

Robust control of longitudinal flight with handling qualities constraints

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Abstract—Classical flight control systems are still widely used in industry because of acquired experience and good understanding of their structure. Nevertheless, with more stringent constraints, it becomes difficult to easily fulfill all the criteria with this classical control laws. This article aims at showing that this problem can be solved by first designing a high order controller satisfying all the constraints, then by reducing and structuring it in order to make it look like a classical controller. Firstly, an H_∞ synthesis is performed in order to get a robust controller versus mass and center of gravity variations, which will satisfy the handling qualities; then it will be reduced by using robust modal control techniques

Index Terms—Handling qualities, Robustness, Modal reduction

I. INTRODUCTION

The development, integration and flight testing of flight control systems are costly and time-consuming. Modern techniques such as H_∞ or μ - *synthesis* provide effective and robust controller design techniques but the main problem remains their high order which prevents them from being easily implemented ([7], [16]). Classical flight control systems are still widely used because of their well-studied and understood architecture [6]. However, they have to deal with stringent performance and robustness requirements over the full flight envelope. It is therefore of interest to keep the simplicity of classical architectures while using modern technique advantages for analysis.

This article expands upon the work initiated in [14] and [13] wherein the authors propose a method to choose the 5 controller gain parameters of two fixed architecture classical control laws. Nevertheless, in [15], for a given flight condition, it is necessary to schedule the controller with the mass and center of gravity measurements to fulfill parametric robust constraints. A unique controller should be sought that would insure, for a chosen flight condition, performance in spite of varying mass and center of gravity location. Moreover, this controller should have a simple structure (low order and physical meaning of the filters)

Firstly, an H_∞ synthesis will be performed in order to get a robust controller which will satisfy the handling qualities in spite of mass and center of gravity variations. Then by using robust modal control techniques, this controller will be reduced to an order as low as possible without losing on performance.

Finally, the controller will be put in a classical form while adding a feedforward gain to improve some particular criteria.

II. MODEL AND CONSTRAINTS

A. Challenger 604 aircraft model

We consider the longitudinal flight model of the *Challenger 604* of *Bombardier Inc.* The open loop order is 35 for a given flight condition: full dynamics of each components are considered. Figure 1 shows a classical pitch attitude hold system. In this structure, the two filters are first order and there are 5 varying gains that must be tuned in order to satisfy the criteria.

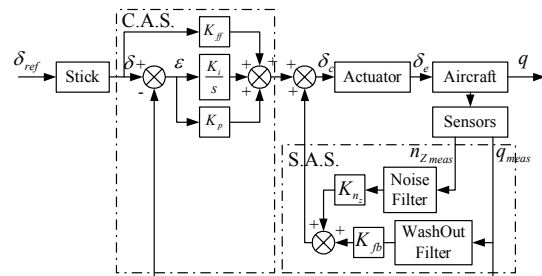


Fig. 1. Classical pitch attitude hold system

B. Handling qualities requirements

The overall performance objective is to track pitch rate commands with predicted Level 1 handling qualities and desired time domain response behavior. The handling quality criteria considered in this article are short period mode damping ratio ζ_{sp} , Gibson's dropback Drb , settling time ST , pitch attitude bandwidth ω_{BW_θ} , phase delay τ_p , gain margin M_G and phase margin M_φ [15]. The boundaries of these criteria are defined by military standards [17]. Table I summarizes the handling quality boundaries being considered in the design procedure.

C. Mass and center of gravity variations

The robustness specifications are based on the variation of total mass (m) and center of gravity along the body X -axis (x_{cg}). For the flight condition at stake, eight models of different masses and centers of gravity are provided by *Bombardier Inc.* and considered in Table II. A unique controller that will fulfill all the requirements is sought.

TABLE I
HANDLING QUALITIES

Handling qualities	Level 1
Short period damping ζ_{sp}	$0.35 < \zeta_{sp}$
θ -bandwidth ω_{BW_θ}	$\omega_{BW_\theta} > 1.5$ (rad/s)
Gibson's dropback Drb	$-0.2 \leq Drb \leq 0.5$
Phase delay τ_p	$\tau_p < 0.2$ (s)
Gain margin M_G	$M_G > 6$ (dB)
Phase margin M_φ	$M_\varphi > 45$ ($^\circ$)
Settling time ST	2% en 3 (s)

TABLE II
GRID POINTS FOR VARYING m AND x_{cg}

m (lbs) \ x_{cg} (% chord)	16	20	35	38	48
30 000		×	×		
32 000					×
39 000		×		×	×
46 000	×			×	

III. H_∞ SYNTHESIS

We seek a controller that satisfies the performance criteria for the 8 configurations. H_∞ synthesis will be used.

A. Theory

H_∞ synthesis was initiated by Zames [18] and further developed by Doyle [4]. H_∞ problem is a stabilization and disturbance rejection problem. A controller is sought that will minimize disturbance effects while stabilizing the system. Theoretical aspect can be found in [19] and [1].

Let us consider the augmented system $P(s)$ (including weight functions or filters) composed by 4 multivariable transfer functions between the inputs u and w and the outputs y and z where:

- u represents the system command
- w represents exogenous inputs (reference and/or disturbance)
- y represents measurements
- z represents regulated outputs

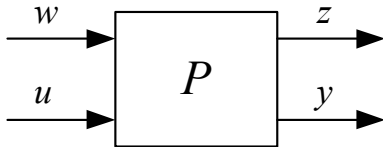


Fig. 2. Augmented model

$P(s)$ can be separated in the following way:

$$\begin{bmatrix} Z(s) \\ Y(s) \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \cdot \begin{bmatrix} W(s) \\ U(s) \end{bmatrix} \quad (1)$$

By closing the loop with the control law $U(s) = K(s)Y(s)$, one can obtain the transfer between the inputs w and the outputs z namely Linear Fractional Transformation (LFT):

$$\begin{aligned} G_{zw}(s) &= \mathcal{F}_l(P(s), K(s)) \\ &= P_{11} + P_{12}K(s)(I - P_{22}K(s))^{-1}P_{21} \end{aligned} \quad (2)$$

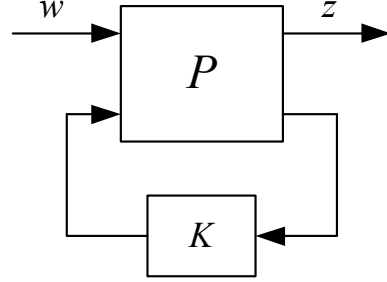


Fig. 3. Standard H_∞ synthesis

The optimal H_∞ problem is the synthesis of a controller $K(s)$ among all internally stabilizing controllers that minimizes the H_∞ norm of $G_{zw}(s) = \mathcal{F}_l(P(s), K(s))$.

H_∞ norm of a transfer function $G(s)$ is defined as:

$$\|G(s)\|_\infty = \sup_{\omega \in \mathbb{R}} \bar{\sigma}(G(j\omega)) \quad (3)$$

Optimal H_∞ problem:

Finding a stabilizing controller $K(s)$ such as $\|\mathcal{F}_l(P(s), K(s))\|_\infty$ is minimal

Knowing the minimal H_∞ norm can be theoretically useful because a limit can be fixed on the reachable performances. Nevertheless, in a practical way, the suboptimal H_∞ problem is defined where the H_∞ is reduced under a positive threshold γ .

Suboptimal H_∞ problem:

Finding a stabilizing controller $K(s)$ such as $\|\mathcal{F}_l(P(s), K(s))\|_\infty \leq \gamma$

Although there are several ways to solve this problem, Doyle *et al.* [5] method will be used as it is based upon a state variable approach. One major problem of H_∞ synthesis is that the controller $K(s)$ is the same order as the augmented plant. The controller has to be reduced in order to simplify it.

The following references are examples of synthesis which gave good results. [7],[2],[8],[9]. Note that in [3], the authors design H_∞ controllers with fixed order *a priori*.

B. Application

An H_∞ synthesis will be performed on our aircraft model. As the controller order is equal to the augmented plant order, the open loop is first reduced to an 11th order. The augmented plant is the one on Figure 4. In order to improve the H_∞

synthesis, a reference model that satisfies most constraints is chosen¹

$$W_{ref}(s) = \frac{e^{-0.1s}}{s^2 + 2(0.75)(4)s + 4^2} \quad (4)$$

The pure time delay is approximated by a second order Pade. W_{perf} is a low-pass filter which weights the difference between the reference model and the aircraft pitch rate q at low frequencies. The other filters are scalar weights which are adjusted till a satisfying synthesis is obtained.

Finally a 16th order controller is designed with a gain margin of $M_G = 6.4$ dB and a phase margin of $M_\varphi = 52^\circ$. Time responses are on Figure 5 and Table III gathers all handling qualities values (most of them are satisfied, except dropback).

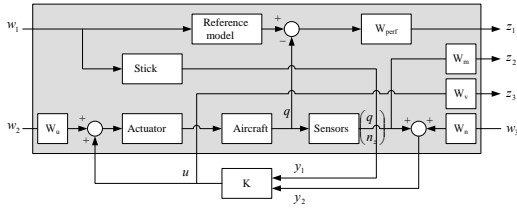


Fig. 4. H_∞ synthesis diagram

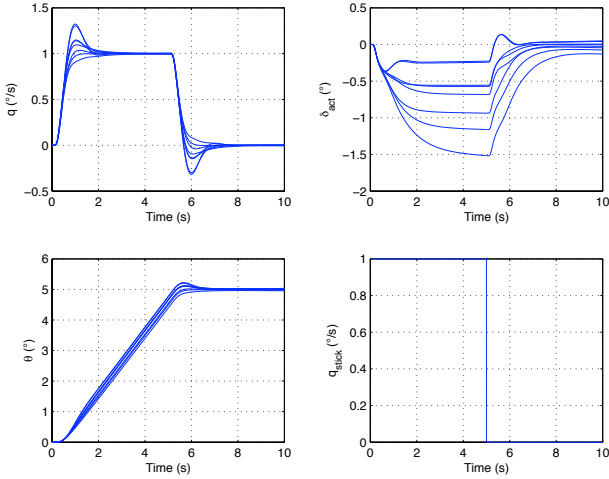


Fig. 5. Time responses of the H_∞ controller

IV. CONTROLLER REDUCTION USING ROBUST MODAL CONTROL METHODS

A. Theoretical aspects

1) *Notations and propositions:* Let us consider the following linear system, with n states, m inputs and p outputs:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad (5)$$

¹Dropback satisfaction is not considered in this reference model as it will be tuned with a specific feedforward gain

TABLE III
HANDLING QUALITY VALUES OF THE H_∞ CONTROLLER

Cas m/x_{cg}	ζ_{sp1}	ζ_{sp2}	ω_{BW_θ}	Drb	τ_p	ST
30 000 /20	0.75	0.64	1.57	-0.5	0.21	1.72
30 000 /35	0.73	0.67	1.7	-0.38	0.22	1.92
39 000 /20	0.74	0.5	1.5	-0.55	0.21	1.21
39 000 /38	0.74	0.53	1.67	-0.36	0.22	2.58
46 000 /16	0.74	0.41	1.49	-0.65	0.19	3.05
46 000 /38	0.74	0.47	1.59	-0.38	0.21	3.05
32 000 /48	0.73	0.5	1.85	-0.29	0.26	2.29
39 000 /48	0.7	0.58	1.76	-0.27	0.25	2.38

where x is the state vector, y the measurement vector and u the input vector, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$ et $D \in \mathbb{R}^{p \times m}$. w_i and v_i are the input directions and right eigenvectors associated to the closed loop eigenvalue λ_i . The control feedback is defined by $u(s) = K(s)y(s)$, and $K(s)$ is obtained using dynamic eigenstructure assignment defined by Proposition 1.

Proposition 1: [11] The triple $T_i = (\lambda_i, v_i, w_i)$ satisfying

$$\begin{bmatrix} A_0 - \lambda_i I & B_0 \end{bmatrix} \begin{bmatrix} v_i \\ w_i \end{bmatrix} = 0 \quad (6)$$

is assigned by the dynamic gain $K(s)$ if and only if

$$K(\lambda_i) \begin{bmatrix} C v_i + D w_i \end{bmatrix} = w_i \quad (7)$$

If the eigenvalue is complex, the equality (7) has to be completed by its conjugate.

The elementary design procedure associated with this proposition is as follows:

- Choose the closed loop eigenvalues λ_i and determine the closed loop admissible eigenvector space using Equation (6). At this step, the triple (λ_i, v_i, w_i) is defined.
- Compute $K(s)$ satisfying Equation (7). This computation is detailed in the next section.

2) *Technical resolution:* The transfer matrix $K(s)$ is assumed to have the following form at each input-output:

$$K_{ij}(s) = \frac{b_{ijq}s^q + \dots + b_{ij1}s + b_{ij0}}{a_{ijq}s^q + \dots + a_{ij1}s + a_{ij0}} \quad (8)$$

Common or different denominators are fixed *a priori* for the matrix $K(s)$. The coefficients a_{ijk} are chosen by choosing the desired roots of each denominator (e.g. the roots of an initial controller) and to identify the corresponding coefficients. The free parameters are the numerator coefficients denoted by b_{ijk} . The structure of the feedback as defined in Equation (8) usually offers a large number of degrees of freedom. For this reason, a quadratic criterion J is considered, for example to keep the controller as close as possible to an initial feedback K_{ref} :

$$J = \sum_{i=1 \dots r} \|K_{ref}(j\omega_i) - K(j\omega_i)\|_F^2 \quad (9)$$

Proposition 2: The problem of computing $K(s)$ satisfying Equation (7) and minimizing criterion (9) consists in solving

for Ξ a LQP problem of the form

$$\Xi A_1 = b_1; J = \Xi H \Xi^T + 2\Xi c \quad (10)$$

where Ξ denotes unknown coefficients of the numerators b_{ijk} .

The main result can be found in [10, 12] for derivation, and [11] for software implementation.

Now we will describe how to combine modal analysis and dynamic eigenstructure assignment to reduce an initial controller while concurrently satisfying the closed loop performances. The method is based on the fact that the system is entirely defined by its closed loop eigenstructure. First the dominant eigenstructure is extracted using modal analysis. In the second step, the dominant eigenstructure of the system will be assigned using a reduced order controller which has a fixed structure obtained from the first step of the procedure. Frequency criteria will be minimized to optimize the efficiency of the reduction.

Step 1: Modal analysis

A modal analysis of the closed loop system is performed in order to identify the dominant modes and their associated eigenvectors.

Step 2: Reduction

- *Choice of the controller poles:* this choice is made *a priori* as a subset of the poles of the original controller. The selection is made using the analysis of the dominant eigenvalues obtained at **Step 1**. At this stage, the coefficients a_{ijk} of Equation (8) are all fixed.
- *Eigenstructure assignment:* the constraints defining the re-assignment of dominant eigenstructure (selected at **Step 1**) are derived; these constraints correspond to Equations (6) and (7). The number of constraints depends on the desired controller order. The higher the order, the higher the number of constraints to be processed. The equations for eigenstructure assignment are linear constraints. At this stage, using Proposition 2, Equation $\Xi A_1 = b_1$ (10) is known.
- *Controller structure constraints:* some coefficients of the reduced controller can be fixed. For example, a desired difference of degree between numerators and denominators means that the relevant b_{ijk} must be set to zero. So, there is an additional equality constraint and new constraints of the form $\Xi A'_1 = b'_1$ are added.
- *Criterion:* a quadratic criterion of the form (9) where $K_{ref}(s)$ is the transfer function of the initial controller is defined. This criterion will fix the degrees of freedom remaining after the above constraints have been taken into account. The computed controller will become as close as possible to the initial controller. The choice of the frequencies ω_j depends on the frequency domain features of the initial controller. At this stage, using the result of Proposition 2, the criterion is written in the following form: $J = \Xi H \Xi^T + 2\Xi c$
- Solve the LQP problem for Ξ , then deduce from Ξ the values of the coefficients b_{ijk} .

Step 3: Final analysis

Performance is evaluated in the time domain, in the frequency domain and in the parametric domain. If some properties are not satisfactory, **Step 2** must be repeated.

B. Application

The initial H_∞ controller order is 16. Before proceeding to the modal analysis, it is sought to reduce in a significant way the order of the controller by using balanced reduction techniques. Moreover, as the controller is a two degree controller, the feedforward part is separated from the feedback part. Again, balanced reduction are performed on each sub-controller. As modal reduction has only interest in the feedback controller, the feedforward controller will remain as it was after the balanced reduction. Fortunately, the feedforward controller order is 3 without any major loss on the temporal, frequency and parametric criteria. The feedback controller order is then 10; if further balanced reduction is performed, there are some significant loss compared to the initial H_∞ controller. New modal reduction is to be used in order to see if the controller can be further reduced.

First a modal analysis is performed to find the dominant poles that must be reassigned. The integral effect must be kept on the q feedback; a pole is set to zero to ensure this. Then a low frequency pole is kept on the two feedback measurements. A roll-off constraint is applied. After a first iteration of the algorithm, expected results are not obtained. By further analyzing the dominant poles, complex conjugate poles are necessary. After trials, a supplementary pole located in -18 is added on the n_z feedback.

Finally, the feedforward controller order is 3, the feedback on q is a 4th order (with an integrator pole) and the feedback on n_z is a 4th order.

$$\frac{\delta_1}{q_{ref}} = \frac{K_1(s^2 + 11s + 65.38)}{s(s^2 + 6s + 16.58)} \quad (11)$$

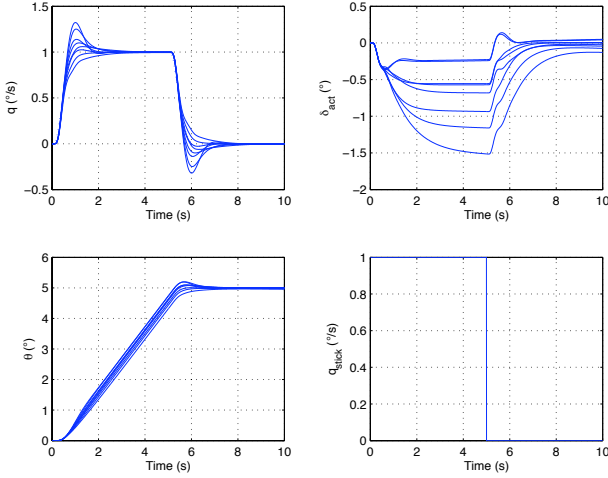
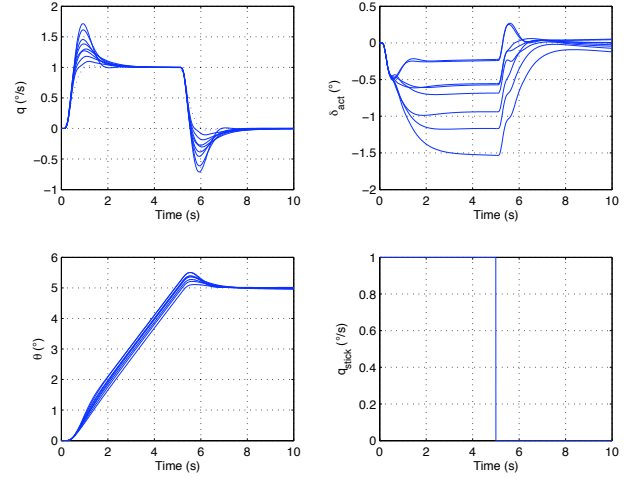
$$\frac{\delta_2}{q} = \frac{-K_2(s + 1.42)(s^2 + 11.65s + 67.5)}{s(s + 2.5)(s^2 + 40s + 1025)} \quad (12)$$

$$\frac{\delta_3}{n_z} = \frac{K_3(s + 2.14)(s^2 + 12.2s + 60.4)}{(s + 2.5)(s + 18)(s^2 + 40s + 1025)} \quad (13)$$

The reduced controller is then tested on the complete high order model and provides similar results to the original one on time responses (Figure 6). Table IV summarizes the handling qualities values.

TABLE IV
HANDLING QUALITY VALUES OF THE REDUCED H_∞ CONTROLLER

Cas m/x_{cg}	ζ_{sp1}	ζ_{sp2}	ω_{BW_θ}	Drb	τ_p	ST
30 000 /20	0.63	0.76	1.5	-0.52	0.23	1.69
30 000 /35	0.66	0.79	1.64	-0.7	0.21	1.94
39 000 /20	0.52	0.59	1.4	-0.42	0.22	1.78
39 000 /38	0.58	0.57	1.52	-0.48	0.21	2.78
46 000 /16	0.55	0.46	1.32	-0.65	0.2	3.22
46 000 /38	0.59	0.4	1.51	-0.38	0.21	3.21
32 000 /48	0.54	0.79	1.82	-0.3	0.25	2.31
39 000 /48	0.52	0.71	1.66	-0.3	0.24	2.52

Fig. 6. Time responses of the reduced H_∞ controllerFig. 8. Time responses of the reduced H_∞ controller with feedforward gain

V. SETTING IN CLASSICAL FORM

In order to get close to the classical structure of Figure 1, the former transfer functions are manipulated. A PI controller can then be extracted and filters on each measurements. A feedforward gain K_{ff} is added (Figure 7).

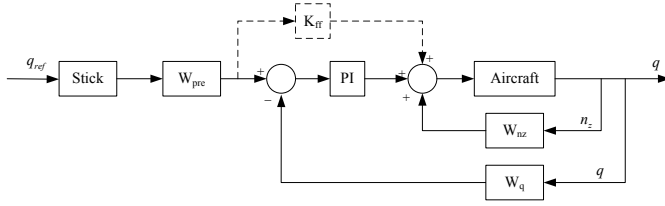


Fig. 7. Classical form setup

By adjusting quickly $K_{ff} = 1.1$, some criteria values can be improved in particular dropback and θ -bandwidth. Time responses have a larger overshoot but satisfying dropback can imply larger overshoots (Figure 8 et Table V)

TABLE V

HANDLING QUALITY VALUES OF THE REDUCED H_∞ CONTROLLER WITH FEEDFORWARD GAIN

m/x_{cg}	ζ_{sp1}	ζ_{sp2}	ω_{BW_θ}	Drb	τ_p	ST
30 000 /20	0.63	0.76	1.55	-0.02	0.23	1.89
30 000 /35	0.66	0.79	1.69	-0.2	0.21	2.05
39 000 /20	0.52	0.59	1.51	0.08	0.22	1.98
39 000 /38	0.58	0.57	1.57	0.02	0.21	2.98
46 000 /16	0.55	0.46	1.49	-0.15	0.2	3.32
46 000 /38	0.59	0.4	1.57	0.12	0.21	3.31
32 000 /48	0.54	0.79	1.89	0.2	0.25	2.61
39 000 /48	0.52	0.71	1.73	0.2	0.24	2.72

VI. CONCLUSIONS

We sought in this article to find a classical control structure from a high order controller issued from H_∞ synthesis, satisfying most handling qualities criteria. Robust modal reduction

lowered the controller order to make it similar to classical structures usually used in industry. Finally, for a given flight condition, we obtained a controller which is robust to varying mass and center of gravity location (at least on the 8 tested configurations) Next step consists in extending this controller structure to the whole flight envelope, by scheduling the PI controller gains and filter gains. This work is presently pursued.

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