GAIN-SCHEDULED FLIGHT CONTROL LAW FOR FLEXIBLE AIRCRAFT: A PRACTICAL APPROACH

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Abstract: This paper presents a gain-scheduling method applied to flight control law design. The method is a stability preserving interpolation technique of existing controllers under observer-state feedback form. Application is made on a flexible civil aircraft example considering multiple scheduling parameters. Although the interpolation technique gives powerful *a priori* stability guarantees, the sufficient condition to satisfy leads to conservative results in practice. We thus use a fixed observer model and check stability and performance thanks to μ-analysis. Provided results are really satisfactory for a final controller of little complexity.

Keywords: gain-scheduling, interpolation, flight control laws

1. INTRODUCTION

"For at least the last few decades, Machines that walk, swim, or fly are gain scheduled" (Rugh and Shamma, 2000, in (Rugh and Shamma, 2000)). This is almost true in the aerospace community. Gain-scheduling is one of the most common way to take into account plants non linearity in controller design. In spite of the wide application field mentioned by (Rugh and Shamma, 2000), gain-scheduling was rarely addressed within the research community before the beginning of the nineties. Two major surveys (Rugh and Shamma, 2000; Leith and Leithead, 2000) give a good overview of the available techniques.

All those methodologies make the controller coefficients continuously varying according to the current value of scheduling signals through the same procedure. Step 1: compute a Linear Parameter Varying (LPV) model of the plant. Step 2: use linear design methods to compute a family of linear controllers. Step 3: implement the controllers family such that the controller coefficients vary according to the scheduling parameters. Step 4: assess performance and return to step 2 if necessary. The first step excepted, we can however distinguish two main classes of gain-scheduling techniques. The first are *a posteriori* scheduling methods based on interpolation of a Linear Time Invariant (LTI) controllers set represented by different ways: zeros, poles, gains of transfer function, riccatti solutions of H_∞ problems, observer and state feedback gains, balanced state space representation.The second are *a priori* scheduling techniques that combine the second and the third steps. Examples are given by LPV or LFT control design.

The Airbus current procedure consists of designing a set of linear controllers (Kubica and Livet, 1994) associated to a system operating domain gridding, interpolating static gains and switching dynamic filters, and finally analyzing stability and performance on the whole operating domain. This technique is closely linked to the controller shape that enables to distinguish the static gain and the dynamic filter parts. Recent works on multi-objective control design (Puyou *et al.*, 2004; Ferreres and Puyou, 2006) lead us to challenge the previous controller structure. We want thus to modify the associated gain scheduling technique in order to handle the general Multi-Input Multi-Ouput (MIMO) controller form in case of multiple scheduling parameters.

In order to stay close to the current scheduling methodology (mainly to apply in-flight tuning techniques), we have here chosen to develop a stability preserving interpolation technique of existing controllers under observer-state feedback form extending (Stilwell and Rugh, 1999) to LMI stability region in the case of vector scheduling variable. We will apply it on the example of aircraft longitudinal feedback control law design.

The paper is organized as follow. First, results on interpolation of observer-state feedback controllers are presented. Then application issues are detailed. We perform application in a third part before concluding.

2. GAIN SCHEDULING TECHNIQUE

Let's define:

- $\theta_k \in [\theta_{k,min};\theta_{k,max}] \subset \mathbb{R}$, $k = 1..l$, the scheduling parameters.
- Θ the scheduling parameter vector such that [1] $(\Theta)_k = \theta_k$.
- $\Gamma = \left\{\Theta \in \mathbb{R}^l, \forall k = 1..l\ (\Theta)_k \in [\theta_{k,min};\theta_{k,max}]\right\}$ the admissible scheduling parameter vector space
- θ_{k,i_k}, $i_k = 1..r_k$, r_k scalar values of θ_k with $k = 1..l$.
- $\Theta_i = \Theta_{(i_1,..,i_l)}$ a scheduling parameter vector value such that $\left(\Theta_{(i_1,..,i_l)}\right)_k = \theta_{k,i_k}$.
- $X_{\Theta_i} = X(\Theta_i)$ the evaluation of the Θ dependent matrix $X(\Theta)$.
- $\mathcal{F}_l$ the *lower LFT*

Let's consider a plant of the form:

$$\left\{\begin{array}{l} \dot{x} = A(\Theta)x + B(\Theta)u \\ y = C(\Theta)x + D(\Theta)u \end{array}\right. \qquad (1)$$

State space representation 1 will be also noted $(A(\Theta), B(\Theta), C(\Theta), D(\Theta))$.

We make the hypothesis that Θ contains slowly varying parameters, so that we can consider (1) as an LTI system.

The aim of the following method is to generate a continuously varying family of controllers given under the observer-state feedback form $(K_c(\Theta), K_f(\Theta), Q(\Theta, s))$, respectively the state feedback gain, the estimator gain and the Youla parameter.

[1] $(.)_k$ represents the kth coordinate of a given vector

Due to the separation principle, the closed-loop eigenvalues can be independently separated into the closed-loop state feedback poles, the closed-loop state estimator poles and the Youla parameter poles. We here consider the case of a static youla parameter, so that we must ensure stability of the state feedback $A(\Theta) + B(\Theta)K_c(\Theta)$ and the state estimator $A(\Theta) + K_f(\Theta)C(\Theta)$ to guarantee the closed-loop stability. In the case of scalar scheduling variable, Stilwell and Rugh (Stilwell and Rugh, 1999) present results on interpolation techniques of initial controllers set. Under stability covering condition, the methodology enables to stabilize the plant on the whole parametrical domain. We are now going to present results that extends Stiwell and Rugh approach (Stilwell and Rugh, 1999) to LMI stability region in the case of vector scheduling variable. Let's first define:

Definition 2.1. (LMI-region). Let $P = P^T$ and Q be real matrix. LMI-region is defined by a region $\mathcal{D}$:

$$\mathcal{D} = \left\{s \in \mathbb{C} \,|\, P + Qs + Q^T\bar{s} < 0\right\}$$

Definition 2.2. ($\mathcal{D}$-stability). Let A be real matrix and $\mathcal{D}$ a LMI-region. A is said $\mathcal{D}$-stable if all its eigenvalues are contained in the region $\mathcal{D}$.

It is then possible to extend the classical Lyapunov stability characterization to the LMI region case (Chilali and Gahinet, 1996).

Proposition 1. ((Chilali and Gahinet, 1996)). Let A be a real matrix, A is $\mathcal{D}$-stable if and only if there exists a symmetric definite-positive matrix X such that [2]:

$$P \otimes X + Q \otimes (AX) + Q^T \otimes (XA^T) < 0 \quad (2)$$

Let's now define the $\mathcal{D}$-stability covering condition.

Definition 2.3. Let $\mathcal{D}$ be a LMI stability region, $A(\Theta)$ and $B(\Theta)$ given as in (1), $\Theta \in \Gamma \subset \mathbb{R}^l$, suppose the state feedback gain $K_{c_{(i_1,\ldots,i_l)}}$ is such that $(A(\Theta_{(i_1,\ldots,i_l)}) + B(\Theta_{(i_1,\ldots,i_l)})\ K_{c_{(i_1,\ldots,i_l)}})$ is stable for all $\{i_k = 1\ldots r_k\}_{k=1\ldots l}$. Let $U_{(i_1,\ldots,i_l)}$, containing $\Theta_{(i_1,\ldots,i_l)}$, be an open neighborhood such that $A(\Theta) + B(\Theta)K_{c_{(i_1,\ldots,i_l)}}$ is $\mathcal{D}$-stable for each fixed $\Theta \in U_{(i_1,\ldots,i_l)}$. If $\Gamma \subset \bigcup_{\{i_k=1\ldots r_k\}_{k=1\ldots l}} U_{(i_1,\ldots,i_l)}$ then we say that the state feedback gains satisfy the *$\mathcal{D}$-stability covering* condition.

We can propose a continuous state feedback gain that ensures the $\mathcal{D}$-stability on Γ.

[2] $\otimes$ represents Kronecker product

Theorem 2.4. Let $P = P^T$ and Q be real matrix defining the LMI-region $\mathcal{D}$. Given $A(\Theta)$ and $B(\Theta)$ as in (1), $\theta \in \Gamma \subset \mathbb{R}^l$, suppose the state feedback gains $K_{c_{(i_1,\ldots,i_l)}}$ corresponding to $\Theta_{(i_1,\ldots,i_l)} \in \Gamma$ with $i_k = 1\ldots r_k$ and $k = 1\ldots l$, satisfy the stability covering condition. If there exists $\gamma > 1$ and symmetric positive-definite matrix $W_{(i_1,\ldots,i_l)}$ such that for all $\{i_k = 1\ldots r_k\}_{k=1\ldots l}$

$$P \otimes W_{(i_1,\ldots,i_l)} + Q \otimes (A(\Theta) - B(\Theta)K_{(i_1,\ldots,i_l)})W_{(i_1,\ldots,i_l)} \\ +Q^T \otimes W_{(i_1,\ldots,i_l)}(A(\Theta) - B(\Theta)K_{(i_1,\ldots,i_l)})^T \le -\gamma I$$

with $\Theta \in U_{(i_1,\ldots,i_l)}$, then there exists intervals

$$[b_{(i_1,\ldots,i_l)}, c_{(i_1,\ldots,i_l)}] \subset U_{(i_1,\ldots,i_l)} \cdots \\ \bigcap U_{(i_1+1,\ldots,i_l+1)} \bigcap [\Theta_{(i_1,\ldots,i_l)}, \Theta_{(i_1+1,\ldots,i_l+1)}]$$

, for all $i_k = 1, \ldots, r_k - 1$ and $k = 1, \ldots, l$, such that the continuous state feedback gain $K_c(\Theta)$ is equal to:

$$\begin{cases} K_{c_{(i_1,\ldots,i_l)}} , \Theta \in [\Theta_{(i_1,\ldots,i_l)}, b_{(i_1,\ldots,i_l)}) \\ \tilde{K}_{c_{(i_1,\ldots,i_l)}}(\Theta)W(\Theta)^{-1} , \Theta \in [b_{(i_1,\ldots,i_l)}, c_{(i_1,\ldots,i_l)}] \\ K_{c_{(i_1+1,\ldots,i_l+1)}} , \Theta \in (c_{(i_1,\ldots,i_l)}, \Theta_{(i_1+1,\ldots,i_l+1)}] \end{cases}$$

where [3]

$$\begin{bmatrix} \tilde{K}_{c_{(i_1,\ldots,i_l)}}(\Theta) \\ W(\Theta) \end{bmatrix} = \sum_{\{j_k = i_k \ldots i_k+1\}_{k=1\ldots l}} \begin{bmatrix} K_{c_{(j_1,\ldots,j_l)}} \\ I \end{bmatrix} \cdots \\ W_{(j_1,\ldots,j_l)} \prod_{h=1\ldots l} \left(\left(\frac{c_{(i_1,\ldots,i_l)} - \Theta}{c_{(i_1,\ldots,i_l)} - b_{(i_1,\ldots,i_l)}} \right)_h \delta_{i_h+1,j_h} \cdots \right. \\ \left. + \left(\frac{\Theta - b_{(i_1,\ldots,i_l)}}{c_{(i_1,\ldots,i_l)} - b_{(i_1,\ldots,i_l)}} \right)_h \delta_{i_h,j_h} \right)$$

guarantees $\mathcal{D}$-stability on Γ.

Remark: Linear interpolation case is obtained when Lyapunov functions $W_{(i_1,\ldots,i_l)}$ are equal to identity.

A continuous state-estimator gain $K_f(\Theta)$ computation method that guarantees $\mathcal{D}$-stability of $A(\Theta) - K_f(\Theta)C(\Theta)$ can be extended in the same way from Stilwell and Rugh results. Concerning the Youla parameter, we will consider in the application the case of a static parameter. Therefore linear interpolation is enough. Nevertheless, if is needed, transfer function interpolation method that guarantees $\mathcal{D}$-stability must be investigated to tackle the dynamic Youla parameter case.

3. INTRODUCTION TO THE APPLICATION

3.1 *Application issue*

The flight control law requirements are widely described in (Puyou *et al.*, 2004). We are dealing in this article with the case of longitudinal aircraft control design. Here are summarized the main specifications:

Handling qualities: they are mainly specified through time domain requirements and are dedicated to feedforward part of the controller. The feedback part of the controller is assumed to enhance the rigid body modes damping to more than 0.5

Comfort: Comfort level improvement is linked to minimisation of the frequency domain maximal response of vertical accelerations at the front of the fuselage on wind input: $\|T_{wind \to Nz_{front}}\|_\infty$

Loads: Loads alleviation is linked to power spectral density reduction of aircraft vertical acceleration response on the wing on wind input: $\|T_{wind \to Nz_{wing}}\|_2$

To fulfill the previous requirements on the whole flight domain two scheduling parameters will be used in the feedback controller computation:

- V_c: the conventional speed [4] varying between 145kt and 330kt.
- ω_{2NW}: the first bending mode (two nodes wing (2NW) bending mode) natural frequency varying between 1Hz and 2Hz.

Remark: We will note δ_{V_c} and $\delta_{\omega_{2NW}}$ the normalized scheduling parameter respectively corresponding to V_c and ω_{2NW}.

3.2 *Control law architecture*

The fly-by-wire architecture enables any kind of control architecture. Fig. 1 we give a conventional one for longitudinal control. The pilot's orders are transmitted by the side stick and correspond to a vertical acceleration Nz objective. The measurements are:

- Nz: vertical acceleration
- q: pitch rate
- Nz_{wing}: vertical acceleration on the wing

The pilot's orders and measurements are mixed through feedforward and feedback controllers to produce orders for the ailerons (d_p) and elevator (d_q). Let's note that we only deal here with feedback part of the controller.

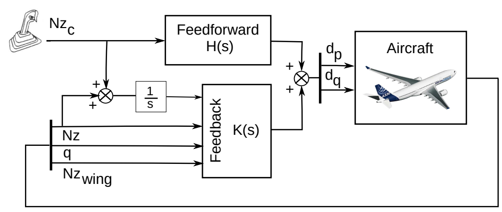

Fig. 1. Longitudinal law architecture

[3] $\delta_{i,j}$ is defined by: $\forall(i,j) \in \mathbb{N}^2$, $\delta_{i,j} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$

[4] expressed in 'kt' for 'knots'

3.3 LFT modeling

One key issue is to compute parametrized model of aircraft with respect to the selected scheduling parameters: V_c and ω_{2NW}. We consider (Cf. Fig. 2) a state variable aircraft representation built by coupling rigid and elastic body models (see (Kubica and Livet, 1994) for more details).

Because aerodynamic coefficients usually depend on V_c^2 term, we chose to build quadratic speed dependent rigid aircraft state-space model. Each matrix of $(A(V_c), B(V_c), C(V_c), D(V_c))$ will thus has the form of $A(V_c)$:

$$A(V_c) = A_0 + A_1 V_c + A_2 V_c^2 \qquad (3)$$

To do so we use three non parametrized state-space models (computed for three different speed values) and apply LFR toolbox (v2.0) (Hecker *et al.*, 2004) routines. The provided rigid aircraft LFT model has so a 8 order variation block Δ_r, which only parameter is δ_{V_c}.

Regarding flexible model, real modal state-space representation is chosen to highlight dependence on bending mode natural frequency. If natural frequency variation is noted : $\omega = (1 + \delta_\omega)\omega_0$, first order variation approximation can be made so that elementary bending mode state-space representation can be written:

$$\dot{x} = \begin{bmatrix} -2\xi\omega & -\omega^2 \\ 1 & 0 \end{bmatrix} x + bu$$

Conversion to LFT of flexible aircraft (using previous approximation) is detailed in (Ferreres, 1999) and implementation is made using LFRT. The provided flexible aircraft LFT model has so a 2 order variation block Δ_f, which only parameter is $\delta_{\omega_{2NW}}$. Let's note that flexible aircraft model dependency on conventional speed is not made explicit. Bending mode natural frequency variations, induced by speed changes, are covered by $\delta_{\omega_{2NW}}$ and amplitude variations, induced by speed changes, are covered by worst case amplitude model (maximum speed model indeed).

4. GAIN SCHEDULING APPLICATION

4.1 Initial controller

We have developed in previous work (Puyou *et al.*, 2004) a multi-objective control design technique based on convex design (Ferreres and Puyou, 2006). This methodology is here used to compute initial controllers. We chose three speed points corresponding to $V_c = 145kt$, $230kt$ and $330kt$, and three pulsation points corresponding to $\omega_{2NW} = 1Hz$, $1.5Hz$ and $2Hz$. We thus generate nine initial controllers $K_{V_c,\omega_{2NW}}$. In order to highlight gain-scheduling interest, we attempt to maximise the robustness to V_c and ω_{2NW} variations under performances constraints. We therefore succeed in producing compensators that still stabilize the plant model on the nearest gridding points, but do not reach performance objective out of design point.

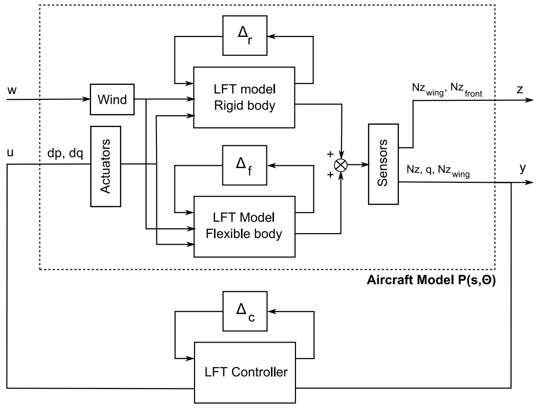

Fig. 2. LFT closed-loop model

4.2 Straightforward approach

4.2.1. Principle The first step is to deduce the observer-state feedback controller form from the initial controllers. We use (Alazard and Apkarian, 1999) to do so. They both have shown that any compensator (which order is higher than the system one) can be put into the observer-state feedback form $(K_c, K_f, Q(s))$. The equivalent representation of the controller $K(s)$ stabilizing the plant $P(s)$, choosing any state-space representation (A, B, C, D) for the observer model, is noted as follow:

$$K(s) = lqg(K_c, K_f, Q(s), A, B, C, D) \qquad (4)$$

Noting that there exists several choice of triplet $(K_c, K_f, Q(s))$, one hot point of this technique is to choose an invariant subspace of the closed loop system matrix by repartition of the closed-loop eigenvalues (between the state feedback closed-loop, the observer state closed-loop and the Youla parameter) that ensures a continuous path connecting observer-based realizations of initial controllers. This problem is addressed by continuation of the selected invariant subspace techniques developed in (Pellanda *et al.*, 2000). We must now try to satisfy stability covering condition (2.3) to be able to apply theorem (2.4).

4.2.2. Difficulties Initial controllers ensure robust stability of the feedback loop between the design points. Therefore, for any design point $\Theta_{(i,j)} = (V_{c_i}, \omega_{2NW_j})$ and associated controller $K_{V_{c_i},\omega_{2NW_j}}$, the feedback loop:

$\mathcal{F}_l\left(P(s,\Theta), K_{V_{c_i},\omega_{2NW_j}}(s)\right)$, is stable for $\Theta \in U$, with $U = [V_{c_{i-1}}, V_{c_{i+1}}] \times [\omega_{2NW_{j-1}}, \omega_{2NW_{j+1}}]$. Therefore if we note $(K_{c_{(i,j)}}, K_{f_{(i,j)}}, Q_{(i,j)}(s))$ the observer-state feedback form of $K_{V_{c_i},\omega_{2NW_j}}$, because of (4) equivalence, initial controller ensures stability of the feedback loop (5) for $\Theta \in U$:

$$\mathcal{F}_l\left(P(s,\Theta), lqg(K_{c_{(i,j)}}, K_{f_{(i,j)}}, Q_{(i,j)}(s), \ldots A_{\Theta_{(i,j)}}, B_{\Theta_{(i,j)}}, C_{\Theta_{(i,j)}}, D_{\Theta_{(i,j)}})\right) \quad (5)$$

To satisfy the stability covering condition, we have to guarantee robust stability of the feedback loop (6) for $\Theta \in U$:

$$\mathcal{F}_l\left(P(s,\Theta), lqg(K_{c_{(i,j)}}, K_{f_{(i,j)}}, Q_{(i,j)}(s), \ldots A(\Theta), B(\Theta), C(\Theta), D(\Theta))\right) \quad (6)$$

Despite of the initial compensators robustness properties, we do not manage to fulfill stability covering condition. In the first case (5), only the plant model is dependent from scheduling parameter Θ, whereas in the second case (6), observer model is also scheduled by Θ (in order to still satisfy the separation principle). It thus clearly appears that initial robustness properties are not directly linked to stability covering condition fulfillment which also depends on $(K_c, K_f, Q(s))$ choice.

4.3 A practical approach

4.3.1. Principle We thus decide to take a fixed observer model $(A_{obs}, B_{obs}, C_{obs}, D_{obs})$ for $\Theta \in U$ whatever the initial controller $K_{V_{c_i},\omega_{2NW_j}}$:

$$K_{(i,j)}(s,\Theta) = lqg(K_{c_{(i,j)}}(\Theta), \ldots K_{f_{(i,j)}}(\Theta), Q_{(i,j)}(s,\Theta), A_{obs}, B_{obs}, C_{obs}, D_{obs})$$

with $(K_{c_{(i,j)}}(\Theta), K_{f_{(i,j)}}(\Theta), Q_{(i,j)}(s,\Theta))$ of the form as $K_{c_{(i,j)}}(\Theta)$ given hereafter as an example:

$$K_{c_{(i,j)}}(\Theta) = \frac{V_c - V_{c_{i+1}}}{V_{c_i} - V_{c_{i+1}}}\left(\frac{\omega_{2NW} - \omega_{2NW_{j+1}}}{\omega_{2NW_j} - \omega_{2NW_{j+1}}} K_{c_{(i,j)}} \ldots + \frac{\omega_{2NW} - \omega_{2NW_j}}{\omega_{2NW_{j+1}} - \omega_{2NW_j}} K_{c_{(i,j+1)}} + \right) \ldots + \frac{V_c - V_{c_i}}{V_{c_{i+1}} - V_{c_i}}\left(\frac{\omega_{2NW} - \omega_{2NW_{j+1}}}{\omega_{2NW_j} - \omega_{2NW_{j+1}}} K_{c_{(i+1,j)}} \ldots + \frac{\omega_{2NW} - \omega_{2NW_j}}{\omega_{2NW_{j+1}} - \omega_{2NW_j}} K_{c_{(i+1,j+1)}}\right)$$

with, for $\tilde{i} = i, i+1$ and $\tilde{j} = j, j+1$:

$$K_{V_{c_{\tilde{i}}},\omega_{2NW_{\tilde{j}}}}(s) = lqg(K_{c_{(\tilde{i},\tilde{j})}}, K_{f_{(\tilde{i},\tilde{j})}}, Q_{(\tilde{i},\tilde{j})}, \ldots A_{obs}, B_{obs}, C_{obs}, D_{obs})$$

This restores the original robustness properties of compensator, but invalidates the separation principle (only true if observer model is equal to the system-to-be-controlled model) and the theoretical stability guarantees that have to be checked *a posteriori*.

4.3.2. Results Results of the application are really satisfying. We only need four initial points (corresponding to the limits of the parametric domain) to generate a scheduled compensator that stabilizes the plant on the whole parametric domain and provides an uniform level of performances. Because computation of observer-state feedback form is now made by linear interpolation, conversion to LFT is straightforward using LFR toolbox. The computed LFT controller has an 18 order variation block Δ_c which parameters are δ_{V_c} (repeated 10 times) and $\delta_{\omega_{2NW}}$ (repeated 8 times).

Stability can be checked by μ-analysis on the global closed-loop LFT model (Cf. Fig. 2). Analysis is implemented through Skew-mu toolbox (Ferreres *et al.*, 2004). Because of high level of uncertainties repetition, μ upper bound computation on the whole parametrical domain return a very conservative result (conservatism is evaluated by considering the gap between lower bound (=0.63) and upper bound (=1.21)). We thus divide the parametrical space into four sub-zones. The four guaranteed stability domains are represented on Fig. 3 (gray boxes). We can check that they cover the whole normalized domain (doted-line box).

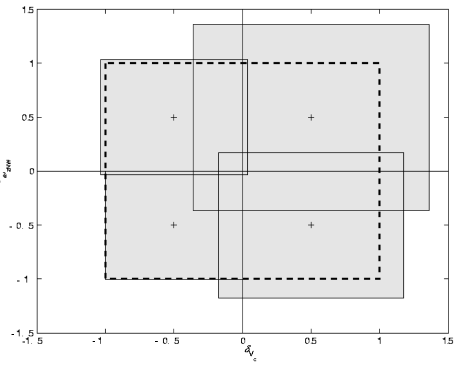

Fig. 3. Robust stability analysis

Handling quality performance can not be validated by robust $\mathcal{D}$-stability because of poor damping of flexible modes. We thus check minimum damping requirement on rigid body mode by cross sweeps on (V_c, ω_{2NW}) values. Root locus visualisation (Fig. 4) shows that short period mode fulfills the 0.5 minimum damping contraint.

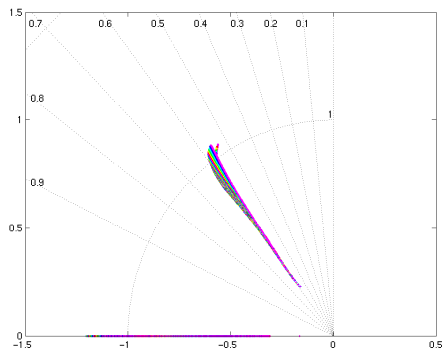

Fig. 4. Rigid body roots locus on the flight domain

Comfort level assessment is defined as an H_∞ performance problem whereas **load alleviation** is an H_2 criteria $\|T_{wind \to Nz_{wing}}\|_2$. Nevertheless load is mainly induced by first bending mode frequency domain response. According to in-house experience, its power spectral density level is linked to its damping and to the H_∞ norm of its frequency domain response. Therefore robust performance can be assessed by $\left\| \begin{matrix} T_{wind \to Nz_{front}} \\ T_{wind \to Nz_{wing}} \end{matrix} \right\|_\infty$ norm. Using main loop theorem, robust H_∞ performance under robust stability condition can be formulated as a classical μ computation problem (see (Ferreres, 1999) for more details) by adding fictitious performance block in the global variation block Δ. μ upper bound computation provides the same result that in the stability assessment case (see Fig. 3), what means performance constraint is not active in the resolved μ problem. In order to compare gain scheduled controller performance to the initial controller we can visualize (see Fig. 5) comfort and load transfer frequency domain response on the middle point ($V_c = 230kt, \omega_{2NW} = 1.5Hz$) - which is the most far away from the points involved in the scheduled controller computation.

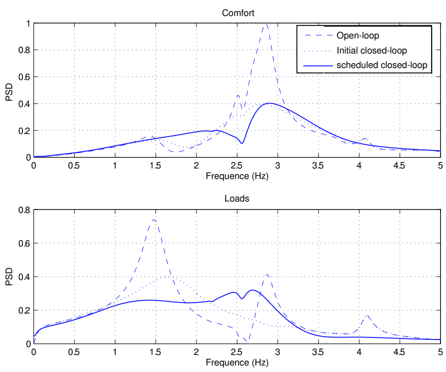

Fig. 5. Comfort and loads for ($V_c = 230kt$,$\omega_{2NW} = 1.5Hz$)

5. CONCLUSION

This paper presents extensions of Stilwell and Rugh stability guarantees on the observer-state feedback controller interpolation to the case of LMI-region stability for a vector scheduling variable. Putting it into practice on a flexible aircraft application was not an easy job. We faced significant difficulties to satisfy the stability covering condition through controller observer-state feedback parametrisation. Although the theoretical guarantees are no more valid, use of a fixed observer model on the whole parametric domain make the scheduling easier. Stability is then *a posteriori* checked by μ-analysis on the LFT closed-loop model. Two way of improvements may be considered in the future. First way is introduction of a non constant Lyapunov matrix (e.g.piecewise linear) in order to reduce conservatism of the method. Second way is development of an *a priori* gain scheduling technique. A first experimentation was made in (Ayache *et al.*, 2006) using robust modal control but further work has to be done in order to include scheduling in a multi-objective synthesis.

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