Paper ID: *CCA02-CCAREG1097*

A Nyquist criterion for time-varying periodic systems, with application to a hydraulic test bench

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Abstract: In this paper, stability results dedicated to sampled periodic systems are applied to a mechanical system whose stiffness exhibits quick variations: a hydraulic test bench used to achieve mechanical test on complex structures. To carry out this application, time-varying w transformation representation of sampled periodic systems are first introduced. An extension of the Nyquist Criterion to sampled periodic systems is then given. Finally, this theorem is applied to evaluate the stability degree of the hydraulic test bench controlled using CRONE control methodology.

Key words: Time-varying system, Periodic systems, CRONE Control, stability degree, hydraulic actuator

1 - Introduction

Transfer functions and associated frequency responses are powerful tools for the analysis and synthesis of stationary systems. Thus, several authors have extended them to time-varying systems. For example, Zadeh defined the system function notion [Zadeh 61] to which the time-varying frequency response can be associated, and Jury defined the time-varying z-transform notion [Jury 64] to which time-varying w-transform notion (TVWT) and time-varying pseudo frequency response (TVPFR) can be associated.

Many aspects of the definition of TVWTs and TVPFRs correspond to the definition of stationary equivalents. However there has been little interest in TVWTs and TVPFRs since Zadeh and Jury.

However these representations of time-varying systems have been recently used by [Garcia 01] to extend, in the case of periodic systems and asymptotically stationary systems:

- many well known theorems such as the initial and final value theorems,
- Nyquist criterion,
- CRONE Control methodology [Oustaloup 99].

In this paper, extensions of Nyquist criterion developed in [Garcia 01] is applied to the analysis of the stability degree of a testing bench constituted of a hydraulic actuator used to achieve mechanical deformations on complex structures. Given parametric variations of the parameters of the structure and given quick variations of the structure stiffness during the test, the testing bench whose velocity is controlled using a robust CRONE controller, behaves as a sampled time-varying system with parametric uncertainties. Given that the velocity of the actuator is controlled on a finite time interval, this time-varying system can be artificially considered as a sampled periodic system.

The paper is organized as follows. Section 2 deals with the representation of continuous time-varying systems using TVWTs and TVPFRs. Section 3 is dedicated to the stability analysis of sampled time-varying systems with periodic coefficients. Section 4 first gives a presentation of the testing bench and of its control using CRONE control methodology. Then this section presents the stability degree analysis of the testing bench using the extension of Nyquist criterion given in section 3.

2 - Continuous-time periodic systems

2.1 - Definitions and hypothesis

A linear time-varying continuous-time system with periodic coefficients, also called periodic continuous-time system, is described by the state variable equation:

$$(\mathrm{H})\begin{cases}\dot{x}(t)=A(t)\,x(t)+B(t)\,u(t)\\ y(t)=C(t)\,x(t)+D(t)\,u(t)\end{cases}\qquad (1)$$

where $A(t)$, $B(t)$, $C(t)$ and $D(t)$ are supposed to be respectively element of $\mathrm{L}_2^{q\times q}[0, T]$, $\mathrm{L}_2^{q\times 1}[0, T]$, $\mathrm{L}_2^{1\times q}[0, T]$ et $\mathrm{L}_2[0, T]$ and verify :

$$A(t+T)=A(t),\; B(t+T)=B(t),\quad C(t+T)=C(t),\; D(t+T)=D(t)\qquad (2)$$

$A(t)$, $B(t)$, $C(t)$ and $D(t)$ and their first derivatives are also supposed to be respectively continuous and piece-wise continuous on $[0, T]$. They can thus be developed in uniformly convergent Fourier series of the form:

$$A(t)=\sum_{k\in\mathbb{Z}}A_k\,\mathrm{e}^{\,\mathrm{j}k\omega_0 t},\quad B(t)=\sum_{k\in\mathbb{Z}}B_k\,\mathrm{e}^{\,\mathrm{j}k\omega_0 t},$$

$$C(t)=\sum_{k\in\mathbb{Z}}\mathrm{C}_k\,\mathrm{e}^{\,\mathrm{j}k\omega_0 t}\quad\text{and}\quad D(t)=\sum_{k\in\mathbb{Z}}\mathrm{D}_k\,\mathrm{e}^{\,\mathrm{j}k\omega_0 t},\qquad (3)$$

with $\omega_0=\dfrac{2\pi}{T}$.

2.2 - System function Representation

In the 1950s, Zadeh [Zadeh 61] demonstrated that linear time-varying systems can be described by system functions $H(s, t)$. System functions are linked to the impulse response of the system, $h(t, \xi)$, which is both a function of the time variable t and of the point in time ξ when the impulse is applied:

$$H(s,t) = e^{-j\omega t}\int_{-\infty}^{\infty} h(t,\xi)e^{s\xi}d\xi \quad . \tag{4}$$

For periodic systems such as system (1), Zadeh also demonstrated that the system function $H(s, t)$ is of the form:

$$H(s,t) = \sum_{k\in Z} H_k(s)e^{jk\omega_0 t} \, . \tag{5}$$

It can be demonstrated [Garcia 01] that Fourier series (5) is uniformly convergent under hypothesis of section 2.1.

3 - Sampled-time periodic systems

3.1 - Time-varying z transfer function

If $h(n, k)$ denotes the response at time nT_e (T_e being the sampling period) of a discrete time-varying system whose input is a Kronecker function, δ_{nk} ($\delta_{nk} = 1$ if $n = k$, $\delta_{nk} = 0$, if $n \neq k$), then by analogy to the stationary case, the time-varying z transfer function of this system can be defined by [Jury 64] :

$$H(n,z) = \mathbf{Z}\left[h(n,k)\right] = \sum_{r=0}^{\infty} h(n,n-r)z^{-r} \qquad z \in \forall, \tag{6}$$

or, using $k = n - r$ (assuming no input before time $kT_e = 0$) :

$$H(n,z) = \sum_{k=-\infty}^{n} h(n,k)z^{-n+k} = z^{-n}\sum_{k=0}^{n} h(n,k)z^{k} \, . \tag{7}$$

3.2 - Time-varying z transfer function representation of system H

System H is represented with two samplers of period T_e in Figure 1 in which signals $u^*(t)$ and $y^*(t)$ are respectively given by:

$$\begin{aligned} u^*(t) &= \sum_{k=0}^{+\infty} u(kT_e)\,\delta(t-kT_e) \\ y^*(t) &= \sum_{k=0}^{+\infty} y(kT_e)\,\delta(t-kT_e) \end{aligned} \quad . \tag{8}$$

Note that Zero Order Holder (ZOH) frequency response can be included in each term of $H_i(p)$.

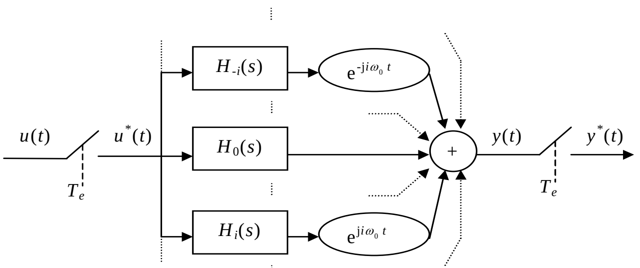

Figure 1 - Representation of sampled system H

If $u(t)$ is a Dirac impulse applied at time kT_e, id $u(t) = \delta(t - kT_e)$, the response of the system of figure 1 is :

$$y(t,k) = \sum_{i\in \mathbb{Z}} h_i(t-kT_e)\mathrm{e}^{\mathrm{j}i\omega_0 t} \;, \tag{9}$$

where $h_i(t)$ represents the impulse response of stationary linear system characterized by Laplace transform $H_i(p)$.

At time nT_e, relation (9) becomes:

$$y(n,k) = \sum_{i\in \mathbb{Z}} h_i(nT_e - kT_e)\mathrm{e}^{\mathrm{j}i\omega_0 nT_e} \;, \tag{10}$$

and so the time-varying z-transfer function of the system of figure 1 is given by:

$$H(n,z) = \sum_{r=0}^{\infty}\left[\sum_{i\in \mathbb{Z}} h_i(nT_e-(n-r)T_e)\mathrm{e}^{\mathrm{j}i\omega_0 nT_e}\right]z^{-r} \;, \tag{11}$$

and, therefore,

$$H(n,z) = \sum_{i\in \mathbb{Z}} H_i(z)\,\mathrm{e}^{\mathrm{j}i\omega_0 nT_e} \;, \tag{12}$$

with

$$H_i(z) \overset{\Delta}{=} \sum_{r=0}^{\infty} h_i(rT_e)z^{-r} \; . \tag{13}$$

Let $M \in$ N denotes the ratio between T (parameter characterizing the system periodicity) and the sampling period T_e:

$$M = \frac{T}{T_e}, \quad M \in \angle^* \, . \tag{14}$$

By combining equations (5), (12) and (14), function $H(n, z)$ becomes:

$$H(n,z) = \sum_{i\in \mathbb{Z}} H_i(z)\,\mathrm{e}^{\mathrm{j}i\frac{2\pi}{M}n} \; . \tag{15}$$

Complex gain $\mathrm{e}^{\mathrm{j}i\frac{2\pi}{M}n}$ being M-periodic, relation (15) can also be written:

$$H(n,z) = \sum_{i=0}^{M-1} H_i'(z)\,\mathrm{e}^{\mathrm{j}i\frac{2\pi}{M}n} \;, \tag{16}$$

with

$$H_i'(z) = \sum_{k\in \mathbb{Z}} H_{k+i}(z) \qquad i \in \{0,1,\ldots,M-1\}. \tag{17}$$

This particular choice of sampling period allows to get a sampled representation of system H of finite dimension with M transfer functions.

3.3 - Time-varying w-transfer function and non stationary pseudo-frequency response

Bilinear transformation or w-transformation defined by:

$$z = \frac{1+w}{1-w} \;, \quad \text{or} \quad z^{-1} = \frac{1-w}{1+w} \;, \tag{18}$$

is applied to each transfer function $H_i'(z)$ to get the time-varying w-transformation of system H. Relation (16) therefore becomes:

$$H(n,w) = \sum_{i=0}^{M-1} H_i'(w)\,\mathrm{e}^{\mathrm{j}i\frac{2\pi}{M}n} \;, \tag{19}$$

and time-varying pseudo-frequency response is defined as:

$$H(n, \mathrm{j}v) = \sum_{i=0}^{M-1} H_i^{'}(\mathrm{j}v)\,\mathrm{e}^{\mathrm{j}i\frac{2\pi}{M}n}, \tag{20}$$

where pseudo-pulsation v corresponds to the imaginary part of w. v is linked to the system real pulsation ω by relation [Sevely 89]:

$$v = \tan\left(\frac{\omega T_e}{2}\right). \tag{21}$$

3.4 - Connection of periodic sampled systems

Series connection

If $L(n,w)$ denotes the time-varying w-transfer function of a system resulting of the series connection of two periodic sampled systems characterized by time-varying w-transfer functions $C(n,w)$ and $G(n,w)$ defined by :

$$C(n,w) = \sum_{i=0}^{M-1} C_i^{'}(w)\mathrm{e}^{ji\frac{2\pi}{M}n},\ G(n,w) = \sum_{i=0}^{M-1} G_i^{'}(w)\mathrm{e}^{ji\frac{2\pi}{M}n} \tag{22}$$

then $L(n,w)$ is given by:

$$L(n,w) = \sum_{i=0}^{M-1} L_i^{'}(w)\mathrm{e}^{ji\frac{2\pi}{M}n}, \tag{23}$$

where w-transfer functions $L_i^{'}(w)$ are issued from the product:

$$\boldsymbol{L}(w) = \boldsymbol{G}(w)\,\boldsymbol{C}(w), \tag{24}$$

in which vectors $\boldsymbol{L}(w)$ and $\boldsymbol{C}(w)$ and matrix $\boldsymbol{G}(w)$ are given by:

$$\boldsymbol{L}(w) = \left[L_0^{'}(w)\ L_1^{'}(w)\ \cdots\ L_{M-1}^{'}(w)\right]^{\mathrm{T}}, \tag{25}$$

$$\boldsymbol{C}(w) = \left[C_0^{'}(w)\ C_1^{'}(w)\ \cdots\ C_{M-1}^{'}(w)\right]^{\mathrm{T}} \tag{26}$$

and

$$\boldsymbol{G}(w) = \begin{bmatrix} G_0^{'}(w_0) & G_{M-1}^{'}(w_1) & \cdots & G_1^{'}(w_{M-1}) \\ G_1^{'}(w_0) & G_0^{'}(w_1) & \ddots & \vdots \\ \vdots & \vdots & \ddots & G_{M-1}^{'}(w_{M-1}) \\ G_{M-1}^{'}(w_0) & G_{M-2}^{'}(w_1) & \cdots & G_0^{'}(w_{M-1}) \end{bmatrix}, \tag{27}$$

parameters w_i being defined by:

$$w_i = \frac{z_i - 1}{z_i + 1} \text{ with } z_i = z\,\mathrm{e}^{\mathrm{j}i\frac{2\pi}{M}n} \text{ and } z = \frac{1+w}{1-w}, \tag{28}$$

and therefore:

$$w_i = \frac{\mathrm{j}\left[\sin i\frac{\pi}{M}\right] + \left[\cos i\frac{\pi}{M}\right]w}{\left[\cos i\frac{\pi}{M}\right] + \mathrm{j}\left[\sin i\frac{\pi}{M}\right]w},\ i \in \{0,1,..,M\text{–}1\}. \tag{29}$$

Feedback connection

Let $C(n, w)$ and $G(n,w)$ be the time-varying w-transfer functions of two periodic sampled systems and $L(n, w)$ the time-varying w-transfer function resulting from the series connection (figure 2). $C(n, w)$, $G(n, w)$ and $L(n,w)$ are given by relations (22) and (23). The time-varying w-transfer function $T(n,w)$ between input $r(n)$ and output $y(n)$ is given by:

$$T(n,w) = \sum_{i=0}^{M-1} T_i^{'}(w)\,\mathrm{e}^{\mathrm{j}i\frac{2\pi}{M}n}, \tag{30}$$

where w-transfer functions $T_i^{'}(w)$ result from the product:

$$\boldsymbol{T}(w) = \left[\,\mathsf{I} + \boldsymbol{\mathsf{L}}(w)\,\right]^{-1}\boldsymbol{L}(w), \tag{31}$$

in which vectors $\boldsymbol{T}(w)$ and $\boldsymbol{L}(w)$ and matrix $\boldsymbol{\mathsf{L}}(w)$ expressed:

$$\boldsymbol{T}(w) = \begin{bmatrix} T_0^{'}(w) \\ T_1^{'}(w) \\ \vdots \\ T_{M-1}^{'}(w) \end{bmatrix}, \boldsymbol{L}(w) = \begin{bmatrix} L_0^{'}(w) \\ L_1^{'}(w) \\ \vdots \\ L_{M-1}^{'}(w) \end{bmatrix}, \tag{32}$$

$$\boldsymbol{\mathsf{L}}(w) = \begin{bmatrix} L_0^{'}(w_0) & L_{M-1}^{'}(w_1) & \cdots & L_1^{'}(w) \\ L_1^{'}(w_0) & L_0^{'}(w_1) & \ddots & \vdots \\ \vdots & \vdots & \ddots & L_{M-1}^{'}(w) \\ L_{M-1}^{'}(w_0) & L_{M-2}^{'}(w_1) & \cdots & L_0^{'}(w) \end{bmatrix} \tag{33}$$

I being the identity matrix of dimension M.

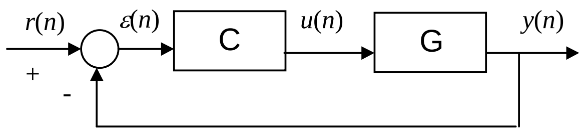

Figure 2 – Feedback connection of two periodic sampled systems in series connection

3.5 - Stability: extension of Nyquist criterion to feedback periodic sampled systems

Let L be a periodic sampled system described by:

$$L(n,w) = \sum_{i=0}^{M-1} L_i^{'}(w)\,\mathrm{e}^{\mathrm{j}i\frac{2\pi}{M}n}. \tag{34}$$

where w-transfer functions $L_i^{'}(w)$ are issued from:

$$\boldsymbol{\mathsf{L}}(w) = \begin{bmatrix} L_0^{'}(w_0) & L_{M-1}^{'}(w_1) & \cdots & L_1^{'}(w) \\ L_1^{'}(w_0) & L_0^{'}(w_1) & \ddots & \vdots \\ \vdots & \vdots & \ddots & L_{M-1}^{'}(w) \\ L_{M-1}^{'}(w_0) & L_{M-2}^{'}(w_1) & \cdots & L_0^{'}(w) \end{bmatrix}. \tag{35}$$

Let Γ' be the path of figure 3 where point A is the origin of w-plane, point B corresponds to point (0, tan(π/M)), point C corresponds to point (1,0) and where curve BC is defined by the relation :

$$w = \frac{R^2 - 1}{R^2 + 2R\cos\frac{2\pi}{M} + 1} + \mathrm{j}\frac{2R\sin\frac{2\pi}{M}}{R^2 + 2R\cos\frac{2\pi}{M} + 1}, \tag{36}$$

with $R \in [1, +\infty[$.

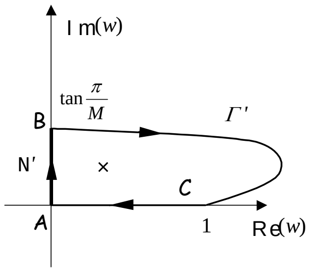

Figure 3 – Path Γ' in w-plane

Theorem 1 – Generalized Nyquist theorem in pseudo-frequency domain [Garcia 01]

System of figure 4 is stable if and only if the Nyquist locus of the eigenvalues of matrix $\mathsf{L}(w)$ (relation 36) encircles P_0 times the point $(-1/K, 0)$, in the counter clockwise direction when w varies on the Γ' path, assuming that there is no hidden unstable modes in the direct loop. P_0 is the number of characteristic multipliers of system L with a modulus greater or equal than 1.

Note that this criterion not only permits to evaluate the stability of system of figure 3 (which can also be done using others methods such as those based on the inspection of the eigenvalues of impulse response matrix over a single period). This criterion also permits to evaluate the stability degree of the control loop and to define stability margins which can be obtained through a measure of the minimal distance between the Nyquist locus with point (-1/K, 0).

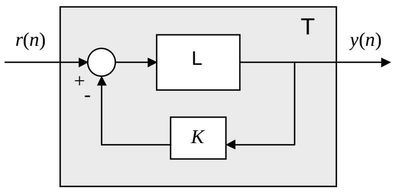

Figure 4 – Feedback periodic sampled system

4 - Application to an electrohydraulic test bench

4.1 - Description of the test bench

The system under study is a electrohydraulic test bench that is used to achieve mechanical tests on structures. The electrohydraulic actuator must be controlled in order to deform the structures with a constant velocity. As the structures are not well identified, the control system must be robust to parametric uncertainties and as the structures may be tested till fracture, it must take into account the time-varying characteristics of the loads. The objective is then to compute a robust control law for a time-varying system. In order not to make the control system too complex, it is chosen to compute a stationary control law and to make sure that the stability of the robustly controlled system is ensured even when time-varying phenomena occur. The results concerning the stability of sampled periodic systems are used to this end.

The structures to be tested are described by a mass-damper-spring set (figure5). The mass M_s and the viscous coefficient b_s are supposed to vary slowly (compared with the system dynamics) in the following limits:

$$0 < M_s < 80 \text{ Kg and } 80 < b_s < 100 \text{ N/m.s}$$

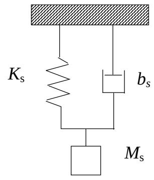

Figure 5 – Model of the structure to be tested

The stiffness of the structure is likely to vary quickly and is supposed to evolve as described in figure 6.

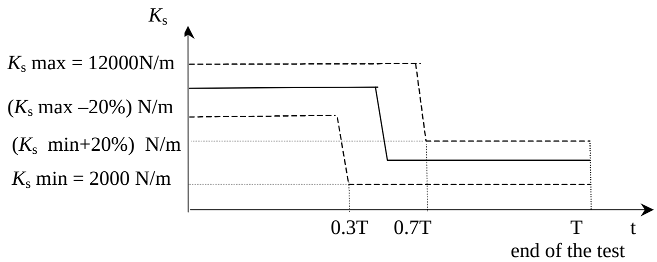

Figure 6 – Evolution of the stiffness during a test

The complete model of the electrohydraulic test bench is given by table 1 and the following non linear state-space model:

$$\begin{cases} \dot{X} = f_1(X) + g(X)u & \text{if } y_u \geq 0 \\ \dot{X} = f_2(X) + g(X)u & \text{if } y_u < 0 \\ Y = h(X) \end{cases}, \qquad (35)$$

with: $X^T = \begin{bmatrix} P_1 & P_2 & v & y & y_u \end{bmatrix}$,

$$f_1(X) = \begin{pmatrix} \dfrac{B}{V_0 + S_0 y}\begin{pmatrix} -S_0 v + y_u \sqrt{|P_s - P_1|}\,sign(P_s - P_1) \\ -\lambda(P_1 - P_2) - \lambda_p(P_1 - P_{P1}) \end{pmatrix} \\ \dfrac{B}{V_0 - S_0 y}\begin{pmatrix} S_0 v - y_u \sqrt{|P_2 - P_r|}\,sign(P_2 - P_r) \\ -\lambda(P_1 - P_2) - \lambda_p(P_2 - P_{P2}) \end{pmatrix} \\ \dfrac{1}{M_s + M_0}\left(S_0 P_1 - S_0 P_2 - K_s(y - y_0) - b_s v - F_f\right) \\ v \\ -\omega_i y_u \end{pmatrix}$$

$$f_2(X) = \begin{pmatrix} \dfrac{B}{V_0 + S_0 y}\begin{pmatrix} -S_0 v + y_u \sqrt{|P_1 - P_r|}\,sign(P_1 - P_r) \\ -\lambda(P_1 - P_2) - \lambda_p(P_1 - P_{P1}) \end{pmatrix} \\ \dfrac{B}{V_0 - S_0 y}\begin{pmatrix} S_0 v - y_u \sqrt{|P_s - P_2|}\,sign(P_s - P_2) \\ -\lambda(P_1 - P_2) - \lambda_p(P_2 - P_{P2}) \end{pmatrix} \\ \dfrac{1}{M_s + M_0}\left(S_0 P_1 - S_0 P_2 - K_s(y - y_0) - b_s v - F_f\right) \\ v \\ -\omega_i y_u \end{pmatrix}$$

$g^T(X) = \begin{pmatrix} 0 & 0 & 0 & 0 & k_u \omega_i \end{pmatrix}$ and $h(X) = v$.

Table 1 – Notations for the electrohydraulic model

P_s	supply pressure	240 bar
P_r	tank pressure	7.5 bar
P_1, P_2	cylinder chamber pressures	bar
P_{P1}	hydrostatic bearings pressure	236 bar
P_{P2}	hydrostatic bearings pressure	212 bar

B	Bulk modulus	10^9 bar
V_1 , V_2	cylinder chamber volumes	m^3
V_0	cylinder half-volume	$245.10^{-7} m^3$
M_0	cylinder rod mass	31.8 kg
S_0	cylinder rod effective area	243.10^{-6} m²
B	mechanical structure viscous coefficient	86 N/m.s
y	cylinder rod position	m
v	cylinder rod velocity	m/s
k_u	amplification stage gain	$1.17*10^{-6}$ m^3/s/A
ω_a	cut-off frequency of the amplification stage	942 rad/s
k_3	mass flow gain	$4.5*10^{-5}$ (m^3/s)/m
λ	cylinder leakage coefficient	1.10^{-11} s^{-1}
λ_p	hydrostatic bearings leakage coefficient	$0,5.10^{-12}$ s^{-1}
F_f	friction force	N

4.2 - Control of the test bench

The aim is to compute a robust control law in order to take into account the structure uncertainty. CRONE (the French acronym of "Commande Robuste d'Ordre Non Entier") control-system design is therefore used [Oustaloup 99]. This is a frequency-domain based methodology using fractional differentiation. It permits the robust control of perturbed plants using the common unity feedback configuration. It consists on determining the nominal and optimal open-loop transfer function that guaranties the required specifications. The controller is then obtained from the ratio of the open-loop frequency response to the nominal plant frequency response.

As CRONE control is to be applied on linear system, it is first necessary to linearize the model of the electrohydraulic bench. To this end, an input-output linearization under diffeomorphism and feedback is achieved. So that this linearization is available whatever the structure and its parameters, the output considered for the linearization is the pressure difference. Indeed, if this output is chosen, the linearization law does not depend on the parameters of the structure [Pommier 01]. Moreover to get a relative degree equal to 1 in order to simplify the linearization and its numerization, the linear model of the amplification stage is not taken into account in the linearization. However it is necessary to consider afterwards an inverse band-limited model of this stage (figure 7).

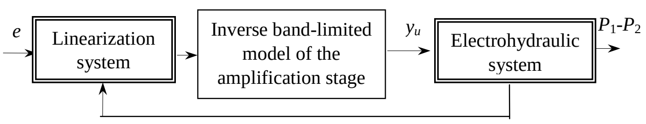

Figure 7 – Scheme of the linearization strategy

Finally the linearized model of the electrohydraulic system between input e and output (P_1-P_2) is described by:

$$\frac{P_1(s)-P_2(s)}{E(s)} = k_p \Big/ \left(\frac{s}{\alpha}+1\right)\left(\frac{s}{5000}+1\right). \qquad (38)$$

with k_p =7.10[8] and α=2000 chosen so that the input-output linearized system has the same behavior than the first-order linearized system around the operating point defined by v= 0 and y=0 and 1/(s+5000) coming from the band-limited model of the amplification stage.

Once the system is linearized, a robust control law is computed to control the velocity by using the model:

$$\begin{cases} \dot{X} = f(X) + g(X)e \\ Y = h(X) \end{cases}, \qquad (39)$$

with: $X^T = [\Delta P \quad \Delta\dot{P} \quad v \quad y]$ where $\Delta P = P_2 - P_1$,

$$f(X) = \begin{pmatrix} \Delta\dot{P} \\ -5000\alpha\Delta P - (5000+\alpha)\Delta\dot{P} \\ \frac{1}{M_s + M_0}\left(S\Delta P - K_s(y-y_0) - b_s v - F_{frot}\right) \\ v \end{pmatrix}$$

$g^T(X) = (0 \quad 5000\alpha\Delta P \quad 0 \quad 0)$ and $h(X) = v$,

this model being sampled at period T_e = 0.2ms.

By considering the uncertain parameters K_s, M_s and b_s and the required specifications:
- magnitude peak of the complementary sensitivity function for the nominal parametric state: 1dB
- maximum of complementary sensitivity function: 3dB
- maximum of sensitivity function: 5dB

the optimal open-loop transfer function is computed in the pseudo-frequency domain, as well as the controller:

$$C(v) = 5.10^{-3}\frac{\left(\frac{v^2}{0{,}00095^2}+\frac{2*0{,}06}{0{,}00095}v+1\right)\left(\frac{v}{0{,}011}+1\right)\left(\frac{v}{0{,}271}+1\right)\left(\frac{v}{0{,}373}+1\right)}{v\left(\frac{v}{0{,}00033}+1\right)\left(\frac{v}{0{,}002}+1\right)\left(\frac{v^2}{0{,}581^2}+\frac{2*0{,}6}{0{,}581}v+1\right)\left(\frac{v}{1{,}756}+1\right)^2}$$

The sampling period used for the implementation of this controller is also T_e = 0.2ms.

4.3 - Study of the stability degree

Control law previously given has be synthesized without taking into account the dynamic variation of parameter K_s (only parametric variations has been considered). The objective is now to evaluate the stability degree of the closed loop in spite of the dynamic variations of parameter K_s described in figure 6. As the final state is to be reached before the end of the test, it is possible to consider periodic (of period T) the evolution of the stiffness. The function that describes the stiffness evolution and its derivative are thus respectively continuous and piece-wise continuous on [0, T]. Therefore the results on stability for periodic systems can be used to conclude on the stability of the electrohydraulic system when the stiffness varies quickly.

On the test bench, the test is supposed to last 220 ms and the velocity is required to follow the profile described in figure 8. Note that the stability analysis is independent of the input applied to the system. This input has thus not to be considered periodic. Only stiffness variations are supposed to be artificially represented by a periodic function and as explained above, the stiffness is considered as a periodic coefficient where period T equals the duration of the test, so T = 220ms. By using these figures, M the integer ratio between T and the sampling period T_e can be computed: M = 1100.

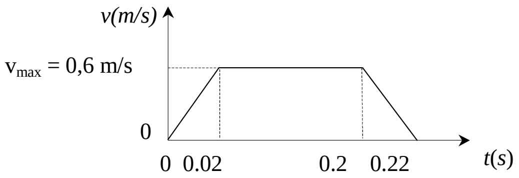

Figure 8 – Profile of the velocity

The controller being a sampled system in series with the electrohydraulic system that is a periodic sampled system, it is possible to compute the time-varying *w*-transfer function of the resulting system L by using (24). Several time varing *w*-transfer functions must be computed for the different values of M_s, b_s and K_s and for different times of fracture. Here two examples are chosen:

- example 1: M_s=40 kg, b=90N/m.s, K_{smax}= 12000N/m, K_{smin}=2000N/m and the fracture that occurs at 0.3T
- example 2: M_s=80 kg, b_s=90N/m.s, K_{smax}= 10000N/m, $K_{smin=}$ 6000N/m and the fracture that occurs at 0.5T

In order to study the stability of the time-varying feedback-controlled system, the extension of the Nyquist criterion described in section 3.4 is used. The Nyquist locus of the eigenvalues of matrix L(w) for the two examples (+++) and the magnitude contour of value 1dB (—) are plotted in figure 9 for w varying along the segment AB of the Γ' path.

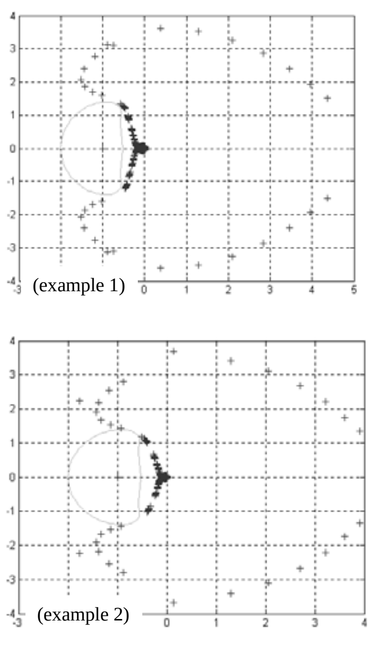

Figure 9 - Nyquist locus of the eigenvalues (+++) of L(w)

There is no multipliers of system L with a module equal or greater than 1. The Nyquist locus encircles 0 times the point (–1, 0). So the electrohydraulic system is stable even when quick variation of the stiffness occurs. The simulation of the system for the example 1 is shown in Figure 10 and tends to confirm this results.

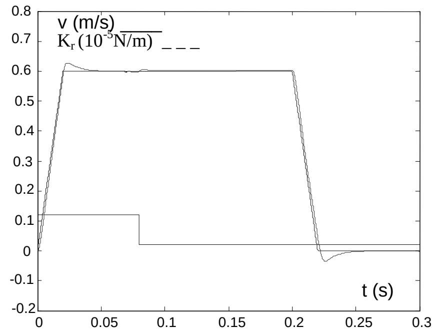

Figure 10 – Simulation of the velocity with a quick variation of the stiffness

Stabity degree variations of the closed loop system can also be evaluated using theorem 1 by computing variations of parameter K (figure 3) for several values of parameters M_s, b_s and for several evolutions of K_s.

5 - Conclusion

In this paper, a control method for a mechanical test bench designed to study the deformation of complex structure with a defined velocity is presented. The control law is based on an input-output linearization of the test bench and on a CRONE controller which ensures the robustness of the velocity control loop in spite of variations in the mass, the viscous coefficient and the stiffness of the structure. However, this synthesis is carried out without taking into account the dynamic variations of the structure stiffness during the test. So, considering these variations as periodic, time-varying pseudo frequency response of the test bench has been computed and used to evaluate the stability degree of the control loop in spite of dynamic variations of the stiffness.

This paper thus provides tools to study the stability of systems submitted to abrupt variations in their parameters.

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