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NUMERICAL ERROR EVALUATION FOR TIP CLEARANCE FLOW CALCULATIONS IN A CENTRIFUGAL COMPRESSOR

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Summary Since globally mesh independent solution are still beyond available computer resources for industrial cases, a method to quantify locally the numerical error is proposed. The design of experiments method helps selecting mesh parameters that influence the tip clearance solution, so that additional meshes are computed to evaluate the numerical error on the shroud friction coefficient.

In the field of CFD applied to turbomachinery, this study results from a partnership between ENSICA, Liebherr-Aerospace Toulouse and Numeca International. This paper focuses on numerical error evaluation for RANS simulations, applied to centrifugal compressor flow field calculations. CFD is now commonly used for centrifugal compressor design optimization, but, as Hutton and Casey develop in [1], there is an urging demand for improved quality and trust in industrial CFD. Indeed, this stresses the need for comprehensive and thorough numerical error evaluation, namely the process of *verification*, as defined for example by Oberkampf and Trucano in [2]. Unfortunately, 3D turbulent calculations for turbomachinery components are still very demanding in computational resources and, to the knowledge of the author, there is no published result concerning comprehensive verification of the entire flow field in centrifugal compressors. As a first step on the way to achieve that, this paper presents a method aiming at the obtention of a numerical solution that can be regarded as *locally* mesh-independent. In other words, the objective is to compute the flow field on a grid such that the solution obtained has a specific region where the numerical error is negligible.

It has long been recognized that the tip clearance of a centrifugal compressor is of paramount importance for aerodynamic performances, which means that accurately predicting the flow field in this region is crucial for accurate prediction of performances by means of CFD codes. Numerous studies have been published that compare numerical and experimental results in the tip region. However, in these studies, numerical error still remains an issue; for instance Basson and Lakshminarayana [3] show excellent comparisons with experiments, but they attribute the remaining discrepancies to insufficient grid resolution. Indeed, accurate predictions of global effects, such as efficiency, require a fine description of flow details. Therefore, friction at the shroud endwall is the concern of the study, since it is a very sensitive indicator of the quality of the velocity profile's prediction at the wall.

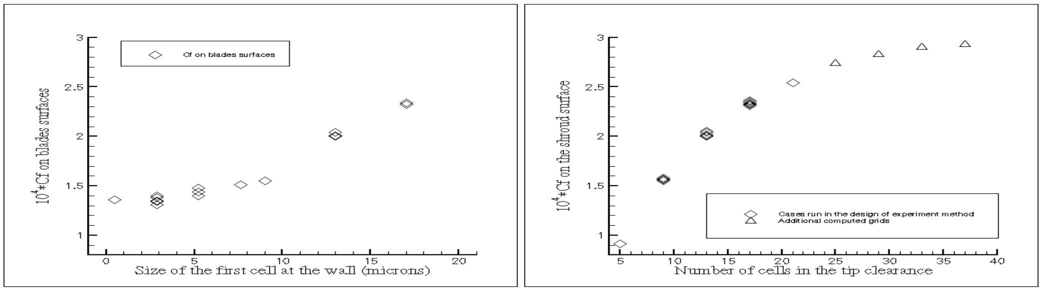
INFLUENCE OF GLOBAL MESH PARAMETERS ON FLOW QUANTITIES IN THE TIP CLEARANCE

Any numerical result is marred with two sources of errors: numerical error, and error due to models. Validation is the process of evaluating the former, and verification assesses the later. The scope of this paper is numerical error.

Although other sources of numerical error can be pointed out, the effects of grid size are dominant in most industrial cases. Therefore, the challenge is to be able to assess the global influence of the mesh on the solution, and in particular on the clearance flow field prediction. The basic idea is to first select relevant parameters that characterize the mesh, and then to evaluate their influence on specific calculated quantities, through the use of the design of experiments method.

The selection of basic parameters and its justification is thoroughly presented in a paper submitted to the ASME Fluid Engineering Summer Conference 2004 (HT-FED2004-56314). It will be sufficient here to recall the 6 selected mesh parameters : (i) the number of points in the spanwise direction (regardless of the cells in the gap); (ii) a multiplicative factor to obtain the number of points in the 2 other directions ; (iii) a parameter quantifying the discretization in the center of the channel; (iv) the size of the first cell on the hub and the blades; (v) a parameter quantifying the discretization of the leading- and trailing-edges; (vi) and the number of cells in the tip clearance.

The design of experiment method used for this study is second order accurate, that is to say it evaluates each parameter influence and 2 by 2 correlations. Therefore, 28 meshes (with total numbers of points ranking from 300.000 to 2.000.000 points) had to be generated and computed on the same test-case. The test case is a 2.7:1 pressure ratio centrifugal impeller, running at 38000 RPM with a mass-flow rate of 0.632 kg/s. Figure 1 presents friction coefficients obtained for the 28 grids. Figure 1(a) is an attempt to correlate the friction coefficient integrated on blade surfaces with the size of the first cell at the wall. The scatter of the data for each cell size demonstrates that this mesh parameter is not the only one influencing the friction value predicted. On the other hand, figure 1(b) demonstrates that the value of the friction coefficient (after scalar integration on the shroud surface) is entirely fixed by the number of cells in the gap. It means that the numerical error due to the influence of other parameters is negligible. This result is confirmed by the analysis of the design of experiment, which allows influence coefficient to be derived that characterize the influence of each mesh parameters on the solution. It shows that 99% of the value of the solution is fixed by the number of cells in the gap. Thus, the results of design of experiment method suggest that, by increasing the number of cells in the gap only and keeping all other parameters to reasonable values, it should possible to derive a mesh such that the numerical error on the shroud friction coefficient tends to a negligible value.



(a) (b)

Figure 1. Friction coefficients. (a) On blades surfaces. (b) On the shroud

DERIVATION OF A "LOCAL BENCHMARK" SOLUTION

Refinement is performed only in the tip region of the grid, and the other parameters are kept to reasonable values (that is, the values centered in the initial range of parameters). This allows rather cheap meshes to be used, so that grid points (*i.e.* computational resources) can be dedicated to the tip gap discretization. Four more meshes are generated and computed, with up to 37 points in the tip clearance.

As shown on figure 1(b), a qualitative convergence of the friction coefficient value is observed. An interesting property of numerical convergence (when the asymptotic range is reached) is that each new refinement should entail a relative change of the solution inferior to the change due to the previous refinement. This is the case here.

NUMERICAL ERROR ESTIMATION FOR THE FRICTION AT THE SHROUD

In the frame of verification procedure, Richardson Extrapolation is commonly used to help evaluating the numerical error with a given set of computed grids. Oberkampf and Trucano [2] use the Taylor development of the numerical solution to derive an *a posteriori* error estimate. Several assumptions need to be made to obtain this estimate: (i) the output considered must be a smooth solution (in the sense of the existence of sufficient derivatives to justify the Taylor Serie expansion); (ii) the formal convergence order of the scheme is known; (iii) the mesh spacing must be small enough for the asymptotic range to be reached.

Assuming all three hypotheses, the use of three grids (Coarse Medium and Fine) yields the following error estimate : $\delta_F = \left(\frac{S_M - S_F}{h_M^p - h_F^p}\right) h_F^p$, where δ_F is the error on the Fine grid, S the computed solution and h a measure of the grid size. The terms neglected in this estimate are of the order $\mathcal{O}(h_F^{p+1}) + \mathcal{O}(h_M^{p+1})$, hence negligible.

Applying this error estimate to the three last computed grids, *i.e.* with 29, 33 and 37 points in the gap, the error on the 37 point case is 4.9% (relative to the estimated exact solution).

To check *a posteriori* for the asymptotic range assumption, Oberkampf and Trucano propose to compare the error on the coarse case (29 points) with the theoretical error. The error in the 29 point case is obtained using the just-computed exact value (using the error estimate and the 37 point case). The theoretical error is deduced from the knowledge of the error in the finest case and the ratio of refinement between the coarse and the fine case, that is to say: $\delta_{C-theoretical} = (37/29)^2$ because it is a second order scheme. The results compare very well: 8.3% for the theoretical error compared to 8% for the computed error. According to Oberkampf, this demonstrates *a posteriori* that all three grids lie in the asymptotic range.

The reliable estimate of the exact solution provided by this method allows for the error to be evaluated on any grid. For instance, if a calculation is run with 21 points in the gap, the error can be estimated to be 17% of the exact solution, and for the 9 point case the error approaches 50%.

To sum up, considering that memory requirements prevent globally mesh independent solutions for industrial cases, this study aimed at generating a locally mesh independent solution. The proposed method enabled the derivation of a grid for accurate tip clearance description, and the numerical error on the friction coefficient was estimated to be less than 5%.

The benefit of the proposed method is to offer a way to assess locally numerical error accurately. This can be regarded as an affordable way to identify local requirements that will still have to be fulfilled when global independence is the concern.

References

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