# Multiobjective Multiproduct Batch Plant Design Under Uncertainty

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## Abstract

This paper addresses the problem of the optimal design of batch plants with imprecise demands and proposes an alternative treatment of the imprecision by using fuzzy concepts. For this purpose, we extended a multiobjective genetic algorithm developed in previous works, taking into account simultaneously maximization of the net present value (NPV) and two other performance criteria, i.e. the production delay/advance and a flexibility criterion. The former is computed by comparing the fuzzy computed production time to a given fuzzy production time horizon and the latter is based on the additional fuzzy demand that the plant is able to produce. The methodology provides a set of scenarios that are helpful to the decision's maker and constitutes a very promising framework for taken imprecision into account in new product development stage.

Keywords: multiobjective optimization, genetic algorithm, fuzzy arithmetic

# 1. Introduction

In recent years, there has been an increased interest in the design of batch processes due to the growth of specialty chemical, pharmaceutical, and related industries, because they are a preferred operating method for manufacturing small volumes of high-value products. The market demand for such products is usually changeable, and at the stage of conceptual design of a batch plant, it is almost impossible to obtain the precise information on the future product demand over the lifetime of the plant. However, decisions must be made on the plant capacity. This capacity should be able to balance the product demand satisfaction and extra plant capacity in order to reduce the loss on the excessive investment cost or that on market share due to the varying demands on products. The design of multiproduct batch plants has been an active area of research over the past decade (see (Shah, 1998) and (Pinto and Grossmann, 1998) for reviews). Most of the work has been yet limited to deterministic approaches, wherein the problem parameters are assumed to be known with certainty. However, in reality there can be uncertainty in a number of factors such as processing times, costs, demands, and not all the requirements placed by the technology of the process and the properties of the substances are defined. To cope with this, there has been increased interest in the development of different types of probabilistic models that explicitly take into account the various uncertainties (Sahinidis, 2003). For example, Wellons and Reklaitis proposed an MINLP model for the design of batch plants under uncertainty with staged

capacity expansions. Based on the structure of multiproduct batch plants, Straub and Grossmann (1992) developed an efficient procedure to evaluate the expected stochastic flexibility, embedded within an optimization framework for selecting the design (size and number of parallel equipment). Two-stage stochastic programming approaches have also been applied for design under uncertainty (Ierapetritou and Pistikopolous (1996); Cao and Yuan (2002).

It must be clearly said that the use of probabilistic models that describe the uncertain parameters in terms of probability distributions in an optimization framework is very greedy in computational time, either because of the large number of scenarios involved in the discrete representation of the uncertainty or the need to use complex integration techniques when uncertainty is modeled by continuous distributions. Besides, the use of probabilistic models is realistic only when a historic data set is available for uncertain parameters, which is rarely the case at the preliminary design stages in new product development.

In this work, fuzzy concepts and arithmetic constitute an alternative to describe the imprecise nature on product demands. Genetic algorithm optimization techniques were retained for both, MINLP and mutliobjective aspects of the optimization problem. For this purpose, we extended a multiobjective genetic algorithm, developed in previous works (Dietz, 2005a, b), taking into account simultaneously the maximization of the net present value (NPV) and two other performance criteria, i.e. the production delay/advance and a flexibility criterion. The paper is organized as follows. Section 2 is devoted to process description and problem formulation. Section 3 presents a brief overview of fuzzy set theory involved in the fuzzy framework within a multiobjective genetic algorithm. The presentation is then illustrated by some typical results. Finally, the conclusions on this work are drawn.

## 2. Process description and problem formulation

The case study is a multiproduct batch plant for the production of proteins taken from the literature (Montagna et al., 2000). This example is used as a test bench since shortcut models describing the unit operations involved in the process are available. The batch plant involves eight stages for producing four recombinant proteins, on one hand two therapeutic proteins, Human insulin (I) and Vaccine for Hepatitis B (V) and, on the other hand, a food grade protein, Chymosin (C) and a detergent enzyme, cryophilic protease (P).

In previous works (Dietz et al. 2005a, b), batch plant design was carried out minimizing the investment cost and the production system was represented using discrete event simulation techniques in order to take into account different production policies. Two strategies for campaign policies were tested, either monoproduct or multiproduct. In this work, only the monoproduct campaign policy was considered, so that the computation of cycle time can be easily implemented using the classical formulation proposed in (Montagna et al., 2000), involving size and time equations as well as constraints. A keypoint of the procedure is the computation of the so-called cycle time  $TL_i$  for each product, which corresponds to the limiting time, i.e., the time between two consecutive batches of the product. The objective is to determine the number and size of parallel equipment units/storage as well as some key process variables in order to satisfy one or several criteria (see (Dietz et al. (2005a)) for a complete description of the problem). Although the minimizing investment (I) is most often considered in the dedicated literature, it is not the most adequate objective for the optimal design problem. In real applications, designers preferentially not only consider to maximize the net present value (NPV), but also to satisfy a due date. The corresponding mathematical expressions of the objective functions (considered as fuzzy with a ~-symbol) are proposed as follows:

$$\begin{aligned} M\tilde{a}x \ (\tilde{NPV}) &= Max \ (\tilde{f}_1) = -I - f + \sum_{p=1}^n \frac{(V_p - D_p - A_p)(1 - a) + A_p}{(1 + i)^n} + \frac{f}{(1 + i)^n} \quad \text{with } I = \sum_{j=1}^M N_j \alpha_j V_j^\beta j \quad (1) \\ M\tilde{i}n \ (\text{Advance / Delay}) &= Min \ (\tilde{f}_2) = \left| \tilde{H} - \sum_{n=1}^p \tilde{H}_i \right| penalization \end{aligned}$$

The penalization term is equal to an arbitrary value of  $1/\omega$  for an advance and  $\omega$  for a delay in order to penalize more delays than advances. A sensitivity analysis leads to adopt a value of 4 for  $\omega$ . Finally, an additional criterion was computed in case of an advance (respectively a delay), representing the additional production that the batch plant is able to produce. Without going further in the detailed presentation of the computation procedure, it can be simply said that a flexibility index (called criterion f<sub>3</sub>) is computed by dividing the potential capacity of the plant by its actual value. *Max* (flexibility index)=*Max* (f<sub>3</sub>) (4)

#### **3.** Overview of fuzzy multiobjective genetic algorithm approach

#### 3.1. Representation of fuzzy demands and time horizon due-date

In this section, only the key concepts from the theory of fuzzy sets that will be used for batch plant design are presented (more detail can be found in (Kaufmann and Gupta, 1988). Different forms can be used for modeling the membership functions of fuzzy numbers. We have chosen to use normalized trapezoidal fuzzy numbers (TrFNs) for modeling product demand. Let us recall that the membership function values of a TrFN range from zero to one with the mode at one. The possibility distribution of TrFNs represented by a four-tuple [ $a_1$ ,  $a_2$ ,  $a_3$ ,  $a_4$ ] with  $a_1 \le a_2 \le a_3 \le a_4$  describes the more or less possible values for a demand. In other words, they can be interpreted as pessimistic or optimistic viewpoints of the designer. Figure 1 presents the typical values adopted in this work which correspond respectively to an imprecision of 10% with mode at one (respectively 15% with mode at zero). We also introduced in the model a fuzzy horizon time with a "rectangular" representation which may be viewed as latest and earliest dates to satisfy, with an imprecision of 10% (see Figure 2).



Figure 1 – Fuzzy representation of product demand (kg/year)

#### 3.2. Fuzzy extension of a multiobjective genetic algorithm

The multiobjective genetic algorithm presented elsewhere (Dietz et al., 2005b) was then extended to take into account the fuzzy nature of both demand and horizon time. Let us mention that the same encoding procedure was adopted since no fuzzy parameter is involved at that stage. The tunable parameters of the GA will also not be discussed here.

Although the GA basic principles will not be recalled, it must be said that arithmetic operations on fuzzy numbers that will be used concern exclusively the objective functions and the constraints.



Figure 2 – Fuzzy representation of horizon time (h)

They involve addition, subtraction, taking the maximum of two fuzzy numbers (mainly at the selection stage and at the Pareto sort procedure), through the extension principle of (Zadeh, 1975). Although there exists a large body of literature that deals with the comparison of fuzzy numbers, the approach proposed by (Liou and Wang, 1992) was finally adopted here. Looking more closely at the selection stage, three cases were considered, as qualitatively shown in Figure 3, corresponding to either unfeasible solutions leading to unacceptable violations of a time horizon constraint ( $f_3=0$ ), or to acceptable solutions sharing a time domain with an horizon constraint ( $f_3=1$ ), or, finally, to solutions for which the computation of the additional demand that the batch plant is able to satisfy is interesting from a flexibility viewpoint ( $f_3>1$ ).In case C, the computed value of the total time necessary to manufacture all the products is shifted to the right so that the highest (respectively lowest) value of the four-tuple of the TrFN corresponds to that of the due date for time horizon.



Figure 3 – Fuzzy evaluation procedure in the GA

## 4. Typical results

#### 4.1. Monocriterion case

GA typical results obtained with NPV as the only criterion to consider are presented in Table 1. Ten runs were performed to guarantee the stochastic nature of the GA. Table 1 presents the mean value of the NPV as well as the right core and support deviation from the mean value. Symmetrical values are obtained since symmetrical values were considered for both product demand and horizon due-date. The order of magnitude of the results is of interest at the design preliminary stages.

Run	Mean Value	Right core deviation	Right support	Run	Mean Value	Right core deviation	Right support
	[M€]		dev.		[M€]		dev.
AG01	4.64	12.5%	18.8%	AG06	4.60	12.6%	18,9%
AG02	4.65	12.5%	18.7%	AG07	4.77	12.9%	19,4%
AG03	4.59	12.6%	19.0%	AG08	4.61	12.6%	18,9%
AG04	4.61	12.6%	18.9%	AG09	4.65	12.5%	18,8%
AG05	4.29	13.5%	20.3%	AG10	4.63	12.5%	18,7%
	Table 1 – Monocriterion (NPV) batch plant design						

## 4.2. Tricriteria case

The three criteria considered here are the NPV, and two batch plant flexibility criteria production Advance/Delay respect the due date and production flexibility criterion defined as the maximal production capacity respect the actual demand. Previous studies showed their antagonist behaviour. An oversized batch plant gives more flexibility in terms of production but is penalising for the NPV criterion. A delay respect to the due data allows increasing the NPV criterion because the Investment can be reduced. Figure 4 displays the results when the three criteria are considered simultaneously after the final Pareto sort procedure over the solutions corresponding to each optimization run. Only, the average value of the involved criteria is reported here. Similar results can be obtained for the other couples of criteria. Although a thorough analysis was performed, only the guidelines that may be useful for the practitioner are given. For instance, this curve may be useful to detect unfeasible regions and to identify the promising regions from the viewpoints of NPV and flexibility index. In the illustrative example, we indicate some regions which may be interesting to explore since they involve high values for the net present value and exhibit a flexibility index greater than 1, corresponding to an acceptable advance in production (not reported here).



Figure 4 – Tricriteria results: NPV-Flexibility results projection.

# 5. Conclusions

In this paper, we have proposed a fuzzy approach to the treatment of imprecise demands in the batch design problem. Its benefits can be summarized as follows:

- Fuzzy concepts allow us to model imprecision in cases where historical data are not readily available, i.e. for demand representation;

- The models do not suffer from the combinatorial explosion of scenarios that discrete probabilistic uncertainty representation exhibit;

- Another significant advantage is that heuristic search algorithms, namely genetic algorithms for combinatorial optimization can be easily extended to the fuzzy case;

- Multiobjective concepts can also be taken into account.

Finally, this framework provides an interesting decision-making approach to design multiproduct batch plants under conflicting goals.

#### Nomenclature

<i>I</i> = Investment cost (M€)	<i>i</i> = Discount rate;	$N_i$ = Number of parallel units
f= Working capital (M€)	<i>n</i> = Number of periods	in stage j
<i>V<sub>P</sub></i> = Revenue (M€/y)	<i>P</i> = Number of products to be	$V_i$ = Required volume of a
$D_P$ = Operation cost	produced	unit in stage <i>j</i>
(M€/y)	M= Number of stages	$\alpha_i = \text{Cost coefficient for unit}$
$A_P$ = Depreciation (M $\in$ /y);	H= Due date (h)	j
a= Tax rate	$H_i$ = Production time for product i (h)	$\beta_i = \text{Cost exponent for unit } j$

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