



FIG. 1. (Color) Temporal development of the secondary (a) and the tertiary (b) Kelvin–Helmholtz instabilities. Successive closeups on the Kelvin–Helmholtz billows [(c)–(e)]. All pictures display the flow density field.

### Fractal Kelvin–Helmholtz breakups

Jérôme Fontane,<sup>1,a)</sup> Laurent Joly,<sup>1</sup>  
and Jean N. Reinaud<sup>2</sup>

<sup>1</sup>Université de Toulouse, ISAE, F-31055 Toulouse, France

<sup>2</sup>School of Mathematics and Statistics, University of St Andrews,  
St Andrews KY16 9SS, United Kingdom

(Received 21 July 2008; published online 25 September 2008)

[DOI: 10.1063/1.2976423]

The Kelvin–Helmholtz billow developing in an infinite-Schmidt number mixing layer at  $Re=1500$  between two density-contrasted fluids (i.e.,  $\rho_{\text{black}}/\rho_{\text{white}}=3$ ) experiences a two-dimensional shear instability. Secondary Kelvin–Helmholtz billows are seen to emerge on the light side of the primary structure, and then are advected towards the core of the main billow as the wave overturns [Fig. 1(a)]. Due to the inertial baroclinic vorticity production, the braid region turns into a sharp vorticity ridge holding high shear levels and is thus sensitized to the Kelvin–Helmholtz instability.<sup>1</sup> We carry out numerical simulations of the temporal development

of the secondary mode when the flow is seeded at  $t=18$  with the perturbation obtained from a linear stability analysis of the primary billow.<sup>2</sup>

If seeded earlier at  $t=13$ , the secondary instability develops on a longer wavelength. The larger central secondary billow breaks up in ternary rollups due to the same mechanism as the previous generation [Fig. 1(b)]. The self-similarity of the density pattern down the scales [Figs. 1(c)–1(e)] prefigures a two-dimensional route to turbulence through a fractal process.

The thinning of the density-gradient layer due to the successive folding of the Kelvin–Helmholtz billows is not compensated by mass diffusion. The corollary of the isovolume stretching is the development of the density field on increasingly smaller scales. This numerical challenge is solved by an adaptive mesh refinement leading to a considerable increase of the spatial resolution up to  $10000 \times 10000$ .

<sup>1</sup>J. N. Reinaud, L. Joly, and P. Chassaing, “The baroclinic secondary instability of the two-dimensional shear layer,” *Phys. Fluids* **12**, 2489 (2000).

<sup>2</sup>J. Fontane and L. Joly, “Stability of the variable-density Kelvin–Helmholtz billow,” *J. Fluid Mech.* **612**, 237 (2008).

<sup>a)</sup>Present address: School of Mathematics and Statistics, University of St Andrews.