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eXtended Finite Element Methods for thin plates

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ABSTRACT

The computation of fracture has been significantly improved thanks to the XFEM (introduced in [1]) and its numerous developments, up to the latest. However, many situations remain challenging. Among all of them, the computation of thin plates is difficult, mainly due to the locking phenomenon.

The aim of our work is to developp numerical methods in the XFEM framework that are optimal, for cracked plates subjected to bending loads. The material is assumed to be homogenous and isotropic, with a through crack. We will take care of the good accuracy of the method, especially for very thin plates. Here is the key problem : finding an enrichment strategy that can be mixed with the usual locking-free finite elements. Very few works have been devoted to this question, we mainly quote [2].

A first proposition is to use the Kirchhoff-Love theory. The corresponding finite element problem is not subjected to locking phenomenon. In addition, this theory provides a realistic description of the stresses and strains, especially for very thin plates (as this theory is the asymptotic limit of the 3-dimensional theory of elasticity), and the crack-tip singularities are simple and well-known (the bilaplacian operator ones [3]). However, this theory has the drawback that the finite element must have the $\mathcal{C}^1$ regularity. This is considered as computational cumbersome, but the reduced HCT/FVS element does the job (HCT for triangles, FVS for quadrilaterals), with only 3 degrees of freedom (dof) per node (for the bending). Using lowest-degree element with Mindlin-Reissner-Naghdi theory leads to the same cost.

Following [4], two enrichment strategies have been considered. In both of them a circular zone is defined around the crack tip. In the first one, named XFEM-standard, each node is enriched with additional degrees of freedom made with the crack tip singular functions. In the second one, the singularities are added globally: there is only one additional dof for each singular function, whose support is the entire enrichment zone. In this second case, an integral matching with the rest of the domain is prescribed.

The method has been tested on a benchmark problem, whose exact solution is known. A convergence study has been carried out, and it shows that the rate of convergence of the XFEM is roughly equal to what is expected with the classical finite element method on a regular problem (in energy norm, for the reduced HCT or FVS, the theoretical error is in $O(h)$, where h is the size of the elements edge, see [5]).

In our experiments, we observe that the deformation of the mesh affects the accuracy, and makes it "oscillatory", but the average level of error does not rise too much. We observe also that the displacement norm is roughly in $O(h^2)$. There is a superconvergence for the "XFEM-standard" on quadrangular meshes. Finally, we can see that the "Integral matching" is more accurate than the "XFEM standard".

In addition, the second strategy ("Integral matching") provides an estimation of the stress intensity factors (SIF) without any additional computation. The relative error on the SIF was less than 3%. This strategy has also significantly reduced the condition number.

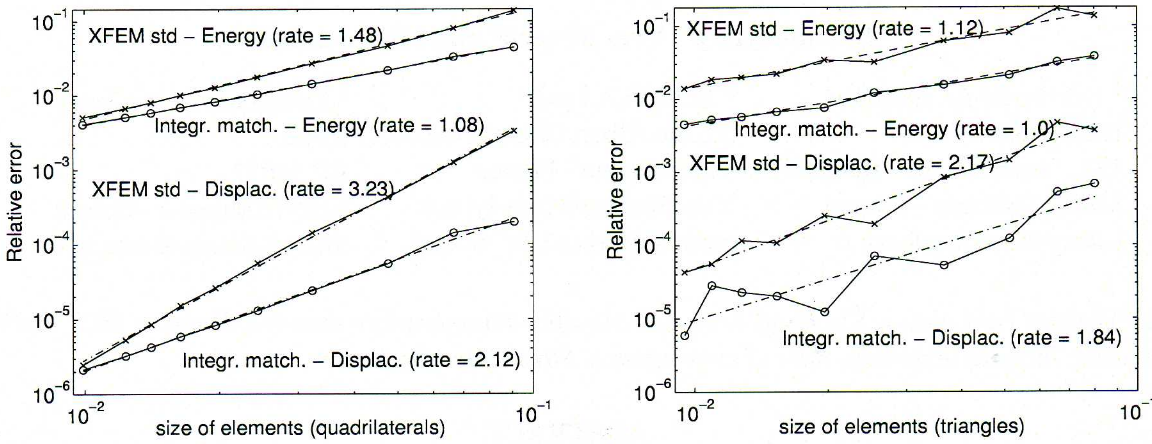

Figure 1: Convergence studies for displacement and energy norms, comparison between both strategies. Left, structured quadrilateral meshes. Right, non-structured triangular meshes.

The results show that our proposition of using the Kirchhoff-Love theory is satisfactory. However, it remains that the Mindlin-Reissner theory is more widely used. So a methodology which could be efficient in order to define some XFEM enrichments compatible with the Mindlin theory will also be discussed.

The numerical experiments were done with the finite element library Getfem++ [6], many thanks to J. Pommier.

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