GLRT-Based Direction Detectors in Homogeneous Noise and Subspace Interference

Francesco Bandiera, *Member, IEEE*, Olivier Besson, *Senior Member, IEEE*, Danilo Orlando, Giuseppe Ricci, *Member, IEEE*, and Louis L. Scharf, *Life Fellow, IEEE*

***Abstract*—In this paper, we derive and assess decision schemes to discriminate, resorting to an array of sensors, between the H_0 hypothesis that data under test contain disturbance only (i.e., noise plus interference) and the H_1 hypothesis that they also contain signal components along a direction which is *a priori* unknown but constrained to belong to a given subspace of the observables. The disturbance is modeled in terms of complex normal random vectors plus deterministic interference assumed to belong to a known subspace. We assume that a set of noise-only (secondary) data is available, which possess the same statistical characterization of noise in the cells under test. At the design stage, we resort to either the plain generalized-likelihood ratio test (GLRT) or the two-step GLRT-based design procedure. The performance analysis, conducted resorting to simulated data, shows that the one-step GLRT performs better than the detector relying on the two-step design procedure when the number of secondary data is comparable to the number of sensors; moreover, it outperforms a one-step GLRT-based subspace detector when the dimension of the signal subspace is sufficiently high.**

***Index Terms*—Adaptive detection, direction detection, distributed targets, generalized likelihood ratio test, interference rejection.**

I. INTRODUCTION

ADAPTIVE detection of multiple-point-like or range-spread (in a word, distributed) targets embedded in Gaussian or non-Gaussian disturbance has received increasing attention from the signal processing community in recent years [1]–[9, and references therein]. More precisely, adaptive detection of distributed targets has been addressed in [1] and [2]; therein, useful target echoes are modeled as signals known up to multiplicative factors, possibly different from one range cell to another, namely supposed to belong to a known one-dimensional subspace of the observables. Noise is modeled in terms of independent, complex normal random vectors with a common covariance matrix up to possibly different power levels. Covariance matrices are unknown at the receiver, and a set of noise-only additional data (the so-called secondary data) is available for estimation purposes. More precisely, detectors based on the generalized-likelihood ratio test (GLRT) and ad hoc decision schemes (relying on the two-step GLRT-based design procedure) have been proposed in [1] for the case that noise vectors share one and the same covariance matrix (homogeneous scenario) or the same covariance matrix up to possibly different power levels between primary data (range cells under test) and secondary ones (partially homogeneous scenario). Proposed detectors possess the constant false alarm rate (CFAR) property under the design assumptions. In [2], an ad hoc detector is adopted in order to address detection of target echoes in a heterogeneous scenario, namely for the more general case that noise returns share the same covariance matrix up to possibly different power levels from one cell to another. Remarkably, the proposed decision scheme guarantees the CFAR property with respect to the covariance matrices of noise returns (under the design assumptions).

Detection of point-like targets, modeled as vectors constrained to belong to a known subspace of the observables, in the presence of interference and noise of unknown power has been considered in [3]; therein, the interference subspace is known and linearly independent of the signal subspace. Modeling useful target echoes in terms of signals belonging to a known subspace of the observables has also been suggested in [4] as a possible means to maintain an acceptable detection loss for slightly mismatched mainlobe targets. Adaptive subspace detection of point-like targets has been addressed in [5]. Detection of distributed targets, modeled in terms of vectors confined to a known subspace, and embedded in noise of unknown power plus deterministic interference, assumed to belong to an unknown subspace, has been considered in [6]. Finally, several detection algorithms are encompassed as special cases of the amazingly general framework and derivations in [7].

In this paper, we address adaptive detection of distributed targets embedded in noise, modeled in terms of complex normal random vectors with unknown covariance matrix, plus interference resorting to the GLRT and the two-step GLRT-based design procedure; interference subspace is known and linearly independent of the signal space. We also assume that a set of noise-only (secondary) data is available at the receiver; noise in primary data and secondary data share the same statistical characterization (homogeneous scenario). The possible useful signals are aligned with an unknown direction constrained to

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F. Bandiera, D. Orlando, and G. Ricci are with the Dipartimento di Ingegneria dell'Innovazione, Università del Salento, I-73100 Lecce, Italy (e-mail: (francesco.bandiera@unile.it; danilo.orlando@unile.it; giuseppe.ricci@unile.it).

O. Besson is with the Department of Avionics Systems, ENSICA, 31056 Toulouse, France (e-mail: besson@ensica.fr).

L. L. Scharf is with the Department of Electrical and Computer Engineering, Colorado State University, Fort Collins, CO 80523-1373 USA (e-mail: scharf@engr.colostate.edu).

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belong to a given subspace of the observables. This model might be a viable means to address adaptive detection in case of mismatched steering vectors. It has been proposed in [8]–[10] where detection in the presence of white noise with a known and unknown power, respectively, has been considered.

The paper is organized as follows. Section II is devoted to the problem formulation while the detector designs are the object of Section III. Section IV contains the performance assessment of the proposed algorithms and, finally, Section V concludes the paper with some remarks and hints for future work.

II. Problem Formulation

Assume that an array of antennas senses K_P range cells and denote by $\boldsymbol{r}_k \in \mathbb{C}^{N\times 1}$, $k \in \Omega_P \equiv \{1,\dots,K_P\}$, the N-dimensional complex vector containing returns from the kth cell. We want to discriminate between the H_0 hypothesis that the $\boldsymbol{r}_k$'s, $k \in \Omega_P$, contain disturbance only and the H_1 hypothesis that they also contain useful target echoes $\boldsymbol{s}_k \in \mathbb{C}^{N\times 1}$.

We assume that the disturbance is the sum of colored noise and interference, modeled as a deterministic signal. Moreover, we suppose that the $\boldsymbol{s}_k$'s can be modeled as $\boldsymbol{s}_k = \alpha_k \boldsymbol{s}$, $\alpha_k \in \mathbb{C}$, $k \in \Omega_P$, with $\boldsymbol{s} \in \mathbb{C}^{N\times 1}$ being, in turn, a linear combination of the columns of the full-column-rank matrix $\boldsymbol{H} \in \mathbb{C}^{N\times r}$; similarly, the interference signals $\boldsymbol{i}_k \in \mathbb{C}^{N\times 1}$, $k \in \Omega_P$, are linear combinations of the q, $q+r \le N$, linearly independent columns of the matrix $\boldsymbol{J} \in \mathbb{C}^{N\times q}$. Succinctly, $\boldsymbol{s}$ and $\boldsymbol{i}_k$, $k \in \Omega_P$, are assumed to belong to $\langle \boldsymbol{H} \rangle$ and $\langle \boldsymbol{J} \rangle$, respectively. Thus, $\boldsymbol{s}$ and $\boldsymbol{i}_k$ can be recast as $\boldsymbol{s} = \boldsymbol{H}\boldsymbol{p}$ and $\boldsymbol{i}_k = \boldsymbol{J}\boldsymbol{q}_k$, $k \in \Omega_P$, where $\boldsymbol{p} \in \mathbb{C}^{r\times 1}$ and $\boldsymbol{q}_k \in \mathbb{C}^{q\times 1}$, $k \in \Omega_P$, are r-dimensional and q-dimensional complex vectors, respectively. In the following, we assume that the subspaces spanned by the columns of the matrices $\boldsymbol{H}$ and $\boldsymbol{J}$ are known and that the matrix $[\boldsymbol{H}\ \boldsymbol{J}]$ is full rank while $\boldsymbol{p}$, the α_k's, and the $\boldsymbol{q}_k$'s are unknown quantities.

The noise vectors $\boldsymbol{n}_k \in \mathbb{C}^{N\times 1}$, $k \in \Omega_P$, are modeled as N-dimensional complex normal vectors with unknown, positive-definite, covariance matrix $\boldsymbol{M} \in \mathbb{C}^{N\times N}$. We also suppose that K_S secondary data, $\boldsymbol{r}_k \in \mathbb{C}^{N\times 1}$, $k \in \Omega_S \equiv \{K_P + 1,\dots,K_P+K_S\}$, containing noise only, namely $\boldsymbol{r}_k = \boldsymbol{n}_k$, $k \in \Omega_S$, are available and that such returns share the same statistical characterization of the noise components in the primary data. Finally, we assume that the $\boldsymbol{n}_k$'s, $k \in \Omega_P \cup \Omega_S$, are independent random vectors.

Summarizing, the detection problem to be solved can be formulated in terms of the following binary hypothesis test:

$$\begin{cases} H_0 : \begin{cases} \boldsymbol{r}_k = \boldsymbol{J}\boldsymbol{q}_k + \boldsymbol{n}_k, & k \in \Omega_P \\ \boldsymbol{r}_k = \boldsymbol{n}_k, & k \in \Omega_S \end{cases} \\ H_1 : \begin{cases} \boldsymbol{r}_k = \alpha_k \boldsymbol{H}\boldsymbol{p} + \boldsymbol{J}\boldsymbol{q}_k + \boldsymbol{n}_k, & k \in \Omega_P \\ \boldsymbol{r}_k = \boldsymbol{n}_k, & k \in \Omega_S \end{cases} \end{cases}$$

where we suppose that $K_S \ge N$ and, as already stated, that $r + q \le N$.

III. Detector Designs

Denote by $\boldsymbol{R} = [\boldsymbol{R}_P\ \boldsymbol{R}_S]$ the overall data matrix, where $\boldsymbol{R}_P = [\boldsymbol{r}_1 \cdots \boldsymbol{r}_{K_P}] \in \mathbb{C}^{N\times K_P}$ is the primary data matrix and $\boldsymbol{R}_S = [\boldsymbol{r}_{K_P+1} \cdots \boldsymbol{r}_{K_P+K_S}] \in \mathbb{C}^{N\times K_S}$ is the secondary data matrix. Moreover, let $\boldsymbol{Q} = [\boldsymbol{q}_1 \cdots \boldsymbol{q}_{K_P}] \in \mathbb{C}^{q\times K_P}$, $\boldsymbol{\alpha} = [\alpha_1 \cdots \alpha_{K_P}] \in \mathbb{C}^{1\times K_P}$, and $K = K_P + K_S$.

A. One-Step GLRT-Based Detector

We now derive the GLRT based upon primary and secondary data, which is tantamount to the following decision rule [11]:

$$\Lambda(\boldsymbol{R}) = \frac{\max\limits_{\boldsymbol{p}} \max\limits_{\boldsymbol{\alpha}} \max\limits_{\boldsymbol{Q}} \max\limits_{\boldsymbol{M}} f_1(\boldsymbol{R}; \boldsymbol{p}, \boldsymbol{\alpha}, \boldsymbol{Q}, \boldsymbol{M})}{\max\limits_{\boldsymbol{Q}} \max\limits_{\boldsymbol{M}} f_0(\boldsymbol{R}; \boldsymbol{Q}, \boldsymbol{M})} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

where $f_j(\boldsymbol{R};\cdot)$ is the probability density function (pdf) of $\boldsymbol{R}$ under the H_j, $j = 0,1$ hypothesis, and γ is the threshold value to be set in order to ensure the desired probability of false alarm (P_{fa}).

The pdf of $\boldsymbol{R}$, under H_0, can be written as

$$f_0(\boldsymbol{R}; \boldsymbol{Q}, \boldsymbol{M}) = \left[\frac{1}{\pi^N \det(\boldsymbol{M})}\right]^K \times \exp\left\{-\mathrm{tr}\left[\boldsymbol{M}^{-1}\left(\boldsymbol{S} + (\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})(\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})^\dagger\right)\right]\right\}$$

where $\boldsymbol{S} = \boldsymbol{R}_S\boldsymbol{R}_S^\dagger \in \mathbb{C}^{N\times N}$ is K_S times the sample covariance matrix based on secondary data,[1] $\det(\cdot)$ and $\mathrm{tr}(\cdot)$ are the determinant and the trace of a square matrix, respectively, and † denotes conjugate transpose. In order to compute the compressed likelihood under H_0, observe that the maximum of $f_0(\boldsymbol{R}; \boldsymbol{Q}, \boldsymbol{M})$ with respect to $\boldsymbol{M}$ is attained by substituting the true covariance matrix with the sample covariance, namely with

$$\widehat{\boldsymbol{M}} = \frac{1}{K}\left[\boldsymbol{S} + (\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})(\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})^\dagger\right].$$

Substitution of $\widehat{\boldsymbol{M}}$ into the $f_0(\boldsymbol{R};\cdot)$ yields

$$f_0(\boldsymbol{R}; \boldsymbol{Q}, \widehat{\boldsymbol{M}}) = \left(\frac{K}{e\pi}\right)^{NK} \times \left\{\frac{1}{\det\left[\boldsymbol{S} + (\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})(\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})^\dagger\right]}\right\}^K.$$

Now maximization over the matrix $\boldsymbol{Q}$ is tantamount to performing the following minimization

$$\begin{aligned} &\min_{\boldsymbol{Q}} \det\left[\boldsymbol{S} + (\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})(\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})^\dagger\right] \\ &\quad = \det(\boldsymbol{S}) \min_{\boldsymbol{Q}} \det\Big[\boldsymbol{I}_N + \boldsymbol{S}^{-1/2}(\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q}) \times (\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})^\dagger \boldsymbol{S}^{-1/2}\Big] \\ &\quad = \det(\boldsymbol{S}) \min_{\boldsymbol{Q}} \det\Big[\boldsymbol{I}_{K_P} + (\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})^\dagger \boldsymbol{S}^{-1}(\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})\Big] \end{aligned}$$

where $\boldsymbol{I}_m$ denotes the m-dimensional identity matrix, and the last equality follows from identity [12]

$$\det(\boldsymbol{I}_m + \boldsymbol{B}\boldsymbol{C}) = \det(\boldsymbol{I}_n + \boldsymbol{C}\boldsymbol{B}) \tag{1}$$

with $\boldsymbol{B} \in \mathbb{C}^{m\times n}$ and $\boldsymbol{C} \in \mathbb{C}^{n\times m}$ rectangular matrices.

[1]Note that the matrix $\boldsymbol{S}$ is invertible if $K_S \ge N$.

As a preliminary step towards the above minimization, observe that

$$\det\left[\boldsymbol{I}_{K_P}+(\boldsymbol{R}_P-\boldsymbol{J}\boldsymbol{Q})^\dagger \boldsymbol{S}^{-1}(\boldsymbol{R}_P-\boldsymbol{J}\boldsymbol{Q})\right] = \det\left[\boldsymbol{A}_1+(\boldsymbol{Q}-\boldsymbol{D})^\dagger \boldsymbol{A}_2(\boldsymbol{Q}-\boldsymbol{D})\right]$$

where

$$\boldsymbol{A}_1=\boldsymbol{I}_{K_P}+\boldsymbol{R}_P^\dagger \boldsymbol{S}^{-1}\boldsymbol{R}_P-\boldsymbol{D}^\dagger \boldsymbol{J}^\dagger \boldsymbol{S}^{-1}\boldsymbol{J}\boldsymbol{D}, \quad \boldsymbol{A}_2=\boldsymbol{J}^\dagger \boldsymbol{S}^{-1}\boldsymbol{J}$$

with, in turn

$$\boldsymbol{D}=\left(\boldsymbol{J}^\dagger \boldsymbol{S}^{-1}\boldsymbol{J}\right)^{-1}\boldsymbol{J}^\dagger \boldsymbol{S}^{-1}\boldsymbol{R}_P$$

and that the matrices $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are positive definite. It follows that

$$\begin{aligned}\min_{\boldsymbol{Q}} \det&\left[\boldsymbol{I}_{K_P}+(\boldsymbol{R}_P-\boldsymbol{J}\boldsymbol{Q})^\dagger \boldsymbol{S}^{-1}(\boldsymbol{R}_P-\boldsymbol{J}\boldsymbol{Q})\right]\\ &=\min_{\boldsymbol{Q}} \det\left[\boldsymbol{A}_1+(\boldsymbol{Q}-\boldsymbol{D})^\dagger \boldsymbol{A}_2(\boldsymbol{Q}-\boldsymbol{D})\right]\\ &=\det\left[\boldsymbol{I}_{K_P}+\boldsymbol{R}_P^\dagger \boldsymbol{S}^{-1}\boldsymbol{R}_P-\boldsymbol{D}^\dagger \boldsymbol{J}^\dagger \boldsymbol{S}^{-1}\boldsymbol{J}\boldsymbol{D}\right]\end{aligned}$$

where we have used equation (2-30) in [7], namely

$$\min_{\boldsymbol{U}} \det(\boldsymbol{A}_1+\boldsymbol{U}^\dagger \boldsymbol{A}_2\boldsymbol{U})=\det(\boldsymbol{A}_1), \tag{2}$$

with $\boldsymbol{U}=\boldsymbol{Q}-\boldsymbol{D}$. In particular, we have that

$$\begin{aligned}\widehat{\boldsymbol{Q}}&=\arg\min_{\boldsymbol{Q}} \det\left[\boldsymbol{S}+(\boldsymbol{R}_P-\boldsymbol{J}\boldsymbol{Q})(\boldsymbol{R}_P-\boldsymbol{J}\boldsymbol{Q})^\dagger\right]\\ &=(\boldsymbol{J}^\dagger \boldsymbol{S}^{-1}\boldsymbol{J})^{-1}\boldsymbol{J}^\dagger \boldsymbol{S}^{-1}\boldsymbol{R}_P.\end{aligned}$$

Finally, the compressed likelihood function under H_0 is given by

$$f_0(\boldsymbol{R};\widehat{\boldsymbol{Q}},\widehat{\boldsymbol{M}})=\left(\frac{K}{e\pi}\right)^{NK} \times\frac{1}{\det^K(\boldsymbol{S})\det^K\left[\boldsymbol{I}_{K_P}+(\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)^\dagger(\boldsymbol{I}_N-\boldsymbol{P}_{\boldsymbol{J}_S})(\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)\right]}$$

where $\boldsymbol{J}_S=\boldsymbol{S}^{-1/2}\boldsymbol{J}\in\mathbb{C}^{N\times q}$ and $\boldsymbol{P}_{\boldsymbol{K}}\in\mathbb{C}^{n\times n}$ is the projection matrix[2] onto the range of the full-column-rank matrix $\boldsymbol{K}\in\mathbb{C}^{n\times m}$, $m\le n$, i.e.,

$$\boldsymbol{P}_{\boldsymbol{K}}=\boldsymbol{K}\left(\boldsymbol{K}^\dagger \boldsymbol{K}\right)^{-1}\boldsymbol{K}^\dagger.$$

[2]For future reference, we denote by $\boldsymbol{P}_{\boldsymbol{K}}^\perp=\boldsymbol{I}_n-\boldsymbol{P}_{\boldsymbol{K}}$ the projection matrix onto the orthogonal complement of the subspace spanned by the columns of $\boldsymbol{K}$.

On the other hand, the pdf of the data matrix under the H_1 hypothesis is given by

$$\begin{aligned}f_1(\boldsymbol{R};\boldsymbol{p},\boldsymbol{\alpha},\boldsymbol{Q},\boldsymbol{M})&=\left[\frac{1}{\pi^N\det(\boldsymbol{M})}\right]^K\\ &\times\exp\left\{-\mathrm{tr}\left[\boldsymbol{M}^{-1}\left(\boldsymbol{S}+(\boldsymbol{R}_P-\boldsymbol{H}\boldsymbol{p}\boldsymbol{\alpha}-\boldsymbol{J}\boldsymbol{Q})\right.\right.\right.\\ &\left.\left.\left.\times(\boldsymbol{R}_P-\boldsymbol{H}\boldsymbol{p}\boldsymbol{\alpha}-\boldsymbol{J}\boldsymbol{Q})^\dagger\right)\right]\right\}.\end{aligned} \tag{3}$$

Again, maximization with respect to $\boldsymbol{M}$ yields

$$\widehat{\boldsymbol{M}}=\frac{1}{K}\left[\boldsymbol{S}+(\boldsymbol{R}_P-\boldsymbol{H}\boldsymbol{p}\boldsymbol{\alpha}-\boldsymbol{J}\boldsymbol{Q})(\boldsymbol{R}_P-\boldsymbol{H}\boldsymbol{p}\boldsymbol{\alpha}-\boldsymbol{J}\boldsymbol{Q})^\dagger\right].$$

Moreover, optimization over the signal and interference vectors $\boldsymbol{\alpha}$ and $\boldsymbol{Q}$, respectively, can be straightforwardly solved after observing that

$$\boldsymbol{H}\boldsymbol{p}\boldsymbol{\alpha}+\boldsymbol{J}\boldsymbol{Q}=[\boldsymbol{H}\boldsymbol{p}\ \boldsymbol{J}]\begin{bmatrix}\boldsymbol{\alpha}\\ \boldsymbol{Q}\end{bmatrix}=\boldsymbol{W}(\boldsymbol{p})\widetilde{\boldsymbol{Q}}$$

where $\boldsymbol{W}(\boldsymbol{p})=[\boldsymbol{H}\boldsymbol{p}\ \boldsymbol{J}]\in\mathbb{C}^{N\times(q+1)}$ is a full-column-rank matrix function of $\boldsymbol{p}$ and $\widetilde{\boldsymbol{Q}}=\left[\boldsymbol{\alpha}^T\ \boldsymbol{Q}^T\right]^T\in\mathbb{C}^{(q+1)\times K_P}$, with ^{T}, in turn, denoting transpose. Thus, optimization with respect to $\boldsymbol{\alpha}$ and $\boldsymbol{Q}$, under the H_1 hypothesis, is formally identical to optimization with respect to $\boldsymbol{Q}$, under the H_0 hypothesis, and the result is given by the equation shown at the bottom of the page, where $\boldsymbol{W}_S(\boldsymbol{p})=\boldsymbol{S}^{-1/2}\boldsymbol{W}(\boldsymbol{p})$.

It still remains to maximize the above equation with respect to $\boldsymbol{p}$ or, equivalently, to minimize

$$\det\left[\boldsymbol{I}_{K_P}+\left(\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\right)^\dagger\left(\boldsymbol{I}_N-\boldsymbol{P}_{\boldsymbol{W}_S(\boldsymbol{p})}\right)\left(\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\right)\right] \tag{4}$$

with respect to $\boldsymbol{p}$. To this end, observe that

$$\begin{aligned}\boldsymbol{S}^{-1/2}\boldsymbol{W}(\boldsymbol{p})&=\boldsymbol{S}^{-1/2}[\boldsymbol{H}\boldsymbol{p}\ \boldsymbol{J}]=[\boldsymbol{S}^{-1/2}\boldsymbol{H}\boldsymbol{p}\ \ \boldsymbol{S}^{-1/2}\boldsymbol{J}]\\ &=[\boldsymbol{H}_S\boldsymbol{p}\ \ \boldsymbol{J}_S]\end{aligned}$$

with $\boldsymbol{H}_S=\boldsymbol{S}^{-1/2}\boldsymbol{H}\in\mathbb{C}^{N\times r}$, and, hence, that [3], [6]

$$\begin{aligned}\boldsymbol{P}_{\boldsymbol{W}_S(\boldsymbol{p})}&=\boldsymbol{P}_{\boldsymbol{J}_S}+\boldsymbol{P}_{\boldsymbol{P}_{\boldsymbol{J}_S}^\perp(\boldsymbol{H}_S\boldsymbol{p})}\\ &=\boldsymbol{P}_{\boldsymbol{J}_S}+\left(\boldsymbol{I}_N-\boldsymbol{P}_{\boldsymbol{J}_S}\right)(\boldsymbol{H}_S\boldsymbol{p})\\ &\quad\times\left[(\boldsymbol{H}_S\boldsymbol{p})^\dagger\left(\boldsymbol{I}_N-\boldsymbol{P}_{\boldsymbol{J}_S}\right)(\boldsymbol{H}_S\boldsymbol{p})\right]^{-1}\\ &\quad\times(\boldsymbol{H}_S\boldsymbol{p})^\dagger\left(\boldsymbol{I}_N-\boldsymbol{P}_{\boldsymbol{J}_S}\right).\end{aligned} \tag{5}$$

Moreover, the projection matrix $\boldsymbol{I}_N-\boldsymbol{P}_{\boldsymbol{J}_S}$ can be recast as $\boldsymbol{I}_N-\boldsymbol{P}_{\boldsymbol{J}_S}=\boldsymbol{Z}\boldsymbol{Z}^\dagger$, where $\boldsymbol{Z}\in\mathbb{C}^{N\times(N-q)}$ is a slice of unitary matrix, i.e., $\boldsymbol{Z}^\dagger\boldsymbol{Z}=\boldsymbol{I}_{N-q}$. Thus, substituting the above-factored form

$$f_1(\boldsymbol{R};\boldsymbol{p},\widehat{\boldsymbol{\alpha}},\widehat{\boldsymbol{Q}},\widehat{\boldsymbol{M}})=\frac{[K/(e\pi)]^{NK}}{\det^K(\boldsymbol{S})\det^K\left[\boldsymbol{I}_{K_P}+\left(\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\right)^\dagger\left(\boldsymbol{I}_N-\boldsymbol{P}_{\boldsymbol{W}_S(\boldsymbol{p})}\right)\left(\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\right)\right]}$$

for $\boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{J}_S}$ into the expression (5) for $\boldsymbol{P}_{\boldsymbol{W}_S(\boldsymbol{p})}$ and it back into (4), after some algebra, yields

$$\begin{aligned}
&\det\left[\boldsymbol{I}_{K_P} + (\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)^\dagger \left(\boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{W}_S(\boldsymbol{p})}\right)(\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)\right]\\
&\quad= \det\Big[\boldsymbol{I}_{K_P} + (\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)^\dagger\\
&\qquad\qquad \times\left(\boldsymbol{I}_{N-q} - \boldsymbol{P}_{\boldsymbol{H}'_S\boldsymbol{p}}\right)(\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)\Big]\\
&\quad= \det\Bigg[\underbrace{\boldsymbol{I}_{K_P} + (\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)^\dagger(\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)}_{\boldsymbol{A}}\\
&\qquad\qquad -(\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)^\dagger\boldsymbol{P}_{\boldsymbol{H}'_S\boldsymbol{p}}(\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)\Bigg]\\
&\quad= \det\left[\boldsymbol{A} - (\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)^\dagger\boldsymbol{P}_{\boldsymbol{H}'_S\boldsymbol{p}}(\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)\right]\\
&\quad= \det(\boldsymbol{A})\det\Big[\boldsymbol{I}_{K_P} - \boldsymbol{A}^{-1/2}\boldsymbol{R}_P^\dagger\boldsymbol{S}^{-1/2}\\
&\qquad\qquad \times\boldsymbol{Z}\boldsymbol{P}_{\boldsymbol{H}'_S\boldsymbol{p}}\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\boldsymbol{A}^{-1/2}\Big] \qquad (6)
\end{aligned}$$

where

$$\begin{aligned}
\boldsymbol{A} &= \boldsymbol{I}_{K_P} + (\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)^\dagger(\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)\\
&= \boldsymbol{I}_{K_P} + (\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)^\dagger(\boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{J}_S})(\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)
\end{aligned}$$

is a positive-definite matrix, $\boldsymbol{H}'_S = \boldsymbol{Z}^\dagger\boldsymbol{H}_S$, and $\boldsymbol{P}_{\boldsymbol{H}'_S\boldsymbol{p}}$ is the projection matrix onto the space spanned by $\boldsymbol{H}'_S\boldsymbol{p} = \boldsymbol{Z}^\dagger\boldsymbol{H}_S\boldsymbol{p}$. Now, $\boldsymbol{P}_{\boldsymbol{H}'_S\boldsymbol{p}}$ can be recast as

$$\boldsymbol{P}_{\boldsymbol{H}'_S\boldsymbol{p}} = \boldsymbol{v}\boldsymbol{v}^\dagger,$$

with $\boldsymbol{v} \in \mathbb{C}^{(N-q)\times 1}$, in turn, a unit-norm vector belonging to the range of $\boldsymbol{H}'_S$, namely to $\langle\boldsymbol{H}'_S\rangle$. Thus, it follows that

$$\begin{aligned}
(6) &= \det(\boldsymbol{A})\det\Big[\boldsymbol{I}_{K_P} - \boldsymbol{A}^{-1/2}\boldsymbol{R}_P^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{Z}\boldsymbol{P}_{\boldsymbol{H}'_S\boldsymbol{p}}\\
&\qquad\times\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\boldsymbol{A}^{-1/2}\Big]\\
&= \det(\boldsymbol{A})\det\Big[\boldsymbol{I}_{K_P} - \boldsymbol{A}^{-1/2}\boldsymbol{R}_P^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{Z}\boldsymbol{v}\boldsymbol{v}^\dagger\\
&\qquad\times\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\boldsymbol{A}^{-1/2}\Big]\\
&= \det(\boldsymbol{A})\left(1 - \boldsymbol{v}^\dagger\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\boldsymbol{A}^{-1}\boldsymbol{R}_P^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{Z}\boldsymbol{v}\right)
\end{aligned}$$

where we have also used identity (1). Eventually, we have shown that minimization of (4) with respect to $\boldsymbol{p}$ is tantamount to solving the following maximization:

$$\max_{\substack{\boldsymbol{v}\in\langle\boldsymbol{H}'_S\rangle\\ \boldsymbol{v}^\dagger\boldsymbol{v}=1}} \boldsymbol{v}^\dagger\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\boldsymbol{A}^{-1}\boldsymbol{R}_P^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{Z}\boldsymbol{v}.$$

To this end, denote by $\boldsymbol{E}$ the matrix

$$\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\boldsymbol{A}^{-1}\boldsymbol{R}_P^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{Z}.$$

The minimization problem at hand can be recast as

$$\begin{aligned}
\max_{\substack{\boldsymbol{v}\in\langle\boldsymbol{H}'_S\rangle\\ \boldsymbol{v}^\dagger\boldsymbol{v}=1}} \boldsymbol{v}^\dagger\boldsymbol{E}\boldsymbol{v} &= \max_{\substack{\boldsymbol{v}\in\langle\boldsymbol{H}'_S\rangle\\ \boldsymbol{v}^\dagger\boldsymbol{v}=1}} \boldsymbol{v}^\dagger\left(\boldsymbol{P}_{\boldsymbol{H}'_S} + \boldsymbol{P}^\perp_{\boldsymbol{H}'_S}\right)\\
&\qquad\times\boldsymbol{E}\left(\boldsymbol{P}_{\boldsymbol{H}'_S} + \boldsymbol{P}^\perp_{\boldsymbol{H}'_S}\right)\boldsymbol{v}\\
&= \max_{\substack{\boldsymbol{v}\in\langle\boldsymbol{H}'_S\rangle\\ \boldsymbol{v}^\dagger\boldsymbol{v}=1}} \boldsymbol{v}^\dagger\boldsymbol{P}_{\boldsymbol{H}'_S}\boldsymbol{E}\boldsymbol{P}_{\boldsymbol{H}'_S}\boldsymbol{v}\\
&= \lambda_{\max}\left(\boldsymbol{P}_{\boldsymbol{H}'_S}\boldsymbol{E}\boldsymbol{P}_{\boldsymbol{H}'_S}\right)\\
&= \lambda_{\max}\Big(\boldsymbol{P}_{\boldsymbol{H}'_S}\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\boldsymbol{A}^{-1}\\
&\qquad\times\boldsymbol{R}_P^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{Z}\boldsymbol{P}_{\boldsymbol{H}'_S}\Big)
\end{aligned}$$

where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of the matrix argument.

Finally, the compressed likelihood function under H_1 is given by the equation shown at the bottom of the page.

Summarizing, the GLRT is given by

$$\Lambda^{1/K}(\boldsymbol{R}) = \frac{1}{1-\lambda_{\max}\left(\boldsymbol{P}_{\boldsymbol{H}'_S}\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\boldsymbol{A}^{-1}\boldsymbol{R}_P^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{Z}\boldsymbol{P}_{\boldsymbol{H}'_S}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma^{1/K}$$

or, equivalently

$$\Lambda_1(\boldsymbol{R}) = \lambda_{\max}\left(\boldsymbol{P}_{\boldsymbol{H}'_S}\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\boldsymbol{A}^{-1}\boldsymbol{R}_P^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{Z}\boldsymbol{P}_{\boldsymbol{H}'_S}\right) \underset{H_0}{\overset{H_1}{\gtrless}} 1 - \frac{1}{\gamma^{1/K}} \qquad (7)$$

with

$$\boldsymbol{A} = \boldsymbol{I}_{K_P} + \left(\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\right)^\dagger\left(\boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{J}_S}\right)\left(\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\right).$$

$$f_1(\boldsymbol{R};\widehat{\boldsymbol{p}},\widehat{\boldsymbol{\alpha}},\widehat{\boldsymbol{Q}},\widehat{\boldsymbol{M}}) = \frac{(K/e\pi)^{NK}}{\det^K(\boldsymbol{S})\det^K\left[\boldsymbol{I}_{K_P} + (\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)^\dagger(\boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{J}_S})(\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)\right]} \times\frac{1}{\left[1-\lambda_{\max}\left(\boldsymbol{P}_{\boldsymbol{H}'_S}\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\boldsymbol{A}^{-1}\boldsymbol{R}_P^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{Z}\boldsymbol{P}_{\boldsymbol{H}'_S}\right)\right]^K}.$$

B. Two-Step GLRT-Based Detector

In this subsection, we derive an ad hoc detector based upon the two-step GLRT-based design procedure. More precisely, we first derive the GLRT detector assuming that $\boldsymbol{M}$ is known. Then, we come up with a fully-adaptive detector by replacing $\boldsymbol{M}$ with $\boldsymbol{S}$.

Under the assumption that the covariance matrix of the noise is known, the GLRT is given by

$$\Lambda(\boldsymbol{R}_P) = \frac{\max_{\boldsymbol{p}} \max_{\boldsymbol{\alpha}} \max_{\boldsymbol{Q}} f_1(\boldsymbol{R}_P; \boldsymbol{p}, \boldsymbol{\alpha}, \boldsymbol{Q})}{\max_{\boldsymbol{Q}} f_0(\boldsymbol{R}_P; \boldsymbol{Q})} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{8}$$

where $f_j(\boldsymbol{R}_P; \cdot)$ is the pdf of $\boldsymbol{R}_P$ under the H_j hypothesis, $j = 0{,}1$; it turns out that $f_0(\boldsymbol{R}_P; \cdot)$ and $f_1(\boldsymbol{R}_P; \cdot)$ are given by

$$f_0(\boldsymbol{R}_P; \boldsymbol{Q}) = \left[\frac{1}{\pi^N \det(\boldsymbol{M})}\right]^{K_P} \times \exp\left\{-\mathrm{tr}\left[\boldsymbol{M}^{-1}(\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})(\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})^\dagger\right]\right\}$$

and

$$f_1(\boldsymbol{R}_P; \boldsymbol{p}, \boldsymbol{\alpha}, \boldsymbol{Q}) = \left[\frac{1}{\pi^N \det(\boldsymbol{M})}\right]^{K_P} \times \exp\left\{-\mathrm{tr}\left[\boldsymbol{M}^{-1}(\boldsymbol{R}_P - \boldsymbol{H}\boldsymbol{p}\boldsymbol{\alpha} - \boldsymbol{J}\boldsymbol{Q})(\boldsymbol{R}_P - \boldsymbol{H}\boldsymbol{p}\boldsymbol{\alpha} - \boldsymbol{J}\boldsymbol{Q})^\dagger\right]\right\}$$

respectively. As to γ, it is the threshold value to be set in order to ensure the desired P_{fa}. Let us begin by solving the optimization problem under the H_0 hypothesis and observe that maximizing $f_0(\boldsymbol{R}_P; \cdot)$ with respect to $\boldsymbol{Q}$ is equivalent to the following minimization problem:

$$\begin{aligned}
&\min_{\boldsymbol{Q}} \mathrm{tr}\left[\boldsymbol{M}^{-1}(\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})(\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})^\dagger\right] \\
&\quad = \min_{\boldsymbol{Q}} \mathrm{tr}\left[(\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})^\dagger \boldsymbol{M}^{-1}(\boldsymbol{R}_P - \boldsymbol{J}\boldsymbol{Q})\right] \\
&\quad = \min_{\boldsymbol{Q}} \mathrm{tr}\left[(\boldsymbol{M}^{-1/2}\boldsymbol{R}_P - \boldsymbol{J}_M\boldsymbol{Q})^\dagger(\boldsymbol{M}^{-1/2}\boldsymbol{R}_P - \boldsymbol{J}_M\boldsymbol{Q})\right]
\end{aligned}$$

where $\boldsymbol{J}_M = \boldsymbol{M}^{-1/2}\boldsymbol{J}$. It is not difficult to show that

$$\begin{aligned}
\widehat{\boldsymbol{Q}} &= \arg\min_{\boldsymbol{Q}} \mathrm{tr}\left[(\boldsymbol{M}^{-1/2}\boldsymbol{R}_P - \boldsymbol{J}_M\boldsymbol{Q})^\dagger(\boldsymbol{M}^{-1/2}\boldsymbol{R}_P - \boldsymbol{J}_M\boldsymbol{Q})\right] \\
&= \left(\boldsymbol{J}_M^\dagger \boldsymbol{J}_M\right)^{-1} \boldsymbol{J}_M^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{R}_P
\end{aligned}$$

and, consequently, that the compressed likelihood function under H_0 is given by

$$f_0(\boldsymbol{R}_P; \widehat{\boldsymbol{Q}}) = \left[\frac{1}{\pi^N \det(\boldsymbol{M})}\right]^{K_P} \times \exp\left\{-\mathrm{tr}\left[\boldsymbol{R}_P^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{P}_{\boldsymbol{J}_M}^\perp \boldsymbol{M}^{-1/2}\boldsymbol{R}_P\right]\right\}.$$

In order to maximize the numerator of (8) with respect to $\boldsymbol{\alpha}$ and $\boldsymbol{Q}$, it is convenient to recast the pdf as follows:

$$f_1(\boldsymbol{R}_P; \boldsymbol{p}, \boldsymbol{\alpha}, \boldsymbol{Q}) = \left[\frac{1}{\pi^N \det(\boldsymbol{M})}\right]^{K_P} \exp\left\{-\mathrm{tr}\left[\left(\boldsymbol{M}^{-1/2}\boldsymbol{R}_P - \boldsymbol{W}_M(\boldsymbol{p})\tilde{\boldsymbol{Q}}\right)^\dagger \left(\boldsymbol{M}^{-1/2}\boldsymbol{R}_P - \boldsymbol{W}_M(\boldsymbol{p})\tilde{\boldsymbol{Q}}\right)\right]\right\}$$

where

$$\boldsymbol{W}_M(\boldsymbol{p}) = \boldsymbol{M}^{-1/2}[\boldsymbol{H}\boldsymbol{p}\ \boldsymbol{J}] \quad \text{and} \quad \tilde{\boldsymbol{Q}} = \begin{bmatrix}\boldsymbol{\alpha}\\ \boldsymbol{Q}\end{bmatrix}.$$

Thus, following the lead of previous maximization with respect to $\boldsymbol{Q}$, yields

$$f_1(\boldsymbol{R}_P; \boldsymbol{p}, \widehat{\boldsymbol{\alpha}}, \widehat{\boldsymbol{Q}}) = \left[\frac{1}{\pi^N \det(\boldsymbol{M})}\right]^{K_P} \times \exp\left\{-\mathrm{tr}\left[\boldsymbol{R}_P^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{P}_{\boldsymbol{W}_M(\boldsymbol{p})}^\perp \boldsymbol{M}^{-1/2}\boldsymbol{R}_P\right]\right\}.$$

It still remains to maximize the pdf under H_1 with respect to $\boldsymbol{p}$, namely to solve the following optimization problem:

$$\min_{\boldsymbol{p}} \mathrm{tr}\left[\boldsymbol{R}_P^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{P}_{\boldsymbol{W}_M(\boldsymbol{p})}^\perp \boldsymbol{M}^{-1/2}\boldsymbol{R}_P\right]. \tag{9}$$

To this end, observe that

$$\begin{aligned}
\boldsymbol{P}_{\boldsymbol{W}_M(\boldsymbol{p})}^\perp &= \boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{W}_M(\boldsymbol{p})} \\
&= (\boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{J}_M}) - (\boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{J}_M})(\boldsymbol{H}_M\boldsymbol{p}) \\
&\quad \times \left[(\boldsymbol{H}_M\boldsymbol{p})^\dagger (\boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{J}_M})(\boldsymbol{H}_M\boldsymbol{p})\right]^{-1} \\
&\quad \times (\boldsymbol{H}_M\boldsymbol{p})^\dagger (\boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{J}_M}) \\
&= \boldsymbol{P}_{\boldsymbol{J}_M}^\perp - \boldsymbol{P}_{\boldsymbol{J}_M}^\perp(\boldsymbol{H}_M\boldsymbol{p})\left[(\boldsymbol{H}_M\boldsymbol{p})^\dagger \boldsymbol{P}_{\boldsymbol{J}_M}^\perp (\boldsymbol{H}_M\boldsymbol{p})\right]^{-1} \\
&\quad \times (\boldsymbol{H}_M\boldsymbol{p})^\dagger \boldsymbol{P}_{\boldsymbol{J}_M}^\perp \\
&= \boldsymbol{Z}_M\left\{\boldsymbol{I}_{N-q} - (\boldsymbol{H}'_M\boldsymbol{p}) \right. \\
&\quad \left. \times \left[(\boldsymbol{H}'_M\boldsymbol{p})^\dagger (\boldsymbol{H}'_M\boldsymbol{p})\right]^{-1} (\boldsymbol{H}'_M\boldsymbol{p})^\dagger\right\}\boldsymbol{Z}_M^\dagger
\end{aligned}$$

where $\boldsymbol{H}_M = \boldsymbol{M}^{-1/2}\boldsymbol{H}$, $\boldsymbol{Z}_M \in \mathbb{C}^{N\times(N-q)}$ is a slice of a unitary matrix satisfying $\boldsymbol{P}_{\boldsymbol{J}_M}^\perp = \boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{J}_M} = \boldsymbol{Z}_M\boldsymbol{Z}_M^\dagger$, and $\boldsymbol{H}'_M = \boldsymbol{Z}_M^\dagger\boldsymbol{H}_M = \boldsymbol{Z}_M^\dagger\boldsymbol{M}^{-1/2}\boldsymbol{H}$. Substituting the above expression for $\boldsymbol{P}_{\boldsymbol{W}_M(\boldsymbol{p})}^\perp$ into (9) yields

$$\mathrm{tr}\left[\boldsymbol{R}_P^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{P}_{\boldsymbol{J}_M}^\perp \boldsymbol{M}^{-1/2}\boldsymbol{R}_P\right] - \max_{\boldsymbol{p}} \mathrm{tr}\left[\left(\boldsymbol{Z}_M^\dagger\boldsymbol{M}^{-1/2}\boldsymbol{R}_P\right)^\dagger \boldsymbol{P}_{\boldsymbol{H}'_M\boldsymbol{p}}\left(\boldsymbol{Z}_M^\dagger\boldsymbol{M}^{-1/2}\boldsymbol{R}_P\right)\right].$$

Again, the projection matrix $\boldsymbol{P}_{\boldsymbol{H}'_M\boldsymbol{p}}$ can be factorized as $\boldsymbol{P}_{\boldsymbol{H}'_M\boldsymbol{p}} = \boldsymbol{v}\boldsymbol{v}^\dagger$, where $\boldsymbol{v} \in \mathbb{C}^{(N-q)\times 1}$ is an orthonormal vector belonging to $\langle \boldsymbol{H}'_M \rangle$, thus leading to the following solution of the maximization problem:

$$\begin{aligned}
&\max_{\boldsymbol{p}} \operatorname{tr}\left[\left(\boldsymbol{Z}_M^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{R}_P\right)^\dagger \boldsymbol{P}_{\boldsymbol{H}'_M\boldsymbol{p}}\left(\boldsymbol{Z}_M^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{R}_P\right)\right]\\
&\quad= \max_{\substack{\boldsymbol{v}\in\left\langle \boldsymbol{H}'_S\right\rangle\\ \boldsymbol{v}^\dagger\boldsymbol{v}=1}} \left[\boldsymbol{v}^\dagger \boldsymbol{Z}_M^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{R}_P\boldsymbol{R}_P^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{Z}_M\boldsymbol{v}\right]\\
&\quad= \max_{\substack{\boldsymbol{v}\in\left\langle \boldsymbol{H}'_S\right\rangle\\ \boldsymbol{v}^\dagger\boldsymbol{v}=1}} \left[\boldsymbol{v}^\dagger\left(\boldsymbol{P}_{\boldsymbol{H}'_M}+\boldsymbol{P}^\perp_{\boldsymbol{H}'_M}\right)\boldsymbol{Z}_M^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{R}_P\right.\\
&\qquad\qquad\left.\times\boldsymbol{R}_P^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{Z}_M\left(\boldsymbol{P}_{\boldsymbol{H}'_M}+\boldsymbol{P}^\perp_{\boldsymbol{H}'_M}\right)\boldsymbol{v}\right]\\
&\quad= \max_{\substack{\boldsymbol{v}\in\left\langle \boldsymbol{H}'_S\right\rangle\\ \boldsymbol{v}^\dagger\boldsymbol{v}=1}} \left[\boldsymbol{v}^\dagger \boldsymbol{P}_{\boldsymbol{H}'_M}\boldsymbol{Z}_M^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{R}_P\boldsymbol{R}_P^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{Z}_M\boldsymbol{P}_{\boldsymbol{H}'_M}\boldsymbol{v}\right]\\
&\quad= \lambda_{\max}\left(\boldsymbol{P}_{\boldsymbol{H}'_M}\boldsymbol{Z}_M^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{R}_P\boldsymbol{R}_P^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{Z}_M\boldsymbol{P}_{\boldsymbol{H}'_M}\right).
\end{aligned}$$

Summarizing, the natural logarithm of the GLRT for known $\boldsymbol{M}$ is given by

$$\lambda_{\max}\left(\boldsymbol{P}_{\boldsymbol{H}'_M}\boldsymbol{Z}_M^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{R}_P\boldsymbol{R}_P^\dagger \boldsymbol{M}^{-1/2}\boldsymbol{Z}_M\boldsymbol{P}_{\boldsymbol{H}'_M}\right) \underset{H_0}{\overset{H_1}{\gtrless}} \ln\gamma. \tag{10}$$

In order to come up with a fully adaptive detector, we can plug $\boldsymbol{S}$ in place of $\boldsymbol{M}$ into (10); such substitution yields

$$\Lambda_2(\boldsymbol{R}) = \lambda_{\max}\left(\boldsymbol{P}_{\boldsymbol{H}'_S}\boldsymbol{Z}^\dagger \boldsymbol{S}^{-1/2}\boldsymbol{R}_P\boldsymbol{R}_P^\dagger \boldsymbol{S}^{-1/2}\boldsymbol{Z}\boldsymbol{P}_{\boldsymbol{H}'_S}\right) \underset{H_0}{\overset{H_1}{\gtrless}} \ln\gamma \tag{11}$$

where $\boldsymbol{Z}$ is such that $\boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{J}_S} = \boldsymbol{Z}\boldsymbol{Z}^\dagger$.

As a final remark, note that receiver (7) and (11) are equivalent for $q+r=N$ and, in fact, the following relation holds true:

$$\Lambda_1(\boldsymbol{R}) = \frac{\Lambda_2(\boldsymbol{R})}{1+\Lambda_2(\boldsymbol{R})}. \tag{12}$$

The proof is given in the Appendix.

IV. Performance Assessment

Since closed-form expressions for the probability of detection (P_d) and the P_{fa} are not available, we resorted to standard Monte Carlo counting techniques. More precisely, in order to evaluate the threshold necessary to ensure a preassigned value of P_{fa} and the P_d's, we resorted to $100/P_{fa}$ and 10^4 independent trials, respectively.

We randomly generated the entries of $\boldsymbol{J}$ and $\boldsymbol{H}$ at each run of the Monte Carlo simulation as independent and identically distributed (i.i.d.) random variables (rv's) taking on values $\pm 1/\sqrt{N}$ with equal probability. Interference coordinates $\boldsymbol{q}_k$, $k \in \Omega_P$, are i.i.d. complex normal vectors with zero mean and covariance matrix given by $\sigma_J^2 \boldsymbol{I}_q$, where σ_J^2 is related to the power of the interference (power per dimension). As to the vector $\boldsymbol{p}$, it is a complex normal vector with zero mean and covariance matrix given by $\boldsymbol{I}_r$. Moreover, $|\alpha_k| = |\alpha|$, $k \in \Omega_P$, where $|\cdot|$ denotes the modulus of a complex number, and the phases of the α_k's are i.i.d. rv's uniformly distributed in (0, 2π). Finally, all of above rv's and random vectors are each other independent.

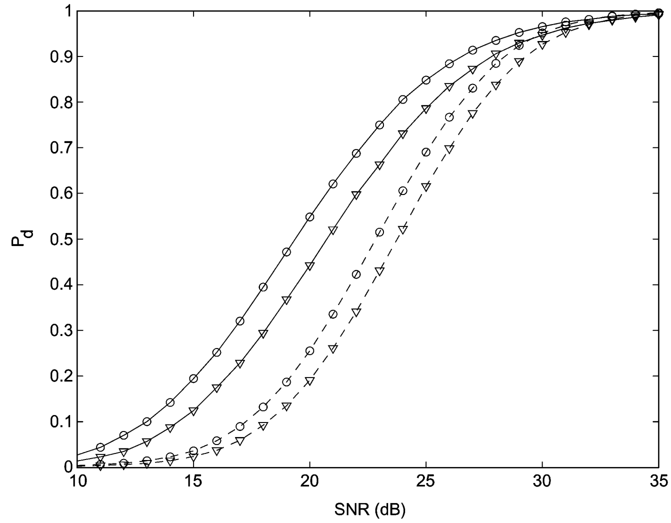

Fig. 1. P_d versus SNR for direction detectors (7) (circle marker) and (11) (triangle marker) with $N=8$, $K_P=8$, and $K_S=8$: $r=2, q=4$ (solid lines); $r=4, q=2$ (dashed lines).

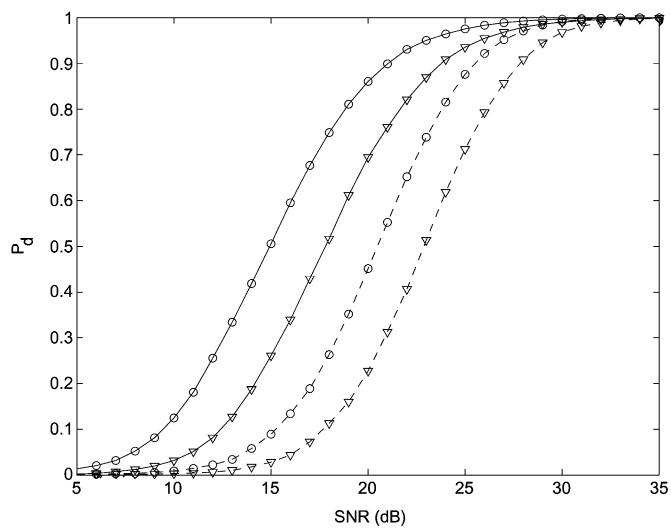

Fig. 2. P_d versus SNR for direction detectors (7) (circle marker) and (11) (triangle marker) with $N=16$, $K_P=8$, and $K_S=16$: $r=2, q=4$ (solid lines); $r=4, q=2$ (dashed lines).

As to the noise, it is modeled as an exponentially correlated complex normal vector with one-lag correlation coefficient ρ, namely the (i,j)th element of the covariance matrix $\boldsymbol{M}$ is given by $\sigma_n^2 \rho^{|i-j|}$, $i,j = 1,\ldots,N$, with $\rho = 0.95$ and $\sigma_n^2 = 1$.

The P_{fa} is set to 10^{-4} and the signal-to-noise ratio (SNR) is defined as

$$\text{SNR} = \frac{1}{N}\sum_{k=1}^{K_P} |\alpha_k|^2 E[\boldsymbol{p}^\dagger \boldsymbol{H}^\dagger \boldsymbol{M}^{-1}\boldsymbol{H}\boldsymbol{p}]$$

where $E[\cdot]$ denotes statistical expectation. Note that

$$E[\boldsymbol{p}^\dagger \boldsymbol{H}^\dagger \boldsymbol{M}^{-1}\boldsymbol{H}\boldsymbol{p}] = \frac{r}{N}\operatorname{tr}(\boldsymbol{M}^{-1}).$$

Figs. 1 and 2 plot P_d versus SNR for the GLRT-based direction detector (7) and the ad hoc direction detector (11), respec-

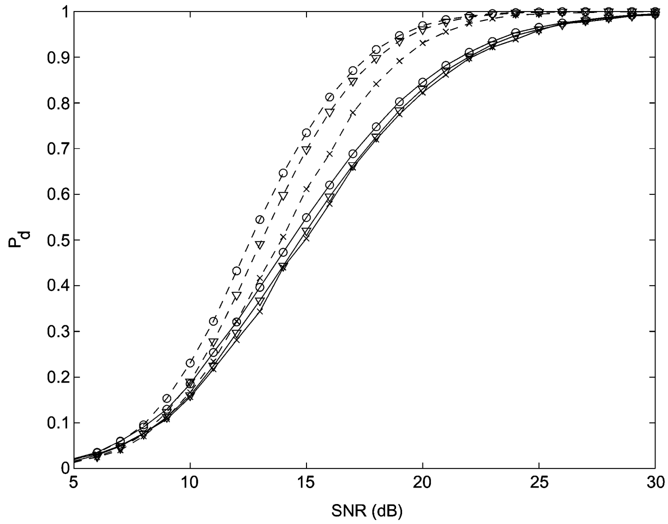

Fig. 3. P_d versus SNR for the direction detectors (7) (circle marker) and (11) (triangle marker), and the subspace detector (cross marker) with $N=8, K_P=8$, and $K_S=16$: $r=2, q=4$ (solid lines); $r=4, q=2$ (dashed lines).

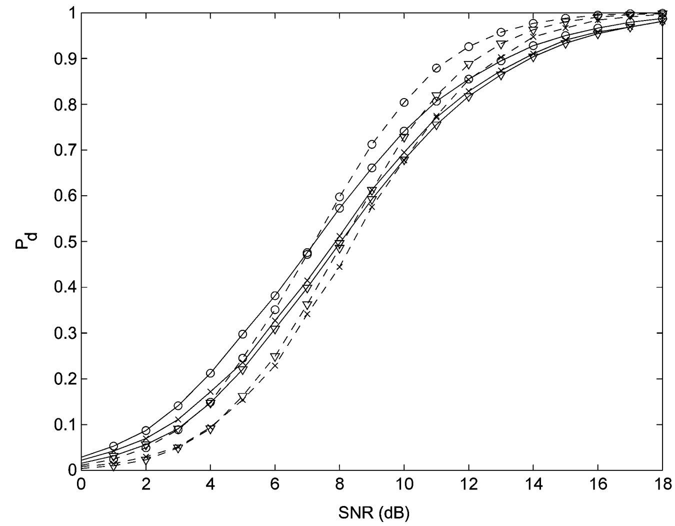

Fig. 4. P_d versus SNR for the direction detectors (7) (circle marker) and (11) (triangle marker), and the subspace detector (cross marker) with $N=16$, $K_P=8$, and $K_S=32$: $r=2, q=4$ (solid lines); $r=4, q=2$ (dashed lines).

tively. Fig. 1 assumes $N=8$, $K_P=8$, and $K_S=8$ while Fig. 2 refers to $N=16$, $K_P=8$, and $K_S=16$; both consider $r=2$, $q=4$ and $r=4$, $q=2$. The figures highlight that the GLRT outperforms the ad hoc detector for the considered parameter values and that the gain is significant for values of r and q such that $r+q$ is small compared to N. Such result is explained by the fact that for $r+q=N$, the two detectors coincide.

Figs. 3 and 4 plot P_d versus SNR for the direction detectors (7) and (11), and the GLRT subspace detector for the homogeneous scenario [13]. More precisely, Fig. 3 assumes $N=8$, $K_P=8$, and $K_S=16$, while Fig. 4 refers to $N=16$, $K_P=8$, and $K_S=32$; both consider $r=2$, $q=4$ and $r=4$, $q=2$. The figures show that the gain of the GLRT direction detector (7) with respect to the ad hoc direction detector (11) is significantly reduced for $K_S=2N$. Other simulation studies, not reported here for the sake of brevity, confirm it for $K_S>2N$. The curves also show that the gain of the GLRT direction detector with respect to the GLRT subspace detector is not negligible for $r=4$, $q=2$ and, more generally, when the dimension of the signal subspace is sufficiently high.

V. Conclusion

In this paper, we have implemented the GLRT direction detector and an ad hoc direction detector to operate in the presence of the homogeneous Gaussian noise with unknown covariance matrix and subspace interference. To this end, we have supposed that a set of noise-only data is available and that the useful target and the interference belong to known subspaces of the observables. The performance assessment shows that the plain GLRT performs better than the ad hoc detector, although at the price of a certain increase of the computational complexity, when the number of secondary data is comparable to the number of sensors. However, simulation studies also indicate that the gains of the former with respect to the latter are in the order of 1 dB (or less) when $K_S \geq 2N$. The comparison with a subspace detector has shown that the GLRT direction detector can guarantee significant gains when the dimension of the signal subspace is sufficiently high. The derivation of GLRT and ad hoc direction detectors for unknown interference subspace and/or a partially homogeneous scenario is part of the current research activity together with a performance analysis of the pros and cons of direction detectors in comparison with other "robust" techniques capable to take into account possible uncertainties on the actual steering vector of the target.

Appendix

The Appendix is aimed at proving that the GLRT (7) and the ad hoc detector (11) are equivalent for $r+q=N$. Since $[\boldsymbol{H}\ \boldsymbol{J}]$ and $\boldsymbol{S}$ have full rank N, the columns of $[\boldsymbol{H}_S\ \boldsymbol{J}_S]$ are a basis for the vector space $\mathbb{C}^{N\times 1}$. It follows that we can write each vector $\boldsymbol{S}^{-1/2}\boldsymbol{r}_k$, $k\in\Omega$, as a linear combination of the columns of $[\boldsymbol{H}_S\ \boldsymbol{J}_S]$. Otherwise stated, the "whitened" version of the primary data matrix $\boldsymbol{R}_P$ can be represented as

$$\boldsymbol{R}_{PS}=\boldsymbol{S}^{-1/2}\boldsymbol{R}_P=[\boldsymbol{H}_S\boldsymbol{J}_S]\begin{bmatrix}\boldsymbol{C}_H\\ \boldsymbol{C}_J\end{bmatrix}=\boldsymbol{H}_S\boldsymbol{C}_H+\boldsymbol{J}_S\boldsymbol{C}_J$$

where $\boldsymbol{C}_H\in\mathbb{C}^{r\times K_P}$ and $\boldsymbol{C}_J\in\mathbb{C}^{q\times K_P}$.

The above premises also imply that in receiver (7), matrix $\boldsymbol{A}$ can be recast as

$$\begin{aligned}\boldsymbol{A}&=\boldsymbol{I}_{K_P}+\left(\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\right)^{\dagger}(\boldsymbol{I}_N-\boldsymbol{P}_{\boldsymbol{J}_S})(\boldsymbol{S}^{-1/2}\boldsymbol{R}_P)\\&=\boldsymbol{I}_{K_P}+(\boldsymbol{H}_S\boldsymbol{C}_H+\boldsymbol{J}_S\boldsymbol{C}_J)^{\dagger}\boldsymbol{P}_{\boldsymbol{J}_S}^{\perp}(\boldsymbol{H}_S\boldsymbol{C}_H+\boldsymbol{J}_S\boldsymbol{C}_J)\\&=\boldsymbol{I}_{K_P}+(\boldsymbol{H}_S\boldsymbol{C}_H)^{\dagger}\boldsymbol{P}_{\boldsymbol{J}_S}^{\perp}(\boldsymbol{H}_S\boldsymbol{C}_H).\end{aligned}$$

In addition, the decision variable of test (7) can be rewritten as

$$\begin{aligned}
\Lambda_1(\boldsymbol{R}) &= \lambda_{\max}\left\{\boldsymbol{P}_{\boldsymbol{H}'_S}\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\boldsymbol{A}^{-1}\boldsymbol{R}_P^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{Z}\boldsymbol{P}_{\boldsymbol{H}'_S}\right\}\\
&= \lambda_{\max}\left\{\boldsymbol{R}_{PS}^\dagger\boldsymbol{Z}\boldsymbol{P}_{\boldsymbol{H}'_S}\boldsymbol{Z}^\dagger\boldsymbol{R}_{PS}\boldsymbol{A}^{-1}\right\}\\
&= \lambda_{\max}\left\{\boldsymbol{R}_{PS}^\dagger\underbrace{\boldsymbol{Z}\boldsymbol{Z}^\dagger}_{\boldsymbol{P}_{\boldsymbol{J}_S}^\perp}\boldsymbol{H}_S\left(\boldsymbol{H}_S^\dagger\underbrace{\boldsymbol{Z}\boldsymbol{Z}^\dagger}_{\boldsymbol{P}_{\boldsymbol{J}_S}^\perp}\boldsymbol{H}_S\right)^{-1}\right.\\
&\qquad\left.\times\boldsymbol{H}_S^\dagger\underbrace{\boldsymbol{Z}\boldsymbol{Z}^\dagger}_{\boldsymbol{P}_{\boldsymbol{J}_S}^\perp}\boldsymbol{R}_{PS}\boldsymbol{A}^{-1}\right\}\\
&= \lambda_{\max}\left\{\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{R}_{PS}\right)^\dagger\boldsymbol{H}_S\right.\\
&\qquad\times\left[\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\right)^\dagger\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\right)\right]^{-1}\\
&\qquad\left.\times\boldsymbol{H}_S^\dagger\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{R}_{PS}\right)\boldsymbol{A}^{-1}\right\}\\
&= \lambda_{\max}\left\{\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\boldsymbol{C}_H+\underbrace{\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{J}_S}_{0}\boldsymbol{C}_J\right)^\dagger\boldsymbol{H}_S\right.\\
&\qquad\times\left[\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\right)^\dagger\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\right)\right]^{-1}\boldsymbol{H}_S^\dagger\\
&\qquad\left.\times\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\boldsymbol{C}_H+\underbrace{\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{J}_S}_{0}\boldsymbol{C}_J\right)\boldsymbol{A}^{-1}\right\}\\
&= \lambda_{\max}\left\{\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\boldsymbol{C}_H\right)^\dagger\boldsymbol{H}_S\right.\\
&\qquad\times\left[\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\right)^\dagger\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\right)\right]^{-1}\\
&\qquad\left.\times\boldsymbol{H}_S^\dagger\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\boldsymbol{C}_H\right)\boldsymbol{A}^{-1}\right\}\\
&= \lambda_{\max}\left\{(\boldsymbol{H}_S\boldsymbol{C}_H)^\dagger\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\right)\right.\\
&\qquad\times\left[\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\right)^\dagger\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\right)\right]^{-1}\\
&\qquad\left.\times\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\right)^\dagger(\boldsymbol{H}_S\boldsymbol{C}_H)\boldsymbol{A}^{-1}\right\}\\
&= \lambda_{\max}\left\{(\boldsymbol{H}_S\boldsymbol{C}_H)^\dagger\boldsymbol{P}_{\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S}(\boldsymbol{H}_S\boldsymbol{C}_H)\boldsymbol{A}^{-1}\right\}. \qquad (13)
\end{aligned}$$

Observe now that

$$\operatorname{rank}\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\right) = \operatorname{rank}(\boldsymbol{H}_S) = r = N-q = \operatorname{rank}\left(\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\right).$$

Hence, the subspaces spanned by the columns of $\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S$ and $\boldsymbol{P}_{\boldsymbol{J}_S}^\perp$ are one and the same. It follows that we can replace $\boldsymbol{P}_{\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S}$ with $\boldsymbol{P}_{\boldsymbol{J}_S}^\perp$ in (13) obtaining

$$\begin{aligned}
&\lambda_{\max}\left\{(\boldsymbol{H}_S\boldsymbol{C}_H)^\dagger\boldsymbol{P}_{\boldsymbol{J}_S}^\perp(\boldsymbol{H}_S\boldsymbol{C}_H)\boldsymbol{A}^{-1}\right\}\\
&\quad= \lambda_{\max}\{\underbrace{(\boldsymbol{H}_S\boldsymbol{C}_H)^\dagger\boldsymbol{P}_{\boldsymbol{J}_S}^\perp(\boldsymbol{H}_S\boldsymbol{C}_H)}_{\boldsymbol{F}}\\
&\qquad\times[\boldsymbol{I}_{K_P}+\underbrace{(\boldsymbol{H}_S\boldsymbol{C}_H)^\dagger\boldsymbol{P}_{\boldsymbol{J}_S}^\perp(\boldsymbol{H}_S\boldsymbol{C}_H)}_{\boldsymbol{F}}]^{-1}\}\\
&\quad= \lambda_{\max}\left\{\boldsymbol{F}\left(\boldsymbol{I}_{K_P}+\boldsymbol{F}\right)^{-1}\right\}.
\end{aligned}$$

Finally, it is easy to check that the maximum eigeinvalue of $\boldsymbol{F}(\boldsymbol{I}_{K_P}+\boldsymbol{F})^{-1}$ can be related to the maximum eigenvalue of $\boldsymbol{F}$; it follows that

$$\Lambda_1(\boldsymbol{R}) = \lambda_{\max}\left\{\boldsymbol{F}\left(\boldsymbol{I}_{K_P}+\boldsymbol{F}\right)^{-1}\right\} = \frac{\lambda_{\max}(\boldsymbol{F})}{1+\lambda_{\max}(\boldsymbol{F})}.$$

Similarly, rewriting the decision statistic of receiver (11) as

$$\begin{aligned}
&\Lambda_2(\boldsymbol{R})\\
&\quad= \lambda_{\max}\left\{\boldsymbol{P}_{\boldsymbol{H}'_S}\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\boldsymbol{R}_P^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{Z}\boldsymbol{P}_{\boldsymbol{H}'_S}\right\}\\
&\quad= \lambda_{\max}\left\{\boldsymbol{R}_P^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{Z}\boldsymbol{P}_{\boldsymbol{H}'_S}\boldsymbol{Z}^\dagger\boldsymbol{S}^{-1/2}\boldsymbol{R}_P\right\}\\
&\quad= \lambda_{\max}\left\{\boldsymbol{R}_{PS}^\dagger\boldsymbol{Z}\boldsymbol{Z}^\dagger\boldsymbol{H}_S\left(\boldsymbol{H}_S^\dagger\boldsymbol{Z}\boldsymbol{Z}^\dagger\boldsymbol{H}_S\right)^{-1}\boldsymbol{H}_S^\dagger\boldsymbol{Z}\boldsymbol{Z}^\dagger\boldsymbol{R}_{PS}\right\}\\
&\quad= \lambda_{\max}\left\{\boldsymbol{R}_{PS}^\dagger\underbrace{\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\left(\boldsymbol{H}_S^\dagger\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S\right)^{-1}\boldsymbol{H}_S^\dagger\boldsymbol{P}_{\boldsymbol{J}_S}^\perp}_{\boldsymbol{P}_{\boldsymbol{P}_{\boldsymbol{J}_S}^\perp\boldsymbol{H}_S}}\boldsymbol{R}_{PS}\right\}\\
&\quad= \lambda_{\max}\left\{(\boldsymbol{H}_S\boldsymbol{C}_H)^\dagger\boldsymbol{P}_{\boldsymbol{J}_S}^\perp(\boldsymbol{H}_S\boldsymbol{C}_H)\right\}\\
&\quad= \lambda_{\max}(\boldsymbol{F})
\end{aligned}$$

proves identity (12).

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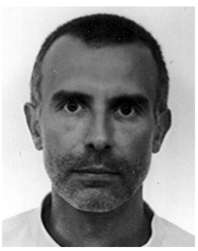
Francesco Bandiera (M'01) was born in Maglie (LE), Italy, on March 9, 1974. He received the Dr. Eng. degree in computer engineering and the Ph.D. degree in information engineering both from the University of Lecce, Italy, in 2001 and 2005, respectively.

From July 2001 to February 2002, he was engaged in a research project at the University of Sannio, Benevento, Italy. From September 2003 to March 2004, he was a Visiting Scholar at the Electrical and Computer Engineering Department of the University of Colorado at Boulder. From December 2004, he is an Assistant Professor at the University of Lecce. His main research interests are in the area of statistical signal processing with emphasis on multiuser communications, adaptive radar detection, and pollution detection on the sea surface based upon SAR imagery.



Olivier Besson (SM'04) received the Ph.D. degree in signal processing and the Habilitation à Diriger des Recherches degree both from INP, Toulouse, France, in 1992 and 1998, respectively.

He is currently an Associate Professor with the Department of Avionics and Systems of ENSICA, Toulouse, France. His research interests are in the general area of statistical signal and array processing with applications in radar and communications.

Dr. Besson is a member of the IEEE Sensor Array and Multichannel (SAM) Technical Committee and served as the Co-Technical Chairman of the IEEE SAM 2004 workshop.

Danilo Orlando was born in Gagliano del Capo (LE), Italy, on August 9, 1978. He received the Dr.Eng. degree in computer engineering from the University of Lecce, Italy, in 2004, where he is currently working towards the Ph.D. degree.

His main research interests are in the field of statistical signal processing with emphasis on adaptive signal processing for radars.

Giuseppe Ricci (M'01) was born in Naples, Italy, on February 15, 1964. He received the Dr. degree and the Ph.D. degree, both in electronic engineering, from the University of Naples "Federico II," Italy, in 1990 and 1994, respectively.

Since 1995, he has been with the University of Lecce, Lecce, Italy, first as an Assistant Professor of Telecommunications and, since 2002, as a Professor. His research interests are in the field of statistical signal processing with emphasis on radar processing and code-division multiple-access (CDMA) systems. More precisely, he has focused on high-resolution radar clutter modeling, detection of radar signals in Gaussian and non-Gaussian disturbance, multiuser detection in overlay CDMA systems, and blind multiuser detection. He has held visiting positions at the University of Colorado at Boulder from 1997 to 1998 and in April/May 2001, at the Colorado State University, Ft. Collins, in July/September 2003 and March 2005, and at Ensica, Toulouse, France, in March 2006.

Louis L. Scharf (S'67–M'69–SM'77–F'86–LF'07) received the Ph.D. degree from the University of Washington, Seattle.

From 1971 to 1982, he served as Professor of electrical engineering and statistics at Colorado State University (CSU), Ft. Collins. From 1982 to 1985, he was Professor and Chairman of electrical and computer engineering at the University of Rhode Island, Kingston. From 1985 to 2000, he was Professor of electrical and computer engineering at the University of Colorado, Boulder. In January 2001, he rejoined CSU as Professor of electrical and computer engineering and statistics. He has held several visiting positions here and abroad, including the Ecole Superieure d'Electricité, Gif-sur-Yvette, France; Ecole Nationale Superieure des Télécommunications, Paris, France; EURECOM, Nice, Italy; the University of La Plata, La Plata, Argentina; Duke University, Durham, NC; the University of Wisconsin, Madison; and the University of Tromsø, Tromsø, Norway. His interests are in statistical signal processing, as it applies to adaptive radar, sonar, and wireless communication. His most important contributions to date are to invariance theories for detection and estimation; matched and adaptive subspace detectors and estimators for radar, sonar, and data communication; and canonical decompositions for reduced dimensional filtering and quantizing. His current interests are in rapidly adaptive receiver design for space–time and frequency–time signal processing in the wireless communication channel.

Prof. Scharf was Technical Program Chair for the 1980 IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP), Denver, CO; Tutorials Chair for ICASSP 2001, Salt Lake City, UT; and Technical Program Chair for the Asilomar Conference on Signals, Systems, and Computers 2002. He is past-Chair of the Fellow Committee for the IEEE Signal Processing Society and serves on its Technical Committees for Theory and Methods and for Sensor Arrays and Multichannel Signal Processing. He has received numerous awards for his research contributions to statistical signal processing, including an IEEE Distinguished Lectureship, an IEEE Third Millennium Medal, and the Technical Achievement Award from the IEEE Signal Processing Society.