CHARACTERIZATION OF IMAGE SETS : THE GALOIS LATTICE APPROACH

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ABSTRACT

This paper presents a new method for supervised image classification. One or several landmarks are attached to each class, with the intention of characterizing it and discriminating it from the other classes. The different features, deduced from image primitives, and their relationships with the sets of images are structured and organized into a hierarchy thanks to an original method relying on a mathematical formalism called Galois (or Concept) Lattices. Such lattices allow us to select features as landmarks of specific classes. This paper details the feature selection process and illustrates this through a robotic example in a structured environment. The class of any image is the room from which the image is shot by the robot camera. In the discussion, we compare this approach with decision trees and we give some issues for future research.

Keywords : Galois Lattices, Visual Landmarks, Image Characterization, Computer Vision, Learning.

1. INTRODUCTION

Characterizing and recognizing a place in a structured environment with several rooms, using only a set of views attached to each place, is a difficult challenge to take up for a machine (computer or robot) today. To do this, the machine needs to find "something" that 1. characterizes a considered place, and 2. distinguishes it from the others. This "something", under specific conditions, is called a (visual) *landmark*. What is a landmark ? How to find it ? And how to select it ?

This article presents a new method to answer these questions. All the images issued from one place are regrouped into a set. Thus, we have to recognize one original place from one image of the associated set. At first, during a learning stage, the relationships between sets of images and features are structured and organized into a hierarchy, through a formalism called *Galois Lattices*, or *Concept lattices*. The use of such mathematical structures allows the machine to determine its own landmarks attached to each place. Subsequently, once this initial characterization has been performed, the machine is able in a second stage to recognize the corresponding place thanks to the landmarks it has learned.

The choice of the application we have done makes the connection between one set of images and one room of a structured environment. Thus we expect that there will be more or less common properties between images of one set. But the theory we have developed here considers only sets of images without any restriction.

This paper is organized as follows : sections 2 and 3 introduce landmarks, primitives and features; section 4 presents Galois lattices ; sections 5 and 6 the way they are used to define and to build landmarks ; section 7 exposes the results and section 8 talks about conclusion, comparison with other techniques and perspectives.

2. LANDMARKS

As defined in the Cambridge Dictionary, a landmark is *a building or place that is easily recognized, especially one which you can use to judge where you are.* This original definition, applied to the mobile robotics field, has several versions such as ”distinctive templates from one image which can be readily recognized in a second image acquired from a different viewpoint” [1], or more simply ”identifiable visual objects in the environment” [2]. Usually landmarks are not introduced according to a formal definition but but through some specific properties as ”easily distinguishable” [3] or ”locally unique” [3]. In concrete terms, a landmark could be an object [4], a color [5], interest points [6], *etc.* There are no universal landmarks. Landmarks are usually tied to one's perceptions and feelings. "Something" chosen by one person as a landmark could be unnoticed, even insignificant, to another one, and vice versa. Nevertheless, it is essential that a landmark checks the two following characteristics : first it should discriminate between locations, and second it should be stable to allow robust identification against variations of the observer position and time [1][7]. Several classifications of landmarks, as static/dynamique [8], already exist, still we propose here another classification based upon the learning ability and the autonomy of the recognition system. We do separate landmarks into three categories :

- fully pre-defined landmarks : the machine is given a database of objects [4][1][9] which are "just" to be recognized ;
- partially pre-defined landmarks : such potential landmarks are specified by a common structure. For instance, in [10], [11], the authors use planar quadrangular forms (typically, posters) they characterize with interest points [12] and Hausdorff distance. Observations which could fit into the specified framework are then dynamically chosen as landmarks ;
- non pre-defined landmarks : no hypothesis is assumed about potential landmarks. The main approaches with such landmarks are biologically inspired [13][14][15].

Our approach deals with the last category : we want the machine to choose the most relevant landmarks in an autonomous and dynamic way. We will develop the landmark definition process further in this article.

3. PRIMITIVES AND FEATURES

Different pictures are extracted from each room of the environment ; thus, a set of images is attached to each room. From these different pictures, *primitives* are extracted to build *features* of images, to help the robot to find *properties* of each place. We do distinguish feature from properties by the fact that features are attached to images, whereas properties are attached to the place. Three kinds of primitives are extracted from the different pictures :

- structural primitives : segments with their size and orientation (they are issued from polynomial contour extraction, interest points [16], . . . ;
- colorimetric primitives : extraction of red, green, blue, cyan, magenta or yellow pixels with joint histograms, . . . ;
- photogrammetric primitives, issued from pixels intensity : a color and texture based segmentation and characterization is applied to the images.

From all these primitives, features are extracted in all set of images. Notice that our definition of feature is extensive and includes any potential feature, whatever it is present in an image or not. For instance, with colorimetric primitives, examples of potential features are *"there is some yellow here"* or *"there is much more green than any other colors here"*. In the same way, *"there is such texture"* could be another feature. Notice that we restrict ourselves to features that are invariant against rotation, translation and scaling. For instance, using segments (primitives) extracted from contours, one feature could be : *"there is a large number of identical (orientation and size) segments"* (Typically, this feature may be issued from a bookcase that is present in the considered place).

4. THE GALOIS LATTICES

Galois Lattices have been used widely in Artificial Intelligence in the past 20 years. This theory has been developped as *FCA : Formal Concept Analysis*, and several lattice building algorithms appeared since then, more and more efficient [17]. Still few concrete applications recently appeared mainly in data mining topics such as machine learning [18, 19] or in the aeronautic field [20]. We outline here an application to image processing in mobile robotics.

4.1. Mathematical Formalism [21][22]

Definition 1 *A* **lattice** *is defined as an ordered set in which two any elements have a least upper bound (lub) and a greatest lower bound (glb). A complete lattice is a lattice where any set has a lub and a glb.*

For instance, the set $\mathcal{P}(\mathcal{O})$ of all subsets of a set $\mathcal{O}$ ordered by the inclusion $\subset$ is a complete lattice.

Definition 2 *A context $\mathcal{K}$ is a triple $(\mathcal{O}, \mathcal{F}, \zeta)$ where $\mathcal{O}$ is a set of objects, $\mathcal{F}$ is a set of attributes and ζ is a mapping from $\mathcal{O} \times \mathcal{F}$ into $\{0, 1\}$.*

In our application the objects are the images of the various sets, the attributes are the features and the mapping ζ is defined by $\zeta(o, f) = 1$ if and only if feature f is present in image o.

Definition 3 *Given a context $(\mathcal{O}, \mathcal{F}, \zeta)$ let us define two mappings from $\mathcal{P}(\mathcal{O})$ into $\mathcal{P}(\mathcal{F})$ and from $\mathcal{P}(\mathcal{F})$ into $\mathcal{P}(\mathcal{O})$ using the same notation $'$ by the formula*

$$\forall \mathcal{A} \subset \mathcal{O}, \mathcal{A}' = \{f \in \mathcal{F} \mid \forall o \in \mathcal{A}, \zeta(o, f) = 1\}$$

and

$$\forall \mathcal{B} \subset \mathcal{F}, \mathcal{B}' = \{o \in \mathcal{O} \mid \forall f \in \mathcal{B}, \zeta(o, f) = 1\}$$

These mappings are called the **Galois connections** *of the context; $\mathcal{A}'$ is called the* **dual** *of $\mathcal{A}$, similarly $\mathcal{B}'$ is called the* **dual** *of $\mathcal{B}$.*

Clearly, $\mathcal{A}'$ is the set of common attributes to all objects of $\mathcal{A}$, and $\mathcal{B}'$ is the set of objects which share any attribute belonging to $\mathcal{B}$.
The properties of the Galois connections can be found in [23]. Let us recall the basic following properties :

Property 1 $\mathcal{A}_1 \subset \mathcal{A}_2 \Rightarrow \mathcal{A}_2' \subset \mathcal{A}_1'$;

Property 2 $\mathcal{A} \subset \mathcal{A}''$;

Property 3 $\mathcal{A}' = \mathcal{A}'''$;

We are now able to state the defintion of a concept :

Definition 4 *Given a context* $\mathcal{K} = (\mathcal{O}, \mathcal{F}, \zeta)$*, the pair* $\mathcal{C} = (\mathcal{A}, \mathcal{B})$ *is called a* **concept** *of* $\mathcal{K}$ *if and only if* $\mathcal{A}' = \mathcal{B}$ *et* $\mathcal{B}' = \mathcal{A}$.

Definition 5 $\mathcal{A}$ *is called the* **extent** *of the concept* $\mathcal{C}$ *and* $\mathcal{B}$ *is called its* **intent**. *One notes* $\mathcal{A} = \texttt{extent}(\mathcal{C})$ *and* $\mathcal{B} = \texttt{intent}(\mathcal{C})$.

The set of all concepts of a context $\mathcal{K}$ is denoted $\mathcal{L}(\mathcal{K})$ or simply $\mathcal{L}$ if the context is clear. One proves [22] the following

Theoreme 1 *Let* $\mathcal{C}_1 = (\mathcal{A}_1, \mathcal{B}_1)$ *and* $\mathcal{C}_2 = (\mathcal{A}_2, \mathcal{B}_2)$ *be a couple of concepts then* $\mathcal{C}_1 \vee \mathcal{C}_2 = ((\mathcal{A}_1 \cup \mathcal{A}_2)'', \mathcal{B}_1 \cap \mathcal{B}_2)$ *and* $\mathcal{C}_1 \wedge \mathcal{C}_2 = (\mathcal{A}_1 \cap \mathcal{A}_2, (\mathcal{B}_1 \cup \mathcal{B}_2)'')$ *are concepts.*

This result may be extended to any set $\mathcal{I}$ of concepts. We shall note $\mathcal{C}_\mathcal{I} = (\mathcal{A}_\mathcal{I}, \mathcal{B}_\mathcal{I}) = \bigvee_{i \in \mathcal{I}} \mathcal{C}_i$ and similarly $\mathcal{C}^\mathcal{I} = (\mathcal{A}^\mathcal{I}, \mathcal{B}^\mathcal{I}) = \bigwedge_{i \in \mathcal{I}} \mathcal{C}_i$
Thus, the set of concepts $\mathcal{L}$ when it is endowed with the order relation $\subset$ of its extents is a complete lattice and we can set

Definition 6 *The complete lattice* $\mathcal{L}(\mathcal{K})$ *of concepts of the context* $\mathcal{K}$ *is called the* **Galois lattice** *or the* **formal concept lattice**.

4.2. Proper context and disjunction

The extent of the smaller element of $\mathcal{L}$ is the set of objects that share all features of $\mathcal{F}$. So it is not generally $\emptyset$. Actually, the formal definition of a context that is given above is too general and includes unnecessary redundancies. It is possible to cancel trivial features which bring no information. By cancelling these trivial features and eventually adding a negative feature it is easy to restrict oneself to proper contexts in the following sense:

Definition 7 *Let* $\mathcal{K} = (\mathcal{O}, \mathcal{F}, \zeta)$ *be a context. If* $(\emptyset, \mathcal{F})$ *and* $(\mathcal{O}, \emptyset)$ *are concepts, then the context* $\mathcal{K}$ *is said* **proper**.

		Ppty 1	Ppty 2	Ppty 3	Ppty 4	Ppty 5	Ppty 6	Ppty 7	Ppty 8	Ppty 9	Ppty 10	Ppty 11	Ppty 12	Ppty 13	Ppty 14	Ppty 15
1	Place 1	x		x											x	
2	Place 1	x	x		x										x	
3	Place 1		x	x							x		x			
4	Place 1		x	x	x			x								x
5	Place 1			x	x					x						x
6	Place 1	x	x		x											
7	Place 2					x	x	x	x				x			
8	Place 2					x		x								x
9	Place 3									x	x					
10	Place 3									x	x				x	
11	Place 3		x				x				x		x		x	
12	Place 3			x						x	x					x
13	Place 3									x					x	
14	Place 4											x	x	x		
15	Place 4								x			x		x	x	
16	Place 4				x			x					x	x		
17	Place 4												x	x		x

Figure 1: *A Simple but Explicit Example*

In case the context is proper, $(\emptyset, \mathcal{F})$ is the smaller concept (the more restrictive), and $(\mathcal{O}, \emptyset)$is the greater concept (the more general). There is no canonical complementary in Galois lattice: if $\mathcal{C} = (\mathcal{A}, \mathcal{B})$ is a concept, generally neither $(\mathcal{A}^c, \mathcal{A}^{c\prime})$ nor $(\mathcal{B}^{c\prime}, \mathcal{B}^c)$ are concepts. However the disjunction property is very useful to build classifiers in standard classification theory [24]. So for learning purpose, it is useful to set

Definition 8 *Let* $\mathcal{C} = (\mathcal{A}, \mathcal{B})$ *be any concept of the context* $\mathcal{K} = (\mathcal{O}, \mathcal{F}, \zeta)$ *be a context. If* $(\mathcal{A}^c, \mathcal{A}^{c\prime})$ *(resp.* $(\mathcal{B}^{c\prime}, \mathcal{B}^c)$*) is a concept then* $\mathcal{A}$ *(resp.* $\mathcal{B}$*) is called a* ***disjunctive*** *concept-based object (resp. feature) set. The concept* $\mathcal{C}$ *is called object-disjunctive (resp. feature disjunctive)*

We can thus substract a disjunctive concept $(\mathcal{C}_2 = \mathcal{A}_2, \mathcal{B}_2)$ from any concept by achieving the set substraction on extents or intents according to the nature of disjunction.
If $\mathcal{I}$ and $\mathcal{J}$ are two subsets of $\mathcal{L}$ it is clear that $\mathcal{B}_\mathcal{I} \cap \mathcal{B}_\mathcal{J}$ is the intent of the least upper bound of $\mathcal{I} \cup \mathcal{J}$. The associate concept is called the *conjunction* of the concepts $\mathcal{C}_\mathcal{I}$ and $\mathcal{C}_\mathcal{J}$. Moreover, if $\mathcal{C}_\mathcal{J}$ is disjunctive one can achieve the disjunction of concepts as defined above. While building the galois lattice of a context, it is useful and amenable to list the disjunctive concepts.

4.3. Lattice Building Algorithms

Two families of lattice building algorithms exist : incremental algorithms and non-incremental algorithms.
Incremental algorithms (*Godin* [25], *Carpineto & Romano* [26], *Norris* [27], . . .) expand the lattice as soon as the objects come in, whereas non-incremental algorithms (*Chen* [28], *Ganter* [29], *Bordat* [30],. . .) build the lattice after the context is fully given.
All these algorithms build a lattice, without considering any specificity. The complexity of the algorithms is exponential *w.r.t.* the number of objects and attributes, and many techniques have been developed with the aim to reduce the complexity in view, as in [31] where attributes

are eliminated from the mapping, with an entropy function.

4.4. Example

Let us consider a simple but explicit example of a context, represented by a *cross table* (figure 1) and the corresponding lattice (figure 4.4).
Actually, the corresponding lattice presented here is a *heritage* lattice, that shows only new object(s) (*resp.* new attribute(s)) of a node *w.r.t.* its lower neighbors (*resp.* upper neighbors). Thus node writing is simplified and does not show redundant elements.
All the nodes of the lattice correspond to a concept. As noticed earlier, if one object is included into the extent of any concept, it is also included into the extent of all upper bounds of this concept. That is why, in this graph (usually called *line diagram* or *Hasse diagram* [22]), objects and attributes appear only once.

5. LANDMARK SELECTION

5.1. Formal concept approach of landmark learning

We want to use the formal concept approach to learn landmarks. For this purpose, the recognition system is provided with a set of images $\mathcal{O}$ and a set of features $\mathcal{F}$. Standard image processing techniques as explained in section 3 help to decide the value of the boolean function ζ. Furthermore, the set of images is the learning set and it is partitioned according to a label set Θ in our application frame : $o \subset \mathcal{O}_\theta$ means that image o has been taken when the robot is in the room $\theta \in \Theta$.
If the $\mathcal{O}_\theta$ are extent of concepts, then each associated feature sets $\mathcal{O}_{\theta'}$ is characteristic of its origin place and we call them *full landmarks*. Practically, this solution is too optimistic and we have to consider more complex situations.

5.2. Partial Landmark Definition

Definition 9 *Given a proper context $(\mathcal{O}, \mathcal{F}, \zeta)$ and a partition $(\mathcal{O}_\theta)_{\theta\in\Theta}$ of its object set θ, we say that a subset $\mathcal{B}$ of $\mathcal{F}$ is a* ***partial landmark*** *of $\theta \in \Theta$ if and only if*

- $\mathcal{B}' \subset \mathcal{O}_\theta$
- $\forall \vartheta \neq \theta, \mathcal{B}' \cap \mathcal{O}_\vartheta = \emptyset$

In other terms, a full landmark is a feature set present in all images of a set $\mathcal{O}_\theta$ whereas a partial landmark is a feature present in some of the images of the set. In both case, such a feature is valid for no image of another image set ϑ.

Proposition 1 *The partial landmark set of a place $\theta \subset \Theta$ is inductive in the sense that the lub of a chain (totally ordered set) of partial landmarks is a partial landmark. So maximal partial landmarks are well defined as maximal elements of the set of partial landmarks.*

Note that for any image set $\mathcal{O}_\theta$, there are maximal partial landmarks. Our decision rule will be based on these maximal partial landmarks.
Note that if all partial landmarks "cover" all images of a set, we can define in a more abstract way a full landmark as the union of such partial landmarks. In this case, the full landmark is not a specific property, but an equation-based structure such as {partial landmark α *or* partial landmark β}.

6. BUILDING A LANDMARK-BASED CLASSIFIER

In this section, we do expose the complete reasoning first to extract landmarks from a set of images, and second to label an image with a set.
Let us detail our basic application. We have at our disposal a set of images from a structured environment. Each image is labelled by the room from it was shot. Our objective is to provide a mobile robot equipped with a camera a decision rule to allow it to find its localization in a topological map[1]. It is basically a supervized classification problem. The decision rule is provided by a maximal partial landmark. Note that we are in a typical learning situation. The decision rule is extracted from a set of labelled exemples, the learning base of images. This rule is formalized for each set by the associated maximal partial landmark. Some images of the learning set may escape from the decision rule. Thus, due to the image preprocessing (primitive extraction) and the complexity of the environment, learning failing may occur.
There are actually two phases : the first phase deals with learning of the landmarks (*learning phase*), and the second phase deals with the use of these landmarks to find the set a new image comes from (*generalization phase*).

6.1. Learning Phase : Extracting the Landmarks

The first step is to extract primitives from each image. The algorithms used to do this are quite classical. For instance, to obtain segments, the contours are extracted with a Canny-Deriche algorithm, from which segments are extracted after contours being approximated with polynomial figures. Other primitives are found through image color or texture segmentation.
The second step is to build features with these primitives, and to fill up the cross table as shown in figure 1. The third step is to built the corresponding lattice. The last step is

[1] A topological map of a structured environment is a graph for which a node is a room and an edge is a connection between two rooms [32]

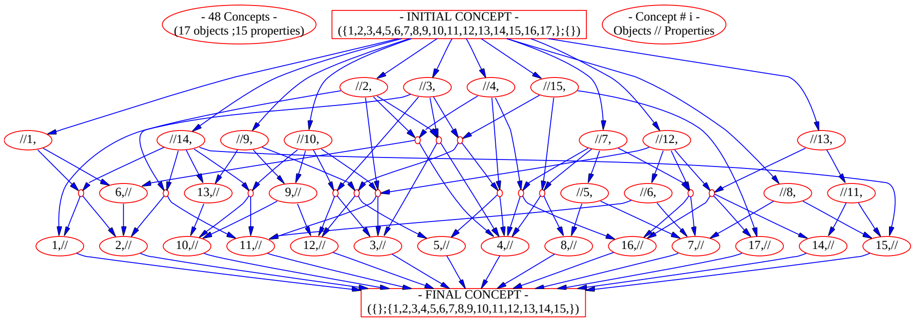

Figure 2: *The Corresponding Lattice*

to "read" the lattice, and to select the landmarks among properties.
The general algorithm of the landmark selection method is presented figure 3.

1. Extract primitives from each image
2. Extract properties of each place and Fill up the cross table
3. Build the corresponding lattice
4. Determine full and partial landmarks among the properties

Figure 3: General Algorithm of the Landmark Selection Method

6.2. Generalization Phase : Set (or image) Retrieval

Once the landmarks selected, we consider now a new image issued from any place. Primitives and properties are extracted from this image. Several cases should be considered :

- if at least one of its properties is a landmark of one set and other properties are not a landmark of another set, the image belongs to the considered set ;
- if no property is a landmark, we cannot conclude ;
- if different properties are landmarks of different sets, we cannot conclude.

7. EXPERIMENTAL RESULTS

We have experimented our approach in the problem of recognizing different places in a structured environment.
A set of 22 images issued from our laboratory has been analyzed, considering 50 potential properties :

- few traces, traces or many traces of red, green, blue, cyan, magenta, yellow ($3 \times 6 = 18$ properties), small, medium or big R-G-B-C-M-Y objects (18) ;

Place	Full Landmark	Partial Landmark
place $\#1$	{#42,#45}	{#22,#28}
place $\#2$	{#5}	{#3,#6,#32}
place $\#3$	$\emptyset$	{#14,#15,#17,#18,#19, #37,#38,#41,#44,#4}
place $\#4$	{#7}	$\emptyset$

Table 1: Les différents amers trouvés

- a large number, a very large number of identical segments in any orientation, horizontally and vertically (2×3), a large number, a very large number of segments with the same orientation (2), horizontally or vertically $(2+2)$, and a large number, a very large number of segments with the same size (2).

The cross table has been filled up and the corresponding lattice has been built (see figure 4).
For each set of images, the supremum of corresponding concepts has been found and we do obtain (table 1) :

- for the first set (first place), {#42,#45} is a full landmark and {#22,#28} is a partial landmark ;
- for the second set, {#5} is a full landmark and {#3,#6,#32} is a partial landmark ;
- for the third set, no full landmark and {#14,#15,#17,#18,#19,#37,# is a partial landmark ;
- and for the last set, {#7} is a full landmark and no partial landmark.

- for the first set, properties #42 and #45 are selected as full landmarks, #22 and #28 as partial landmarks ;
- for the second set, properties #5 is selected as a full landmark, #3,6 and #32 as partial landmarks ;
- for the third set, properties #14,15,17,18,19,37,38,41,44 and #49 as partial landmarks (no one as full landmark) ;

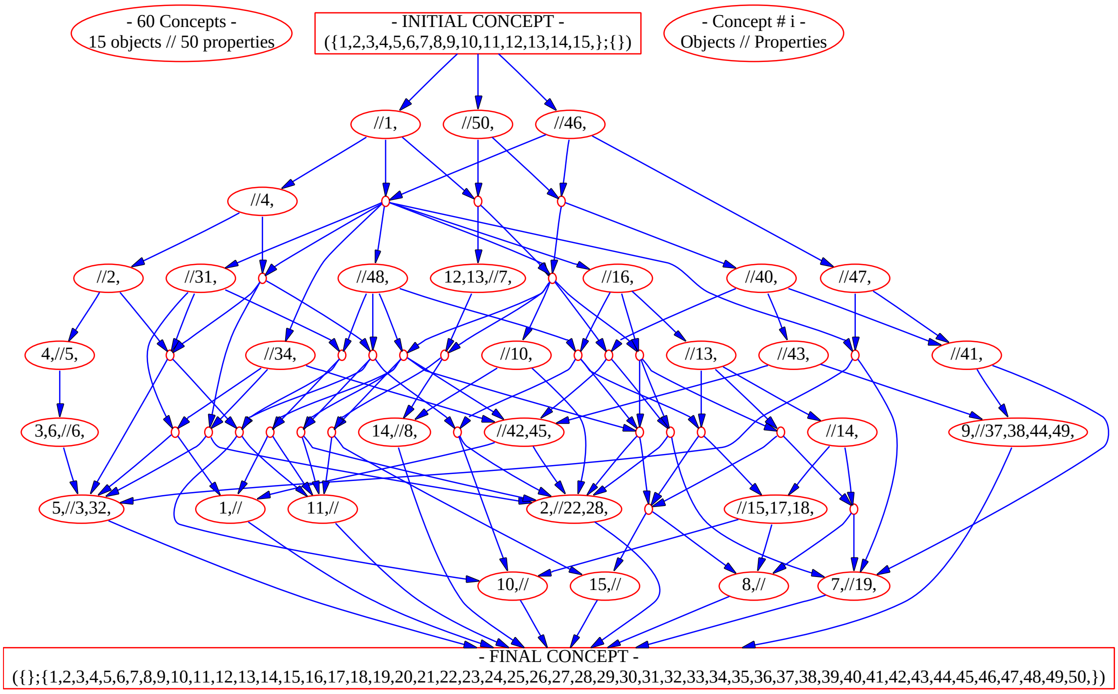

Figure 4: *Lattice Corresponding to the real application*

- finally, for the fourth set, properties #7 as full landmark, and no one as partial landmark.

A new set of 20 images has been taken in the different places of the environment. 18 of them have been correctly situated. The two last images belong to the fourth place, and the property #7 has not been revealed in these images.

8. DISCUSSION AND PERSPECTIVES

We have described in this paper a new application of the Galois lattice theory through a learning approach to solve the problem of characterizing sets of images. This work is motivated by actual issues in the autonomous mobile robotics field : our goal is to endow a robot with the ability to learn its environment and to recognize the place it is situated.

We have validated our general framework through experimentations with sets of real images. The approach turns out to provide very good results in small contexts (less than 30 objects/properties). However, the building lattice algorithms have an exponential complexity (see [17]). This prevents us from realizing a larger application with hundreds of images and properties. Moreover, in a real context, the exploitation of such a lattice will be ineffective, owing to the complexity of relationships between concepts. There could be so much concepts, and so much connections between them, that most of the time no feature set is selected to be a landmark.

Nevertheless, we have identified these flaws and started to elaborate a solution, the *decentralized approach* that will be soon implemented to find landmarks. This approach consists in building as many lattices as sets (or places). Thus the size of this context will be divided, but interaction between lattices have to be modelized to have good results.

Other supervized classification methods can use the same primitive data base to solve these classification problems. Neural networks and decision trees seem appropriate and have to be tested in the next future. Notably, decision trees have been recently used for characterizing handwritten characters with *tags* which can be compared to our features [24]. These techniques have their own limitations.

Notice that learning localization by a mobile robot is generally an on-line process. Thus it's better to get failing than classification errors which are common in statiscal approaches. If failing occurs, the robot will slightly change its location and a new snapshot will provide another opportunity to find the correct topological localization. If there are contradictory informations from the landmarks, the landmark semi-lattice has to be pruned to restore coherence. This ability is specific to our approach and is allowed by redundancy of the concepts. In the decision tree construction of [24], the relevant node of the trees (features) are selected by maximizing the entropy of the queries. So a new contradictory information may be difficult to process. Actually there is a compromise to establish between redundancy and robustness of the symbolic learning approach of Galois lattice and controlled complexity of statistical approaches. It may be interesting to allow moderate pruning of the concept lattice according to entropy rules to reduce the volume of computation in real applications [31]. This issue will be considered at the next step of the project.

Eventually, the landmarks which are considered in this paper are ”virtual landmarks” in the sense that they are constituted by perceptual features and do not label physical objects. The passage from landmarks to properties have to be considered to improve environment learning.

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