Adaptive Detection in Nonhomogeneous Environments Using the Generalized Eigenrelation

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***Abstract*—This letter considers adaptive detection of a signal in a nonhomogeneous environment, more precisely under a covariance mismatch between the test vector and the training samples, due to an interference that is not accounted for by the training samples, e.g., a sidelobe target or an undernulled interference. We assume that the covariance matrices of the test vector and the training samples verify the so-called generalized eigenrelation. Under this assumption, we derive the generalized likelihood ratio test and show that it coincides with Kelly's detector.**

***Index Terms*—Adaptive detection, covariance mismatch, generalized eigenrelation, generalized likelihood ratio test, undernulled interference.**

I. Problem Statement

In a conventional adaptive detection problem, it is desired to detect the presence of a target signal, whose presumed array response is $\boldsymbol{v}$, in a $N \times 1$ observation vector $\boldsymbol{z}$, contaminated by Gaussian noise whose covariance matrix $\boldsymbol{R}_T$ is unknown. Additionally, it is assumed that L training samples $\boldsymbol{z}_\ell$ are available, which share the same covariance matrix $\boldsymbol{R}$. When the environment is homogeneous, i.e., when $\boldsymbol{R}_T = \boldsymbol{R}$, the reference detectors are Kelly's generalized likelihood ratio test (GLRT) [1], the adaptive matched filter (AMF) [2], and the adaptive coherence estimator (ACE) [3]–[5]. These detectors have been thoroughly studied, and their performances are now well understood in this canonical case, as well as in the practically important case of steering vector mismatch (i.e., the actual array response differs from $\boldsymbol{v}$) or covariance matrix mismatch (i.e., $\boldsymbol{R}_T \neq \boldsymbol{R}$); see, e.g., [6]–[9]. All detectors have to face a conventional tradeoff between robustness to slightly mismatched signals and rejection of unwanted signals, such as sidelobe targets or undernulled interference. Indeed, assume that a signal is present with signature $\boldsymbol{s}$. Then there are mainly two possibilities. Either $\boldsymbol{s}$ is a perturbed version of $\boldsymbol{v}$—which can occur, e.g., with non perfectly calibrated arrays—and then a detection should be declared, even if $\boldsymbol{s} \neq \boldsymbol{v}$, since there is a target in the look direction. Another possibility is that $\boldsymbol{s}$ corresponds to a sidelobe target, in which case no detection should be declared. The performance of Kelly's GLRT, the AMF, and the ACE have been assessed in the case of mismatched signals, and they behave quite differently. The AMF, which is an energy detector, has excellent mainlobe sensitivity but is highly sensitive to sidelobe signals. On the contrary, the ACE that measures the angle between the received signal and $\boldsymbol{v}$ has excellent sidelobe rejection capabilities at the price of a lower detection for slightly mismatched signals. Kelly's GLRT appears to offer an excellent tradeoff in terms of robustness versus sidelobe rejection. Ideally, it would be highly desirable to control the rate at which the probability of detection falls when $\boldsymbol{s}$ departs from $\boldsymbol{v}$. To this end, some strategies have been proposed. Reference [10] proposes a blend of the GLRT and the AMF; a single scalar sensitivity parameter is used to control the degree to which unwanted signals are rejected. In [11] and [12], the actual target's signature is assumed to belong to a cone, whose axis is the presumed signature. The cone angle is a user parameter that can be selected so as to ensure the desired tradeoff between mainlobe sensitivity and sidelobes energy rejection. Another solution is to use a two-stage detection scheme: the first stage uses a detection scheme that should let pass most signals—so as not to miss a slightly mismatched target signal—while the second stage is more selective and rejects signals that are deemed not to impinge from the look direction; see, e.g., [13] and [14]. A third solution consists in modifying the hypothesis testing problem, so as to include a fictitious signal under the null hypothesis, which is sufficiently far from $\boldsymbol{v}$; see, e.g., [15] and [16]. Hence, if $\boldsymbol{s}$ is too far from $\boldsymbol{v}$, no detection will be declared.

However, the previous references consider the case where $\boldsymbol{R}_T = \boldsymbol{R}$ and a single signal is impinging. In some situations, such as undernulled interference, there might be an additional signal in the test vector, which is not accounted for by secondary data, resulting in $\boldsymbol{R}_T = \boldsymbol{R} + \boldsymbol{q}\boldsymbol{q}^H$, where $\boldsymbol{q}$ is unknown. In [17], we considered the following hypothesis testing problem:

$$H_0 : \begin{cases} \boldsymbol{z} = \boldsymbol{n} \\ \boldsymbol{z}_\ell = \boldsymbol{n}_\ell, \quad \ell = 1, \cdots, L \end{cases}$$
$$H_1 : \begin{cases} \boldsymbol{z} = \alpha\boldsymbol{v} + \boldsymbol{n} \\ \boldsymbol{z}_\ell = \boldsymbol{n}_\ell, \qquad \ell = 1, \cdots, L \end{cases} \tag{1}$$

where $\boldsymbol{n}$ and $\boldsymbol{n}_\ell$ are independent, zero-mean complex-valued Gaussian distributed random vectors with respective covariance matrices

$$\mathcal{E}\left\{\boldsymbol{n}_\ell\boldsymbol{n}_\ell^H\right\} = \boldsymbol{R}, \quad \mathcal{E}\{\boldsymbol{n}\boldsymbol{n}^H\} = \boldsymbol{R}_T = \boldsymbol{R} + \boldsymbol{q}\boldsymbol{q}^H \tag{2}$$

where $\boldsymbol{q}$ is unknown and arbitrary. In this letter, we propose a modification of the detection problem (1), where $\boldsymbol{R}_T$ and $\boldsymbol{R}$ are now constrained to satisfy the so-called generalized eigenrelation (GER) [7], [18], which states that $\boldsymbol{R}^{-1}\boldsymbol{v} = \lambda\boldsymbol{R}_T^{-1}\boldsymbol{v}$.

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In the case where $\boldsymbol{R}_T = \boldsymbol{R} + \boldsymbol{q}\boldsymbol{q}^H$, this amounts to enforce that $\boldsymbol{q}^H \boldsymbol{R}^{-1} \boldsymbol{v} = 0$, i.e., that $\boldsymbol{q}$ falls in a null of the asymptotic optimal beampattern. The GER was primarily introduced by Richmond as a means to simplify the analysis of conventional adaptive detectors in nonhomogeneous environments and to derive closed-form expressions for the distribution of the corresponding test statistics. However, as convincingly argued by Richmond, the GER, even though it is not likely to be perfectly satisfied in practice, is a viable approach to modelling undernulled interference [7], [18].

Another motivation for studying the type of detection problem described in (1) is that it can also be helpful to discriminate between a slightly mismatched target signal and a sidelobe target. Consider first the case where $\boldsymbol{q}$ is arbitrary. If a single signal with response $\boldsymbol{s}$ is present, and if $\boldsymbol{s}$ is not close to $\boldsymbol{v}$, then it is likely that $\boldsymbol{s}$ will be assimilated to $\boldsymbol{q}$, and thus, H_0 will be in force. Therefore, we can expect this test to have excellent sidelobe rejection capabilities. Indeed, we showed that the GLRT for the detection problem (1) with arbitrary $\boldsymbol{q}$ is the ACE, which provided a new explanation of the behavior of ACE. However, the price to be paid is a lower detection capability of slightly mismatched signals. In order to remedy this problem, introducing the constraint $\boldsymbol{q}^H \boldsymbol{R}^{-1} \boldsymbol{v} = 0$ may be a solution. Indeed, only those signals $\boldsymbol{s}$ that are sufficiently "far" from $\boldsymbol{v}$ can be assimilated to $\boldsymbol{q}$, and therefore, one can expect a better detection of slightly mismatched target signals. In other words, enforcing the GER may offer a good tradeoff between robustness to steering vector uncertainties and sidelobe rejection capabilities.

In the next section, we derive the GLRT for the hypothesis testing problem (1), assuming that the GER holds, i.e., that $\boldsymbol{q}^H \boldsymbol{R}^{-1} \boldsymbol{v} = 0$.

II. Generalized Likelihood Ratio Test

Let $\boldsymbol{Z} = [\boldsymbol{z}_1 \dots \boldsymbol{z}_L]^T$ denote the secondary data matrix. Under the stated assumptions, the joint likelihood function of $\boldsymbol{z}$ and $\boldsymbol{Z}$, under hypothesis H_k, is given by [17]

$$\begin{aligned} f_k(\boldsymbol{z}, \boldsymbol{Z}) = {} & \pi^{-N(L+1)} |\boldsymbol{R}|^{-(L+1)} \mathrm{etr}\{-\boldsymbol{R}^{-1}\boldsymbol{S}_1\} \\ & \times [1 + \boldsymbol{q}^H \boldsymbol{R}^{-1} \boldsymbol{q}]^{-1} \exp\left\{ \frac{\left| \boldsymbol{y}_k^H \boldsymbol{R}^{-1} \boldsymbol{q} \right|^2}{1 + \boldsymbol{q}^H \boldsymbol{R}^{-1} \boldsymbol{q}} \right\} \end{aligned} \tag{3}$$

where

$$\boldsymbol{y}_k = \boldsymbol{z} - \mu_k \alpha \boldsymbol{v} \tag{4}$$

with $\mu_0 = 0$, $\mu_1 = 1$, and

$$\boldsymbol{S}_1 = \boldsymbol{S} + \boldsymbol{y}_k \boldsymbol{y}_k^H \tag{5}$$

$$\boldsymbol{S} = \sum_{\ell=1}^{L} \boldsymbol{z}_\ell \boldsymbol{z}_\ell^H. \tag{6}$$

In (3), $\mathrm{etr}\{.\}$ stands for the exponential of the trace of the matrix between braces. We maximize the likelihood function (LF) successively with respect to (w.r.t.) $\boldsymbol{q}$—under the constraint $\boldsymbol{q}^H \boldsymbol{R}^{-1} \boldsymbol{v} = 0$—$\boldsymbol{R}$ and w.r.t. α under H_1. In order to maximize the LF w.r.t. $\boldsymbol{q}$, we need to maximize

$$J_k = [1 + \boldsymbol{q}^H \boldsymbol{R}^{-1} \boldsymbol{q}]^{-1} \exp\left\{ \frac{\left| \boldsymbol{y}_k^H \boldsymbol{R}^{-1} \boldsymbol{q} \right|^2}{1 + \boldsymbol{q}^H \boldsymbol{R}^{-1} \boldsymbol{q}} \right\} \tag{7}$$

under the constraint that $\boldsymbol{q}^H \boldsymbol{R}^{-1} \boldsymbol{v} = 0$. Let $\tilde{\boldsymbol{y}}_k = \boldsymbol{R}^{-1/2} \boldsymbol{y}_k$, $\tilde{\boldsymbol{v}} = \boldsymbol{R}^{-1/2} \boldsymbol{v}$, $\tilde{\boldsymbol{q}} = \boldsymbol{R}^{-1/2} \boldsymbol{q}$ and let $\tilde{\boldsymbol{V}} \in \mathbb{C}^{N \times (N-1)}$ be an orthonormal basis for $\mathcal{R}(\tilde{\boldsymbol{v}})^\perp$. The GER constraint implies that $\tilde{\boldsymbol{q}} = \tilde{\mathbf{V}} \boldsymbol{\theta}$ for some unknown vector $\boldsymbol{\theta} \in \mathbb{C}^{(N-1) \times 1}$. J_k can thus be rewritten as

$$J_k = [1 + \boldsymbol{\theta}^H \boldsymbol{\theta}]^{-1} \exp\left\{ \frac{\left| \tilde{\boldsymbol{y}}_k^H \tilde{\boldsymbol{V}} \boldsymbol{\theta} \right|^2}{1 + \boldsymbol{\theta}^H \boldsymbol{\theta}} \right\}. \tag{8}$$

Using arguments similar to those presented in [17], it can be shown that

$$\max_{\boldsymbol{q}/\boldsymbol{q}^H \boldsymbol{R}^{-1} \boldsymbol{v} = 0} J_k = \left(\tilde{\boldsymbol{y}}_k^H \tilde{\boldsymbol{V}} \tilde{\boldsymbol{V}}^H \tilde{\boldsymbol{y}}_k \right)^{-1} \exp\left\{ \tilde{\boldsymbol{y}}_k^H \tilde{\boldsymbol{V}} \tilde{\boldsymbol{V}}^H \tilde{\boldsymbol{y}}_k - 1 \right\}. \tag{9}$$

Let us now observe that

$$\begin{aligned} g(\boldsymbol{y}_k, \boldsymbol{R}) &= \tilde{\boldsymbol{y}}_k^H \tilde{\boldsymbol{V}} \tilde{\boldsymbol{V}}^H \tilde{\boldsymbol{y}}_k = \| \boldsymbol{P}_{\tilde{\boldsymbol{V}}} \tilde{\boldsymbol{y}}_k \|^2 \\ &= \| \tilde{\boldsymbol{y}}_k - \boldsymbol{P}_{\tilde{\boldsymbol{v}}} \tilde{\boldsymbol{y}}_k \|^2 = \left\| \tilde{\boldsymbol{y}}_k - \tilde{\boldsymbol{v}} \frac{\tilde{\boldsymbol{v}}^H \tilde{\boldsymbol{y}}_k}{\tilde{\boldsymbol{v}}^H \tilde{\boldsymbol{v}}} \right\|^2 \\ &= \left[\boldsymbol{y}_k - \frac{\boldsymbol{v}^H \boldsymbol{R}^{-1} \boldsymbol{y}_k}{\boldsymbol{v}^H \boldsymbol{R}^{-1} \boldsymbol{v}} \boldsymbol{v} \right]^H \\ &\quad \times \boldsymbol{R}^{-1} \left[\boldsymbol{y}_k - \frac{\boldsymbol{v}^H \boldsymbol{R}^{-1} \boldsymbol{y}_k}{\boldsymbol{v}^H \boldsymbol{R}^{-1} \boldsymbol{v}} \boldsymbol{v} \right] \end{aligned} \tag{10}$$

where $\boldsymbol{P}_{\tilde{\boldsymbol{V}}}$ and $\boldsymbol{P}_{\tilde{\boldsymbol{v}}}$ are the orthogonal projectors on $\mathcal{R}(\tilde{\boldsymbol{V}})$ and $\mathcal{R}(\tilde{\boldsymbol{v}})$, respectively. It ensues that

$$\begin{aligned} \max_{\boldsymbol{q}/\boldsymbol{q}^H \boldsymbol{R}^{-1} \boldsymbol{v} = 0} f_k(\boldsymbol{z}, \boldsymbol{Z}) = {} & \pi^{-N(L+1)} e^{-1} |\boldsymbol{R}|^{-(L+1)} \\ & \times \mathrm{etr}\{-\boldsymbol{R}^{-1}\boldsymbol{S}_1\} [g(\boldsymbol{y}_k, \boldsymbol{R})]^{-1} \exp\{g(\boldsymbol{y}_k, \boldsymbol{R})\}. \end{aligned} \tag{11}$$

In order to obtain $\boldsymbol{R}$, we need now to maximize

$$h(\boldsymbol{y}_k, \boldsymbol{R}) = |\boldsymbol{R}|^{-(L+1)} \mathrm{etr}\{-\boldsymbol{R}^{-1}\boldsymbol{S}_1\} [g(\boldsymbol{y}_k, \boldsymbol{R})]^{-1} e^{g(\boldsymbol{y}_k, \boldsymbol{R})} \tag{12}$$

with respect to $\boldsymbol{R}$. Using the readily verified fact that

$$\frac{\partial g(\boldsymbol{y}_k, \boldsymbol{R})}{\partial \boldsymbol{R}} = -\boldsymbol{R}^{-1} (\boldsymbol{y}_k - \beta \boldsymbol{v})(\boldsymbol{y}_k - \beta \boldsymbol{v})^H \boldsymbol{R}^{-1} \tag{13}$$

with

$$\beta = \frac{\boldsymbol{v}^H \boldsymbol{R}^{-1} \boldsymbol{y}_k}{\boldsymbol{v}^H \boldsymbol{R}^{-1} \boldsymbol{v}} \tag{14}$$

it follows that

$$\begin{aligned} \frac{\partial \ln h(\boldsymbol{y}_k, \boldsymbol{R})}{\partial \boldsymbol{R}} = & -(L+1)\boldsymbol{R}^{-1} + \boldsymbol{R}^{-1}\boldsymbol{S}_1\boldsymbol{R}^{-1} \\ & + \frac{\boldsymbol{R}^{-1}(\boldsymbol{y}_k - \beta\boldsymbol{v})(\boldsymbol{y}_k - \beta\boldsymbol{v})^H \boldsymbol{R}^{-1}}{(\boldsymbol{y}_k - \beta\boldsymbol{v})^H \boldsymbol{R}^{-1}(\boldsymbol{y}_k - \beta\boldsymbol{v})} \\ & - \boldsymbol{R}^{-1}(\boldsymbol{y}_k - \beta\boldsymbol{v})(\boldsymbol{y}_k - \beta\boldsymbol{v})^H \boldsymbol{R}^{-1} \end{aligned} \tag{15}$$

from which we infer that $\boldsymbol{R}$ satisfies the following implicit equation:

$$(L+1)\boldsymbol{R} = \boldsymbol{S}_1 + \gamma(\boldsymbol{y}_k - \beta\boldsymbol{v})(\boldsymbol{y}_k - \beta\boldsymbol{v})^H \tag{16}$$

with

$$\gamma = \left[(\boldsymbol{y}_k - \beta\boldsymbol{v})^H \boldsymbol{R}^{-1}(\boldsymbol{y}_k - \beta\boldsymbol{v})\right]^{-1} - 1. \tag{17}$$

We now show that if $\boldsymbol{R}$ satisfies (16), then β and γ can be expressed as functions of $\boldsymbol{y}_k$ and $\boldsymbol{S}_1$ only. In order to determine β, note from (16) that

$$\begin{aligned} \boldsymbol{S}_1^{-1} = & (L+1)^{-1}\boldsymbol{R}^{-1} \\ & + \frac{\gamma(L+1)^{-2}\boldsymbol{R}^{-1}(\boldsymbol{y}_k - \beta\boldsymbol{v})(\boldsymbol{y}_k - \beta\boldsymbol{v})^H \boldsymbol{R}^{-1}}{1 - \gamma(L+1)^{-1}(\boldsymbol{y}_k - \beta\boldsymbol{v})^H \boldsymbol{R}^{-1}(\boldsymbol{y}_k - \beta\boldsymbol{v})}. \end{aligned} \tag{18}$$

Since from (14) $(\boldsymbol{y}_k - \beta\boldsymbol{v})^H \boldsymbol{R}^{-1}\boldsymbol{v} = 0$, post-multiplying (18) by $\boldsymbol{v}$ leads to

$$\boldsymbol{S}_1^{-1}\boldsymbol{v} = (L+1)^{-1}\boldsymbol{R}^{-1}\boldsymbol{v}. \tag{19}$$

Therefore, if $\boldsymbol{R}$ satisfies (16), β in (14) is necessarily given by

$$\beta = \frac{\boldsymbol{v}^H \boldsymbol{S}_1^{-1} \boldsymbol{y}_k}{\boldsymbol{v}^H \boldsymbol{S}_1^{-1} \boldsymbol{v}}. \tag{20}$$

Accordingly, (16) yields

$$(L+1)^{-1}\boldsymbol{R}^{-1} = \boldsymbol{S}_1^{-1} - \frac{\gamma \boldsymbol{S}_1^{-1}(\boldsymbol{y}_k - \beta\boldsymbol{v})(\boldsymbol{y}_k - \beta\boldsymbol{v})^H \boldsymbol{S}_1^{-1}}{1 + \gamma(\boldsymbol{y}_k - \beta\boldsymbol{v})^H \boldsymbol{S}_1^{-1}(\boldsymbol{y}_k - \beta\boldsymbol{v})}. \tag{21}$$

Pre-multiplying the previous equation by $(\boldsymbol{y}_k - \beta\boldsymbol{v})^H$ and post-multiplying it by $(\boldsymbol{y}_k - \beta\boldsymbol{v})$, and after simple calculations, it can be shown that

$$\gamma = \frac{1 - (L+1)\xi_k}{L\xi_k} \tag{22}$$

where

$$\xi_k = (\boldsymbol{y}_k - \beta\boldsymbol{v})^H \boldsymbol{S}_1^{-1}(\boldsymbol{y}_k - \beta\boldsymbol{v}). \tag{23}$$

Therefore, the matrix $\boldsymbol{R}$ that maximizes $h(\boldsymbol{y}_k, \boldsymbol{R})$ is given by (16), with β and γ given in (20) and (22), respectively. Reporting this value in the LF, and after some tedious but straightforward derivations, we obtain

$$\max_{\boldsymbol{q},\boldsymbol{R}} f_k(\boldsymbol{z}, \boldsymbol{Z}) \propto |\boldsymbol{S}_1|^{-(L+1)} \frac{(1-\xi_k)^{-L}}{\xi_k}. \tag{24}$$

We need now to derive a simpler expression for (24) that involves $\boldsymbol{S}$ and $\boldsymbol{y}_k$ only. For that, we notice that

$$\begin{aligned} |\boldsymbol{S}_1| &= |\boldsymbol{S}|\left(1 + \boldsymbol{y}_k^H \boldsymbol{S}^{-1} \boldsymbol{y}_k\right) \\ \boldsymbol{S}_1^{-1} &= \boldsymbol{S}^{-1} - \frac{\boldsymbol{S}^{-1}\boldsymbol{y}_k \boldsymbol{y}_k^H \boldsymbol{S}^{-1}}{1 + \boldsymbol{y}_k^H \boldsymbol{S}^{-1} \boldsymbol{y}_k}. \end{aligned}$$

Using the previous equation, it is not difficult to show that

$$\begin{aligned} \xi_k &= 1 - \frac{1}{1 + \left(\boldsymbol{y}_k - \frac{\boldsymbol{v}^H \boldsymbol{S}^{-1} \boldsymbol{y}_k}{\boldsymbol{v}^H \boldsymbol{S}^{-1} \boldsymbol{v}}\boldsymbol{v}\right)^H \boldsymbol{S}^{-1} \left(\boldsymbol{y}_k - \frac{\boldsymbol{v}^H \boldsymbol{S}^{-1} \boldsymbol{y}_k}{\boldsymbol{v}^H \boldsymbol{S}^{-1} \boldsymbol{v}}\boldsymbol{v}\right)} \\ &= 1 - \frac{1}{1 + \left(\boldsymbol{z} - \frac{\boldsymbol{v}^H \boldsymbol{S}^{-1} \boldsymbol{z}}{\boldsymbol{v}^H \boldsymbol{S}^{-1} \boldsymbol{v}}\boldsymbol{v}\right)^H \boldsymbol{S}^{-1} \left(\boldsymbol{z} - \frac{\boldsymbol{v}^H \boldsymbol{S}^{-1} \boldsymbol{z}}{\boldsymbol{v}^H \boldsymbol{S}^{-1} \boldsymbol{v}}\boldsymbol{v}\right)}. \end{aligned} \tag{25}$$

Consequently

$$\max_{\boldsymbol{q},\boldsymbol{R}} f_k(\boldsymbol{z}, \boldsymbol{Z}) \propto \left(1 + \boldsymbol{y}_k^H \boldsymbol{S}^{-1} \boldsymbol{y}_k\right)^{-(L+1)}. \tag{26}$$

Under H_0, the LF maximization is terminated. Under H_1, it remains to minimize $(\boldsymbol{z} - \alpha\boldsymbol{s})^H \boldsymbol{S}^{-1}(\boldsymbol{z} - \alpha\boldsymbol{s})$. However, it is known that [1]

$$\min_{\alpha} (\boldsymbol{z} - \alpha\boldsymbol{v})^H \boldsymbol{S}^{-1}(\boldsymbol{z} - \alpha\boldsymbol{v}) = \boldsymbol{z}^H \boldsymbol{S}^{-1}\boldsymbol{z} - \frac{|\boldsymbol{z}^H \boldsymbol{S}^{-1}\boldsymbol{v}|^2}{\boldsymbol{v}^H \boldsymbol{S}^{-1}\boldsymbol{v}} \tag{27}$$

and therefore the GLR is

$$\mathrm{GLR} = \left(\frac{1 + \boldsymbol{z}^H \boldsymbol{S}^{-1}\boldsymbol{z}}{1 + \boldsymbol{z}^H \boldsymbol{S}^{-1}\boldsymbol{z} - \frac{|\boldsymbol{z}^H \boldsymbol{S}^{-1}\boldsymbol{v}|^2}{\boldsymbol{v}^H \boldsymbol{S}^{-1}\boldsymbol{v}}}\right)^{L+1}. \tag{28}$$

Finally, the GLRT is equivalent to

$$\mathrm{GLRT} \equiv \frac{|\boldsymbol{z}^H \boldsymbol{S}^{-1}\boldsymbol{v}|^2}{[1 + \boldsymbol{z}^H \boldsymbol{S}^{-1}\boldsymbol{z}](\boldsymbol{v}^H \boldsymbol{S}^{-1}\boldsymbol{v})} \underset{H_0}{\overset{H_1}{\gtrless}} \eta \tag{29}$$

which is nothing but Kelly's detector [1]. It is amazing to see that Kelly's detector, which is one of the most celebrated adaptive detectors, whose good properties and performances have been thoroughly studied, emerges as the solution of this new detection problem. The result presented here partly explains why Kelly's detector performs well in the case of undernulled interference [7]. Moreover, as we intuitively discussed in the introduction, it explains why it is less selective than ACE (in terms of sidelobe rejection capabilities) but more performant for detection of slightly mismatched signals, hence providing a good solution for trading robustness against sidelobe rejection.

III. Conclusion

We considered a variation on our previous work [17] on adaptive detection in the case of a covariance mismatch due to an interfering signal that contaminates the test vector but not the training samples. In contrast to [17], where the interference's signature could be arbitrary, we assumed here it is such that the generalized eigenrelation between $\boldsymbol{R}_T$ and $\boldsymbol{R}$ is satisfied. As we discussed, this constraint may also help to offer a good tradeoff between robustness to slightly mismatched signals and rejection of unwanted signals. We showed that the GLRT for this problem is Kelly's detector, providing one more argument to use it.

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