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Controller robustification using equivalent observer based structure

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Abstract. This paper deals with equivalent observer based structure and controller robustification. The purpose of the method is to find an observer based controller equivalent to an initial controller (designed using H_∞ or μ technics for example). Matrices of the equivalent "observer plus feedback" are synthesized to guarantee the same closed loop eigenstructure and input/output behaviors than the initial controller. Contrary to other methods there is no restrictions on the initial controller order. The relations coming from this equivalence permits to point out the fundamental matrices of observer and static feedback which will be used to initialize parametric robustification design procedure [12]. Due to the observer structure, this one permits to improve the parametric robustness of the initial controller without paying attention on the controller closed loop dynamic. Finally the global method (equivalent observer plus robustification) is applied on the robust control of the space shuttle described in μ-analysis and synthesis toolbox [1].

Keywords. Observer based structure, parametric robustness improvement, multi-model approach, modal control.

I. Introduction

Observer based structure presents many advantages in control theory. The most significant one is that controller states are meaningful as they are similar to the plant states estimates. Moreover observer dynamic are uncontrollable and unobservable. The separation principle states that closed loop spectrum is made up by assigned eigenvalues and observer ones. Consequently observer based formulation can avoid problems due to closed loop eigenstructure part introduced when dynamic controller is used. His synthesis reduces to static output feedback design on the augmented plant (physical plant + observer).

The other advantage of observer based controller synthesis lies in the fact that controller can be directly expressed from the system representation. In the case of uncertain system control, the observer dynamic will be directly scheduled by mesurable uncertain parameters. This property is very significant in the context of robust control (see ref. [2], [3], [4] for discussion about theoretical complexity and practical interests of scheduling implantation) .

Many works have been done on observer based structure for arbitrary observers. The main contributions are those of Schumacher [5], Bender and Fowell [6]. Shumacher has shown that in theory the observer based equivalent structure of an arbitrary controller under condition of rank always exists. Bender and Fowell followed this idea and found a method based on linear state space transformation in the case of full order compensator. In this case the computation matrix are obtained from the resolution of non-symmetric algebraic equation. This resolution, initially introduced by Kokotovic in the context of singularly perturbed systems has been shown to have unique solution in some special situation. Recently Alazard and Apkarian [7] have generalized these results to augmented order compensator and, under some conditions, to reduced order controller. This generalization has been done using LQG formulation and Q-parametrization (Youla Parametrization). It is shown that the Q-parametrization of the controllers and Luenberger observers formulation can be exploited to derive equivalent observer based state space representation with an explicitly separated structure.

In this paper we propose a method based on modal formulation of the observer structure (generalization of the Luenberger approach [8], [9]). This formulation shows that observer based controllers are characterized by three matrices U_Π, T_Π, Π for the observer and two static matrices K_y, K_z for the feedback. In the sequel, we'll shown that any controller can be put under observer based structure using an appropriate choice of the matrices $U_\Pi, T_\Pi, \Pi, K_y, K_z$. Comparing to the aforementioned methods the one proposed here :

- is entirely algebraic,
- can be applied on any initial controller (no controller order restriction),
- leads to an observer based formulation that can directly be used for global robustification design procedure developed by [10], [11], [12].

Considering this last point it will be possible to increase the parametric robustness of any frequency oriented controller (synthesized using H_∞ method) by initializing the aforementioned multimodel design procedure [13] with the observer based formulation. During the second phase it is even more possible to take into account explicit temporal specification using additional modal constraints.

This paper is organized as follows. We'll first describe the observer modal structure and we'll point out his characterizing matrices. Afterward the main result on the synthesis of equivalent observer based controller is given. We'll briefly present the modified robust design procedure of [13] aimed at taking into account initialization from initial controller. In a fourth part method will be applied on the space shuttle control extracted from the Mu-analysis and Synthesis Toolbox [14] .

II. Notation

The considered system, with n states, m inputs, p outputs, will be denoted:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \tag{1}$$

where x is the vector of states, y the vector of measurements, and u the vector of inputs, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and $D \in \mathbb{R}^{p\times m}$. Rank$(B) = m$, Rank$(C) = p$. The associated transfer matrix is $G(s) = C(sI - A)^{-1}B + D$.

This system (1) is controlled by a dynamic feedback $K(s) = C_c(sI - A_c)^{-1}B_c + D_c$ ($A_c \in \mathbb{R}^{n_c\times n_c}$, $B_c \in \mathbb{R}^{n_c\times p}$, $C_c \in \mathbb{R}^{m\times n_c}$ and $D_c \in \mathbb{R}^{m\times p}$). The n-dimensional state space system will be denoted $\mathcal{X}$ and the n_c-dimensional feedback dynamic extension will be denoted $\mathcal{X}_c$ (this vector space is viewed over $\mathbb{R}$ for state vectors and over $\mathbb{C}$ for eigenvectors). The state space corresponding to the connection of the system and of the dynamic feedback with $y = u_c$ and $u = y_c$ is $\mathcal{X} \oplus \mathcal{X}_c$. In the sequel we will use the notation : $\Phi_1 = (I_m - D_cD)^{-1}, \Phi_2 = (I_p - DD_c)^{-1}$ Let note these two matrices satisfy to the following properties :

$$\begin{array}{ll} D_c\Phi_1 = \Phi_1 D & \Phi_2 D = D\Phi_1 \\ D\Phi_2 = \Phi_2 D_c & D_c\Phi_2 = \Phi_1 D_c \end{array}$$

Considering these notation the closed loop state matrix is given by :

$$A_{cl} = \begin{bmatrix} A + B\Phi_1 D_c C & B\Phi_1 C_c \\ B_c\Phi_2 C & A_c + B_c\Phi_2 DC_c \end{bmatrix} \tag{2}$$

and the corresponding closed loop eigenstructure [1], *i.e.g. eigenvalues, left eigenvectors and right eigenvectors* will be noted :

$$\begin{array}{l} \Lambda_{cl} = \begin{bmatrix} \Lambda & 0 \\ 0 & \Lambda_c \end{bmatrix}, U_{cl} = \begin{bmatrix} U & U_c \\ U_{\Lambda_c} & U_{\Lambda_{c,c}} \end{bmatrix} \\ \text{and } V_{cl} = \begin{bmatrix} V & V_{\Lambda_c} \\ V_c & V_{\Lambda_{c,c}} \end{bmatrix} \end{array} \tag{3}$$

where Λ is the part of the closed loop spectrum assigned by the dynamic feedback and Λ_c is the part which correspond to the closed loop controller dynamic. These matrices satisfy $U_{cl}A_{cl} = \Lambda_{cl}U_{cl}$, $A_{cl}V_{cl} = V_{cl}\Lambda_{cl}$ and $U_{cl}V_{cl} = I$.

III. Observer eigenstructure

A modal approach to observer design is proposed in [8] and [9], see also [15]. This modal approach is based on the Lemma III.1.

Lemma III.1: The system defined by (see Figure 1):

$$\dot{\hat{z}}_i = \pi_i\hat{z}_i - t_{\pi_i}y + u_{\pi_i}Bu + t_{\pi_i}Du \tag{4}$$

where $u_{\pi_i} \in \mathbb{C}^n$, $t_{\pi_i} \in \mathbb{C}^p$ and $\pi_i \in \mathbb{C}$ satisfy :

$$u_{\pi_i}A + t_{\pi_i}C = \pi_i u_{\pi_i} \tag{5}$$

is an observer of the variable $z_i = u_{\pi_i}x$ and the observation error $\epsilon_i = \hat{z}_i - u_{\pi_i}x$ satisfies:

$$\dot{\epsilon}_i = \pi_i\epsilon_i$$

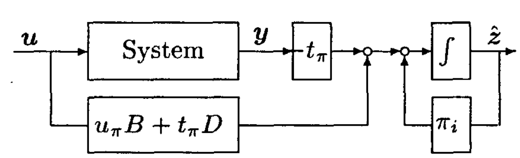

Fig. 1. Elementary observer of $z = u_\pi x$.

[1]In the following eigenvalues are considered has being distinct.

Figure 1 can be modified in such a way that the transfer matrix $G(s)$ of the system appears explicitly ([?]). When the dependency on varying parameters (say Δ) of the system will be considered the parameterized form of the transfer matrix $G(s)$ (say $G_\Delta(s)$) being used observer scheduling will be straightforward.

IV. Observer based feedback

When a bank of elementary observers is considered, the "measurements" at our disposal are y and $\hat{z}$. Designing a controller for this system consists of

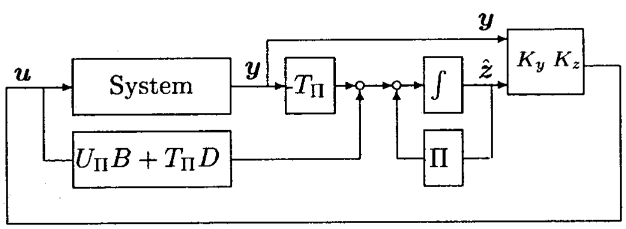

Fig. 2. Observer based closed-loop.

finding K_y and K_z such that the system

$$\left\{ \begin{array}{l} \dot{x} = Ax + Bu \\ \dot{\hat{z}} = \Pi\hat{z} + (U_\Pi B + T_\Pi D)u - T_\Pi y \\ y = Cx + Du \end{array} \right. \tag{6}$$

controlled by $u = K_y y + K_z\hat{z}$ have the expected dynamics. This problem is usually divided into two sub-problems by considering the separation principle.

Theorem IV.1: Separation Principle. Let us consider three matrices $U_\Pi \in \mathbb{R}^{n_c \times n}$, $T_\Pi \in \mathbb{R}^{n_c \times p}$ and $\Pi \in \mathbb{R}^{n_c \times n_c}$ designed such that:

$$U_\Pi A + T_\Pi C = \Pi U_\Pi$$

It is equivalent to assign (by the static feedback $[K_y \quad K_z]$) the eigenstructure of the system (6) and the one of the system (7).

$$\begin{array}{rl} \dot{x} & = Ax + Bu \\ \begin{bmatrix} y \\ z \end{bmatrix} & = \begin{bmatrix} C \\ U_\Pi \end{bmatrix} x + \begin{bmatrix} D \\ 0 \end{bmatrix} u \end{array} \tag{7}$$

The resulting closed loop system eigenstructure is made up by:

- The assigned closed loop poles plus the open loop controller dynamic, says Π.
- The closed loop eigenvectors defined by the assigned eigenvectors of (7) for the upper part, and some u_i^* for the lower part.

Proof. See[13]

Equivalent dynamic compensator. Observer-based controllers are dynamic compensators.

Lemma IV.2: The dynamic compensator equivalent to the observer-based controller of Figure 2 can be characterized by a feedback part :

$$\left\{ \begin{array}{l} A_{fbck} = \Pi + (U_\Pi B + T_\Pi D)K_z \\ B_{fbck} = -T_\Pi + (U_\Pi B + T_\Pi D)K_y \\ C_{fbck} = K_z \\ D_{fbck} = K_y \end{array} \right. \tag{8}$$

and a feedforward part :

$$\left\{ \begin{array}{l} A_{fwd} = \Pi + (U_\Pi B + T_\Pi D)K_z \\ B_{fwd} = U_\Pi B + T_\Pi D \\ C_{fwd} = K_z \\ D_{fwd} = \mathrm{I} \end{array} \right. \tag{9}$$

Proof.

V. Equivalent observer based controller

The purpose of this section is to find an observer based controller $(U_\Pi, T_\Pi, \Pi, K_z, K_y)$ and feedforward $(A_{fwd}, B_{fwd}, C_{fwd}, D_{fwd})$ equivalent to a given feedback A_c, B_c, C_c, D_c.

Lemma V.1: Let us consider the current controller A_c, B_c, C_c, D_c and the associated notation of section (II). The system made up by the observer and the feedforward :

$$\left\{ \begin{array}{l} U_\Pi = -U_{\Lambda_c} \\ T_\Pi = -T_{\Lambda_c} \\ K_z = C_c U_{\Lambda_{c,c}}^{-1} \\ K_y = D_c \end{array} \right. \quad \left\{ \begin{array}{l} A_{fwd} = \Pi \\ B_{fwd} = -U_{\Lambda_c}B - T_{\Lambda_c}D \\ C_{fwd} = -C_c U_{\Lambda_{c,c}}^{-1} \\ D_{fwd} = \mathrm{I} \end{array} \right. \tag{10}$$

is equivalent to the controller A_c, B_c, C_c, D_c.

Proof. The system controlled by the observer (6) has the following closed loop matrix:

$$A_{obs,cl} = \begin{bmatrix} A + B\Phi_1 K_y C & B\Phi_1 K_z \\ -T_\Pi C + U_\Pi B K_y \Phi_2 C & \Pi + U_\Pi B\Phi_1 K_z \end{bmatrix} \tag{11}$$

Let us consider the traditional change of variable $\epsilon = z - Ux$. It is equivalent to pre and post multiply the state matrix by $\begin{bmatrix} \mathrm{I} & 0 \\ -U & \mathrm{I} \end{bmatrix}$ and $\begin{bmatrix} \mathrm{I} & 0 \\ U & \mathrm{I} \end{bmatrix}$. The resulting matrix is given by

$$\begin{bmatrix} A + B\Phi_1 K_y C + B\Phi_1 K_z U & B\Phi_1 K_z \\ -T_\Pi C + U_\Pi A - \Pi U_\Pi & \Pi \end{bmatrix}.$$

It is obvious that the searched controller have : an observer structure (observer dynamic is given by Π) if $-T_\Pi C + U_\Pi A - \Pi U_\Pi = 0$, *i.e.g* $T_\Pi = T_{\Lambda_c}, U_\Pi = U_{\Lambda_c}$, closed loop poles and eigenvectors similar to the ones assigned by the initial feedback if $A + B\Phi_1 K_y C + B\Phi_1 K_z U = A + B\Phi_1 K_y C + B\Phi_1 K_z U$, *i.e.g* $K_z = D_c, K_z = C_c U_{\Delta_c}^{-1}$. These equation permits to obtain the feedback part of system (10). From Lemma IV.2 the aforementioned observer is a dynamic compensator (feedforward and feedback parts). Consequently if we want to have the input/output equivalence between the initial controller and the observer based one it is necessary to compute a feedforward aimed at cancelling the observer feedforward contribution. The inverse of this feedforward is defined by:

$$\left\{\begin{array}{l} A_{fwd} = \Pi \\ B_{fwd} = U_\Pi B + T_\Pi D \\ C_{fwd} = -K_z \\ D_{fwd} = \mathrm{I} \end{array}\right. \qquad (12)$$

Comment V.1: Choice of the observer dynamic. The observer dynamic can be chosen using the root locus of the system closed by the initial feedback using the same tuned gain on each output. The selected eigenvalues are those corresponding to the controller open loop ones. A second possible choice, consist in finding the set of n_c eigenvectors which maximize the condition number of the matrix $U_{\Lambda_{c,c}}$.

VI. Design procedure for controller robustification

The main advantage of the method of §V is that any controller can be put under observer based form and used in the global design procedure of (μ-Mu)-iteration [10]. It permits the designer to automatically fix the equivalent closed loop controller dynamic. In fact the initial controller is put under observer formulation and iterations concern the augmented equivalent output feedback. The resulting design procedure from [12] is as follows.

Procedure VI.1: Controller robustification using observer based formulation.

Step A Synthesis of the equivalent observer based controller and extraction of the dominant closed loop eigenstructure.

Step A.1 Synthesis of the equivalent observer based controller. Let (A_c, B_c, C_c, D_c) denotes the initial controller synthesized on the nominal model (A, B, C, D) by any kind of method (H_∞ synthesis, μ synthesis etc ...). At this step we look for an observer based controller $U_\Pi, T_\Pi, \Pi, K_z, K_y$ equivalent to the initial controller using the technic described in Lemma V.1.

Step A.2 Extraction of the dominant eigenstructure. This is based on modal simulation of the closed loop system and permits to point out the significant eigenstructure which has to be preserved on the nominal model during **Step B**.

The Step B will permit to refine the matrices K_z, K_y in vue of increasing the parametric robustness.

Step B Robustification of the observer based controller.[13]

The current equivalent based controller satisfy:

$$\left[\begin{array}{cc} K_{z,k} & K_{y,k} \end{array}\right] \left[\begin{array}{ccc} \gamma_1 & \cdots & \gamma_k \\ v_1 & \cdots & v_k \end{array}\right] = \left[\begin{array}{ccc} w_1 & \cdots & w_k \end{array}\right] \qquad (13)$$

where $\gamma_k = (\lambda_k - \Pi)^{-1}((U_\Pi B + T_\Pi D)w_k - T_\Pi C v_k)$ and U_Π, T_Π, Π defined at **Step A**.

Step B.1 Analysis. Proceed to a μ analysis of the system closed by the current observer based controller. If robustness is achieved stop the algorithm. Else let matrices $(A(\Delta_i), B(\Delta_i), C(\Delta_i), D(\Delta_i))$ denote the worst case model and λ_i the critical mode.

Step B.2 Multimodel synthesis. Add to the system of equations (13) the equation:

$$\left[\begin{array}{cc} K_{z,k+1} & K_{c,k+1} \end{array}\right] \left[\begin{array}{c} \gamma_{k+1} \\ v_{k+1} \end{array}\right] = \left[\begin{array}{c} w_{k+1} \end{array}\right] \qquad (14)$$

where $\gamma_{k+1} = (\lambda_{k+1} - \Pi)^{-1}((U_\Pi B + T_\Pi D)w_{k+1} - T_\Pi C v_{k+1})$, U_Π, T_Π, Π defined at **Step A** and $(\lambda_{k+1}, v_{k+1}, w_{k+1})$ the assigned triple. Go to **Step B.1**. Compute the gain $[K_{z,k+1} \;\; K_{y,k+1}]$ minimizing the difference between this gain and a reference gain (initial observer based controller for example).

VII. Application to the Space Shuttle Robust controller

A. Problem Description

The system. This example is extracted from the Mu-analysis and synthesis toolbox [1]. The considered system is a simplified model of the Space Shuttle, in the final stages of landing as it transitions from supersonic to subsonic speeds (see [16] for details on this application). The four states of the rigid body of the aircraft at Mach 0.9 are the slideslip angle (β), the roll rate (p), the yaw rate (r) and the bank angle (ϕ). The three inputs are the angular deflection of the elevon surface (θ_{ele}), the deflection of the rudder surface (θ_{rud}) and the lateral wind gust disturbance input (d_{gust}). Output is a part of the state (p, r and ϕ) plus the lateral acceleration at the pilot location (n_y). This rigid body model is completed by actuators model, weighting functions used to take into account the desired frequency performances in term of exogenous disturbances, noises and commands. Weighting functions are also added to take into account the desired performances on the outputs. The aircraft has two controlled inputs, ruder and elevon commands. Each actuator is modeled with a second order transfer function as well as a second order delay approximation to model the effects of the digital implementation. The final LFT formulation of the aircraft is built including the aero-coefficient uncertainties. Finally, the global model used for synthesis has 23 states, 2 command input, 5 regulated output. This model also has 9 inputs and 9 outputs corresponding the the real uncertainties, 6 inputs for the exogenous disturbances, 3 output for the error on performances and 6 output for the error on actuators.

The initial controller. In [1] three controllers has been synthesized. The first one kh (order 23) is designed to optimize H_∞ performance under the assumption that there is no model uncertainty. The second one kmu (order 16) is designed with the $D-K$ iteration approach to μ-synthesis. The third one kx (order 3) is a tradeoff between the two controllers.

B. Design procedure and results

In this section we shall apply the design procedure VI.1.

Step A Equivalent observer based controller and dominant eigenstructure.

Step A.1 The considered controller is the controller noted kx page 7-57 of [1]. It is a tradeoff between H_∞ and μ controller. The controller order is three. The equivalent observer is synthesized considering the choice of the observer dynamic minimizing the condition number of the matrix $U_{\Lambda_{c,c}}$ as explained in Comment V.1. Matrices U_Π, T_Π, K_z, K_y are obtained using formula (8). The dynamic of the computed observer are $-15.3277 \pm 12.8793i$ and -1.3985. Closed loop eigenvalues and frequency responses obtained from both initial and observer based controller similar. The closed loop poles and frequency input/output response show that both controller are strictly equivalent. In the following the nominal configuration will be noted *model0*.

Step A.2 The dominant eigenstructure of the nominal closed loop system is extracted using modal simulation (see [17] for details). Three triples (and conjugated ones) associated to eigenvalues : -0.23,$-0.61 \pm 0.44i$, and $-0.71 \pm 1.38i$ are pointed out. These triple has to be assigned on the nominal system during the **Step B** to preserve the good performances on the nominal configuration (they can be moved on the left to increase the performances of the nominal system).

Step B Controller robustification.

Step B.1 Real μ analysis (lower bound proposed by in [18] is performed on the system closed by the equivalent controller. Two picks are pointed out : the first one at frequency 0 correspond to the μ value 1.3, the second one at frequency 0.35 correspond to the μ value 0.88. The two corresponding worst case models are named *model1* and *model2*. This technics also permits to exhibit the critical eigenvalues of these models at 0.0 on *model1* and $0.0 + 0.3i$ on *model2* (not treated here).

Step B.2 At this step we shall attempt to control the destabilized eigenvalue of *model1* minimizing the effects on the nominal *model0*. For this purpose, system is closed by the initial controller and new (K_z, K_y) is synthesized considering multimodel objectives : stabilization of model 1 and preservation of the performances of model 0. This leads to the following eigenstructure assignment:

$$Model0 \left\{ \begin{array}{l} -0.72 \pm 1.38i \\ -0.61 \pm 0.44i \\ -0.23 \end{array} \right. \rightarrow \left\{ \begin{array}{l} -0.72 \pm 1.38i \\ -0.61 \pm 0.44i \\ -0.33 \end{array} \right.$$

$$Model1 \quad 0.0 \quad \rightarrow \quad -0.11$$

Note that the eigenvalue -0.23 is moved on the left to respect the coherence between the both models. Eigenvectors are chosen using orthogonal projection (see [12]).

Step B.1 (bis) The resulting pole are correctly assigned and well damped on *model0* and *model1* without degradation on *model2*. The μ analysis is performed on the system closed by the controller computed at **Step B**. We can see on Figure 3 that the pick of μ at the frequency 0 has been reduced from 1.3 to 0.95. The pick at frequency 0.3 doesn't vary. Figure 4 shows the μ plot corresponding to the nominal performance (exogenous signals and error attenuation, see [1] for more details) obtained from the three controllers. It shows that results are similar or better with the observer based controller. Note that μ plots corresponding to robust performances of our controller and the kx controller are quite similar.

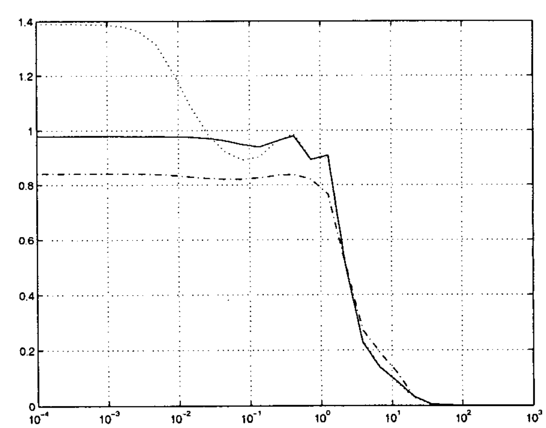

Fig. 3. Real μ curves obtained from the three controllers k_x (dot) , k_{mu} (dash dot) and the observer based one

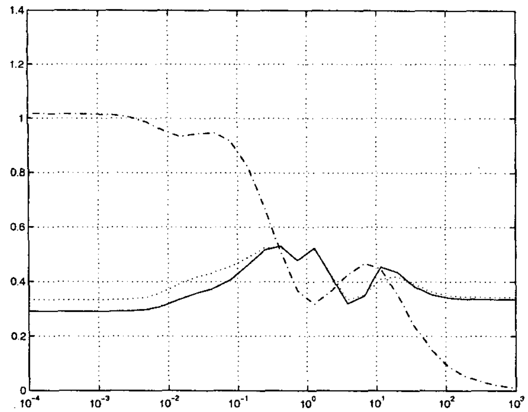

Fig. 4. μ perf curve obtained from the three controllers k_x (dot) , k_{mu} (dash dot) and the observer based one

Furthermore we can note that open loop time responses are quite similar (performances are better with our controller than with the μ one). We can conclude that our results are satisfactory until order of observer is 3 (the μ one is 16).

A second step may be used to decrease the second pick of μ but in practice results can't be improved without significant deterioration on the robust performances.

VIII. Conclusion

In this paper we have propose a new design approach based on the observer based formulation. The main contribution leads in the proposed method used to find an equivalent observer based controller. This method consists in finding five matrices $U_\Pi, T_\Pi, \Pi, K_z, K_y$ insuring the initial closed loop eigenstructure. It does not depends on the system or controller order. The choice of the observer dynamic is based on the optimization of matrices condition number or multivariable root locus. This starting point permits to use this equivalent observer in a more global iterative design procedure aimed at controller robustification. Feedback matrices K_z, K_y are updated considering worst case analysis and multimodel modal control to increase the parametric robustness.

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