ADAPTIVE DETECTION FOR A DIFFERENTIAL CHAOS-BASED MULTIPLE ACCESS SYSTEM ON UNKNOWN MULTIPATH FADING CHANNELS

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ABSTRACT

This paper addresses the problem of bit detection for a chaos-based multiple-access system. In particular, one considers the Differential Chaos Shift Keying modulation. It is assumed that the transmission channels are frequency-selective. Moreover, the channel coefficients and the channel delays are unknown to the receiver. It is only assumed that vague estimates of the minimum and maximum channel delays are available for the user of interest. In this context, the detection is achieved using a training sequence from which an LMS detector is derived. The theoretical performance results are compared to those of the optimal detector for which all channel characteristics are known. Simulation results are given, which confirm the theoretical study.

Index Terms— Chaos, Adaptive signal detection, Multiple-access, Multipath channels, Multiuser channels.

1. INTRODUCTION

Chaos-based digital communications have received much attention during these last two decades [1], [2]. In particular, their unpredictable behavior makes them very attractive for secure transmissions. Two main classes of chaotic systems can be considered: coherent [3] and non-coherent [4] chaotic systems. Coherent systems require perfect synchronization between transmitter and receiver. For practical signal-to-noise ratios used in digital transmissions, this synchronization is a very difficult task, and one then resorts to non-coherent systems for which synchronization is not necessary. Among these non-coherent systems, this paper considers in particular a Differential Chaos Shift Keying (DCSK) system as the one described in [4]. The different users are characterized by their chaotic signatures.

This paper addresses the problem of multiple-access DCSK-based transmission on frequency-selective channels. Previous works considered the case of synchronous transmissions [4] and asynchronous transmissions [5], [6] on Gaussian channels. Two kind of detectors were studied: those based on an LMMSE approach, and those based on a preliminary estimation of the user chaotic sequence. It has been shown in [5], [6] that this latter approach, which is well suited for synchronous transmission on Gaussian channels [4], is not efficient in the asynchronous case, compared to the LMMSE-based receivers. Consequently, we focus in this paper on the LMMSE approach. Two receivers are then studied: the theoretical LMMSE detector, and its adaptive version, i.e. the LMS detector. Obviously, the LMMSE detector is not suited for practical applications, due to the important a priori knowledge it requires. However, it can be used as a reference with which the LMS detector can be compared. For the LMS detector, the existence of a training sequence is needed. Consequently, this LMS detector is better suited for practical applications. Two cases are considered in this paper concerning the channel delays. First, it is assumed that the delays of the kth-user channel are known to the receiver corresponding to this user. This assumption requires then perfect estimation of the delays, which is a difficult problem in practice. On the other hand, we address the case where only a rough estimation of the minimum and the maximum delays are available to the receiver. These two approaches are investigated in a common framework. They are compared through simulation results.

The paper is organized as follows. Section 2 presents the main characteristics of the DCSK-based multiple access system. Section 3 considers the sampling of this received signal. It provides the general form of the sampled vector used for detection. Section 4 addresses the LMMSE and LMS detectors. Section 5 gives the theoretical performance results. Some simulation results are presented in section 6.

2. CHARACTERISTICS OF THE CHAOTIC MULTI-USER SIGNAL

2.1. Chaos generator

Each user is characterized by a unique N_{S}-sample chaotic sequence, denoted by $\mathbf{s}_k$, computed from a particular chaotic map and a given initial condition. In this paper, there is no restriction concerning this map, so that all chaotic maps can be considered. Low cross-correlations between chaotic sequences is obtained by simply choosing different initial conditions.

2.2. Frame structure for the MA DCSK system

As in [4], [5] and [6], the signal transmitted by each user is structured in frames, as illustrated in fig. 1. Each frame is subdivided into time slots of N_{S} samples, which contains the chaotic sequence of the user. The first part of the frame, referred to as the Training Sequence (TS), is composed by L_{T} time slots, during which a sequence of bits (in the set $\{-1;+1\}$) known to the receiver is transmitted. The information-bearing bits are transmitted during the second part of the frame. In this paper, $b_{k,j}$ denotes the bit transmitted by user k in the jth time slot, during which the signal $b_{k,j}\mathbf{s}_k$ is transmitted.

2.3. Channel model

The channel model of the kth user is a frequency-selective channel whose discrete impulse response is

$$c_k(n) = \sum_{l=0}^{L_k-1} c_{k,l}\,\delta(n-\tau_{k,l})$$

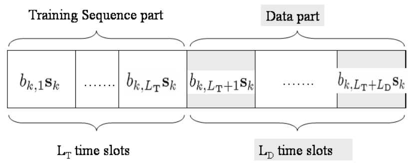

Fig. 1. DCSK frame structure.

where L_k is the number of paths, $c_{k,l}$ and $\tau_{k,l}$ are the gain and the propagation delay (in number of samples) of the lth path of the kth user, respectively, and δ is the digital Dirac function. The digital multi-user received signal is then expressed as:

$$r(n) = \sum_{k=1}^{K} \sum_{j} \sum_{l=0}^{L_k-1} c_{k,l} b_{k,j} \mathbf{s}_k(n - jN_{\mathrm{S}} - \tau_{k,l}) + \nu(n)$$

where K is the number of users and $\nu(n)$ is a white Gaussian noise with variance σ^2. It is supposed for convenience that $\mathbf{s}_k(n) = 0$ for $n \notin \{1, \ldots, N_{\mathrm{S}}\}$.

3. SAMPLING OF THE MULTI-USER RECEIVED SIGNAL

The samples corresponding to bit $b_{k,j}$ belong to the index set $\mathcal{I}_k^j \triangleq \bigcup_{l=0}^{L_k-1}\{\tau_{k,l} + jN_{\mathrm{S}} + 1; \tau_{k,l} + (j+1)N_{\mathrm{S}}\}$. In order to detect the bit $b_{k,j}$, the kth-user receiver must consider a set of samples $\mathbf{\Theta}_k^j$ of the received signal $r(n)$, i.e. the vector $\mathbf{r}_k^j$ used to detect $b_{k,j}$ is defined by

$$\mathbf{r}_k^j \triangleq \left\{ r(n) | n \in \mathbf{\Theta}_k^j \right\}. \tag{1}$$

Two different assumptions are addressed in this paper concerning the delays, which lead to two different definitions of the set $\mathbf{\Theta}_k^j$.

3.1. Known delays

All delays $(\tau_{k,l})_{l=0,\ldots,L_k-1}$ are known to the kth-user receiver (which however does not know the delays of the other users). One can then detect bit $b_{k,j}$ by considering the samples in the set $\mathcal{I}_k^j$, i.e. $\mathbf{\Theta}_k^j \triangleq \mathcal{I}_k^j$.

3.2. Unknown delays

The receiver for user k does not know the delays $\tau_{k,l}$. Instead, only rough estimates $\widehat{\tau}_{k,\text{min}}$ and $\widehat{\tau}_{k,\text{max}}$ of $\min_l\{\tau_{k,l}\}$ and $\max_l\{\tau_{k,l}\}$, respectively, are available. More generally, one can define $\widehat{\tau}_{k,\text{min}}$ and $\widehat{\tau}_{k,\text{max}}$ such that the main amount of energy transmitted by user k for the jth bit is (assumed to be) received between samples $\widehat{\tau}_{k,\text{min}} + jN_{\mathrm{S}} + 1$ and $\widehat{\tau}_{k,\text{max}} + (j+1)N_{\mathrm{S}}$. These estimates could be obtained for instance from the position of the user in the cell. However, this problem is still open and is beyond the scope of this paper. Samples which contain a part of the transmitted energy for $b_{k,j}$ lie then (more precisely, are supposed to lie) between $\widehat{\tau}_{k,\text{min}} + jN_{\mathrm{S}} + 1$ and $\widehat{\tau}_{k,\text{max}} + (j+1)N_{\mathrm{S}}$. One defines therefore: $\mathbf{\Theta}_k^j \triangleq \{\widehat{\tau}_{k,\text{min}} + jN_{\mathrm{S}} + 1, \ldots, \widehat{\tau}_{k,\text{max}} + (j+1)N_{\mathrm{S}}\}$. An important advantage of this sampling method is that the knowledge of the number of paths L_k is not required. Note that none of these choices for $\mathbf{\Theta}_k^j$ is optimal. Indeed, an optimal strategy would consist of defining a much wider sample set $\mathbf{\Theta}_k^j$ to take into account more inter-symbol- and multi-user- interferences. Obviously, this solution is not practically tractable. The advantage of the first proposed approach is that it focuses only on the samples which contain direct information on $b_{k,j}$. On the other hand, it may not consider important samples which would contain information on the bits which interfered $b_{k,j}$. On the contrary, the second solution may manage more interference, but includes signal energy which is not directly related to $b_{k,j}$. These two approaches will be addressed in a common framework by considering the general form (1). Let γ_k denote the number of elements of $\mathbf{\Theta}_k^j$ (there is no superscript j since it actually does not depend on j). It can then be shown that $\mathbf{r}_k^j$ is expressed as

$$\mathbf{r}_k^j = \boldsymbol{\xi}_{k,k}^{j,j} b_{k,j} + \text{ISI}(k) + \sum_{k' \neq k} \text{MUI}(k') + \boldsymbol{\nu}_k^j \tag{2}$$

where $\boldsymbol{\nu}_k^j \triangleq (\nu(n))_{n \in \mathbf{\Theta}_k^j}$. In (2), $\text{ISI}(k)$ denotes the inter-symbol interference term for user k due to the multi-path channel, and $\text{MUI}(k')$ denotes the multi-user interference term due to user k'. For clarity of presentation, the detailed definitions of $\boldsymbol{\xi}_{k,k}^{j,j}$, $\text{ISI}(k)$ and $\text{MUI}(k')$ are given in appendix 8. The objective of the detector consists of mitigating the ISI, the MUI, and the additive noise, in order to detect the transmitted bit.

4. LMMSE-BASED DETECTORS

4.1. The theoretical LMMSE detector

The Linear Minimum Mean-Squared Error (LMMSE) approach consists of defining bit estimates $\widehat{b}_{k,j}$ by [1]

$$\widehat{b}_{k,j} \triangleq \text{sign}(\mathbf{h}_{k,j}^{\mathrm{T}} \mathbf{r}_k^j) \tag{3}$$

where $.^{\mathrm{T}}$ denotes transposition. In (3), $\mathbf{h}_{k,j}$ is the vector which minimizes the mean-squared error (MSE) $E\left[(b_{k,j} - \mathbf{h}^{\mathrm{T}}\mathbf{r}_k^j)^2\right]$ with respect to vector $\mathbf{h}$. The optimal $\mathbf{h}_{k,j}$ is given by [7]

$$\mathbf{h}_{k,j} = \mathbf{\Sigma}_{k,j}^{-1} \boldsymbol{\rho}_{k,j} \tag{4}$$

where $\mathbf{\Sigma}_{k,j}$ is the covariance matrix of vector $\mathbf{r}_k^j$, and $\boldsymbol{\rho}_{k,j} \triangleq E\left[\mathbf{r}_k^j b_{k,j}\right]$. Using the fact that the bits are independent and lie in the set $\{-1; +1\}$, it can easily be shown that

$$\boldsymbol{\rho}_{k,j} = \boldsymbol{\xi}_k \tag{5}$$

and

$$\mathbf{\Sigma}_{k,j} = \sum_{k'=1}^{K} \sum_{p \in \Delta_{k,k'}^0} \left(\boldsymbol{\xi}_{k,k'}^p\right)^{\mathrm{T}} \boldsymbol{\xi}_{k,k'}^p + \sigma^2 \mathbf{I}_{\gamma_k} \tag{6}$$

where $\mathbf{I}_\alpha$ denotes the identity matrix of dimension α, and $\Delta_{k,k'}^0$ is defined in appendix 8. Note that, from (5) and (6), $\boldsymbol{\rho}_{k,j}$, $\mathbf{\Sigma}_{k,j}$, and $\mathbf{h}_{k,j}$ do not depend on j and will now be denoted by $\boldsymbol{\rho}_k$, $\mathbf{\Sigma}_k$, and $\mathbf{h}_k$, respectively.

4.2. The LMS detector

The previous section shows that the optimal detection, in the MSE sense, requires the knowledge of all channel gains, all chaotic sequences, and the noise variance. Now, in DCSK systems, the receiver does not know the chaotic sequences of the different users (including the one of the user of interest). Consequently, the detection (3) cannot be applied. However, it is still possible, assuming the transmission of a training sequence, to derive for each user an LMS

[1] In this paper, the terms "bit estimation" and "bit detection" are used as synonymous, since the problem of estimating a bit b in the set $\{-1; +1\}$ can be seen as the binary detection problem $b = +1$ vs. $b = -1$.

algorithm [7], which will adaptively converge to the solution of the LMMSE detector. More precisely, the algorithm iteratively builds the sequence of vectors $\mathbf{h}_k^j$ (initially set to the null vector) which converges, for a sufficiently long training sequence to the optimal vector $\mathbf{h}_k$. The well-known LMS update equations are not recalled here and can be found in [7]. Obviously, the resort to the LMS algorithm is not a novel idea. We simply propose to use its properties to solve the detection problem in the transmission context addressed in this paper, which, to the best of our knowledge, has not been investigated so far.

5. THEORETICAL PERFORMANCE RESULTS

5.1. LMMSE performance

Define $\zeta_k \triangleq \mathbf{h}_k^{\mathrm{T}} \boldsymbol{\xi}_k$, and $\zeta_{k,k'}^{i-j} \triangleq \mathbf{h}_k^{\mathrm{T}} \boldsymbol{\xi}_{k,k'}^{i-j}$ (see appendix 8 for the definition of $\boldsymbol{\xi}_{k,k'}^{i-j}$). Note that ζ_k is positive since $\zeta_k = (\boldsymbol{\xi}_k)^{\mathrm{T}} \boldsymbol{\Sigma}_k^{-1} \boldsymbol{\xi}_k$ and Σ_k is a positive definite matrix. Denote Γ_k as the set $\{(k', i) | k' \in \{1, \dots, K\}, i \in \Delta_{k,k'}^0$ and $(k', i) \neq (k, 0)\}$. The decision variable $\mathbf{h}_k^{\mathrm{T}} \mathbf{r}_k^j$ can be expressed as

$$\mathbf{h}_k^{\mathrm{T}} \mathbf{r}_k^j = \zeta_k b_{k,j} + \sum_{(k',p) \in \Gamma_k} \zeta_{k,k'}^p b_{k',p+j} + \mathbf{h}_k^{\mathrm{T}} \boldsymbol{\nu}_j^k \qquad (7)$$

The second term in (7) is formed by the MUI and the ISI terms. The third term is a zero-mean Gaussian random variable with variance $\widetilde{\sigma}_k^2 \triangleq \sigma^2 \|\mathbf{h}_k\|^2$. Using the Bayes'rule, it is possible to obtain an exact expression of the bit error rate (BER). However, due to the huge number of terms involved in this expression, the exact BER cannot be practically computed. Instead, we propose to give an estimated BER. Indeed, using the Central Limit theorem, the interference term in (7) can be considered as a zero-mean Gaussian variable with variance

$$\sigma_{k,\text{interf}}^2 \triangleq \sum_{(k',p) \in \Gamma_k} \left(\zeta_{k,k'}^p \right)^2$$

The estimated BER $\widetilde{P}_k$ is then given by

$$\widetilde{P}_k = Q \left(\zeta_k \Big/ \sqrt{\sigma_{k,\text{interf}}^2 + \widetilde{\sigma}_k^2} \right)$$

5.2. LMS performance

To derive the performance of the LMS detector, one must consider the random behavior of the LMS coefficients around the optimal LMMSE coefficients. Similarly as [5], it can be observed that the covariance matrix of the coefficient error vector for the kth user is a diagonal matrix, whose elements are all equal to a particular value denoted by $\sigma_{\text{LMS},k}^2$. Define the matrix $\overline{\boldsymbol{\Sigma}}_k \triangleq \boldsymbol{\Sigma}_k - \boldsymbol{\xi}_k \boldsymbol{\xi}_k^{\mathrm{T}}$. One can then show that the exact BER for the LMS receiver can be approximated by (still using the Central Limit theorem)

$$\widetilde{P}_k^{\text{lms}} = Q \left(\zeta_k \Big/ \sqrt{\overline{\sigma}_k^2} \right)$$

with $\overline{\sigma}_k^2 \triangleq \mathbf{h}_k^{\mathrm{T}} \overline{\boldsymbol{\Sigma}}_k \mathbf{h}_k + \sigma_{\text{LMS},k}^2 (\|\boldsymbol{\xi}_k\|^2 + \text{trace} \overline{\boldsymbol{\Sigma}}_k)$. Note that for brevity reasons, details of this derivation cannot be given here. However, similar computations can be found in [5].

6. SIMULATIONS

The figures shown in this paper have been obtained with the same system characteristics, which are the following. The bit period is $T_{\text{b}} = 4.88.10^{-7}$s. The number of users is $K = 4$. For each of them, a transmission channel has been simulated from a static Typical-Urban channel model. More precisely, each channel contains $L_k = 5$ paths, whose delays have been randomly drawn around mean delays equal respectively to 0, 2.10^{-7}, 5.10^{-7}, 16.10^{-7}, and 23.10^{-7} seconds (therefore, the delays of different users are different). The gains of the five paths have been generated according to Rayleigh distributions whose means are respectively equal to -3dB, 0dB, -2dB, -6dB, and -8dB. The chaotic sequences have $N_{\text{S}} = 50$ samples, generated form the chaotic map: $x_{n+1} = 4x_n^3 - 3x_n$, as in [4], [5]. The length of the training period is $L_{\text{T}} = 1000$. Figure 2 presents the performance results obtained for user 1 when the channel delays are known to the receiver, as a function of the signal-to-noise ratio (SNR). The SNR is defined as the ratio between the power of the un-noisy multi-user received signal and the power of the additive Gaussian noise. Compared to the usual E_b/N_0 ratio, the SNR is a better measure of the importance of the noise with respect to the received multi-user signal, since it takes into account the gains of the channels, contrary to the E_b/N_0 ratio. It can clearly be seen on this figure that the theoretical and the simulated LMS BERs are very close, which confirms the theoretical derivations. Moreover, one can note that these BERs are around 0.5dB higher than the theoretical LMMSE BERs. This difference is due to the fact that the LMS coefficients oscillate around the optimal LMMSE coefficients with a particular covariance matrix. Figure 3 presents the corresponding results obtained with the second sampling strategy. In that case, variables $\widehat{\tau}_{k,\min}$ and $\widehat{\tau}_{k,\max}$ have been randomly generated using an uniform distribution with width $2T_{\text{b}}$, centered around $\min_l\{\tau_{k,l}\}$ and $\max_l\{\tau_{k,l}\}$, respectively. The previous comments given for figure 1 are globally also valid here. Note however that for SNR $= -2dB$, the estimated and theoretical LMS BERs are very different. This is due to the fact that the LMS algorithm has diverged, such that the LMS detection is completely random. Moreover, for the other SNR values, the difference between theoretical and estimated LMS BERs are a bit greater than that obtained previously. This can be explained by the fact that the convergence of the LMS algorithm was more difficult in that case: indeed, the length of the coefficient vector is higher here ($\gamma_1 = 419$) than for the previous case ($\gamma_1 = 171$). Finally, comparing figure 2 with figure 3, one can see that the theoretical LMS performance are hardly better for the first sampling strategy. This is due to the fact that, in that case, the receiver only focuses on the samples which contains a part of the energy of the transmitted bit, which is not true for the second strategy. Similar performance comparisons have been found for all simulations. One can conclude that the second sampling method, which is much more practical, gives quasi-identical performance results than the first one. Fig. 4 gives a comparison of the detector performance

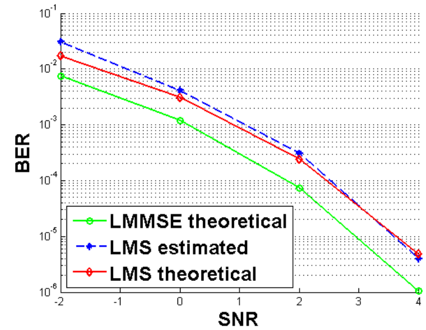

Fig. 2. BERs for user 1 with known channel delays.

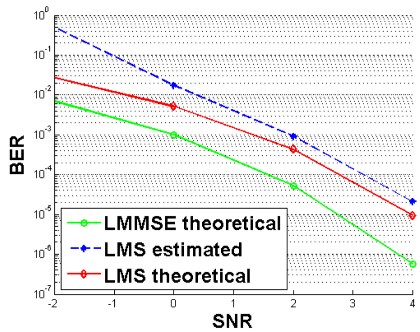

Fig. 3. BERs for user 1 with estimated minimum and maximum channel delays.

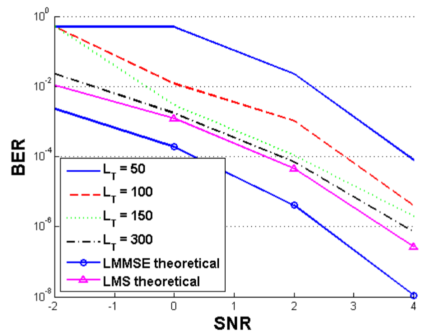

Fig. 4. BERs for different lengths L_{T} of the training sequence.

for different values of the length of the training sequence, in the case of estimated minimum and maximum channel delays, with the same system parameters as for fig. 3. Obviously, one can observe that the results improve when L_{T} increases, in the sense where, for too short training sequences, it may happens that the LMS algorithm does not converge (note however that this divergence did not occur at each instance of the detector). Thus, for low SNRs, larger values of L_{T} are required to ensure convergence. Note that this need for longer training sequences is due to the very small amount of a priori knowledge available at the receiver (i.e. only the estimated minimum and maximum channel delays). Now, once the convergence is obtained, the performance are similar to those obtained with higher values of L_{T}, since in that case all these results approach the theoretical LMS results.

7. CONCLUSION

This paper addressed the problem of multi-user detection for chaos-based transmission on multipath channels. The DCSK modulation system has been considered, for which two receivers have been studied. The first one was an LMMSE receiver, which is mainly theoretical since it requires many a priori information, which are not available for chaos-based systems. However, it served as a reference for the LMS-based detector, which is much better fitted for practical applications. Two different hypotheses concerning the channel delays have been considered. First, it was assumed that these delays were known to the receiver. In the second hypothesis, more realistic, only rough estimates of the minimum and maximum delays are available. Simulation examples have shown that the performance results are not really degraded with respect to the first hypothesis. Moreover, the LMS detector presented in this paper can also be used for time-varying channels. The analysis of the behavior of the receiver in such a case is under investigation.

8. APPENDIX

Define $\Delta^j_{k,k'} \triangleq \{i | \mathbf{\Theta}^j_k \cap \mathcal{I}^i_{k'} \neq \emptyset\}$, and $\Omega^{j,i}_{k,k'} \triangleq \{l = 0, \ldots, L_{k'} - 1 | \mathbf{\Theta}^j_k \cap \{\tau_{k',l} + iN_{\mathrm{S}} + 1; \tau_{k',l} + (i+1)N_{\mathrm{S}}\} \neq \emptyset\}$ In other words, $\Delta^j_{k,k'}$ denotes the set of indexes i of bits $b_{k',i}$ transmitted by the k'th user which interfere with $\mathbf{\Theta}^j_k$; $\Omega^{j,i}_{k,k'}$ is the set of paths l such that the lth component of the signal transmitted for bit $b_{k',i}$ interferes with $\mathbf{\Theta}^j_k$. Define then vector $\mathbf{s}^{j,i}_{k\cap k',l}$ as follows: $\mathbf{s}^{j,i}_{k\cap k',l}(n) = \mathbf{s}_{k'}(m)$ for m such that $\mathcal{I}^j_k(n) = \tau_{k',l} + iN_{\mathrm{S}} + m$, and $\mathbf{s}^{j,i}_{k\cap k',l}(n) = 0$ otherwise. Then, for $i \in \Delta^j_{k,k'}$, define

$$\boldsymbol{\xi}^{j,i}_{k,k'} \triangleq \sum_{l \in \Omega^{j,i}_{k,k'}} c_{k',l} \mathbf{s}^{j,i}_{k\cap k',l}.$$

It can then be shown that

$$\mathrm{ISI}(k) = \sum_{i \in \Delta^j_{k,k}} \boldsymbol{\xi}^{j,i}_{k,k} b_{k,i} \text{ and } \mathrm{MUI}(k') = \sum_{i \in \Delta^j_{k,k'}} \boldsymbol{\xi}^{j,i}_{k,k'} b_{k',i}$$

Note that, as long as the channel delays do not change, one has $\mathbf{\Theta}^j_k = \mathbf{\Theta}^0_k + \{j\}$ [2], which implies that $\Delta^j_{k,k'} = \Delta^{j'}_{k,k'} + \{j - j'\}$, and $\Omega^{j,i}_{k,k'} = \Omega^{j',i'}_{k,k'}$ for $i' - j' = i - j$. This means that the value of $\boldsymbol{\xi}^{j,i}_{k,k'}$ does not depend on the particular values of j and i, but rather on the difference $j - i$. Consequently, one can more simply denote: $\boldsymbol{\xi}_k \triangleq \boldsymbol{\xi}^{j,j}_{k,k}$ and $\boldsymbol{\xi}^{i-j}_{k,k'} \triangleq \boldsymbol{\xi}^{j,i}_{k,k'}$.

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[2]The addition between sets A and B is defined by: $A + B \triangleq \{a + b | a \in A, b \in B\}$.