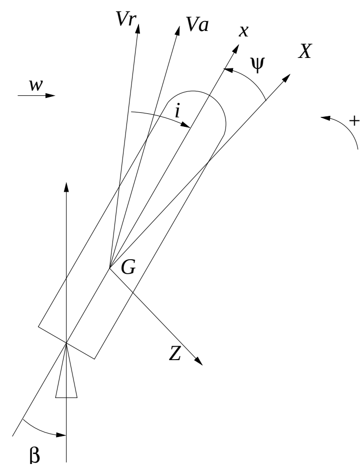

Figure 1: Launcher simplified representation.

The discrete-time full order validation model $G_f(z)$) considered in this paper (that includes the rigid dynamics, the dynamics of thrusters (order 2), of sensors (order 2) and the first 5 bending modes (order 10). The launcher is aerodynamically unstable. Finally, the characteristics of bending modes are uncertain (4 uncertain parameters per mode).

2.2 Objectives

The available measurements are the attitude angle ψ and rate $\dot{\psi}$. The control signal is the thruster deflection angle β. The control objectives for the whole atmospheric flight are as follows:

- performance with respect to disturbances (wind): the angle of attack peak, in response to the typical wind profile $w(t)$, must stay within a narrow band ($\pm\, i_{max}$). This wind profile is plotted in Figure 4 (dashed plot) and corresponds to a worst case wind encountered during launches with a strong gust when aerodynamic pressure is maximal (at time T_1),
- closed-loop stability with prescribed margins for both rigid and flexible dynamics. These specifications can be interpreted as a template on the NICHOLS locus of the open loop transfer $L = KG$ (see Figure 5 as an example): *(i)* the locus must cross the axis above the critical point with a Low Frequency Gain Margin $LFGM_{dB}$; *(ii)* it must cross the same axis under the critical point with a High Frequency Gain Margin $HFGM_{dB}$ (negative value); *(iii)* resonance associated with flexible modes 2 to 5 must be gain controlled and must stay below a specified level X_{dB} (roll-off specification). Note that the first flexible mode is "naturally" phase controlled (resonance phase around $0\,deg$) due to the collocation between sensors and actuator and that the flexible modes are not taken into account in the synthesis model. But a roll-off behavior with a cut-off frequency between the first and the second flexible modes must be specified in the synthesis,
- delay margin must be greater than one sampling period (T_s).

All these objectives must be achieved for all configurations in the uncertain parameter domain (22 uncertain parameters including aerodynamics coefficient, propulsion efficiency and bending modes characteristics). particularly in a number of identified worst cases, where the combination of parameter extremal values is particularly critical. In this paper, the robustness analysis is limited to these worst cases as the experience has shown that they are quite representative of the robustness problem. A complete μ-analysis is presented in [1].

3 Stationary H_∞ design

The design is based on the CSF (Cross Standard Form) [2] presented as a generalization of the LQ inverse problem to the H_2 and H_∞ inverse problem. The CSF enables to formulate a standard problem from which an initial compensator can be obtained by H_2 or H_∞ synthesis. The CSF is used to mix various synthesis techniques in order to satisfy the different specifications of the launcher control problem. The general idea is to perform a first synthesis achieving some specifications, mainly time-domain performance specifications. This first solution is then used to initialize a standard problem which is gradually completed to handle frequency-domain or parametric robustness specifications.

This approach is detailed in [2] and [3]. The standard problem set up for the final H_∞ is depicted in Figure 2. This standard problem depends on:

- the 4 state space matrices (A_d^a, $B_{2_d}^a$, C_2^a, D_{22}) of the discrete-time rigid model augmented with a first order wind model,
- the state feedback and estimator gains K_d^a and G_d^a of the LQG/LTR[1] design proposed to fulfill the specifications regarding the rigid dynamics,
- the frequency weighting $F(z)$ introduced to attempt to fulfill the frequency-domain specifications on flexible modes 2 to 5 (X_{dB} constraint). $F(z)$ is a high pass second order filter with a wide resonance including flexible modes 2 and 3 and their variations (see Figure 3). Note that flexible modes 4 and 5 are not significant regarding the X_{dB} specification.

The 8 tuning parameters (gathered in a vector p) for the whole design are displayed in Table 1[2].

Of course, at each step of this design (the first step is the LQ control law, the second step is the introduction of the KALMAN filter and the last one is the introduction of the filter $F(z)$ for the final H_∞ synthesis), the specifications satisfied at the previous step are perturbed by the new ones taken into account in the following step. This problem is particularly relevant as this stationary design is built up at the flight instant T_1 where the aerodynamic pressure is maximal and where the performance/robustness trade off is the more stringent. Figures 4 and 5 show results obtained with a rough tuning (see Table 1) which was established without any trial and error tuning. We note $K_1(z)$ this LTI (Linear Time Invariant) controller. One can notice that the constraint i_{max} on the angle-of-attack response is violated. It also can be shown that the time delay margin (T_s) is also violated while others specifications ($LFGM_{dB}$, $HFGM_{dB}$ and X_{dB}) are met.

[1] LTR: Loop Transfer Recovery.

[2] the LQ weighting on the angle of attack i is normalized to 1, that is $J_{LQ} = \int_0^{+\infty}(i^2 + p(1)\dot{z} + p(2)\beta^2)dt$.

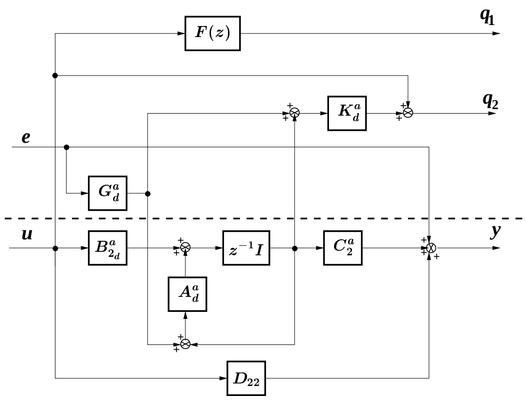

Figure 2: Set-up for final H_∞ synthesis.

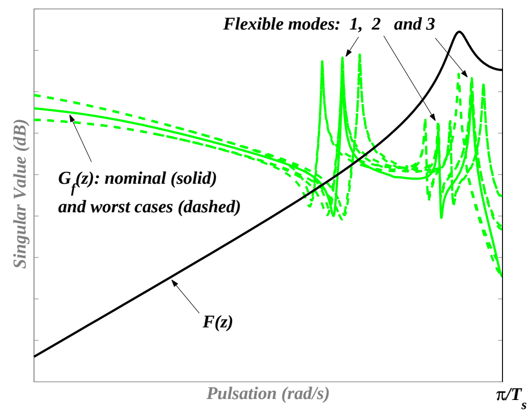

Figure 3: Singular values: $F(z)$ (black) and $G_f(z)$ (grey).

parameter	signification	initial rough tuning (at flight instant T_1)	final tuning at flight instant T_1	final tuning at flight instant T_2
p(1)	LQ weighting on $\dot{z}$	10^{-4}	1.29×10^{-4}	1.16×10^{-4}
p(2)	LQ weighting on $u = \beta$	1	0.693	0.543
p(3)	wind model dynamics	-0.1	-0.0795	-0.215
p(4)	LTR weighting	10^{-5}	6.58×10^{-6}	8.96×10^{-6}
p(5)	general state to measurement noise covariance ratio	10^{-7}	1.06×10^{-7}	1.86×10^{-7}
p(6)	rate to position measurement noise covariance ratio	10	12.0	21.6
p(7)	static gain of $F(z)$	30	27.6	20.0
p(8)	central resonance frequency of $F(Z)$	30	29.4	28.7

Table 1: Vector p of tuning parameters: signification and numerical values (normalized units).

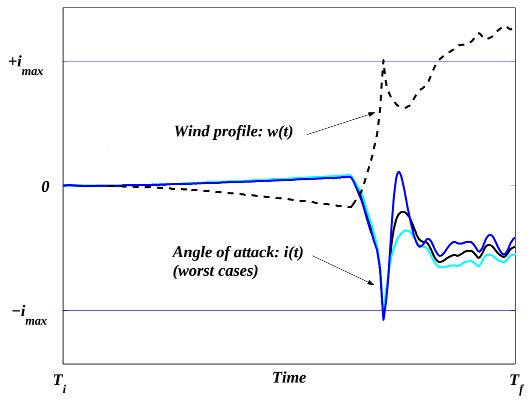

Figure 4: Angle of attack $i(t)$ (solid) obtained with $K_1(z)$ and wind profile $w(t)$ (dashed, normalized unit).

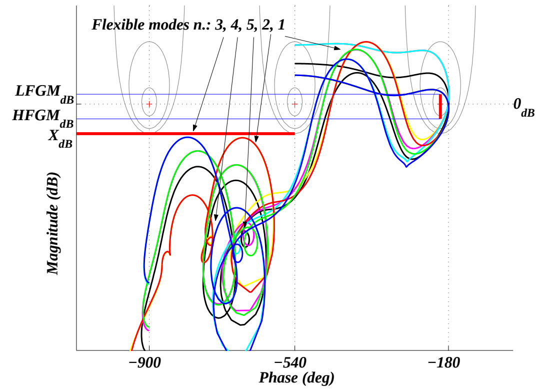

Figure 5: $K_1(z)G_f(z)$: NICHOLS plots for worst cases.

4 Fine tuning

Instead of a long trial and error procedure to fulfill exactly all the requirements, a multi-constraint satisfaction procedure is proposed. The specific features of such a procedure w.r.t. a pure criterion optimization are the followings :

- one can take into account the end-to-end requirements computed on full order validation model, that is the certification specifications,
- of course, these specifications can not be optimized with this procedure. One can only check if a set of constraints can be achieved. But this enables the trade-off between specifications to be evaluated. Note also that the definition of a scalar index function including the various specifications is always a tedious task,
- such a procedure is quite interesting when a good initialization can be provided as it is the case in our problem: the rough solution previously presented is not so far from the final objective,
- this procedure is also applied to propagate the nominal tuning found at flight instant T_1 to the others operating flight instants where new specification values are prescribed.

Five performance indexes are considered in this problem and gathered in the vector c:

- $c(1)$: absolute value of the angle of attack time-response peak,
- $c(2)$: low frequency gain margin,
- $c(3)$: high frequency gain margin,
- $c(4)$: peak of the frequency response of the open loop transfer $L(z) = K(z)G(z)$ computed around flexible mode 2, 3, 4 and 5,

- $c(5)$: delay margin.

This vector c is a function of the tuning parameters $c = \mathcal{C}(p)$ and must satisfy the following constraints:

$$c(1) < i_{max}, \quad c(2) > LFGM_{dB}, \quad c(3) < HFGM_{dB} < 0, \quad c(4) < X_{dB} < 0, \quad c(5) > T_s \,. \tag{1}$$

Considering the current vector of tuning parameters p, one can derive numerical value of the sensitivity matrix $J(p)$ around p (that is the local derivative matrix of the vectorial function $\mathcal{C}(p)$): this matrix gives the relative variations of the performance indexes w.r.t. the relative variations of the tuning parameter [3]:

$$\delta_c = J(p)\delta_p \,.$$

The procedure aims at satisfying all the constraints (1) by a gradient type exploration. The 2 scalar tuning parameters of this procedure are N_{max}, the maximal iteration number, and ε, the step length in the gradient direction:

initialization: $p = p_0$, $N_{max} = 100$, $\varepsilon = 0.03$, $i = 1$, $c = \mathcal{C}(p)$, $J = J(p)$,

while (1) is not met and $i < N_{max}$*:*

- $\delta_c^d(1) = (i_{max} - c(1))/i_{max}$, $\delta_c^d(2) = (LFGM_{dB} - c(2))/LFGM_{dB}$, $\delta_c^d(i) = (LFGM_{dB} - c(3))/LFGM_{dB}$, $\delta_c^d(4) = (X_{dB} - c(4))/X_{dB}$, $\delta_c^d(5) = (T_s - c(5))/T_s$,
- $\delta_c^d = 1.02\,\delta_c^d$ *(this factor* 1.02 *is introduced to ensure that the procedure will overpass the constraint),*
- $\delta_p = J^T(JJ^T)^{-1}\delta_c^d$,
- $p \leftarrow p(1 + \varepsilon\delta_p)$ *(*ε *is the step length along the gradient direction),*
- *hard constraint verification* [4]*,*
- $i \leftarrow i + 1$, $c = \mathcal{C}(p)$, $J = J(p)$.

end while

This procedure is applied to fulfill the specifications at time T_1 from the rough tuning. The relative evolution of the 5 performance index versus the iteration number is depicted in Figure 8: one can notice that the both violated constraints (that is the angle of attack peak and the delay margin) are satisfied in 47 iterations [5]. This diagram enables also the trade-off with the 3 others specifications to be evaluated. This controller is noted $K_2(z)$. Figures 6 and 7 highlight that the specification are met (to be compared with Figures 4 and 5).

[3] this calculus is done considering 5% relative variation for each of the 8 parameters.

[4] The hard constraints on the vector p aim to ensure that:

- the wind model dynamics $p(3)$ is stable,
- the LQ weighting $p(2)$ is strictly positive,
- the measurement noise covariance $p(5)$ is strictly positive.

[5] In Figure 8 all the relative performance must be positive to solve the multi-constraint problem.

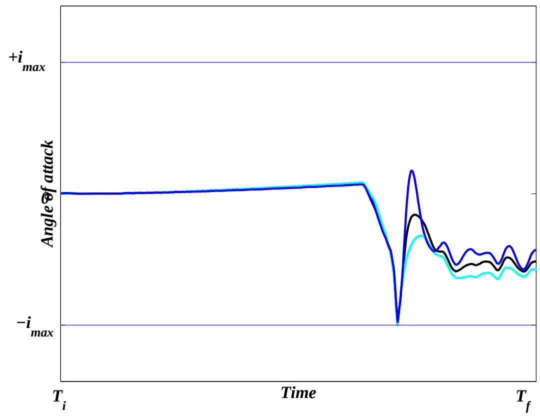

Figure 6: Angle of attack $i(t)$ obtained with $K_2(z)$.

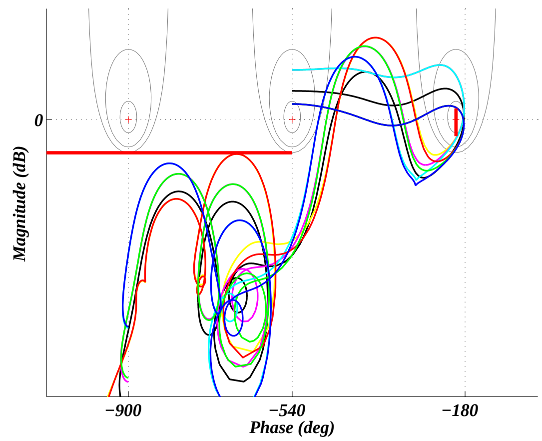

Figure 7: $K_2(z)G_f(z)$: NICHOLS plots for worst cases.

5 Linear Time Variant (LTV) design

The previous procedure was applied forward from T_1 to T_f (last instant of atmospheric flight) and backward from T_1 to T_i (initial time of atmospheric flight) to fulfill all the specifications defined for a set of operating points regularly spaced between T_i and T_f. At each operating point, the tuning parameter vector is initialized on the solution found at the previous computed point. Figure 9 presents, for instance, the evolution of the performance indexes at time T_2 from the tuning parameter vector previously found at time T_1 (see table 1). All the specifications are fulfilled within 26 iterations.

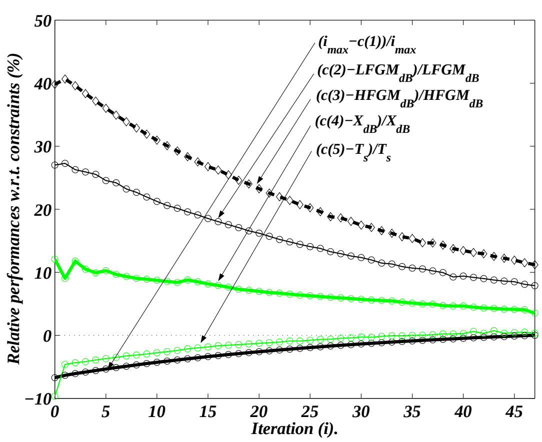

Figure 8: Performance convergence during the tuning computation at flight instant T_1 from the rough tuning p_0.

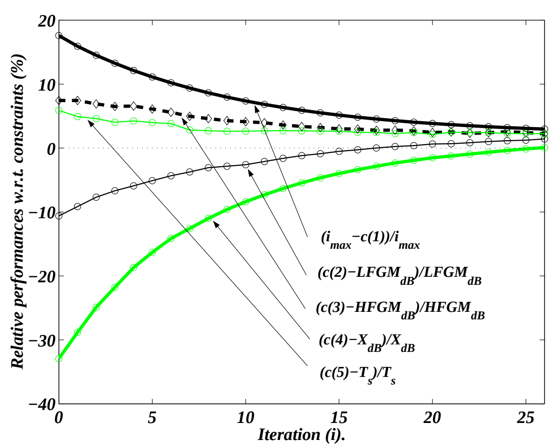

Figure 9: Performance convergence during the tuning computation at flight instant T_2 from the tuning at flight instant T_1.

This procedure enables to find a LTI controller at each point avoiding long trial and error designs. The last problem is the gain-scheduling of these LTI controllers w.r.t. time.

It is important to notice that the final H_∞ synthesis on the problem described by Figure 2 is performed using the LMI solver of Matlab. The advantage of this solver is that it provides the best H_∞ performance index among all the available solvers. The drawback is that the state space representation of the resulting controller has no physical meaning and cannot be mastered due to internal changes of variable in this solver to optimize numerical calculus. Therefore the direct linear interpolation of those state matrices provide a LTV controller $K(z,t)$ with a chaotic behavior on intermediate point. This is highlighted in Figure 10 where the singular value of the LTV controller is plotted versus the frequency (between 0 and the half sampling frequency) and versus time , the gain scheduling variable (between T_i and T_f).

To solve this problem, we propose to compute the observer-based realization of each LTI controller using the procedure described in [4]. In this new realization, the controller states become meaningful variables (that is: estimates of plant states) and the linear interpolation of new state space matrices provides a new LTV controller $K_{LQG}(z,t)$ with smooth transition between various controllers as it can be seen in Figure 11 (see also [3] for more details).

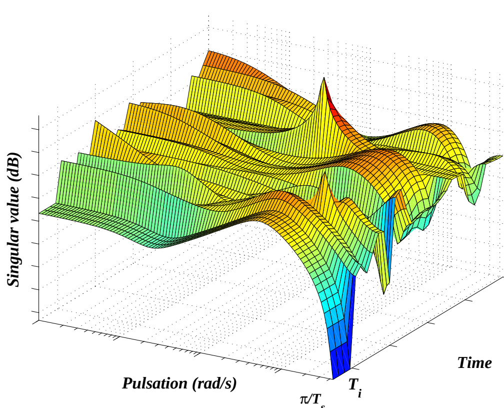

Figure 10: $K(z,t)$: singular value w.r.t. time.

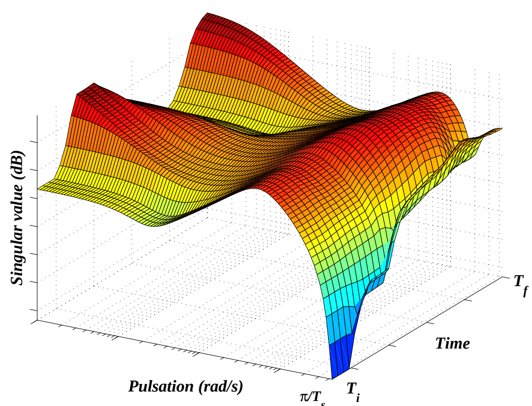

Figure 11: $K_{LQG}(z,t)$: singular value w.r.t time.

6 Conclusions

The conjonction of an H_∞ design based on the Cross Standard Form and a multi-constraint satisfaction problem solver provides efficient tools for the design of non-stationary launcher pilots. From a practical point of view, the main advantage of such an approach is that the trade-off between the various specifications can be handled. This property will be used in the next future to evaluate if new control architectures (involving new sensors) would make possible to push back the limits in the trade-off tuning.

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