An ABORT-Like Detector With Improved Mismatched Signals Rejection Capabilities

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***Abstract*—In this paper we present a GLRT-based adaptive detection algorithm for extended targets with improved rejection capabilities of mismatched signals. We assume that a set of secondary data is available and that noise returns in primary and secondary data share the same statistical characterization. To increase the selectivity of the detector, similarly to the ABORT formulation, we modify the hypothesis testing problem at hand introducing fictitious signals under the null hypothesis. Such unwanted signals are supposed to be orthogonal to the nominal steering vector in the whitened observation space. The performance assessment, carried out by Monte Carlo simulation, shows that the proposed dectector ensures better rejection capabilities of mismatched signals than existing ones, at the price of a certain loss in terms of detection of matched signals.**

***Index Terms*—Adaptive beamformer orthogonal rejection test (ABORT), constant false alarm rate (CFAR), detection, generalized likelihood ratio test (GLRT), rejection of mismatched signals.**

I. Introduction

In the last decades many papers have addressed adaptive radar detection of point-like targets embedded in Gaussian or non-Gaussian disturbance. Most of these papers follow the lead of the seminal paper by Kelly [1], where the generalized likelihood ratio test (GLRT) is used to conceive an adaptive decision scheme capable of detecting coherent pulse trains in presence of Gaussian disturbance with unknown spectral properties. The case of point-like targets (possibly modeled as stochastic signals) assumed to belong to a known subspace of the observables has been addressed in [2], [3]. Detection of possibly extended targets in Gaussian and non-Gaussian noise has been dealt with in [4]–[6]. In addition, several detection algorithms for point-like or extended targets embedded in Gaussian disturbance are encompassed as special cases of the amazingly general framework and derivation presented in [7]. All of the above papers rely on the assumption that a set of secondary data, namely returns free of signals components, but sharing certain properties of the noise in the data under test, is available. Such secondary data are used to come up with fully adaptive detection schemes.

However, previously cited detectors have been designed without taking into account the possible presence of mismatched signals. In practice, instead, the actual signal backscattered from a target (or target's scattering centers) can be different from the nominal one. A mismatched signal may arise due to several reasons as, for example, [8] and [9]:

- coherent scattering from a direction different to that in which the radar system is steered (sidelobe target);
- imperfect modeling of the mainlobe target by the nominal steering vector, where the mismatch may be due to multipath propagation, array calibration uncertainties, beampointing errors, etc.

Thus, it might be important to trade detection performance of mainlobe targets for rejection capabilities of sidelobe ones. In [8], the adaptive beamformer orthogonal rejection test (ABORT) is proposed; such detector takes into account rejection capabilities at the design stage. The idea of the ABORT is to modify the null hypothesis, which usually states that data under test contains noise only, so that it possibly contains a fictitious signal which, in some way, is orthogonal to the assumed target's signature. Doing so, if a mismatched signal is present, the detector will be less inclined to declare a detection, as the null hypothesis will be more plausible than in the case where, under the null hypothesis, the test vector contains noise only. As customary, in [8] it is assumed that a set of noise only (secondary) data is available at the receiver. The extension of this idea to the case of signals belonging to known subspaces of the observables has been dealt with in [9], as a possible means to maintain an acceptable detection loss for slightly mismatched mainlobe targets. Moreover, in [10] the ABORT rationale together with the so-called two-step GLRT design procedure [11] has been used to derive detection strategies for extended targets capable of working without a distinct set of secondary data and guaranteeing good capabilities of rejection of sidelobe targets. It is important to stress that in the detector proposed in [8] the fictitious signal is assumed to be orthogonal to the nominal steering vector in the *quasi-whitened* space, i.e., after whitening of the data through the sample covariance matrix computed over the secondary data set. The same approach is also proposed in [9], where the quasi-whitening transformation is presented as a way to face with the absence of knowledge about the interference subspace.

Following the aforementioned approach, we attack the design of ABORT-like algorithms; however, differently from [8],

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we assume that the useful and the fictitious signals are orthogonal in the *truly* whitened observation space, i.e., after whitening with the true noise covariance matrix. At the design stage we derive both the GLRT and the *ad hoc* detector based upon the two-step GLRT design procedure for the problem at hand, although the main novelty of this work is the derivation of the (one step) GLRT. Remarkably, proposed detectors possess the constant false alarm rate (CFAR) property with respect to the noise covariance matrix (this point will be better clarified in Section III-C). Finally, a preliminary performance assessment, carried out with simulated data, indicates that the newly proposed (one step) GLRT possesses better selectivity properties of previously proposed detectors, although at the price of a certain loss in detection performance of matched signals.

The reminder of the paper is organized as follows: next section is devoted to the problem formulation while the design of the detectors is the object of Section III. The performance assessment is attacked in Section IV while some concluding remarks are given in Section V; finally, in order not to burden too much the main body of the paper, some mathematical derivations are reported in the Appendices.

II. Problem Formulation

Assume that an array of antennas senses K_P range cells and denote by $\boldsymbol{z}_k \in \mathbb{C}^{N\times 1}$, $k \in \Omega_P = \{1,\ldots,K_P\}$, the N-dimensional complex vector containing returns from the kth cell, $k\in\Omega_P$. We assume that each return $\boldsymbol{z}_k$ is corrupted by an additive noise vector $\boldsymbol{n}_k \in \mathbb{C}^{N\times 1}$, $k\in\Omega_P$, modeled as a complex normal vector with unknown, positive definite, covariance matrix $\boldsymbol{R}\in\mathbb{C}^{N\times N}$.

Similarly to the ABORT formulation [8], we want to discriminate between the H_1 hypothesis that the $\boldsymbol{z}_k$'s, $k\in\Omega_P$, contain useful target echoes $\boldsymbol{v}_k \in \mathbb{C}^{N\times 1}$, and the H_0 hypothesis that they contain fictitious signals $\boldsymbol{p}_k\in\mathbb{C}^{N\times 1}$, which herein are assumed orthogonal to the useful ones in the whitened observation space.

We suppose that the $\boldsymbol{v}_k$'s, $k\in\Omega_P$, can be modeled as $\boldsymbol{v}_k = \alpha_k\boldsymbol{v}$, $\boldsymbol{v}\in\mathbb{C}^{N\times 1}$, where $\boldsymbol{v}$ is the (known) nominal steering vector and the α_k's are unknown deterministic complex scalars accounting for both target and channel effects; the unwanted signals $\boldsymbol{p}_k$'s, $k\in\Omega_P$, can be expressed as $\boldsymbol{p}_k = \boldsymbol{U}\boldsymbol{q}_k$, $\boldsymbol{q}_k\in\mathbb{C}^{(N-1)\times 1}$, i.e., as linear combinations[1] of the $N-1$ linearly independent columns of an unknown deterministic matrix $\boldsymbol{U}\in\mathbb{C}^{N\times(N-1)}$ such that

$$\langle \boldsymbol{R}^{-1/2}\boldsymbol{v}\rangle^{\perp} = \langle \boldsymbol{R}^{-1/2}\boldsymbol{U}\rangle. \tag{1}$$

In other words, we assume that the range spaces of the arrays $\boldsymbol{v}$ and $\boldsymbol{U}$ are orthogonal after a whitening transformation. As customary, we also suppose that K_S secondary data, $\boldsymbol{z}_k\in\mathbb{C}^{N\times 1}$, $k\in\Omega_S = \{K_P+1,\ldots,K_P+K_S\}$, containing noise only, namely $\boldsymbol{z}_k = \boldsymbol{n}_k$, $k\in\Omega_S$, are available and that such returns share the same statistical characterization of the noise components in the primary data (the so-called homogeneous environment [4]). Finally, we assume that the $\boldsymbol{n}_k$'s, $k\in\Omega_P\cup\Omega_S$, are independent random vectors.

[1]For a given matrix $\boldsymbol{A}\in\mathbb{C}^{n\times m}$, we will denote by $\langle\boldsymbol{A}\rangle$ the space spanned by the columns of $\boldsymbol{A}$ and by $\langle\boldsymbol{A}\rangle^{\perp}$ its orthogonal complement.

We stress again that the ABORT [8] has been derived assuming orthogonality in the quasi-whitened observation space, i.e., it uses condition (1) with $\boldsymbol{R}$ replaced by the sample covariance matrix based upon secondary data $\boldsymbol{z}_k$, $k\in\Omega_S$.

Summarizing, the detection problem to be solved can be formulated in terms of the following binary hypothesis test

$$\begin{cases} H_0: \begin{cases} \boldsymbol{z}_k = \boldsymbol{U}\boldsymbol{q}_k + \boldsymbol{n}_k, & k\in\Omega_P,\\ \boldsymbol{z}_k = \boldsymbol{n}_k, & k\in\Omega_S,\end{cases}\\ H_1: \begin{cases} \boldsymbol{z}_k = \alpha_k\boldsymbol{v} + \boldsymbol{n}_k, & k\in\Omega_P,\\ \boldsymbol{z}_k = \boldsymbol{n}_k, & k\in\Omega_S\end{cases}\end{cases} \tag{2}$$

where we suppose that $K_S \geqslant N$ and, as already stated, that $\langle\boldsymbol{R}^{-1/2}\boldsymbol{v}\rangle^{\perp} = \langle\boldsymbol{R}^{-1/2}\boldsymbol{U}\rangle$.

III. Detector Designs

Denote by $\boldsymbol{Z} = [\boldsymbol{Z}_P\boldsymbol{Z}_S]$ the overall data matrix, where $\boldsymbol{Z}_P = [\boldsymbol{z}_1\cdots\boldsymbol{z}_{K_P}]\in\mathbb{C}^{N\times K_P}$ is the primary data matrix and $\boldsymbol{Z}_S = [\boldsymbol{z}_{K_P+1}\cdots\boldsymbol{z}_{K_P+K_S}]\in\mathbb{C}^{N\times K_S}$ is the secondary data matrix. Moreover, let $K = K_P+K_S$, $\boldsymbol{Q} = [\boldsymbol{q}_1\cdots\boldsymbol{q}_{K_P}]\in\mathbb{C}^{(N-1)\times K_P}$, and $\boldsymbol{\alpha} = [\alpha_1\cdots\alpha_{K_P}]^T\in\mathbb{C}^{K_P\times 1}$, where the superscript ^{T} denotes transpose.

A. One-Step GLRT-Based Detector

We now derive the GLRT based upon primary and secondary data, which is tantamount to the following decision rule [12]

$$\frac{\max\limits_{\boldsymbol{\alpha}}\max\limits_{\boldsymbol{R}} f_1(\boldsymbol{Z};\boldsymbol{R},\boldsymbol{\alpha})}{\max\limits_{\boldsymbol{Q}}\max\limits_{\boldsymbol{R}} f_0(\boldsymbol{Z};\boldsymbol{R},\boldsymbol{Q})} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{3}$$

where $f_j(\boldsymbol{Z};\cdot)$ is the probability density function (pdf) of $\boldsymbol{Z}$ under the H_j hypothesis, $j=0,1$, and γ is the threshold value to be set in order to ensure the desired probability of false alarm (P_{fa}). Note that $f_0(\boldsymbol{Z};\boldsymbol{R},\boldsymbol{Q})$ is not explicitly indexed by $\boldsymbol{U}$ in (3). In fact, by (1), $\boldsymbol{U}$ is a function of $\boldsymbol{v}$ and $\boldsymbol{R}$ and, as a consequence, it is not an independent parameter to be jointly estimated with $\boldsymbol{R}$ and $\boldsymbol{Q}$.

The pdf of $\boldsymbol{Z}$, under H_0, can be written as

$$\begin{aligned} f_0(\boldsymbol{Z};\boldsymbol{R},\boldsymbol{Q}) &= \left[\pi^N\det(\boldsymbol{R})\right]^{-K}\\ &\times\exp\left\{-\mathrm{tr}\left[\boldsymbol{R}^{-1}\left(\boldsymbol{S}+(\boldsymbol{Z}_P-\boldsymbol{U}\boldsymbol{Q})(\boldsymbol{Z}_P-\boldsymbol{U}\boldsymbol{Q})^{\dagger}\right)\right]\right\}\end{aligned} \tag{4}$$

where $\boldsymbol{S} = \boldsymbol{Z}_S\boldsymbol{Z}_S^{\dagger}\in\mathbb{C}^{N\times N}$ is K_S times the sample covariance matrix based on secondary data,[2] $\det(\cdot)$ and $\mathrm{tr}(\cdot)$ are the determinant and the trace of a square matrix, respectively, and the superscript † denotes conjugate transpose. In order to compute the compressed likelihood under H_0, observe that the maximum of $f_0(\boldsymbol{Z};\boldsymbol{R},\boldsymbol{Q})$ with respect to $\boldsymbol{Q}$ can be obtained as follows [10]:

$$\begin{aligned}\widehat{\boldsymbol{Q}} &= \arg\max_{\boldsymbol{Q}} f_0(\boldsymbol{Z};\boldsymbol{R},\boldsymbol{Q})\\ &= \arg\min_{\boldsymbol{Q}}\mathrm{tr}\left\{\boldsymbol{R}^{-1}(\boldsymbol{Z}_P-\boldsymbol{U}\boldsymbol{Q})(\boldsymbol{Z}_P-\boldsymbol{U}\boldsymbol{Q})^{\dagger}\right\}\\ &= \arg\min_{\boldsymbol{Q}}\mathrm{tr}\left\{(\boldsymbol{X}_P-\boldsymbol{U}_w\boldsymbol{Q})^{\dagger}(\boldsymbol{X}_P-\boldsymbol{U}_w\boldsymbol{Q})\right\}\\ &= \left(\boldsymbol{U}_w^{\dagger}\boldsymbol{U}_w\right)^{-1}\boldsymbol{U}_w^{\dagger}\boldsymbol{X}_P\end{aligned} \tag{5}$$

[2]Note that the matrix $\boldsymbol{S}$ is invertible (with probability one) if $K_S\geqslant N$.

where $\boldsymbol{X}_P = \boldsymbol{R}^{-1/2}\boldsymbol{Z}_P$ and $\boldsymbol{U}_w = \boldsymbol{R}^{-1/2}\boldsymbol{U}$. If we now substitute this estimate into the expression of the pdf (4), after some algebra, we get

$$f_0(\boldsymbol{Z};\boldsymbol{R},\widehat{\boldsymbol{Q}}) = \left[\pi^N \det(\boldsymbol{R})\right]^{-K} \exp\left\{-\mathrm{tr}(\boldsymbol{R}^{-1}\boldsymbol{S})\right\} \times \exp\left\{-\mathrm{tr}\left[\boldsymbol{X}_P^\dagger \left(\boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{U}_w}\right)\boldsymbol{X}_P\right]\right\} \quad (6)$$

where

$$\boldsymbol{P}_{\boldsymbol{U}_w} = \boldsymbol{U}_w\left(\boldsymbol{U}_w^\dagger \boldsymbol{U}_w\right)^{-1}\boldsymbol{U}_w^\dagger$$

is the projector onto the range space of $\boldsymbol{U}_w = \boldsymbol{R}^{-1/2}\boldsymbol{U}$ and $\boldsymbol{I}_N$ denotes the N-dimensional identity matrix. If we now recall condition (1), we have that $\boldsymbol{I}_N - \mathbf{P}_{\boldsymbol{U}_w}$ can be replaced with a projection matrix onto $\langle \boldsymbol{R}^{-1/2}\boldsymbol{v}\rangle$, i.e.,

$$\boldsymbol{R}^{-1/2}\boldsymbol{v}(\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{v})^{-1}\boldsymbol{v}^\dagger\boldsymbol{R}^{-1/2}$$

which, substituted into (6), after some algebra, yields

$$f_0(\boldsymbol{Z};\boldsymbol{R},\widehat{\boldsymbol{Q}}) = \left[\pi^N \det(\boldsymbol{R})\right]^{-K} \exp\left\{-\mathrm{tr}(\boldsymbol{R}^{-1}\boldsymbol{S})\right\} \times \exp\left\{-\frac{\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{Z}_P\boldsymbol{Z}_P^\dagger\boldsymbol{R}^{-1}\boldsymbol{v}}{\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{v}}\right\}. \quad (7)$$

In order to maximize $f_0(\boldsymbol{Z};\boldsymbol{R},\widehat{\boldsymbol{Q}})$ with respect to $\boldsymbol{R}$, let us begin with the following proposition [13].

Proposition 1: Let $\mathcal{D}_+$ be the set of all positive definite Hermitian matrices over the complex field, then

- the function $\ln f_0(\boldsymbol{Z};\boldsymbol{R},\widehat{\boldsymbol{Q}})$ admits maximum over $\mathcal{D}_+$;
- such a maximum occurs at a stationary point.

Proof: See Appendix I. ■

In the light of previous proposition, we have to search for the stationary points of $f_0(\boldsymbol{Z};\boldsymbol{R},\widehat{\boldsymbol{Q}})$. This can be accomplished by setting to zero the derivative[3] of $\ln f_0(\boldsymbol{Z};\boldsymbol{R},\widehat{\boldsymbol{Q}})$, with respect to the (h,l)th entry, $r_{h,l}$ say, $h \leqslant l$, of the Hermitian matrix $\boldsymbol{R}$, i.e.,

$$\begin{aligned}\frac{\partial \ln f_0(\boldsymbol{Z};\boldsymbol{R},\widehat{\boldsymbol{Q}})}{\partial r_{h,l}} &= -K[\boldsymbol{R}^{-1}]_{h,l} + [\boldsymbol{R}^{-1}\boldsymbol{S}\boldsymbol{R}^{-1}]_{h,l} \\ &\quad - \sum_{k\in\Omega_P}\left[\boldsymbol{R}^{-1}\left[\left(\boldsymbol{z}_k - \frac{\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{z}_k}{\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{v}}\boldsymbol{v}\right)\right.\right. \\ &\quad \left.\left.\times\left(\boldsymbol{z}_k - \frac{\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{z}_k}{\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{v}}\boldsymbol{v}\right)^\dagger - \boldsymbol{z}_k\boldsymbol{z}_k^\dagger\right]\boldsymbol{R}^{-1}\right]_{h,l} \\ &= 0\end{aligned}$$

where $[\cdot]_{h,l}$ denotes the (h,l)-th entry of the matrix argument. Such conditions are equivalent to the following matrix equation

$$K\boldsymbol{R}^{-1} = \boldsymbol{R}^{-1}\boldsymbol{S}\boldsymbol{R}^{-1} - \sum_{k\in\Omega_P}\boldsymbol{R}^{-1}\left[\left(\boldsymbol{z}_k - \frac{\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{z}_k}{\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{v}}\boldsymbol{v}\right) \times \left(\boldsymbol{z}_k - \frac{\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{z}_k}{\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{v}}\boldsymbol{v}\right)^\dagger - \boldsymbol{z}_k\boldsymbol{z}_k^\dagger\right]\boldsymbol{R}^{-1}$$

which can be rewritten in a more compact form as

$$K\boldsymbol{R} = \boldsymbol{S}_0 - (\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\beta}^\dagger)(\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\beta}^\dagger)^\dagger \quad (8)$$

where the Hermitian matrix $\boldsymbol{S}_0 \in \mathbb{C}^{N\times N}$ is defined as $\boldsymbol{S}_0 = \boldsymbol{S} + \boldsymbol{Z}_P\boldsymbol{Z}_P^\dagger$ and $\boldsymbol{\beta} \in \mathbb{C}^{K_P\times 1}$ is given by

$$\boldsymbol{\beta} = \frac{\boldsymbol{Z}_P^\dagger\boldsymbol{R}^{-1}\boldsymbol{v}}{\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{v}}. \quad (9)$$

If we pre- and post-multiply both sides of (8) by $\boldsymbol{R}^{-1}$ and $\boldsymbol{R}^{-1}\boldsymbol{v}$, respectively, we obtain

$$K\boldsymbol{R}^{-1}\boldsymbol{v} = \boldsymbol{R}^{-1}\left[\boldsymbol{S}_0 - (\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\beta}^\dagger)(\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\beta}^\dagger)^\dagger\right]\boldsymbol{R}^{-1}\boldsymbol{v}.$$

Now note that

$$\begin{aligned}K\boldsymbol{R}^{-1}\boldsymbol{v} &= \boldsymbol{R}^{-1}\left[\boldsymbol{S}_0 - (\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\beta}^\dagger)(\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\beta}^\dagger)^\dagger\right]\boldsymbol{R}^{-1}\boldsymbol{v} \\ &= \boldsymbol{R}^{-1}\boldsymbol{S}_0\boldsymbol{R}^{-1}\boldsymbol{v} - \boldsymbol{R}^{-1}(\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\beta}^\dagger) \\ &\quad \times \underbrace{\left(\boldsymbol{Z}_P^\dagger\boldsymbol{R}^{-1}\boldsymbol{v} - \boldsymbol{\beta}\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{v}\right)}_{=0} \\ &= \boldsymbol{R}^{-1}\boldsymbol{S}_0\boldsymbol{R}^{-1}\boldsymbol{v}\end{aligned}$$

i.e.,

$$K\boldsymbol{R}^{-1}\boldsymbol{v} = \boldsymbol{R}^{-1}\boldsymbol{S}_0\boldsymbol{R}^{-1}\boldsymbol{v}. \quad (10)$$

By premultiplying both sides of (10) by $\boldsymbol{S}_0^{-1}\boldsymbol{R}$ we obtain[4]

$$\boldsymbol{R}^{-1}\boldsymbol{v} = K\boldsymbol{S}_0^{-1}\boldsymbol{v}$$

which substituted into the expression of $\boldsymbol{\beta}$ gives

$$\boldsymbol{\beta} = \frac{\boldsymbol{Z}_P^\dagger\boldsymbol{S}_0^{-1}\boldsymbol{v}}{\boldsymbol{v}^\dagger\boldsymbol{S}_0^{-1}\boldsymbol{v}}.$$

Using this $\boldsymbol{\beta}$ into (8) we find the unique stationary point of $f_0(\boldsymbol{Z};\boldsymbol{R},\widehat{\boldsymbol{Q}})$, $\widehat{\boldsymbol{R}}_0$ say,

$$\widehat{\boldsymbol{R}}_0 = \frac{1}{K}\left[\boldsymbol{S}_0 - \left(\boldsymbol{Z}_P - \frac{\boldsymbol{v}\boldsymbol{v}^\dagger\boldsymbol{S}_0^{-1}\boldsymbol{Z}_P}{\boldsymbol{v}^\dagger\boldsymbol{S}_0^{-1}\boldsymbol{v}}\right)\left(\boldsymbol{Z}_P - \frac{\boldsymbol{v}\boldsymbol{v}^\dagger\boldsymbol{S}_0^{-1}\boldsymbol{Z}_P}{\boldsymbol{v}^\dagger\boldsymbol{S}_0^{-1}\boldsymbol{v}}\right)^\dagger\right].$$

Based upon Proposition 1, we have that $\widehat{\boldsymbol{R}}_0$ is a positive definite Hermitian matrix and that it corresponds to a maximum

[3]We make use of the following definition for the derivative of a real function $f(\boldsymbol{R})$ with respect to a complex variable

$$\frac{\partial f(\boldsymbol{R})}{\partial r_{h,l}} = \frac{1}{2}\left[\frac{\partial f(\boldsymbol{R})}{\partial x_{h,l}} + j\frac{\partial f(\boldsymbol{R})}{\partial y_{h,l}}\right]$$

where $r_{h,l} = x_{h,l} + jy_{h,l}$ is the (h,l)th entry of $\boldsymbol{R}$, $h < l$; see [14], [15] for more details.

[4]Observe that matrix $\boldsymbol{S}_0 = \boldsymbol{S} + \boldsymbol{Z}_P\boldsymbol{Z}_P^\dagger$ is invertible since $\boldsymbol{S}$ is invertible and $\boldsymbol{Z}_P\boldsymbol{Z}_P^\dagger$ is positive semidefinite [16].

of $f_0(\boldsymbol{Z};\boldsymbol{R},\widehat{\boldsymbol{Q}})$; summarizing, it is the maximum likelihood estimate (MLE) of $\boldsymbol{R}$ under the H_0 hypothesis. Now it only remains to compute the compressed likelihood under H_0; to this end, note that from (8) it follows that:

$$K\boldsymbol{I}_N = \boldsymbol{R}^{-1}\boldsymbol{S} + \boldsymbol{R}^{-1}\boldsymbol{Z}_P\boldsymbol{Z}_P^\dagger - \boldsymbol{R}^{-1}(\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\beta}^\dagger)(\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\beta}^\dagger)^\dagger$$

from which we have

$$\boldsymbol{R}^{-1}\boldsymbol{S} = K\boldsymbol{I}_N - \boldsymbol{R}^{-1}\boldsymbol{Z}_P\boldsymbol{Z}_P^\dagger + \boldsymbol{R}^{-1}(\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\beta}^\dagger)(\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\beta}^\dagger)^\dagger.$$

By taking the trace at both sides of previous equation and using (9), we obtain

$$\begin{aligned}\mathrm{tr}(\boldsymbol{R}^{-1}\boldsymbol{S}) &= KN - \mathrm{tr}\left(\boldsymbol{R}^{-1}\boldsymbol{Z}_P\boldsymbol{Z}_P^\dagger\right) \\ &\quad + \mathrm{tr}\left[\boldsymbol{R}^{-1}(\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\beta}^\dagger)(\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\beta}^\dagger)^\dagger\right] \\ &= KN - \frac{\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{Z}_P\boldsymbol{Z}_P^\dagger\boldsymbol{R}^{-1}\boldsymbol{v}}{\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{v}}\end{aligned} \tag{11}$$

which, substituted into the compressed likelihood (7), with $\widehat{\boldsymbol{R}}_0$ in place of $\boldsymbol{R}$, yields the final expression for the denominator of (3)

$$f_0(\boldsymbol{Z};\widehat{\boldsymbol{R}}_0,\widehat{\boldsymbol{Q}}) = (\pi e)^{-NK}\left[\det(\widehat{\boldsymbol{R}}_0)\right]^{-K}.$$

The solution to the optimization problem under the H_1 hypothesis is well known (see, for instance, [4], [7]) and the compressed likelihood is given by

$$f_1(\boldsymbol{Z};\widehat{\boldsymbol{R}}_1,\widehat{\boldsymbol{\alpha}}) = (\pi e)^{-NK}\left[\det(\widehat{\boldsymbol{R}}_1)\right]^{-K}$$

where

$$\widehat{\boldsymbol{R}}_1 = \frac{1}{K}\left[\boldsymbol{S} + \left(\boldsymbol{Z}_P - \frac{\boldsymbol{v}\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}\right)\left(\boldsymbol{Z}_P - \frac{\boldsymbol{v}\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}\right)^\dagger\right].$$

Thus, we conclude that the one-step GLRT for problem (2) is equivalent to

$$\frac{\det(\widehat{\boldsymbol{R}}_0)}{\det(\widehat{\boldsymbol{R}}_1)} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{12}$$

where γ is a proper modification of the original threshold in (3). Test (12) can also be expressed in an alternative form as follows

$$\frac{1}{(\ell_{\mathrm{GLRT}} - 1)^2 \det\left(\boldsymbol{I}_{K_P} + \boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{13}$$

where

$$\ell_{\mathrm{GLRT}} = \frac{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\left(\boldsymbol{I}_{K_P} + \boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\right)^{-1}\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}$$

is the Kelly GLRT (see [4] and [7]). For the special case $K_P = 1$, i.e., the case of a single cell under test, expression (13) simplifies to

$$\frac{1}{(\ell_{\mathrm{GLRT}} - 1)^2\left(1 + \boldsymbol{z}_1^\dagger\boldsymbol{S}^{-1}\boldsymbol{z}_1\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

where ℓ_{GLRT} can be written as in [1]:

$$\ell_{\mathrm{GLRT}} = \frac{|\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{z}_1|^2}{\left(1 + \boldsymbol{z}_1^\dagger\boldsymbol{S}^{-1}\boldsymbol{z}_1\right)(\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v})}.$$

Details on the derivation of expression (13) for the proposed GLRT can be found in the Appendix II.

B. Two-Step GLRT-Based Detector

This section is devoted to the derivation of an *ad hoc* detector for the hypothesis test (2) based upon the two-step GLRT design criterion. As a matter of fact, it can be straightforwardly derived using results in [10] and will be reviewed here only for the sake of clarity.

The rationale of the design procedure is the following: first assume that the covariance matrix $\boldsymbol{R}$ is known and derive the GLRT based on primary data $\boldsymbol{Z}_P$. Then, a fully adaptive detector is obtained by replacing the unknown matrix $\boldsymbol{R}$ with $\boldsymbol{S}$, i.e., K_S times the sample covariance matrix based on secondary data only.

Step 1) The GLRT, under the assumption that $\boldsymbol{R}$ is known, is given by

$$\frac{\max_{\boldsymbol{\alpha}} f_1(\boldsymbol{Z}_P;\boldsymbol{\alpha})}{\max_{\boldsymbol{Q}} f_0(\boldsymbol{Z}_P;\boldsymbol{Q})} \underset{H_0}{\overset{H_1}{\gtrless}} \eta \tag{14}$$

where $f_j(\boldsymbol{Z}_P;\cdot)$ is the pdf of $\boldsymbol{Z}_P$ under the H_j hypothesis, $j = 0, 1$, and η the threshold value to be set according to the desired P_{fa}.

If we write the explicit expression of the pdf's, we come up with the decision rule that is shown in the equation at the bottom of the page. Minima over $\boldsymbol{\alpha}$

$$\frac{\max_{\boldsymbol{\alpha}}\left[\pi^N\det(\boldsymbol{R})\right]^{-K}\exp\left\{-\mathrm{tr}\left[\boldsymbol{R}^{-1}(\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\alpha}^T)(\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\alpha}^T)^\dagger\right]\right\}}{\max_{\boldsymbol{Q}}\left[\pi^N\det(\boldsymbol{R})\right]^{-K}\exp\left\{-\mathrm{tr}\left[\boldsymbol{R}^{-1}(\boldsymbol{Z}_P - \boldsymbol{U}\boldsymbol{Q})(\boldsymbol{Z}_P - \boldsymbol{U}\boldsymbol{Q})^\dagger\right]\right\}} = \frac{\exp\left\{-\min_{\boldsymbol{\alpha}}\mathrm{tr}\left[\boldsymbol{R}^{-1}(\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\alpha}^T)(\boldsymbol{Z}_P - \boldsymbol{v}\boldsymbol{\alpha}^T)^\dagger\right]\right\}}{\exp\left\{-\min_{\boldsymbol{Q}}\mathrm{tr}\left[\boldsymbol{R}^{-1}(\boldsymbol{Z}_P - \boldsymbol{U}\boldsymbol{Q})(\boldsymbol{Z}_P - \boldsymbol{U}\boldsymbol{Q})^\dagger\right]\right\}} \underset{H_0}{\overset{H_1}{\gtrless}} \eta.$$

and $\boldsymbol{Q}$ can be computed using the result given in (5) to obtain

$$\frac{\exp\left\{-\mathrm{tr}\left[\boldsymbol{X}_P^\dagger\left(\boldsymbol{I}_N-\boldsymbol{P}_{\boldsymbol{v}_w}\right)\boldsymbol{X}_P\right]\right\}}{\exp\left\{-\mathrm{tr}\left[\boldsymbol{X}_P^\dagger\left(\boldsymbol{I}_N-\boldsymbol{P}_{\boldsymbol{U}_w}\right)\boldsymbol{X}_P\right]\right\}}\underset{H_0}{\overset{H_1}{\gtrless}}\eta$$

where $\boldsymbol{P}_{\boldsymbol{v}_w}=\boldsymbol{v}_w(\boldsymbol{v}_w^\dagger\boldsymbol{v}_w)^{-1}\boldsymbol{v}_w^\dagger$ is the projection matrix onto the range space of the vector $\boldsymbol{v}_w=\boldsymbol{R}^{-1/2}\boldsymbol{v}$. Condition (1) implies that $\boldsymbol{I}_N-\mathbf{P}_{\boldsymbol{U}_w}=\boldsymbol{P}_{\boldsymbol{v}_w}$ and, hence, the GLRT becomes

$$\begin{aligned}\exp&\left\{-\mathrm{tr}\left[\boldsymbol{X}_P^\dagger\left(\boldsymbol{I}_N-\boldsymbol{P}_{\boldsymbol{v}_w}\right)\boldsymbol{X}_P\right]+\mathrm{tr}\left[\boldsymbol{X}_P^\dagger\boldsymbol{P}_{\boldsymbol{v}_w}\boldsymbol{X}_P\right]\right\}\\&=\exp\left\{-\mathrm{tr}\left[\boldsymbol{X}_P^\dagger\boldsymbol{X}_P\right]+2\mathrm{tr}\left[\boldsymbol{X}_P^\dagger\boldsymbol{P}_{\boldsymbol{v}_w}\boldsymbol{X}_P\right]\right\}\underset{H_0}{\overset{H_1}{\gtrless}}\eta.\end{aligned}$$

Taking the natural logarithm and substituting for $\boldsymbol{X}_P$ and $\boldsymbol{P}_{\boldsymbol{v}_w}$ their expressions as functions of the original quantities, we come up with the following *ad hoc* detector:

$$\frac{\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{Z}_P\boldsymbol{Z}_P^\dagger\boldsymbol{R}^{-1}\boldsymbol{v}}{\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{v}}-\frac{1}{2}\mathrm{tr}\left(\boldsymbol{Z}_P^\dagger\boldsymbol{R}^{-1}\boldsymbol{Z}_P\right)\underset{H_0}{\overset{H_1}{\gtrless}}\eta \qquad (15)$$

where η is a proper modification of the original threshold in (14).

Step 2) Detector (15) can be made fully adaptive by plugging $\boldsymbol{S}$ in place of $\boldsymbol{R}$ to obtain

$$\frac{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}-\frac{1}{2}\mathrm{tr}\left(\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\right)\underset{H_0}{\overset{H_1}{\gtrless}}\eta \qquad (16)$$

or, equivalently

$$\ell_{\mathrm{GAMF}}-\frac{1}{2}\mathrm{tr}\left(\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\right)\underset{H_0}{\overset{H_1}{\gtrless}}\eta$$

where ℓ_{GAMF} is the generalized adaptive matched filter (GAMF) proposed in [4], i.e.,

$$\ell_{\mathrm{GAMF}}=\frac{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}.$$

C. On the CFAR Property of the Proposed Solutions

It is of interest to investigate the CFAR behavior of the proposed algorithms. To this end, we follow the usual assumption and define the P_{fa} as the probability to declare that a useful target echo is present (i.e., to accept the H_1 hypothesis) when data under test contain noise only, i.e., $\boldsymbol{z}_k=\boldsymbol{n}_k$, $k\in\Omega_P\cup\Omega_S$. With this definition in mind, we have that both the GLRT (13) and the *ad hoc* detector (16) possess the CFAR property with respect to the unknown covariance matrix $\boldsymbol{R}$ (see Appendix III for the proof).

IV. Performance Assessment

This section is devoted to a performance assessment of the presented algorithms in terms of probability of detection (P_d) and selectivity, also in comparison to previously proposed solutions. More precisely, we compare our detectors to the GLRT, the GAMF, and the generalized adaptive subspace detector (GASD) [4], and to the ABORT[5] proposed in [8]. For the sake of clarity, such competitors are repeated in the following:

$$\ell_{\mathrm{GLRT}}=\frac{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\left(\boldsymbol{I}_{K_P}+\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\right)^{-1}\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}\underset{H_0}{\overset{H_1}{\gtrless}}\gamma \qquad (17)$$

$$\ell_{\mathrm{GAMF}}=\frac{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}\underset{H_0}{\overset{H_1}{\gtrless}}\gamma \qquad (18)$$

$$\ell_{\mathrm{GASD}}=\frac{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}{(\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v})\mathrm{tr}\left(\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\right)}\underset{H_0}{\overset{H_1}{\gtrless}}\gamma. \qquad (19)$$

For the case of a single snapshot under test (i.e., $K_P=1$), previous detectors reduce to well-known detection schemes; precisely, the GLRT reduces to that presented in [1], the GAMF to the adaptive matched filter (AMF) [11], and the GASD to the adaptive normalized matched filter (ANMF) [17], also known as adaptive coherence estimator (ACE) [18, and references therein]; in addition, the ABORT [8] is given by

$$\ell_{\mathrm{ABORT}}=\frac{1+\frac{|\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{z}_1|^2}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}}{2+\boldsymbol{z}_1^\dagger\boldsymbol{S}^{-1}\boldsymbol{z}_1}\underset{H_0}{\overset{H_1}{\gtrless}}\gamma. \qquad (20)$$

Moreover, it is worth pointing out here that all of the aforementioned detectors guarantee the CFAR property with respect to the noise covariance matrix $\boldsymbol{R}$.

Analysis is carried out by resorting to standard Monte Carlo counting techniques. More precisely, in order to evaluate the thresholds necessary to ensure a preassigned value of P_{fa} and the P_d's, we resort to $100/P_{fa}$ and 10^4 independent trials, respectively. For simulation purposes we also set $N=20$, $P_{fa}=10^{-4}$, and

$$\boldsymbol{v}=\frac{1}{\sqrt{N}}[1\cdots1]^T.$$

Moreover, we assume that all of the range cells in Ω_P contain, under the H_1 hypothesis, target returns generated with one and the same non-fluctuating radar cross section. The signal-to-noise ratio (SNR) is defined as

$$\mathrm{SNR}=\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{v}\sum_{k\in\Omega_P}|\alpha_k|^2 \qquad (21)$$

where $|\cdot|$ is the magnitude of a complex number. As to the noise, it is modeled as an exponentially correlated complex normal random vector with one-lag correlation coefficient $\rho=0.95$; namely, the (i,j)th element of the noise covariance matrix $\boldsymbol{R}$ is given by $\sigma^2\rho^{|i-j|}$, $i,j=1,\ldots,N$, with $\sigma^2=1$.

We conduct the analysis in two phases: first, we compare the performance of the different detectors for matched mainlobe targets (Figs. 1–4); second, we study the selectivity properties of the detectors (Figs. 5–9). For this last case, detector (13) is assumed as a benchmark and the comparison is made only with detector (16), the GASD, and the ABORT, since the GAMF and the GLRT do not provide high selectivity (see also [8]).

[5]Comparison with the ABORT can be made only for the case of a single cell under test, i.e., $K_P=1$.

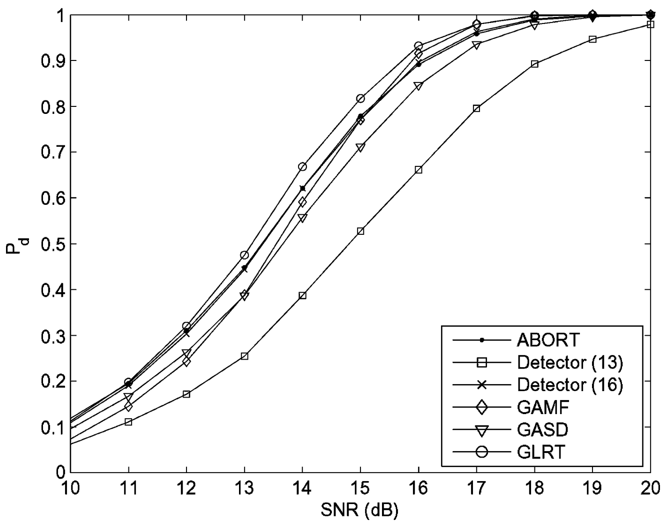

Fig. 1. P_d versus SNR for the ABORT, the proposed detectors (13) and (16), the GAMF, the GASD, and the GLRT, $N = 20, K_P = 1, K_S = 40, P_{fa} = 10^{-4}$, and $\rho = 0.95$.

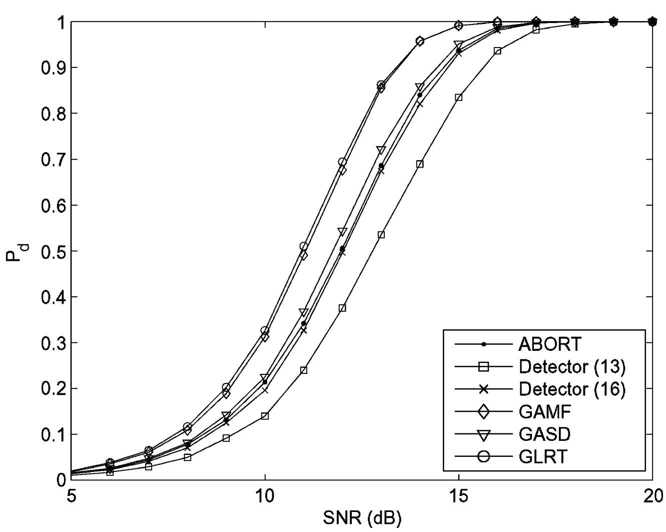

Fig. 2. P_d versus SNR for the ABORT, the proposed detectors (13) and (16), the GAMF, the GASD, and the GLRT, $N = 20, K_P = 1, K_S = 80, P_{fa} = 10^{-4}$, and $\rho = 0.95$.

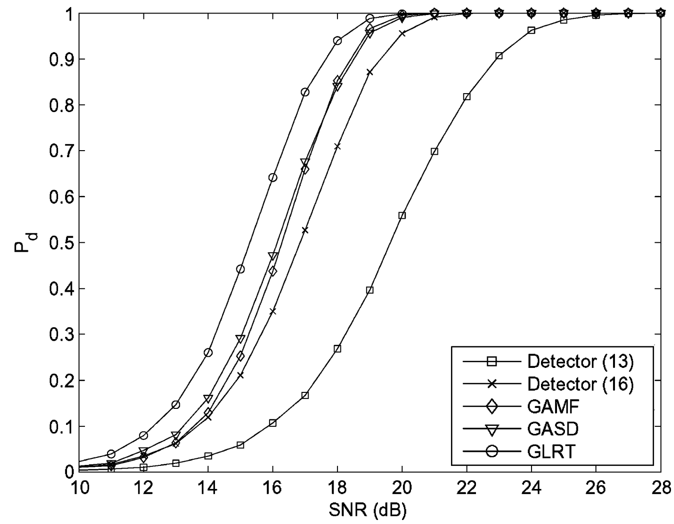

Fig. 3. P_d versus SNR for the proposed detectors (13) and (16), the GAMF, the GASD, and the GLRT, $N = 20, K_P = 5, K_S = 40, P_{fa} = 10^{-4}$, and $\rho = 0.95$.

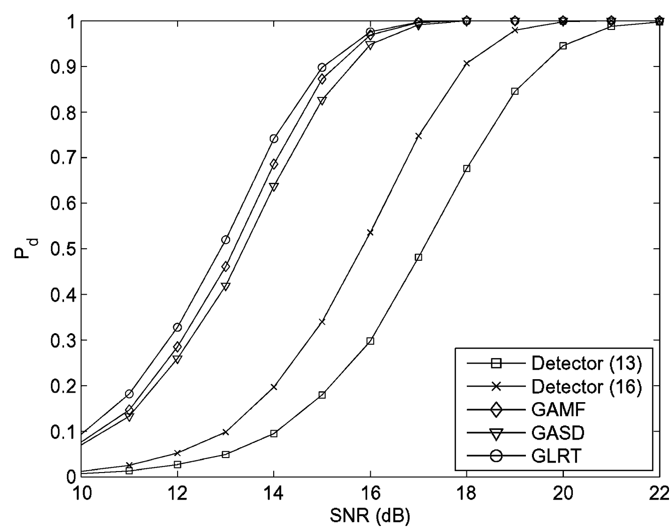

Fig. 4. P_d versus SNR for the proposed detectors (13) and (16), the GAMF, the GASD, and the GLRT, $N = 20, K_P = 5, K_S = 80, P_{fa} = 10^{-4}$, and $\rho = 0.95$.

In Fig. 1 we plot the curves of P_d versus SNR of detectors (13) and (16) proposed herein together with those of the GLRT (17), the GAMF (18), the GASD (19), and the ABORT (20), for $K_P = 1$ and $K_S = 40$. As it can be seen, the best performance is attained by the GLRT and the GAMF (for high SNR), while detector (16) and the ABORT experience a loss of less than 0.5 dB at $P_d = 0.9$; the GASD looses about 1 dB with respect to the GLRT and, finally, the horizontal displacement between detector (13) and the GLRT is about 2.5 dB. In Fig. 2 we have the same system parameters as in Fig. 1, but for $K_S = 80$. Results indicate that the hierarchy of the six considered detectors remains approximately the same, but for the fact that the GASD has now practically the same performance of the ABORT and of detector (16).

In Fig. 3 we plot the P_d's of the same detectors considered in Figs. 1 and 2 (but for the ABORT), for $K_P = 5$, $K_S = 40$. It is seen that the best performance is still attained by the GLRT, while both the GAMF and the GASD experience a loss of about 1 dB at $P_d = 0.9$. The losses of detectors (13) and (16) are larger and amount to 2 and 5 dB, respectively. In Fig. 4 we plot the P_d's for $K_S = 80$ and the remaining parameters as in Fig. 3; again, the increase of the number of secondary data does not significantly affect the ranking of the considered detectors, but for the fact that the GAMF and the GASD are closer to the GLRT.

In Figs. 5–9 we present a selectivity analysis of the detectors for the case of mismatch between design and operating steering vector, for $K_S = 80$ and different values of K_P. More precisely, we plot contours of constant P_d, as functions of the SNR (21), with $\boldsymbol{v}$ replaced by the actual steering vector, $\boldsymbol{v}_m$ say. To this end, we define

$$\cos^2\theta = \frac{|\boldsymbol{v}^\dagger \boldsymbol{R}^{-1}\boldsymbol{v}_m|^2}{(\boldsymbol{v}^\dagger \boldsymbol{R}^{-1}\boldsymbol{v})\left(\boldsymbol{v}_m^\dagger \boldsymbol{R}^{-1}\boldsymbol{v}_m\right)}$$

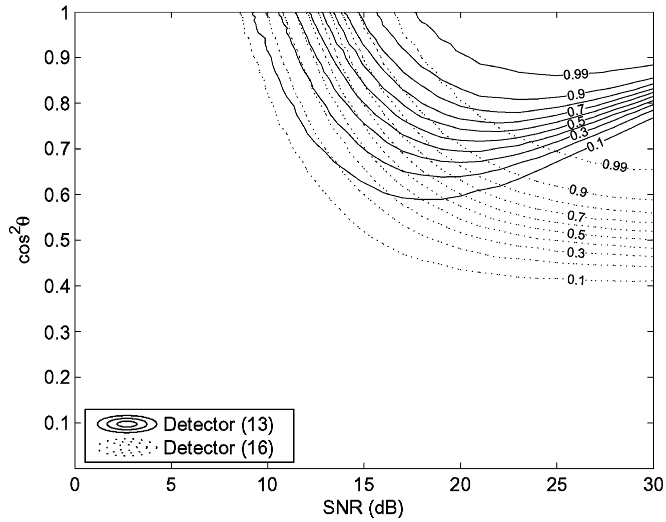

Fig. 5. Contours of constant P_d for detectors (13) and (16), $N = 20, K_P = 1, K_S = 80, P_{fa} = 10^{-4}$, and $\rho = 0.95$.

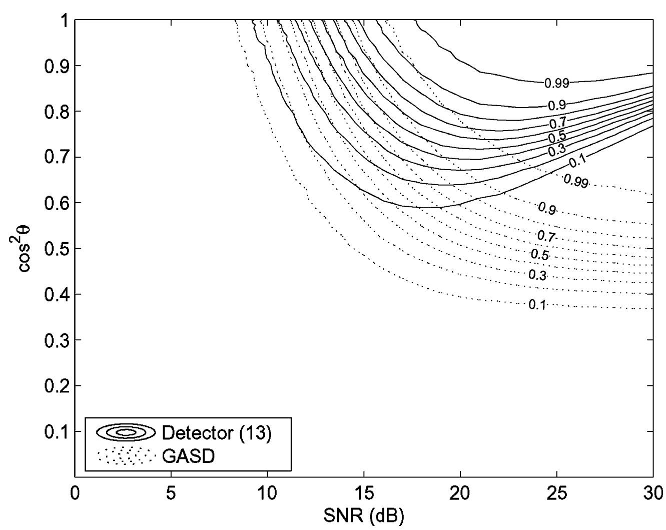

Fig. 6. Contours of constant P_d for detector (13) and GASD (equivalently, ACE), $N = 20, K_P = 1, K_S = 80, P_{fa} = 10^{-4}$, and $\rho = 0.95$.

where θ is the mismatch angle between the nominal steering vector $\boldsymbol{v}$ and its mismatched version $\boldsymbol{v}_m$ in the whitened observation space. Observe that $\cos^2\theta = 1$ corresponds to perfect match (H_1 hypothesis), while $\cos^2\theta = 0$ corresponds to the H_0 hypothesis. Figs. 5, 6, and 7 contain the performance of detector (13) as it compares to detector (16), the GASD, and the ABORT, respectively, for $K_P = 1$. Inspection of the figures shows that, for the considered system parameters, detector (13) exhibits the strongest performance in terms of mismatched signals rejection, at the price of a certain detection loss for matched ones; notice also that detector (13) is even stronger than the GASD, which, for the case at hand, coincides with the ACE (Fig. 6). Moreover, we can see that detector (16), the GASD, and the ABORT have basically the same performance. In Figs. 8 and 9 we plot the selectivity of detector (13) as it compares to detector (16) and the GASD, respectively, for $K_P = 5$ and remaining parameters as in Figs. 5–7. It is seen that in case of multiple cells under test,

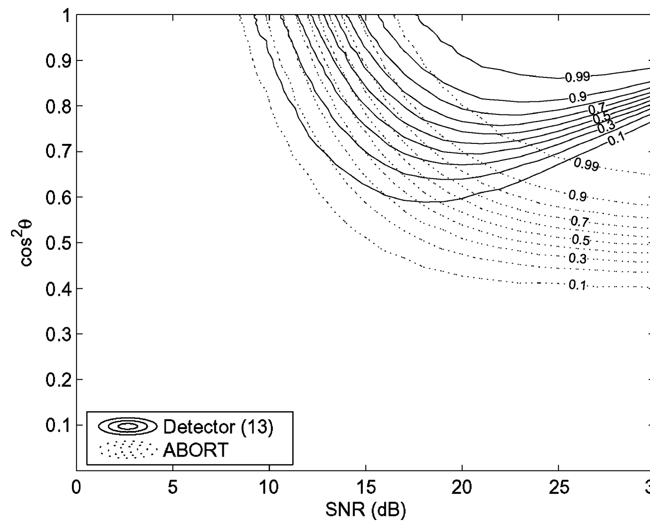

Fig. 7. Contours of constant P_d for detector (13) and ABORT, $N = 20, K_P = 1, K_S = 80, P_{fa} = 10^{-4}$, and $\rho = 0.95$.

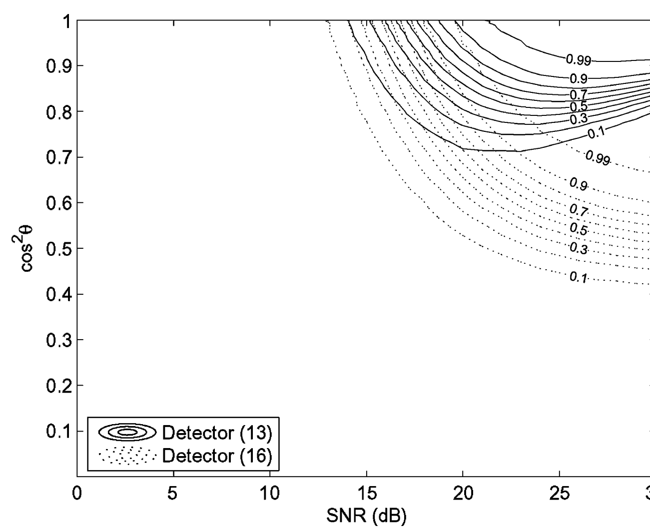

Fig. 8. Contours of constant P_d for detectors (13) and (16), $N = 20, K_P = 5, K_S = 80, P_{fa} = 10^{-4}$, and $\rho = 0.95$.

detector (13) still ensures better capabilities of mismatched signals rejection.

V. Conclusion

In this paper we have proposed GLRT-based adaptive detection schemes for possibly distributed targets capable of providing improved rejection capabilities against mismatched signals. We have considered the homogeneous Gaussian environment, i.e., the case where noise returns into primary and secondary data possess the same (unknown) covariance matrix. The capability to reject unwanted mismatched signals has been achieved by following the lead of the ABORT formulation, namely by adding fictitious signals under the null hypothesis. The novelty stems from the fact that in this paper we assume, under the H_0 hypothesis, the possible presence of signals orthogonal to the nominal steering vector in the whitened observation space, i.e., the space obtained after whitening the

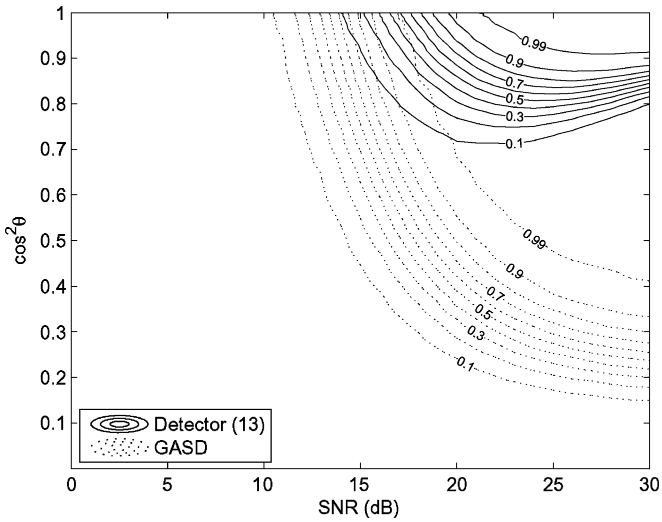

Fig. 9. Contours of constant P_d for detector (13) and GASD, $N = 20, K_P = 5, K_S = 80, P_{fa} = 10^{-4}$, and $\rho = 0.95$.

observables with the true covariance matrix. Proposed detectors possess the CFAR property with respect to the unknown covariance matrix of the noise. Finally, some simulation studies have been presented to assess the performance of the newly introduced detectors with respect to existing ones; such studies have shown that the (one-step) GLRT herein proposed can guarantee better rejection capabilities of mismatched signals, although at the price of a certain detection loss for matched signals.

A further way to apply results contained in this work might rely on the possibility to use two-stage algorithms [8], [19], [20], i.e., detection structures formed by two detectors: a first stage with poor selectivity properties to identify signals that deserve further attention and a second stage much more selective to discriminate whether detected signals are to be considered useful target echoes or unwanted mismatched signals; in the light of previous considerations, the proposed solution might be a good candidate for the second stage of detection.

Appendix I
Proof of Proposition 1

In this Appendix, we give the proof of Proposition 1. Our proof parallels that reported in [13] and is reviewed here for the sake of completeness. For notational convenience, let $J(\boldsymbol{R}) = \pi^{NK} f_0(\boldsymbol{Z}; \boldsymbol{R}, \widehat{\boldsymbol{Q}})$; Proposition 1 is thus equivalent to

Proposition 2: Let $\mathcal{D}_+$ be the set of all $N \times N$-dimensional, positive definite, and Hermitian matrices over the complex field, then:

- the function $\ln J(\boldsymbol{R})$ admits maximum over $\mathcal{D}_+$;
- such a maximum occurs at a stationary point.

Proof: To begin with, observe that an Hermitian matrix $\boldsymbol{R} \in \mathbb{C}^{N\times N}$ can be represented by $2N(N+1)/2 - N = N^2$ real numbers; hence, it is possible to map complex Hermitian matrices onto the space $\mathbb{R}^{N^2\times 1}$, where $\mathbb{R}$ is the real field. This can be accomplished by considering the real and imaginary part of all independent elements of $\boldsymbol{R} \in \mathbb{C}^{N\times N}$ as the components of a vector in $\mathbb{R}^{N^2\times 1}$.

Now let $\mathcal{D}$ be the set of all positive semidefinite Hermitian matrices. Then, $\mathcal{D}$ corresponds to a closed and unbounded subset of $\mathbb{R}^{N^2\times 1}$. The boundary between $\mathcal{D}$ and its complement corresponds to the set of singular positive semidefinite matrices. Our first step is to show that $\ln J(\boldsymbol{R})$ approaches minus infinity when $\boldsymbol{R}$, assumed to belong to $\mathcal{D}_+$, approaches a singularity. To this end, observe that

$$\begin{aligned} J(\boldsymbol{R}) &= [\det(\boldsymbol{R})]^{-K} \exp\left\{-\mathrm{tr}(\boldsymbol{R}^{-1}\boldsymbol{S})\right\} \\ &\quad \times \exp\left\{-\sum_{k\in\Omega_P} \frac{\boldsymbol{v}^\dagger \boldsymbol{R}^{-1}\boldsymbol{z}_k\boldsymbol{z}_k^\dagger \boldsymbol{R}^{-1}\boldsymbol{v}}{\boldsymbol{v}^\dagger \boldsymbol{R}^{-1}\boldsymbol{v}}\right\} \\ &\leqslant [\det(\boldsymbol{R})]^{-K} \exp\left\{-\mathrm{tr}(\boldsymbol{R}^{-1}\boldsymbol{S})\right\} \quad \forall \boldsymbol{R} \in \mathcal{D}_+. \end{aligned}$$

Moreover, by the arithmetic-geometric mean inequality [16], we have that

$$\mathrm{tr}(\boldsymbol{R}^{-1}\boldsymbol{S}) \geqslant N\left[\det(\boldsymbol{R}^{-1}\boldsymbol{S})\right]^{1/N} = N\frac{[\det(\boldsymbol{S})]^{1/N}}{[\det(\boldsymbol{R})]^{1/N}}, \ \forall \boldsymbol{R} \in \mathcal{D}_+.$$

Therefore, the following inequality holds true

$$\begin{aligned} \ln J(\boldsymbol{R}) &\leqslant -K\ln\det(\boldsymbol{R}) - \mathrm{tr}(\boldsymbol{R}^{-1}\boldsymbol{S}) \\ &\leqslant -K\ln\det(\boldsymbol{R}) - N\frac{[\det(\boldsymbol{S})]^{1/N}}{[\det(\boldsymbol{R})]^{1/N}}, \ \forall \boldsymbol{R} \in \mathcal{D}_+. \end{aligned} \quad (22)$$

Hence, when $\det(\boldsymbol{R})$ tends to zero (i.e., matrix $\boldsymbol{R}$ tends to be singular), $\ln J(\boldsymbol{R})$ tends to minus infinity. In other words, this implies that for each $\varepsilon < 0$ there exists a positive number a such that for each $\boldsymbol{R}$ in $\mathcal{D}_+$, such that $\det(\boldsymbol{R}) < a$, we have that $\ln J(\boldsymbol{R}) < \varepsilon$. Now let $\boldsymbol{R}_0$ be a positive definite Hermitian matrix (i.e., $\boldsymbol{R}_0$ belongs to $\mathcal{D}_+$). It follows that $\ln J(\boldsymbol{R}_0)$ exists and it is some finite value. Choose $\varepsilon_0 < \ln J(\boldsymbol{R}_0)$, it follows that there exists $a > 0$ such that for each $\boldsymbol{R}$ in $\mathcal{D}_+$, whose determinant is less than a, we have $\ln J(\boldsymbol{R}) < \ln J(\boldsymbol{R}_0)$.

We can thus define $\mathcal{C}_a$ to be the set of positive definite Hermitian matrices whose determinant is greater than or equal to a. Clearly we have that $\mathcal{C}_a$ is a closed and unbounded subset of $\mathcal{D}_+$, and that $\boldsymbol{R}_0$ belongs to $\mathcal{C}_a$. Moreover, the boundary between $\mathcal{C}_a$ and its complement consists of positive definite Hermitian matrices whose determinant is equal to a. The search for the possible maximum of $\ln J(\boldsymbol{R})$ over $\mathcal{D}_+$ can, thus, be restricted to $\mathcal{C}_a$.

Now let $b > 0$ such that each entry of $\boldsymbol{R}_0$ is less than or equal to b (in magnitude), and let $\mathcal{C}_b$ be the set of positive semidefinite Hermitian matrices whose elements are less than or equal to b (in magnitude). It is clear that $\mathcal{C}_b$ is compact in $\mathbb{R}^{N^2\times 1}$ and it contains $\boldsymbol{R}_0$ by construction. Let $\mathcal{D}_{ab}$ be the intersection between $\mathcal{C}_a$ and $\mathcal{C}_b$. Then, $\mathcal{D}_{ab}$ is compact and nonempty, since it contains at least $\boldsymbol{R}_0$. The boundary with its complement consists of positive definite Hermitian matrices with determinant equal to a and/or with some (diagonal) element equal to b in magnitude.[6] Since $\ln J(\boldsymbol{R})$ is a continuously differentiable function of $\boldsymbol{R}$, it is clear that it has a maximum on $\mathcal{D}_{ab}$ and such a maximum

[6]Recall that, as a consequence of the Cauchy-Schwarz inequality, for a positive semidefinite matrix $\boldsymbol{A} \in \mathbb{C}^{N\times N}$, whose entries are denoted by $a_{m,n}$, $m, n \in \{1, \ldots, N\}$, we have that

$$|a_{m,n}| \leqslant \max_i |a_{i,i}|.$$

is equal to or greater than $\ln J(\boldsymbol{R}_0)$. To complete the proof of the first statement, we have to show that for a b large enough, each point outside $\mathcal{D}_{ab}$ (but still in $\mathcal{D}_+$) does not provide values of $\ln J(\boldsymbol{R})$ greater than $\ln J(\boldsymbol{R}_0)$. To this end, let us consider the eigenvalue decomposition of the matrix $\boldsymbol{R}$, i.e.

$$\boldsymbol{R} = \boldsymbol{W}\boldsymbol{\Lambda}\boldsymbol{W}^\dagger$$

where $\boldsymbol{\Lambda} \in \mathbb{C}^{N\times N}$ is a diagonal matrix whose diagonal entries are the eigenvalues of $\boldsymbol{R}$ and $\boldsymbol{W} \in \mathbb{C}^{N\times N}$ is a unitary matrix whose columns are the corresponding orthonormal eigenvectors. We have

$$\mathrm{tr}(\boldsymbol{R}^{-1}\boldsymbol{S}) = \mathrm{tr}(\boldsymbol{W}^\dagger\boldsymbol{R}^{-1}\boldsymbol{W}\boldsymbol{W}^\dagger\boldsymbol{S}\boldsymbol{W}) = \sum_{i=1}^{N}\frac{q_i}{\lambda_i}$$

where $\lambda_1,\ldots,\lambda_N$ are the eigenvalues of $\boldsymbol{R}$ arranged in decreasing order and $q_1,\ldots,q_N$ are the diagonal elements of $\boldsymbol{W}^\dagger\boldsymbol{S}\boldsymbol{W}$. If s is the minimum eigenvalue of $\boldsymbol{S}$, then $q_i \geqslant s$, for all $i \in \{1,\ldots,N\}$. Then,

$$\begin{aligned}\ln J(\boldsymbol{R}) &\leqslant -\sum_{i=1}^{N}\left[K\ln(\lambda_i) + \frac{s}{\lambda_i}\right]\\ &\leqslant -K\ln(\lambda_1) - \frac{s}{\lambda_1}\\ &\quad - K(N-1)\left[\ln\left(\frac{s}{K}\right)+1\right] \qquad (23)\end{aligned}$$

where the last inequality follows from the fact that for $x > 0$

$$-K\ln(x) - \frac{s}{x} \leqslant -K\ln\left(\frac{s}{K}\right) - K.$$

Now, let b be the maximum value of $\boldsymbol{R}$ (in magnitude), it follows that

$$b < \mathrm{tr}(\boldsymbol{R}) = \sum_{i=1}^{N}\lambda_i \leqslant N\lambda_1 \quad \Rightarrow \quad \lambda_1 > \frac{b}{N}.$$

Therefore, as b goes to plus infinity, λ_1 goes to plus infinity and $\ln J(\boldsymbol{R})$ goes to minus infinity by (23).

As a consequence, for each $\varepsilon < 0$, there exists a $b > 0$ such that, for each $\boldsymbol{R}$ whose maximum element is greater than b, $\ln J(\boldsymbol{R}) < \varepsilon$. Now choose $\varepsilon_1 < \ln J(\boldsymbol{R}_0)$, we have that there exists a b such that $\ln J(\boldsymbol{R}) < \ln J(\boldsymbol{R}_0)$ for each positive definite $\boldsymbol{R}$ outside $\mathcal{C}_b$.

We have, thus, obtained that $\ln J(\boldsymbol{R})$ admits a maximum over $\mathcal{C}_{ab}$ and that each point in the intersection between $\mathcal{D}_+$ and the complement of $\mathcal{C}_{ab}$ in $\mathcal{D}_+$ does not provide values greater than $\ln J(\boldsymbol{R}_0)$. As a consequence, the maximum of $\ln J(\boldsymbol{R})$ over $\mathcal{C}_{ab}$ is the maximum over the open set $\mathcal{D}_+$, too.

The second statement is straightforward; since $\ln J(\boldsymbol{R})$ is continuously differentiable over the open set $\mathcal{D}_+$, the maximum of $\ln J(\boldsymbol{R})$ has to be a stationary point of $\mathcal{D}_+$. The proof is thus finished. ■

From Proposition 2 and the fact that $\widehat{\boldsymbol{R}}_0$ is the unique stationary point of $J(\boldsymbol{R})$ we can conclude that $\widehat{\boldsymbol{R}}_0$ is the maximizer of $J(\boldsymbol{R})$ and, hence, the maximum likelihood estimate of $\boldsymbol{R}$ under H_0.

Appendix II
Derivation of Equation (13)

The aim of this Appendix is the derivation of an alternative form for the GLRT given by (12). To this end, observe that

$$\begin{aligned}&\det(K\widehat{\boldsymbol{R}}_0)\\ &=\det\left[\boldsymbol{S}_0-(\boldsymbol{Z}_P-\boldsymbol{v}\boldsymbol{\beta}^\dagger)(\boldsymbol{Z}_P-\boldsymbol{v}\boldsymbol{\beta}^\dagger)^\dagger\right]\\ &=\det(\boldsymbol{S}_0)\det\left[\boldsymbol{I}_N-\boldsymbol{S}_0^{-1/2}(\boldsymbol{Z}_P-\boldsymbol{v}\boldsymbol{\beta}^\dagger)(\boldsymbol{Z}_P-\boldsymbol{v}\boldsymbol{\beta}^\dagger)^\dagger\boldsymbol{S}_0^{-1/2}\right]\\ &=\det(\boldsymbol{S}_0)\det\Big[\boldsymbol{I}_N-(\boldsymbol{S}_0^{-1/2}\boldsymbol{Z}_P-\boldsymbol{P}\boldsymbol{S}_0^{-1/2}\boldsymbol{Z}_P)\\ &\qquad\qquad\qquad\times(\boldsymbol{S}_0^{-1/2}\boldsymbol{Z}_P-\boldsymbol{P}\boldsymbol{S}_0^{-1/2}\boldsymbol{Z}_P)^\dagger\Big]\\ &=\det(\boldsymbol{S}_0)\det\left[\boldsymbol{I}_{K_P}-\boldsymbol{Z}_P^\dagger\boldsymbol{S}_0^{-1/2}(\boldsymbol{I}_N-\boldsymbol{P})\boldsymbol{S}_0^{-1/2}\boldsymbol{Z}_P\right]\end{aligned}$$

where $\boldsymbol{P} = \boldsymbol{S}_0^{-1/2}\boldsymbol{v}(\boldsymbol{v}^\dagger\boldsymbol{S}_0^{-1}\boldsymbol{v})^{-1}\boldsymbol{v}^\dagger\boldsymbol{S}_0^{-1/2}$ is the projector onto $\langle\boldsymbol{S}_0^{-1/2}\boldsymbol{v}\rangle$. Hence, $\det(K\widehat{\boldsymbol{R}}_0)$ can be rewritten as

$$\det(K\widehat{\boldsymbol{R}}_0) = \det(\boldsymbol{S}_0)\det\left[\boldsymbol{I}_{K_P} - \boldsymbol{Z}_P^\dagger\boldsymbol{S}_0^{-1}\boldsymbol{Z}_P + \frac{\boldsymbol{Z}_P^\dagger\boldsymbol{S}_0^{-1}\boldsymbol{v}\boldsymbol{v}^\dagger\boldsymbol{S}_0^{-1}\boldsymbol{Z}_P}{\boldsymbol{v}^\dagger\boldsymbol{S}_0^{-1}\boldsymbol{v}}\right]. \quad (24)$$

Recall now that $\boldsymbol{S}_0 = \boldsymbol{S} + \boldsymbol{Z}_P\boldsymbol{Z}_P^\dagger$; as a consequence, we have that

$$\begin{aligned}\det(\boldsymbol{S}_0) &= \det\left(\boldsymbol{S}+\boldsymbol{Z}_P\boldsymbol{Z}_P^\dagger\right)\\ &= \det(\boldsymbol{S})\det\left[\boldsymbol{I}_N+\boldsymbol{S}^{-1/2}\boldsymbol{Z}_P\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1/2}\right]\\ &= \det(\boldsymbol{S})\det\left[\boldsymbol{I}_{K_P}+\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\right] \qquad (25)\end{aligned}$$

and [16]

$$\boldsymbol{S}_0^{-1} = \boldsymbol{S}^{-1}-\boldsymbol{S}^{-1}\boldsymbol{Z}_P\left(\boldsymbol{I}_{K_P}+\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\right)^{-1}\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}. \quad (26)$$

$$\begin{aligned}\det(K\widehat{\boldsymbol{R}}_0) &= \det(\boldsymbol{S})\det\left(\boldsymbol{I}_{K_P}+\boldsymbol{A}\right)\\ &\quad\times\det\left[\boldsymbol{I}_{K_P}-\boldsymbol{A}+\boldsymbol{A}\left(\boldsymbol{I}_{K_P}+\boldsymbol{A}\right)^{-1}\boldsymbol{A}+\frac{\left[\boldsymbol{I}_{K_P}-\boldsymbol{A}\left(\boldsymbol{I}_{K_P}+\boldsymbol{A}\right)^{-1}\right]\boldsymbol{b}\boldsymbol{b}^\dagger\left[\boldsymbol{I}_{K_P}-\left(\boldsymbol{I}_{K_P}+\boldsymbol{A}\right)^{-1}\boldsymbol{A}\right]}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}-\boldsymbol{b}^\dagger\left(\boldsymbol{I}_{K_P}+\boldsymbol{A}\right)^{-1}\boldsymbol{b}}\right]\end{aligned}$$

If we substitute (25) and (26) into (24), after some algebra, we come up with the expression shown at the bottom of the previous page, where

$$\boldsymbol{A} = \boldsymbol{Z}_P^\dagger \boldsymbol{S}^{-1}\boldsymbol{Z}_P \quad \text{and} \quad \boldsymbol{b} = \boldsymbol{Z}_P^\dagger \boldsymbol{S}^{-1}\boldsymbol{v}.$$

Observe now that

$$\begin{aligned} &\boldsymbol{I}_{K_P} - \boldsymbol{A} + \boldsymbol{A}(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{A} \\ &\quad = \boldsymbol{I}_{K_P} - \boldsymbol{A}(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}(\boldsymbol{I}_{K_P}+\boldsymbol{A}) + \boldsymbol{A}(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{A} \\ &\quad = \boldsymbol{I}_{K_P} - \boldsymbol{A}(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1} \end{aligned} \tag{27}$$

and, hence, that

$$\left[\boldsymbol{I}_{K_P} - \boldsymbol{A} + \boldsymbol{A}(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{A}\right](\boldsymbol{I}_{K_P}+\boldsymbol{A}) = \boldsymbol{I}_{K_P} \tag{28}$$

which, in turn, implies that

$$\boldsymbol{I}_{K_P} - \boldsymbol{A} + \boldsymbol{A}(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{A} = (\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}. \tag{29}$$

We also have that

$$\begin{aligned} &\boldsymbol{I}_{K_P} - \boldsymbol{A} + \boldsymbol{A}(I_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{A} \\ &\quad = \boldsymbol{I}_{K_P} - (\boldsymbol{I}_{K_P}+\boldsymbol{A})(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{A} + \boldsymbol{A}(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{A} \\ &\quad = \boldsymbol{I}_{K_P} - (\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{A}. \end{aligned} \tag{30}$$

Using (27), (29), and (30) into the expression of $\det(K\widehat{\boldsymbol{R}}_0)$ we get

$$\begin{aligned} \det(K\widehat{\boldsymbol{R}}_0) &= \det(\boldsymbol{S})\det(\boldsymbol{I}_{K_P}+\boldsymbol{A}) \\ &\quad \times \det\Bigg[(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1} \\ &\qquad + \frac{(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{b}\boldsymbol{b}^\dagger(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v} - \boldsymbol{b}^\dagger(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{b}}\Bigg] \\ &= \det(\boldsymbol{S})\det\left[\boldsymbol{I}_{K_P} + \frac{(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{b}\boldsymbol{b}^\dagger}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v} - \boldsymbol{b}^\dagger(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{b}}\right] \\ &= \det(\boldsymbol{S})\left[1 + \frac{\boldsymbol{b}^\dagger(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{b}}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v} - \boldsymbol{b}^\dagger(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{b}}\right] \\ &= \det(\boldsymbol{S})\frac{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v} - \boldsymbol{b}^\dagger(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{b}}. \end{aligned}$$

Following the lead of previous derivation, it is easy to check that $\det(K\widehat{\boldsymbol{R}}_1)$ can be expressed as

$$\begin{aligned} \det(K\widehat{\boldsymbol{R}}_1) &= \det(\boldsymbol{S})\det(\boldsymbol{I}_{K_P}+\boldsymbol{A}) \\ &\quad \times \frac{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v} - \boldsymbol{b}^\dagger(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{b}}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}} \end{aligned}$$

and, therefore, that the GLRT is given by

$$\frac{1}{\det(\boldsymbol{I}_{K_P}+\boldsymbol{A})}\left[\frac{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v} - \boldsymbol{b}^\dagger(\boldsymbol{I}_{K_P}+\boldsymbol{A})^{-1}\boldsymbol{b}}\right]^2 \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

which can also be expressed as a function of the original quantities as shown in the equation at the bottom of the page, which is exactly the expression given by (13).

Appendix III
CFARness of the Proposed Detectors

The aim of this Appendix is to show that detectors (13) and (16) guarantee the CFAR property with respect to covariance matrix $\boldsymbol{R}$. As already stated, we follow the usual assumption to define the P_{fa} as the probability to decide that the hypothesis H_1 is true when data under test contain noise only, i.e., $\boldsymbol{z}_k = \boldsymbol{n}_k$, $k \in \Omega_S \cup \Omega_P$.

The proof parallels results presented in [1], but it is reported here for the sake of completeness. To begin with, let us define the following quantities

$$\begin{aligned} \boldsymbol{X}_P &= \boldsymbol{R}^{-1/2}\boldsymbol{Z}_P, \\ \boldsymbol{X}_S &= \boldsymbol{R}^{-1/2}\boldsymbol{Z}_S, \\ \boldsymbol{S}_w &= \boldsymbol{R}^{-1/2}\boldsymbol{S}\boldsymbol{R}^{-1/2} = \boldsymbol{X}_S\boldsymbol{X}_S^\dagger, \\ \boldsymbol{v}_w &= \boldsymbol{R}^{-1/2}\boldsymbol{v} \end{aligned}$$

where, in particular, $\boldsymbol{X}_P \in \mathbb{C}^{N\times K_P}$ and $\boldsymbol{X}_S \in \mathbb{C}^{N\times K_S}$ are the whitened data. As to the decision statistic (13), it can be rewritten as (31) shown at the top of the next page. Now denote by $\boldsymbol{V} \in \mathbb{C}^{N\times N}$ a unitary transformation aimed at rotating the vector $\boldsymbol{v}_w$ onto the direction of $\boldsymbol{e}_1 = [1\ 0\ \cdots\ 0]^T$; in particular, we can write

$$\boldsymbol{v}_w = \sqrt{\boldsymbol{v}^\dagger\boldsymbol{R}^{-1}\boldsymbol{v}}\boldsymbol{V}^\dagger\boldsymbol{e}_1. \tag{32}$$

$$\begin{aligned} &\frac{1}{\det\left(\boldsymbol{I}_{K_P}+\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\right)}\left[\frac{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v} - \boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\left(\boldsymbol{I}_{K_P}+\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\right)^{-1}\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}\right]^2 \\ &\quad = \frac{1}{\det\left(\boldsymbol{I}_{K_P}+\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\right)}\left[\frac{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\left(\boldsymbol{I}_{K_P}+\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\right)^{-1}\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}}{\boldsymbol{v}^\dagger\boldsymbol{S}^{-1}\boldsymbol{v}} - 1\right]^{-2} \\ &\quad = \frac{1}{\det\left(\boldsymbol{I}_{K_P}+\boldsymbol{Z}_P^\dagger\boldsymbol{S}^{-1}\boldsymbol{Z}_P\right)(\ell_{\text{GLRT}}-1)^2} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \end{aligned}$$

$$\frac{1}{\det\left(\boldsymbol{I}_{K_P}+\boldsymbol{X}_P^\dagger\boldsymbol{S}_w^{-1}\boldsymbol{X}_P\right)}\left[\frac{\boldsymbol{v}_w^\dagger\boldsymbol{S}_w^{-1}\boldsymbol{v}_w}{\boldsymbol{v}_w^\dagger\boldsymbol{S}_w^{-1}\boldsymbol{v}_w-\boldsymbol{v}_w^\dagger\boldsymbol{S}_w^{-1}\boldsymbol{X}_P\left(\boldsymbol{I}_{K_P}+\boldsymbol{X}_P^\dagger\boldsymbol{S}_w^{-1}\boldsymbol{X}_P\right)^{-1}\boldsymbol{X}_P^\dagger\boldsymbol{S}_w^{-1}\boldsymbol{v}_w}\right]^2. \quad (31)$$

$$\frac{1}{\det\left(\boldsymbol{I}_{K_P}+\boldsymbol{W}_P^\dagger\boldsymbol{C}_S^{-1}\boldsymbol{W}_P\right)}\left[\frac{\boldsymbol{e}_1^\dagger\boldsymbol{C}_S^{-1}\boldsymbol{e}_1}{\boldsymbol{e}_1^\dagger\boldsymbol{C}_S^{-1}\boldsymbol{e}_1-\boldsymbol{e}_1^\dagger\boldsymbol{C}_S^{-1}\boldsymbol{W}_P\left(\boldsymbol{I}_{K_P}+\boldsymbol{W}_P^\dagger\boldsymbol{C}_S^{-1}\boldsymbol{W}_P\right)^{-1}\boldsymbol{W}_P^\dagger\boldsymbol{C}_S^{-1}\boldsymbol{e}_1}\right]^2$$

Thus, plugging (32) into (31) and performing straightforward manipulations we have the unnumbered equation shown at the top of the page, where

$$\boldsymbol{C}_S=\boldsymbol{W}_S\boldsymbol{W}_S^\dagger, \text{ with } \boldsymbol{W}_S=\boldsymbol{V}\boldsymbol{X}_S \text{ and } \boldsymbol{W}_P=\boldsymbol{V}\boldsymbol{X}_P$$

$\boldsymbol{W}_P = [\boldsymbol{w}_1\cdots\boldsymbol{w}_{K_P}] \in \mathbb{C}^{N\times K_P}$ and $\boldsymbol{W}_S = [\boldsymbol{w}_{K_P+1}\cdots\boldsymbol{w}_{K_P+K_S}] \in \mathbb{C}^{N\times K_S}$ being matrices whose columns are independent and identically distributed complex normal random vectors with zero mean and identity covariance matrix, i.e.

$$\boldsymbol{w}_k\sim\mathcal{CN}_N(\boldsymbol{0},\boldsymbol{I}_N), \quad k\in\Omega_P\cup\Omega_S.$$

It is then apparent that under the noise-only hypothesis the decision statistic (13) can be expressed as a function of random variables whose distribution does not depend on $\boldsymbol{R}$. The claimed CFAR property of detector (13) is thus proved.

Similarly, the decision statistic of the *ad hoc* detector (16) can be recast as

$$\frac{\boldsymbol{e}_1^\dagger\boldsymbol{C}_S^{-1}\boldsymbol{W}_P\boldsymbol{W}_P^\dagger\boldsymbol{C}_S^{-1}\boldsymbol{e}_1}{\boldsymbol{e}_1^\dagger\boldsymbol{C}_S^{-1}\boldsymbol{e}_1}-\frac{1}{2}\mathrm{tr}\left(\boldsymbol{W}_P^\dagger\boldsymbol{C}_S^{-1}\boldsymbol{W}_P\right)$$

whose distribution is apparently independent of $\boldsymbol{R}$ under the noise-only hypothesis.

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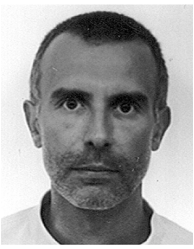

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