BUCKLING OF FOAM STABILISED COMPOSITE STRUCTURES

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ABSTRACT

An analytical modelling of the symmetrical wrinkling is proposed : from original assumptions on displacements within the core, and from an energy minimisation method, it is possible to predict critical loads and buckling modes better than traditional models do, and to distinguish the influence of each structure component.

Compression tests were carried out on sandwich structures to validate the model. Little curved structures were also tested to estimate the influence of skin curvature on rupture and buckling mode.

A finite elements analysis has been achieved in parallel : a fine modelling allows to find results close to experimental ones.

LOCAL BUCKLING, WRINKLING, COMPOSITE, FOAM, FINITE ELEMENTS, ANALYTICAL MODEL.

INTRODUCTION

In the field of aeronautical engineering, composite sandwich structures are widespread. The use of foam core instead of Nida to stabilise the skin of such structures present an economic benefit, but their weak mechanical characteristics poses the problem of local buckling. Thus, to design such structures, it is important to be able to evaluate the critical loads of buckling. Many theoretical studies were carried out for plane structures, but there are few experimental results and thus very few correlation.

First of all, an analytical study of the local buckling of skins on elastic soil is made. The use of new functions to represent the elastic foundations allows to analyse and to underscore the influence of the different component of the structure, and to improve the physical understanding of the phenomenon. This model is compared with finite elements calculations and other buckling models.

Various compression tests were carried out on sandwich beams made of foam core and glass-epoxy skin. These tests give experimental results to validate the analytical model. Tests were also realized on slightly curved sandwich beams, to study the influence of a skin curvature on rupture and buckling phenomenon.

A finite elements modelling is made to analyse the experiments. It shows the necessity of a precise representation of skin-foam interface to correlate plane specimen buckling tests. Modelling of curved structures permits to find results close to experiments, and to have a better understanding of the rupture phenomenon.

ANALYTICAL MODELLING

The study of symmetrical local buckling of sandwich type structures is equivalent to the study of the instability of a skin on an elastic foundation (figure 1). The notations used are usual ; c and s refer respectively to the core and the skin. b is the depth of the skin.

Existing modelling

One of the first wrinkling model developed, is the Winkler type model, which considers the elastic foundation as a succession of springs. Springs allow to represent linear transverse displacements in the core, but the model do not consider transverse shearing. The critical buckling load is (Timoshenko, 1966 ; Allen, 1969) :

$F_c = 2 \cdot \sqrt{K_y \cdot E_s \cdot I_z}$, with the foundation stiffness $K_y = \dfrac{b \cdot E_c}{h}$

This model does not suit to the case of short wavelength buckling. It is possible to improve it by adding Kxy springs to take into account shear stress, but the identification of the stiffness becomes difficult (Aiello and Ombres, 1997).

Hoff and Mautner (Hoff and Mautner, 1945), and then Allen (Allen, 1969), worked on models with a continuous representation of the core. They used Airy functions to represent stress in the core. This leads to the classical, extensively used expression :

$$\sigma_{buckling} = Q \cdot \sqrt[3]{E_s \cdot E_c \cdot G_c} + Q_1 \cdot G_c \qquad \text{(eq 1)}$$

Allen determines Q et Q_1 in function of Poisson's ratio in the core : $0.780 < Q < 0.794$ and $0.200 < Q_1 < 0.333$.

Recently, in a study on the local and global buckling coupling, Léotoing (Léotoing, 2001) proposed an original method, based on displacements. He determines the equilibrium equations of the problem from the potential energy of the system, and from the virtual works principle. He linearizes the equilibrium equations, which leads to an eigenvalues problem. The buckling critical strength is :

$$F_{critique} = 2 \cdot \sqrt{\frac{E_c \cdot b \cdot E_s \cdot I_z}{h}} + \frac{G_c \cdot b \cdot h}{3} \qquad \text{(eq 2)}$$

More specific models can be mentioned : Starlinger (Starlinger, 1990) take in account the orthotropy of the core, and Niu and Talrejas (Niu and Talreja, 1999) use Airy functions in the core, in forms of Fourier series.

Most of these models do not give a precise representation of displacements within the core. Besides, these models do not allow to visualise the influence of the different structure components in the buckling phenomenon.

Hypotheses

The structure studied is a plane skin on an elastic foundation (figure 1).

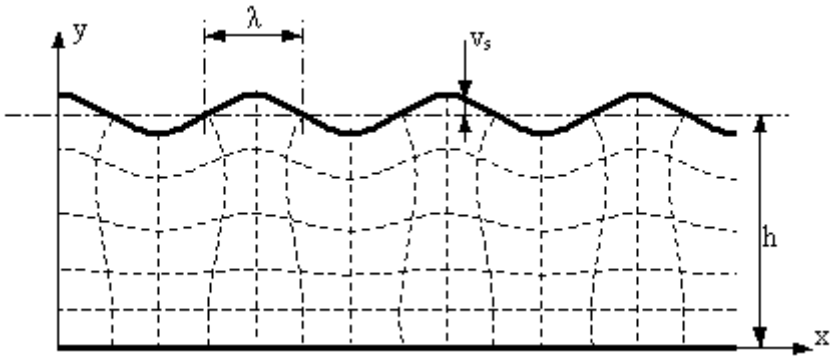

Figure 1 : wrinkling model : beam on elastic foundation

The structure is a 2D model. A compressive load is applied on the beam which represents the skin. The structure is in a compression state, until the buckling appears.

qi are the parameters of the system : qi = qi°+dqi, where qi° are parameters at equilibrium, and dqi the perturbations around equilibrium.

After buckling, the deformed skin is supposed to be sinusoidal (figure 1). If λ is the deformed skin wavelength, the transverse displacement of the skin is supposed to be :

$$\delta v_s = \delta A \cdot \sin(\frac{\pi \cdot x}{\lambda})$$

Deformations within the core are complex. To analyse displacements in the foam, a finite elements calculation is achieved, that allows to observe the form of displacements (figure 1). For short wavelengths in comparison to core thickness, perturbations are localised in a restricted area, under skin. Therefore linear transverse displacements in the core are not satisfactory to represent reality, except while adjusting the stiffness according to the wavelength. It is necessary to represent displacements by functions which permit a high decrease versus y. In this study, we chose to consider either polynomial of order higher than 3, or piecewise polynomial.

Besides, it might be important not to impose a null displacement in the x direction, to avoid shear stiffening in the core.

By consideration of symmetry on the infinite-length structure, it is possible to demonstrate the periodicity of displacements dv_c and du_c in the x direction. Besides, dv_c and du_c increase with dA, which allows to write the following hypothesis :

$$\delta v_c = \delta A \cdot P(y) \cdot \sin(\frac{\pi \cdot x}{\lambda}), \quad \delta u_c = \delta A \cdot Q(y) \cdot \cos(\frac{\pi \cdot x}{\lambda})$$

with $P(y) = \sum a_i \cdot x^i$ et $Q(y) = \sum b_i \cdot x^i$

and $P(0) = 0,\ P(h) = 1$, to be cinematically admissible; $Q(h) = 0$.

(P and Q might be piece-polynomials).

The simplified behaviour law used in the foam core is the following :

$$\sigma_x = E_c \cdot \varepsilon_x, \quad \sigma_y = E_c \cdot \varepsilon_y, \quad \sigma_{xy} = 2 \cdot G_c \cdot \varepsilon_{xy}$$

Energetic approach

Total potential energy of the structure is the sum of deformation energy and external energy. The calculation is done for a fixed wavelength (λ):

$$E_T = U_{def} + V$$

A stable equilibrium state corresponds to the minimum of the total potential energy : $\delta E_T = 0,\ \delta^2 E_T > 0$.

Buckling happens when equilibrium becomes unstable : $\delta^2 E_T = 0$.

Energies are expressed below (Barrau, 1987) :

Elastic energy in the skin (transverse shearing is neglected) :

$$W_{skin} = W_{normal} + W_{bending} = \int_0^{\lambda} \frac{1}{2} \cdot A_{11} \cdot (e_1^{\circ})^2 \cdot dx + \int_0^{\lambda} \frac{1}{2} \cdot C_{11} \cdot (k_1^{\circ})^2 \cdot dx$$

with $e_1^{\circ} = \varepsilon_x = \frac{du}{dx} + \frac{1}{2} \cdot \left(\frac{dv}{dx}\right)^2,\ \ k_1^{\circ} = -\frac{d^2 v}{dx^2},\ \ u = u^{\circ} + \delta u,\ \ v = v^{\circ} + \delta v$

Elastic energy in the core :

$$W_{core} = \iint_{core} \frac{1}{2} \cdot A_{mij} \cdot e_i \cdot e_j \cdot dx \cdot dy$$

with $\begin{cases} e_1 = \varepsilon_x = \frac{du}{dx} + \frac{1}{2}\left(\frac{dv}{dx}\right)^2,\ e_2 = \varepsilon_y = \frac{dv}{dy} + \frac{1}{2}\left(\frac{du}{dy}\right)^2 \\ e_3 = 2 \cdot \varepsilon_{xy} = \frac{du}{dy} + \frac{dv}{dx},\ u = u^{\circ} + \delta u,\ v = v^{\circ} + \delta v \end{cases}$

External energy (punctual load F) :

$$V = \int_{skin} F \cdot \frac{du}{dx} \cdot dx$$

The total potential energy is written in second-order Taylor series. $\delta^2 ET=0$ leads to the following equation, according to the problem parameters :

$$0 = \int_0^{\lambda} \frac{E \cdot S}{2} \cdot \left(\frac{d\delta u_s}{dx} \right)^2 \cdot dx + \int_0^{\lambda} \frac{E \cdot I_z}{2} \cdot \left(\frac{d^2 \delta v_s}{dx^2} \right)^2 \cdot dx - \int_0^{\lambda} F \cdot \left(\frac{d\delta u_s}{dx} \right) \cdot dx$$
$$+ \iint_{core} \left\{ \frac{E_c}{2} \cdot \left[\left(\frac{d\delta u_c}{dx} \right)^2 + \left(\frac{d\delta v_c}{dy} \right)^2 \right] + G_c \cdot \left(\frac{d\delta u_c}{dy} + \frac{d\delta v_c}{dx} \right)^2 \right\} \cdot dx \cdot dy$$

The critical load for a fixed wavelength is then :

$$F = \frac{\pi^2 \cdot E \cdot I_z}{\lambda^2} + \frac{4 \cdot \lambda}{\pi^2} \cdot \iint_{core} \left\{ \begin{array}{l} \frac{E_c}{2} \cdot \sin^2 \left(\frac{\pi \cdot x}{\lambda} \right) \left[\left(\frac{\pi}{\lambda} \right)^2 Q(y)^2 + \left(\frac{dP(y)}{dy} \right)^2 \right] \\ + G_c \cdot \cos^2 \left(\frac{\pi \cdot x}{\lambda} \right) \left[\left(\frac{dQ(y)}{dy} \right)^2 + \left(\frac{\pi}{\lambda} \right)^2 P(y)^2 \right] \end{array} \right\} \cdot dx \cdot dy$$

Critical buckling load and deformation mode are calculated by minimisation of the load, in function of λ.

Calculation of shape functions in the core

The previous calculation is made for a fixed wavelength and fixed shape functions. The critical effort has to be minimal, therefore it is possible to determine polynomial coefficients of the shape function by minimisation of elastic energy in the core, for a given wavelength.

A matrix formulation of the energy is used to simplify calculation. The energy is quadratic in function of the unknown coefficient vector. It permits to have a simple expression of the energy, and an expression of the vector that reaches the minimum (Kelley, 1999): (Kelley, 1999) :

$$W_{core} = {}^t[Coef] \cdot [Quad] \cdot [Coef] + [Lin] \cdot [Coef] + Cste$$
$$[Coef]_{\min i} = -\frac{1}{2} \cdot [Quad]^{-1} \cdot [Lin]$$

In any case, it is possible to express the displacements in the core with an analytical function.

No restriction is given on materials : the core, in foam, can be either isotropic or orthotropic ; and skin can be made of composite, the expression of bending energy being expressed according to bending stiffness Iz.

Comparison with a finite element calculation

To validate the modelling, a 2D linear buckling finite element analysis have been carried out. Skins are represented by beams, and core is represented by membrane elements. Boundary conditions have few influence on wrinkling (local phenomenon), so the load can be applied through a rigid body, at the extremities of the model. The mesh must be fine so that a wavelength contains about ten elements; the length of the model must be great enough to neglect boundary conditions (more than 5 wavelengths).

Table 1 gives the critical load for several models, with different structure configurations. The reference configuration is the following: E_c=50 MPa, E_s=50000 MPa, h=20 mm, t=1 mm (thickness of skin), nu_c=0.3, b=30 mm. The other configurations only present a variation of one of the parameter around the reference. Several shape functions are used to represent the core : linear, cubic, 2-piece cubic spline, and 2-piece cubic spline plus a 3-order polynomial which takes count of the longitudinal displacement (x direction).

Model	Ref.	E_c= 10	E_c= 200	E_s= 10000	E_s= 100000	t= 0.25	t= 5	h= 50
Linear	13%	7%	21%	23%	10%	103%	3%	50%
Cubic	5%	5%	0%	-2%	5%	3%	3%	1%
2-piece cubic	5%	5%	0%	-2%	5%	-1%	3%	0%
2-piece cubic + δu_m	1%	1%	-3%	-4%	2%	-1%	0%	-1%
Classic	-2%	-10%	-3%	-5%	-4%	-5%	-38%	-3%
Leotoing	13%	7%	21%	23%	10%	103%	3%	50%

Table 1 : Critical load : comparison between analytical models and FE

In the case of simple shape functions, it is possible to express the critical strength with an analytical function. It is the case of linear model, which permits to recover the expressions of load and wavelength given by Léotoing symmetrical buckling model (eq 2). In the other cases, the problem of minimisation became complex, and need to be solved by numerical calculation.

Table 1 shows that linear model can lead to 100% errors. The hypothesis of linearity is only valid in a domain in which wavelengths are greater than thickness of the soul (λ>2.h).

Even if it does not perfectly represent the field of displacements (figure 3), the cubic model permits to have a good precision (5% in the studied domain). It is simple to use, and can easily be implanted in a software like Excel. The critical load is obtained by minimisation of the load in function of λ :

$$F = \frac{\pi^2 E I_z}{\lambda^2} + \frac{b}{15}\frac{\pi^6 G_c{}^3 h^6 + 135\pi^4 G_c{}^2 h^4 \lambda^2 E_c + 2880\pi^2 G_c h^2 \lambda^4 E_c{}^2 + 6300\lambda^6 E_c{}^3}{h\pi\left(420\lambda^4 E_c{}^2 + 52\lambda^2 E_c \pi^2 G_c h^2 + \pi^4 G_c{}^2 h^4\right)}$$

For small values of λ/h, it is necessary to consider a more complex shape function : for example, a 2-piece cubic spline. The calculation of polynomials and critical load becomes complex, but transverse displacements are better estimated (fig 2).

Taking account of the longitudinal displacement in the core (du_c) permits to recover the form of displacements given by finite elements analysis, and improve a little bit the critical strength calculation.

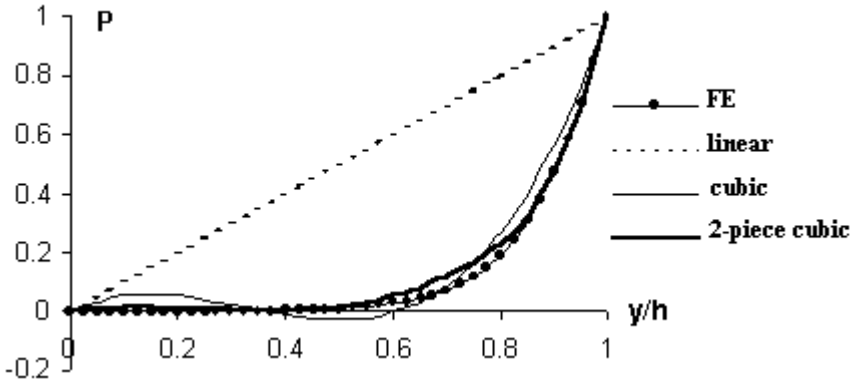

Figure 2 : δv_c displacement shape function in the foam (λ=5mm, h=20 mm)

Influence of structure components

From the energetic approach, it is possible to determine precisely the influence of structure components by analysing the energy stored in skin and core, in function of λ.

Figure 3a shows the distribution of energies in the core : the energy due to normal stress becomes high in comparison to shear energy for high wavelengths. For short λ, it is necessary to consider shear in the core.

Figure 3b shows the distribution of the energies between skin and core, and their evolutions when E_s increases.

Generally, energy in skin decreases when λ increases.

In the core, it is the opposite : if λ is short, only a local region under the skin is concerned by the perturbation. When λ increases, the perturbation spreads to a larger region, and the energy increases.

An evolution of E_s provokes an evolution of λ and the critical effort in the same direction. The evolution of λ is significant, whereas the variation of the effort can be small.

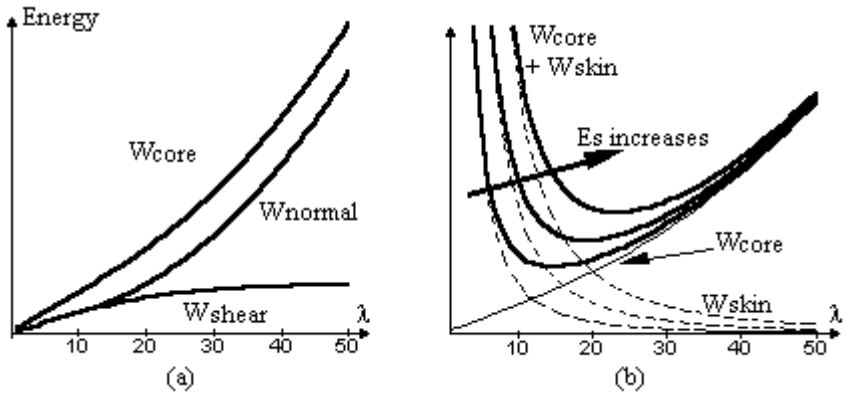

Figure 3 : (a) Repartition elastic energy in the core

(b) Evolution of energies with E_s

If the core thickness (h) is important, it is possible to show, with the 2-piece cubic spline model, that energy in core is linear with λ. Stress in the skin is then thickness independent (for metallic, or through thickness homogeneous skins). Contrary to the classic model results (eq 1), when core thickness is small, the stress can vary strongly with the thickness of skin (fig 4).

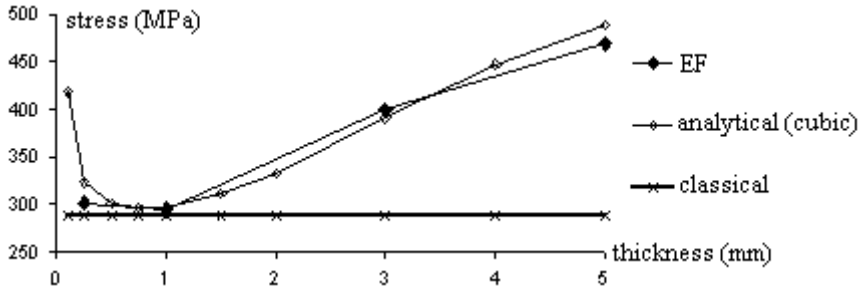

Figure 4 : Skin thickness influence on critical stress in skin (h=20 mm)

EXPERIMENTS

Compression tests have been achieved on simple structures, to correlate experiments, analytical model, numerical method (finite elements), and to improve the phenomenon comprehension.

The chosen structures are represented on figure 5. They are thick and short sandwich beams, made of a polyurethane foam core, covered by glass fibber and epoxy resin skins. The introduction of the compressive load is made by steel pieces at the extremities. To study the effect of skin curvature, three different skin curvatures are

tested : C-type (0.75 mm offset), D-type (1.5 mm) and E-type (2.5 mm). Plane skin structures are B-type specimens.

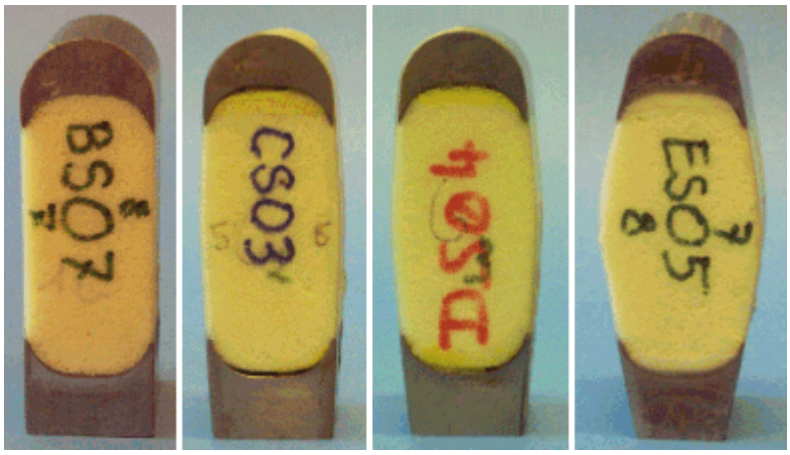

Figure 5 : structures tested

Material characterisation

Characterisation of foam core and skin were made.

The foam is slightly anisotropic : E_{cx}=10 Mpa, E_{cy}=12 Mpa, G_c=6 Mpa. Tensile rupture stress is about 0.7 MPa.

The characteristics of glass-epoxy tissue are: E_s=21000 Mpa, G_s=3000 Mpa, for a 0.16 mm theoretical thickness.

The glass tissue is manually impregnated with epoxy resin.

Tests

Tests have been achieved on a traction-compression machine, with an adapted set-up which permits to assure the applied compression load direction.

During tests, displacement is imposed (0.5mm/min). The acquisitions allows to get the applied load, the displacement, and deformations at the centre of each skin (gages).

Specimens whose skin is made of one or two plies of glass have been tested.

Results

For plane skin specimens, tests show that for one-ply tissue skin, debonding is local, and a forcing of skin in the foam can be observed on a few specimens (fig 6). For two-plies skin structures, debonding is global : it immediately propagates (less than a microsecond) along the specimen length, even if displacement is stopped.

For curved skin structures, forcing never appears. Rupture is a debonding of the skin, which can either be local (one-ply) or global (two-ply) (fig 6). Debonding seems to appear at the middle of the skin length, were curvature is the highest.

After eliminating non-valid tests, dispersion of rupture load values is under 25%, which is reasonable, considering the specimen production method, and the dispersion observed on rupture load in the foam.

In all cases, rupture load increases with the skin thickness, but decreases when curvature increases.

The deformation versus load diagram of plane skin specimens is linear till the rupture (fig 8), and looks like the compression diagram of a single skin (the foam Young modulus is only about 10 Mpa). This permits to say that rupture is due to an instability : the local buckling of the skin.

For curved skins, deformation is not linear with load. The rupture phenomenon is different. When load increases, bending in the skin increases immediately, up to rupture (fig 8).

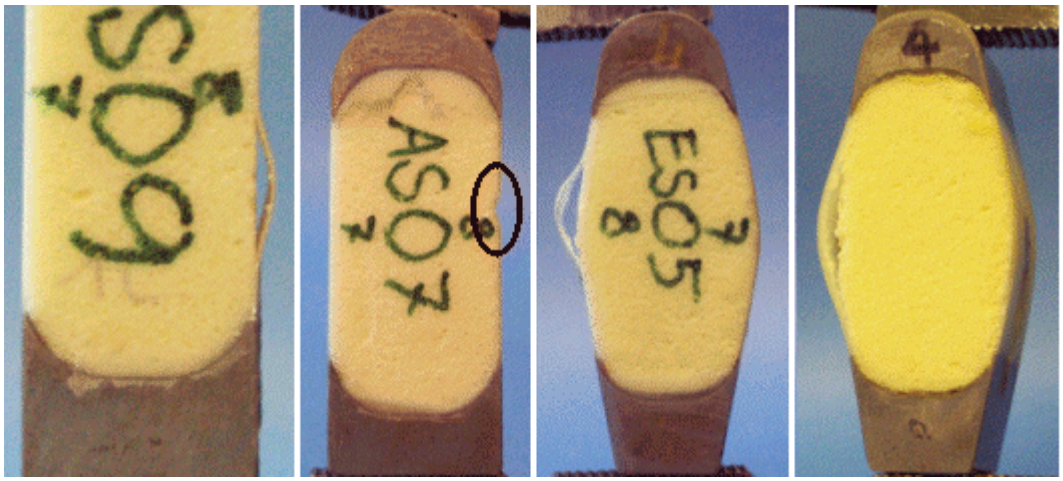

Figure 6 : rupture of the specimen

COMPARISON : TESTS - ANALYTICAL MODEL - FE MODELLING

Plane specimen

The calculation of critical load with the analytical model, while representing skin only by a glass and resin epoxy tissue gives results lower than experiments. Indeed, when skins are thin, the contribution of the

rigidity of the resin interface between skin and foam is non negligible, since it increases in a meaningful manner the bending rigidity.

SEM (Scanning Electron Microscope) observations permits to reveal the state of this interface (fig 7). Foam core is made of cells, so that the surface is unsteady, even at a 1 mm scale. The resin used to impregnate the glass tissue enter the open cells at the surface, and the skin is then made of a 0.16 mm glass tissue plus an unsteady resin layer. It is difficult to control both the thickness of this layer, and its geometry, and then it is not possible to have a precise estimation of skin bending rigidity. Thickness measured during the test is between 0.25 mm and 0.40. Calculations are achieved with a minimal and a maximal interface thickness.

Table 2 shows a few results for experiments, analytical model and EF linear buckling calculations. The resin interface thickness is taken in account. Calculations give a minimal and a maximal buckling load. Experimental results are between these values.

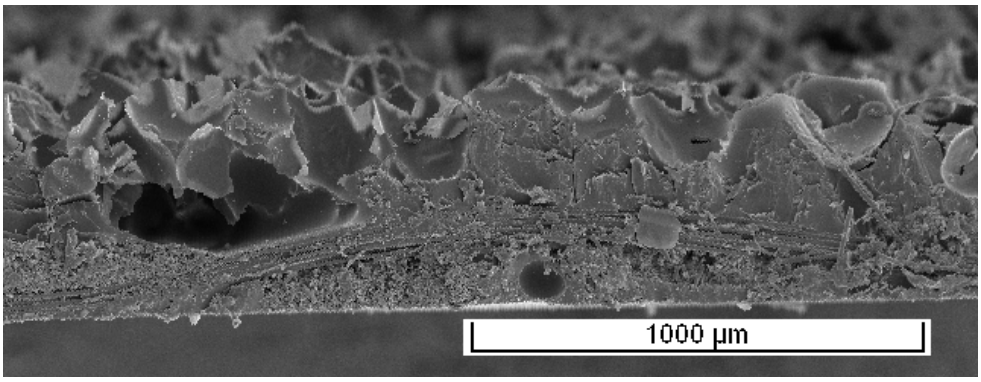

Figure 7 : skin and interface SEM analysis

Plane skin structures	FE		Analytical model		Experiments
	Resin thickness	load	Resin thickness	load	
2-plies	-	-	0.20 mm	2260 N	2500 to 3000 N
	0.28 mm	2670 N	0.28 mm	2600 N	
	0.40 mm	3450 N	0.40 mm	3075 N	
1-ply	0.16 mm	1400	0.16 mm	1400	1250 to 1500 N
	0.24 mm	1750	0.24 mm	1740	
	0.34 mm	2240	0.34 mm	2180	

Table 2 : buckling load – comparison between FE, analytical model and experiments

For plane specimens, a non linear finite elements modelling of the structure permits to confirm that rupture appears in a buckling mode. The FE load versus deformation graph (fig 8) shows that behaviour is linear up to the buckling load. Deformation is measured with gages, at the middle of the skin. Gage 1 on a side of the

specimen, gage 2 on the other side. Skin debonding is due to a local rupture of the foam, in traction and shearing, under a stress concentration due to buckling.

Results from FE model are slightly superior to analytical model results, because analytical model does not take in account the boundary conditions.

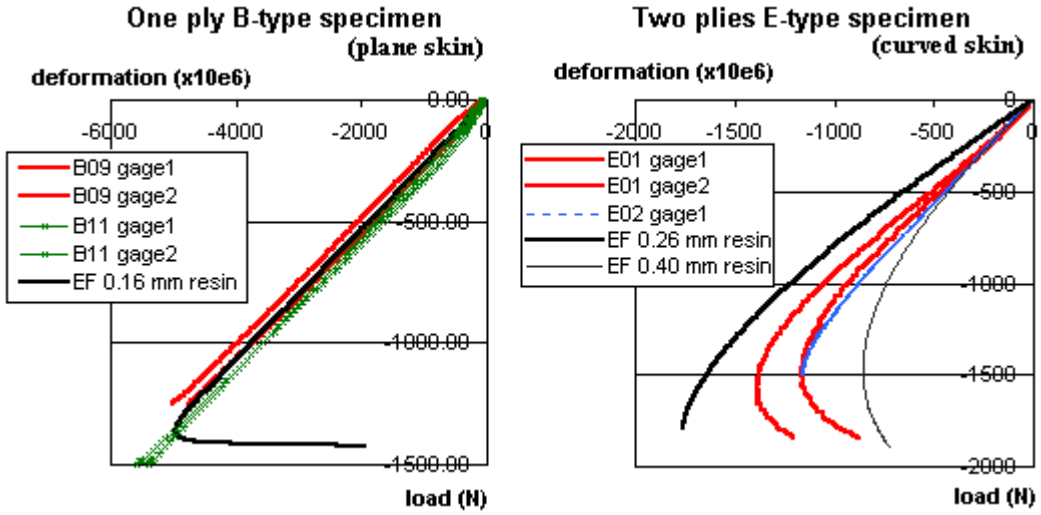

Figure 8 : load versus deformation graph – experiments and FE

Curved skins

To understand the phenomenon that rules the rupture in this case, a finite elements analysis is realised. Linear buckling calculations show that the curvature increases the critical buckling load (about 5% in these cases), whereas experiments show that rupture load decrease. The rupture is not due to buckling.

The mean rupture values for 2-plies specimens are given in table 3. B-type is a plane skin specimen. The others are curved skin specimen.

	B-type	C-type	D-type	E-type
Rupture load	2800 N	2515 N	2125 N	1670 N
curvature	0	0.75 mm offset	1.5 mm offset	2.5 mm offset

Table 3 : Two plies specimens - evolution of rupture load with curvature

Non linear EF calculations, with linear materials permits to show that rupture occurs when stress reaches its limits in the foam. Figure 8 show that behaviour is non-linear. Bending appears in the skin when load increases, and stress concentrations appear in the foam core (fig 9-10). The stress is the highest under the skin, at the middle of the specimen. Figure 10 represents the σ_x stress in the foam versus applied load, at this critical point. This

diagram shows that, for the tested specimen, tensile stress limit (about 0.7 MPa) is reached in the foam before buckling appears. Debonding is due to tensile foam rupture.

To improve the structure, it is possible to use foam with highest rupture limits. Rupture load can be increased up to critical buckling load.

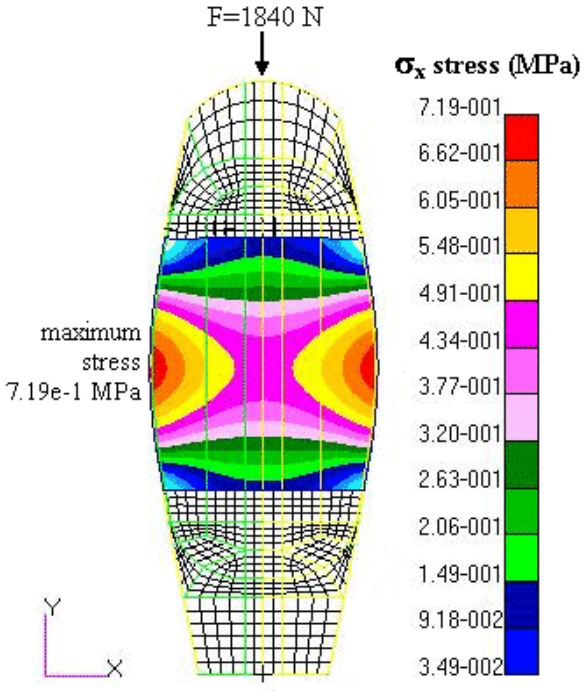

Figure 9 : σ_x stress concentration in the core (two plies specimen, 0.32mm resin)

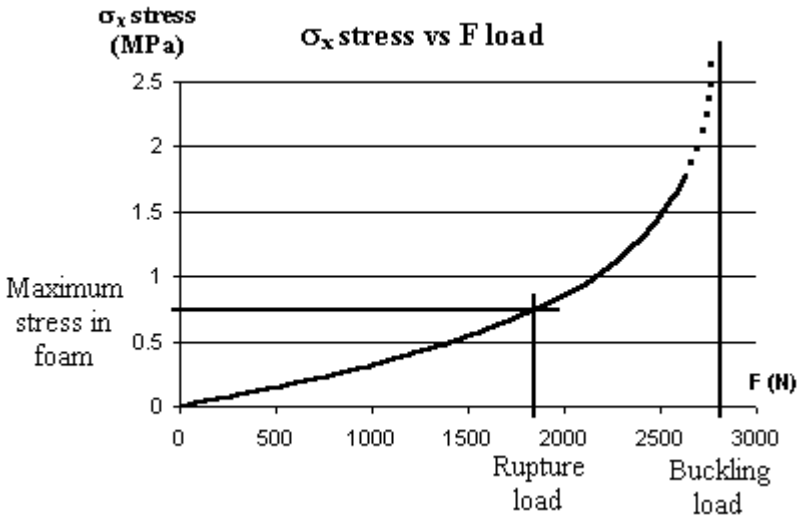

Figure 10 : σ_x stress vs applied load at critical point of the foam core (two plies, 0.26 mm resin)

Limitations

The analytical model can only be used for plane sandwich structures, and only take in account symmetrical buckling. It is therefore necessary to verify that the buckling mode is not antisymmetric. Critical load would then be lower.

The rigidity of skin is a dominant factor in the calculation of the buckling. For thin skins, it is therefore important to estimate the influence of the interface, and to take it in account if necessary.

The analytical model doesn't take in account structure length, and boundary conditions. To estimate this influence, EF calculations have been achieved with different length. With the shortest structures, buckling happens for 5 to 10 % superior loads.

CONCLUSION

The analytical model, based on an energetic method and original shape functions, permits better estimations of the buckling loads, deformations and stress in the core that classic models. It permits a good representation of deformations in the score, even for short wavelengths ($\lambda<h$). The model can be used with composite skins. It permits to visualise the influence of the different components of the structure.

Finite elements calculations and tests allows to validate the model, and permit to underscore the necessity to take in account the resin interface, between core and skin to have a valid estimation of buckling loads.

The model presents some limitations. The boundary conditions are not taken in account in the analytical model. Critical loads calculated with the model might be overestimated for configurations where the structure length is high ($L>5\lambda$). It is however interesting to note that in any case, results are conservative.

Tests achieved on curved structures show that the phenomenon is different and more complex when skins present a curvature. The curvature increase theoretical linear buckling loads, but the weak mechanical properties of foam can lead to rupture before reaching buckling. This can decrease the structure resistance and decrease the rupture load. When designing such structures, the choice of foam used for the core is essential.

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