H_∞ control design for generalized second order systems based on acceleration sensitivity function

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***Abstract*— This article presents an H_∞ control design method based on the Acceleration Sensitivity (AS) function. This approach can be applied to any fully actuated generalized second order system. In this framework, classical modal specifications (pulsations / damping ratios) are expressed in terms of H_∞ templates allowing other frequency domain specifications to be taken into account. Finally, a comparison between AS with a more classical H_∞ approach and with the Cross Standard Form (CSF) is presented. A 2 degrees of freedom spring-damper-mass academic example is used to illustrate the properties of the AS, though this method was developed and is used for atmospheric reentry control design.**

I. Introduction

Many systems can be represented by a generalized second-order equation (1). In particular mechanical (sub)systems whom positions and/or orientations are controlled by means of accelerations and/or torques naturally lead to such generalized second-order equations. Acceleration-sensitivity H_∞ control design can be applied to any system that can be expressed as a generalized second order system, but does not need such a realization: any other realization of such a system can be used. For simplicity, we will consider hereafter that the system is written as a generalized second order, that is on the form introduced in [7], [9] and given in (1).

$$M\ddot{q} + D\dot{q} + Kq = Fu \tag{1}$$

$q \in \mathbb{R}^n$ is the n degrees of freedom (dof.) vector and M, D, K respectively the $n \times n$ mass, damping and stiffness matrices. $u \in \mathbb{R}^m$ is the input vector and F the input matrix (of size $n \times m$). We also suppose that each degree of freedom is actuated (that is F is full-row rank).

Although many system can be represented as generalized second-order system, they usually are represented by a classical state-space equation $\dot{x} = A\ x + B\ u$. This has been pointed out in [3], [4], [6] in which their respective authors demonstrate that taking into account the second-order structure of the system simplifies some control design and implementation tasks. In all these references, control design methods are based on pole placement or eigenstructure assignment techniques.

Basic specifications for such a system are often expressed as follows:

- each dof. q_i must have a second-order dynamic characterized by a pulsation ω_i and a damping ratio ξ_i
- dynamic decoupling between dof. is required.

In this article, an acceleration sensitivity based H_∞ control design method for such a system with such specifications is presented (section II). This method is compared to a more classical H_∞ control design method and to the Cross Standard Form on sections III-B and III-C. A 2 d.o.f. mass-damper-spring academic example (see section III-A) is used in this comparison. Although acceleration sensitivity H_∞ control design can be applied for output feedback synthesis, only state feedback control design (that is all positions q_i and velocities $\dot{q}_i$ are measured) will be considered hereafter.

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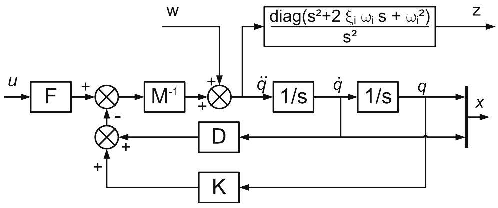


Fig. 1: Acceleration-sensitivity based standard problem $P(s)$

II. H_∞ Acceleration sensitivity control design

A. Description

Acceleration-sensitivity H_∞ control design method is based on Fig. 1 standard form. This standard form weights the acceleration-sensitivity function, that is the transfer from disturbance w on acceleration and the acceleration $\ddot{q}$. The weight W_q is the inverse of desired template on acceleration sensitivity: W_q is diagonal, of order $2n$ and only depends on closed-loop desired dynamic (ω_i, ξ_i) , $i = [\![1, n]\!]$.

$$W_q = \begin{bmatrix} \frac{s^2+2\xi_1\omega_1+\omega_1^2}{s^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{s^2+2\xi_n\omega_n+\omega_n^2}{s^2} \end{bmatrix} \tag{2}$$

One can easily demonstrate that a minimal realization of $P(s)$ reads:

$$P(s) = \left[\begin{array}{cc|c|c} 0_n & I_n & 0_n & 0_{n\times m} \\ -M^{-1}K & -M^{-1}D & I_n & M^{-1}F \\ \hline diag(\omega_i^2)\ldots & diag(2\xi_i\omega_i)\ldots & I_n & M^{-1}F \\ -M^{-1}K & -M^{-1}D & & \\ \hline I_n & 0_n & 0_n & 0_{n\times m} \\ 0_n & I_n & 0_n & 0_n \end{array}\right] \tag{3}$$

B. Properties

Direct feedthrough on the $T_{w\to z}$ transfer is the identity matrix. As a consequence, for any controller K, we have $\|F_l(P,K)\|_\infty \geq 1$. One particular optimal solution to this problem (see [5]) is the static feedback K_0 defined in (4).

$$K_0 = F^+ \left[K - Mdiag(\omega_i^2) \qquad D - Mdiag(2\xi_i\omega_i)\right] \quad (4)$$

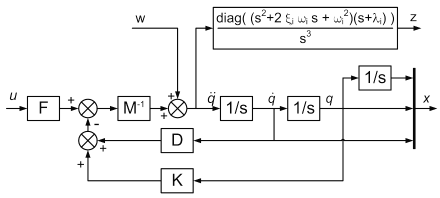
Indeed, the closed-loop reads:

$$F_l(P,K_0) = \left[\begin{array}{cc|c} 0_n & I_n & 0_n \\ -diag(\omega_i^2) & -diag(2\xi_i\omega_i) & I_n \\ \hline 0_n & 0_n & I_n \end{array}\right] = I_n \quad (5)$$

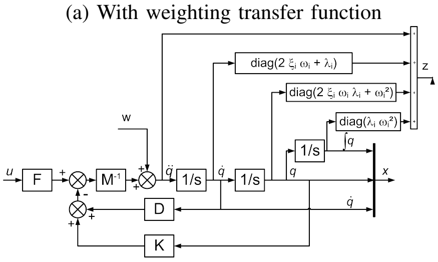
K_0 is the solution obtained using eigenstructure assignment with decoupling constraints as done in [8]. The same solution K_0 can also be obtained by means of a LQ problem with reference model.

As explained in [5], the solution K_∞ to the H_∞ control problem is not unique in the general case and there is no possibility to ensure that a solution provided by the H_∞ solver will coincide with K_0. It means that the frequency response of $F_l(P,K_\infty)$ can be below the W_q^{-1} template for some frequencies. From a practical and a numerical point of view, H_∞ solvers provide a controller whose order is equal to the standard problem order (which is $2n$). In fact, in the state feedback case the poles of this controller are very fast and this controller can be reduced to its DC gain. Readers will find in *http://www.onecert.fr/dcsd/THESES/nfezans/MED08/-demo.html* a MATLAB demo showing how the nominal solution K_0 can be obtained using MATLAB macro-function `hinflmi`

Although it ensures good closed-loop disturbance rejection, the methodology does not permit to ensure robust reference tracking at this stage.

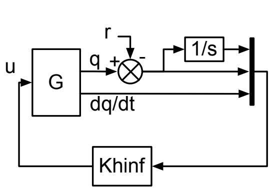


(a) With weighting transfer function

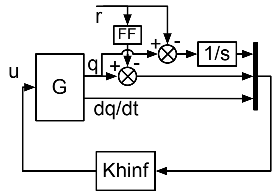


(b) Minimal realization

Fig. 2: Equivalent bloc diagrams of the acceleration sensitivity control design problem with integrator



(a) With integrator



(b) With integrator and feedforward

Fig. 3: Closed-loop

C. Use with an integrator

1) Control design: In order to ensure robust reference tracking, an integral term is directly taken into account in the standard form of Fig. 2a whom structure is basically constituted by Fig.1, an integrator and an adequate weighting function of order 3. Again, a minimal realization (Fig. 2b) is used in order to prevent failures of H_∞ control design algorithms.

2) Controller implementation: The controller K_{hinf} designed using this approach will be implemented according to Fig.3a. The input reference r is introduced on the position q. Note that input reference r does not appear in design schemes of Fig.2, whereas it appears in the more classical output-sensitivity approach introduced later (Fig.7). In fact, models are generally obtained by linearization around an operating point. State $\begin{bmatrix} q \\ \dot{q} \end{bmatrix}$ of the linearized model is the variation around the operating point $\begin{bmatrix} q_{op} \\ \dot{q}_{op} \end{bmatrix}$. The linearized model is thus supposed to be trimmed such that the operating point is an equilibrium point. Input reference r in Fig.3 is the variation between the absolute reference q_{ref} and q_{op}.

Closed-loop is obtained as shown in Fig. 3a or alternatively as shown in Fig. 3b with FF defined as follows:

$$FF = (T_{r\to q}^{1/s\to 0}(\omega = 0))^{-1} \quad (6)$$

where $T_{r\to q}^{1/s\to 0}$ is the transfer between r and q on the diagram of Fig. 3a replacing the integrator by a zero gain. In this case, if the additional dynamics (due to the integral effet) is slow wrt. closed-loop dynamics (that is $\forall i \in [\![1,n]\!],\ \lambda_i \ll \omega_i$), then the integral effect only compensates low-frequency domain model error. In a stabilized state of the system the integrator compensates the trim error; it ensures that chosen operating point is always an equilibrium point.

D. Link with Cross Standard Forms

In this section, acceleration-sensitivity standard problem will be compared to the cross-standard form (see [2]).

Definition 1: Cross standard form
If the standard plant $P(s)$ is such that the 4 conditions

- C1: $P_{yu}(s) = G(s)$,
- C2: K_0 stabilizes $P(s)$,
- C3: $F_l(P(s), K_0(s)) = 0$,
- C4: K_0 is the unique solution of the optimal H_2 or H_∞ problem $P(s)$,

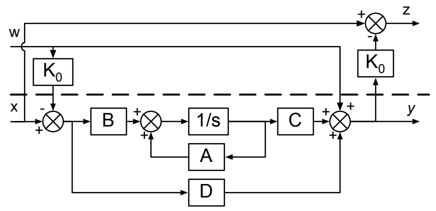


Fig. 4: Cross standard form CSF_{K_0} based on K_0

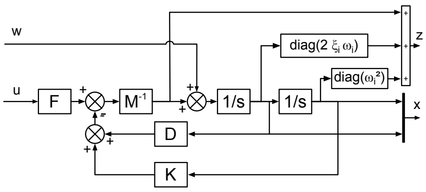


Fig. 5: Cross standard form CSF_{ASSF} derived from acceleration sensitivity standard form

are met, then $P(s)$ is called the CSF associated with the system $G(s)$ and the controller $K_0(s)$.

Acceleration sensitivity standard form is not a cross standard form (CSF) as defined in [2] and in Definition 1. Due to the unitary direct transfer on performance channel conditions C3 and C4 (see Definition 1) are not met. Note that condition C3, implies that K_0 is such that closed-loop input to output transfer is null for all frequencies. As a consequence, optimal performance of a CSF H_2 or H_∞ problem is always 0. Standard problem $P1$ of Fig.5 can be built by removing this direct transfer : direct transfer is the only difference between $P1$ standard problem and AS standard problem.

Proposition 1.
Standard problem $P1$ (see Fig.5) with F a full rank square matrix is a cross standard form.

Sketch of the proof. Properties C1, C2 and C3 can easily be verified. C4 is also easy to prove writing that for any optimal controller K_∞, it exists ΔK_∞ such that $K_\infty = K_0 + \Delta K_\infty$ and then showing that ΔK_∞ cannot be non null.

Remark: In case F is not a n-by-n full rank matrix but a n-by-m matrix of rank n with $m > n$ there are multiple solution to $P1$ and to $P2$. In this case $dim(Ker(F)) = m - n > 0$, so any matrix whom columns are either null or elements of $Ker(F)$ is a valid candidate for ΔK_∞.

Direct transfer between w and z (that is on the performance channel) is unitary in the acceleration-sensitivity standard form of Fig. 1 and null in the cross standard form of Fig. 4. As a consequence, performances of optimal controllers in the acceleration sensitivity problem is $\gamma_{OPT} = 1$ whereas by definition $\gamma_{OPT} = 0$ for a cross standard form. Thus, solutions to acceleration-sensitivity control design problems are not unique. A major advantage of AS standard problem on CSF is that sub-optimal controllers (that is $\gamma > 1$) performance index γ has a physical meaning and can be interpreted as a distance to the objective. With the cross standard form such an interpretation is not possible: a

$m_1 = 15\ kg$	$d_1 = 0.40\ N.s/m$	$k_1 = 9.0\ N/m$	$f_1 = 1.0\ N$
$m_2 = 25\ kg$	$d_2 = 0.55\ N.s/m$	$k_2 = 2.0\ N/m$	$f_2 = 1.5\ N$

TABLE I: Numerical values of the nominal system

Eigenvalues	Damping ratios	Freq. (rad/s)
$-3.55e-002+8.65e-001i$	$4.10e-002$	$8.65e-001$
$-3.55e-002-8.65e-001i$	$4.10e-002$	$8.65e-001$
$-7.20e-003+2.53e-001i$	$2.84e-002$	$2.53e-001$
$-7.20e-003-2.53e-001i$	$2.84e-002$	$2.53e-001$

TABLE II: Modes of the nominal system (open-loop)

performance index $\gamma > 0$ cannot be used neither to reject nor to accept a controller.

III. Comparison and results

A. Academic example

In order to compare the Acceleration-Sensitivity H_∞ control design method introduced with other approaches, academic example of Fig. 6 is considered. Like many mechanical systems, this example can be represented by a generalized second order equation (1). Let $\overline{x_1}$ and $\overline{x_2}$ be the positions of masses m_1 and m_2 at equilibrium, $\hat{x_1}$ and $\hat{x_2}$ be the instantaneous positions of the same masses and x_1 and x_2 the difference between their instantaneous positions and their equilibrium, that is: $x_1 = (\hat{x_1} - \overline{x_1})$ and $x_2 = (\hat{x_2} - \overline{x_2})$. f_1 and f_2 are the actuator gains, whereas u_1 and u_2 are dimensionless inputs. Thus $(f_i\ u_i), i \in [1, 2]$ are forces.

With these notations, the system can be written as follows:

$$\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{M} \ddot{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}} + \underbrace{\begin{bmatrix} d_1 + d_2 & -d_2 \\ -d_2 & d_2 \end{bmatrix}}_{D} \dot{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}} + \dots$$
$$\dots + \underbrace{\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}}_{K} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \underbrace{\begin{bmatrix} f_1 & 0 \\ 0 & f_2 \end{bmatrix}}_{F} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{7}$$

Numerical values chosen for this 2-dof. academic example are given in table I and corresponding open-loop modes are given in table II: this system is a poorly-damped coupled second order system. Specifications for closed-loop modes were defined as follows: $\omega_i = 5\ rad/s,\ \xi_i = 0.6, i = 1, 2$.

B. Comparison with a more classical H_∞ method

A more classical position sensitivity problem (Fig. 7) is introduced here to compare reference tracking properties. The weight W_e (8) is chosen diagonal, without uncontrollable mode on the imaginary-axis and such that it ensures static error is small enough (ie. low-frequency gain must be high

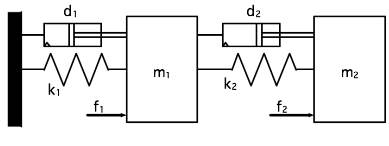


Fig. 6: Academic example with 2 degrees-of-freedom

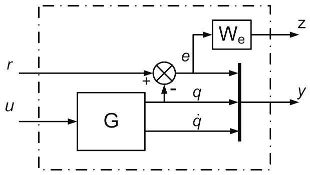


Fig. 7: Position sensitivity standard problem.

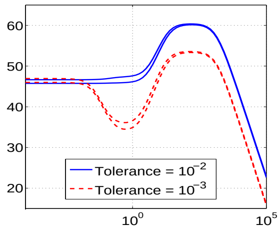
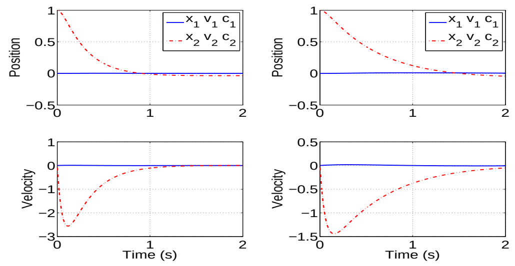


Fig. 8: Singular values of controllers (tol = 10^{-2} and 10^{-3}).



(a) with tolerance = 10^{-2} (b) with tolerance = 10^{-3}

Fig. 9: Response to the initial condition $x_2(0) = 1$.

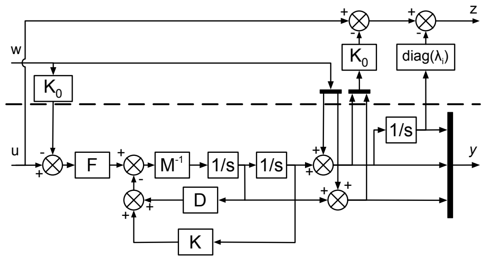


Fig. 10: CSF$_{K_0}$ with integrator standard problem

enough). Though it has not been considered here, measuring $\int e$ would permit cancel static error.

$$W_e = \begin{bmatrix} \frac{s+0.15}{s+0.002} & 0 \\ 0 & \frac{s+0.15}{s+0.002} \end{bmatrix} \quad (8)$$

1) Reference tracking properties: Dynamics of reference tracking are directly and fully controlled with the position sensitibity problem, whereas only indirect control of these dynamics is done with the acceleration-sensitivity with integrator standard problem. Another way to improve these dynamics without any change of closed-loop dynamics is to modify input matrices (using a feedforward for instance). Both approaches have very good robustness properties. In the acceleration sensitivity with integrator and with feedforward (Fig. 3b) case, if $\lambda_i << \omega_i$ step responses can be decomposed into 2 components: nominal response and model error correction.

2) Closed-loop modes: On the one hand, as mixed-sensitivity H_∞ control design methods provide many tuning parameters, they may appear very flexible and suitable for many applications. On the other hand, it has been shown in [10], [1] that such methods do not permit to modify system modes.

The position sensitivity standard form used in this comparison is not a mixed-sensitivity control design method. It allows to modify system modes and with the LMI formulation of H_∞ control design problem seems to place them in an acceptable region of the complex plane. In fact, we observe that slight modifications of numerical tolerances do strongly impact resulting closed-loop modes. Fig.8 shows the singular values of two controllers obtained with two differents tolerance parameter values (10^{-2} and 10^{-3}). Consequences on closed-loop modes and perturbation rejection is shown on Fig.9. Finding a coherent set of controller designed for a set of operating points with such a method will thus be difficult.

C. Comparison with a CSF based on optimal controller K_0

In this section, acceleration-sensitivity standard form (ASSF) will be compared to the cross-standard forms (CSFs) of Fig.4 (CSF$_{K_0}$) and Fig.5 (CSF$_{ASSF}$). All these standard problems permit to obtain good disturbance rejection properties (results showing these good rejection properties are not presented in this article) and nominal reference tracking can be obtained by means of a feedforward. Robustness of reference tracking often involve using an integrator.

1) Closed-loop modes and reference tracking properties with an integrator: The way to add an integrator in the ASSF has previously been presented. It can also be done in the CSF$_{K_0}$ and the CSF$_{ASSF}$. Standard form obtained after adding an integrator to the CSF$_{ASSF}$ is derived from the one presented in Fig.2b: only disturbance w entry point is modified. Fig.10 presents the standard form CSF$_{K_0}$ with integrator.

Using an integrator may modify the disturbance rejection properties of the closed-loop. Effect of the integral term for various values of $\lambda = [\lambda_1 \ \lambda_2]$ is presented in Fig.11-14. Step response with ASSF + integrator problem (Fig.12) are good: reference tracking dynamics is directly related to λ (it is not a first order dynamic though). Closed-loop initial conditions responses (see Fig.11) are not always second order like responses with $\omega = 5 \ rad/s$ and $\xi = 0.6$, but is a third order response when $\lambda \not\ll \omega$. Depending on the strictness of the specifications related to closed-loop modes (see Introduction), it may be interesting to consider lower values of λ and a feedforward gain. In this case, reference tracking behavior and performance is a consequence of closed-loop modes.

Responses obtained with the CSF + integrator problem (respectively Fig.13 and Fig.14) are not acceptable. Although evolution of λ seems to be be related with an evolution of

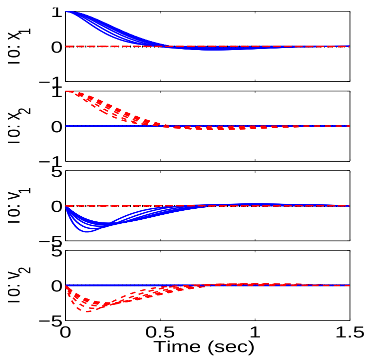


Fig. 11: Initial conditions response : ASSF with integrator, λ logarithmically varying from $[0.01\ 0.01]$ to $[10\ 10]$. Blue: $x_1(0) = 1$, red-dashed: $x_2(0) = 1$.

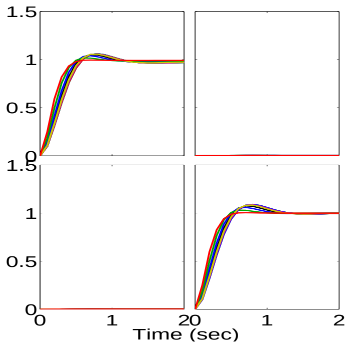


Fig. 12: Step response : ASSF with integrator, λ logarithmically varying from $[0.01\ 0.01]$ to $[10\ 10]$.

system behavior, acceptable values of λ cannot be predicted. Moreover, as all specifications are symmetrical and as $\lambda_1 = \lambda_2$ a visibly poor performance solution (see Fig.14), it is obvious that λ does not only depend on specifications in the CSF + integrator case. As a consequence, a try and error method has to be used to find correct values for λ in this case.

Initial conditions responses of Fig.11 and 13 are responses of a third order system. To strictly meet a second order specification with the ASSF on each dof. small value of λ (wrt. pulsation of the second order specification) has to be used. In this case, in order to have good reference tracking properties a feedforward can be used as shown in section II-C.2. A similar technique can be used with the CSF; here again acceptable values for λ have to be found with a try and error procedure.

2) Physical meaning of the H_∞ norm: These standard forms are particularly interesting in case additional frequency-domain specifications must be taken into account: otherwise we could have used eigenstructure assignment or LQ with reference model methods. We now consider that an additional roll-off criterion must be satisfied. Any standard form P previously defined in this article may be augmented as shown in Fig.15.

Direct transfer between w and z_1 is unitary in the acceleration-sensitivity standard form of Fig.1 and null in the cross standard form of Fig.4. As a consequence, performances of optimal controllers in the acceleration sensitivity problem is $\gamma_{OPT} = 1$ whereas by definition $\gamma_{OPT} = 0$ for a cross standard form. After adding a new specification, optimal performance on the agmented problem may be greater than the optimal performance on the nominal problem (0 with a CSF and 1 with the ASSF).

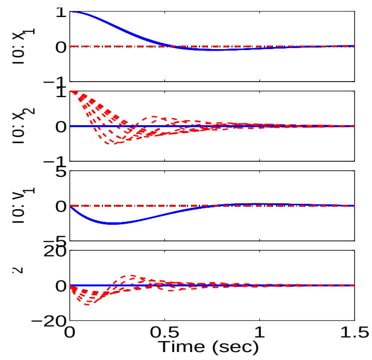


Fig. 13: Initial conditions response : CSF with integrator, λ logarithmically varying from $[0.01\ 0.01]$ to $[10\ 10]$. Blue: $x_1(0) = 1$, red-dashed: $x_2(0) = 1$.

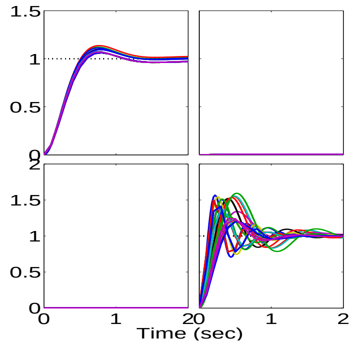


Fig. 14: Step response : CSF with integrator, λ logarithmically varying from $[0.01\ 0.01]$ to $[10\ 10]$.

If the new specification is already satisfied by the optimal controller of the CSF, $||T_{w\rightarrow z_1}||_\infty$ on the augmented problem will still be equal to 0; this norm will be greater than 0 otherwise. In case $\gamma > 0$ value of γ cannot be used used neither to reject nor to accept a controller.

As there are various optimal controllers in the ASSF, it may happen that there exists a least an optimal solution of the augmented problem with a performance index $\gamma = 1$ and that the solution obtained on the ASSF without additional specification is not optimal on the augmented problem. In the general case when adding a new specification, we have $\gamma_{OPT} > 1$. On a augmented ASSF problem, performance index $\gamma > 1$ can still be interpreted: it is a distance to the objective.

$$Roll-Off_{weight} = \begin{bmatrix} \frac{1}{\omega_{RO}}\frac{s}{1+\tau s} & 0 \\ 0 & \frac{1}{\omega_{RO}}\frac{s}{1+\tau s} \end{bmatrix} \quad (9)$$

Considering a roll-off criterion of order 1 (9), we analyze the evolution of the following H_∞ norms : $N1 = ||F_l(P_{ASwRO}, K_{CSF})||_\infty$,

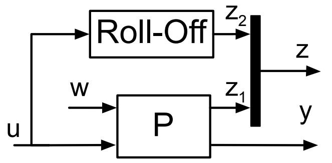


Fig. 15: Augmented standard problem: a roll-off is added

$N2 = ||F_l(P_{CSFwRO}, K_{CSF})||_\infty$ and $N3 = ||F_l(P_{ASwRO}, K_{AS})||_\infty$. N_1 is the performance of the controller designed by means of the CSF and used on the ASSF, N_2 the performance of the controller designed by means of the CSF and analyzed on the CSF, N_3 is the performance of the controller designed with the ASSF and analyzed on the ASSF.

Comparison is made using the academic example with nominal values and a roll-off criterion of order 1 (9). Behavior of the CSF and the ASSF wrt. roll-off frequency ($1/\tau$) is analyzed: the lower the frequency of the roll-off criterion is, the lower the cut-off frequency is and the stronger the effect of the roll-off criterion is. The CSF is based on controller K_0 which is an optimal solution of the acceleration-sensitivity standard form : being an optimal solution of ASSF is the property of K_0 that has to be preserved while adding a new specification (roll-off). Thus, comparing N_1 and N_3 is meaningful and does correspond to the way performance of these methods must be evaluated.

CSF behavior on mid-frequency range is not as good as the behavior of AS standard problem behavior. For $\omega_{RO} \geq 1/\tau (= 10^5 \text{here})$, roll-off criterion becomes inefficient and thus controllers obtained with the CSF can reach nominal performance on the acceleration sensitivity criterion. Physical interpretation of low-frequency range trends is immediate: lowering cut-off frequency leads to performance degradation. For $\omega_{RO} = 14\ rad/s$, the following numerical results were computed: $N_1 = \begin{bmatrix} 2.37 & 0.58 \end{bmatrix}$, $N_2 = \begin{bmatrix} 415.4 & 6.29 \times 10^5 \end{bmatrix}$ and $N_3 = \begin{bmatrix} 1.08 & 0.45 \end{bmatrix}$. First component of N_3 (ie. 1.08) directly gives us that the trade-off between satisfaction of the roll-off criterion were not difficult to make: performance loss is very low. Superiority of acceleration sensitivity on cross standard form seems evident here. Nevertheless, performance gap shown in Fig.16 and by numerical results presented here would be narrower if we were considering a cross standard form based on a dynamic controller.

ASSF has better behavior and properties than the cross standard form when applied to a generalized second order system with pulsations / damping ratio specifications and an additional frequency domain specification. Although ASSF is a CSF plus a unitary direct transfer (Fig.5) and ASSF provides better results on generalized second order systems, CSF can be used to adress some problems that cannot be adressed with an ASSF. In particular, cross-standard forms are very powerful tools for controller improvement; inverse H_2 and H_∞ problem can be solved building a CSF.

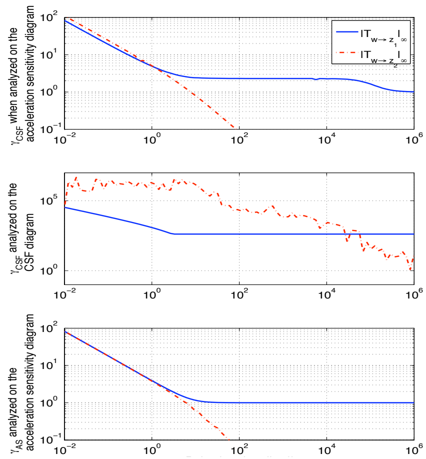


Fig. 16: Comparison between AS and CSF behavior wrt. roll-off pulsation (blue: $|T_{w\to z_1}|_\infty$, red-dot: $|T_{w\to z_2}|_\infty$)

IV. Conclusion

In this article, an acceleration-sensitivity control design method has been presented. This method can be applied to generalized second order systems which is a large class of dynamical systems. Expressing second order specifications with H_∞ templates permits to have more flexibility than with eigenstructure assignement or LQ with reference model method. Properties of this method were compared to a more classical H_∞ method and to the cross standard form. Better results were obtained using acceleration-sensitivity control and in a more convenient way, in particular when using an integral term. Physical meaning of the H_∞ norm makes it posible to accept/reject a controller by means of its performance index γ.

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