A truncated low approach of intrinsic linear and nonlinear damping in thin structures

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Abstract

An adaptive approach of vibrating thin structures is proposed here. The method consists in applying an equivalent adimensional damping ratio to each potential resonance. This ratio is deduced from experimental data obtained in vacuum facility, in relation with frequencies, for several structural technologies. Consequently, it is possible to calculate the structure in a linear non-dissipative context, valid out of resonance bands, and truncated in those bands. Thus, the equivalent damping ratio is directly used to define adimensional resonance truncation bandwith and level. The contribution consists in tested and applied modal methodology and algebraic representations of damping including several dissipations - viscous and internal microfrictions - inducing non-monotonous model. The here aim is to provide realistic recommendations for simple vibrational analysis of aerospace thin structures - panels and stiffeners.

Keywords: Vibrations, Eigenmodes, Intrinsic Damping, Nonlinear Damping, Modal Truncation.

Notation

ω_0	Fundamental Eigenpulsation (rd/s)
f_0	Fundamental Eigenfrequency (Hz)
m	Mass or Inertial Parameter
k	Stiffness or Rigidity Parameter
c	Rayleigh's Viscous Factor
ζ	Adimensional Damping Ratio
χ	Solid Friction Force Module
x	Reference Abscissa
$sign$	Heaviside sign function
Re	Real part of Complex Number
γ	Adimensional Frequency of Excitation
$\{q\}$	Column Vector of Lagrange's Parameters
$\{Q\}$	Column Vector of Lagrange's Generalized Forces
$[M]$	Square Matrix of Masses and Inertias
$[K]$	Square Matrix of Rigidities
m_j	Effective Mass in j-th mode
$d(t)$	Imposed Deflection
$F(t)$	Imposed Force
$\|F(f)\|$	Factor of Dynamic Amplification

1. Introduction

The dynamic identification of thin structures for their qualification and certification requires a robust knowledge of vibrating intrinsic dissipation. For this type of structure, usually constituting aerospace or terrestrial vehicles, the structural materials themselves, combined with accurate technology of assembly lead, to low ratios of intrinsic damping. This paper aims at proposing a low approach based on a modified linear model, as it appears that in operational and optimized structures, the vibrational non-linearity does not occur for elastic terms, but rather for damping, which implies intricate phenomena, even - and sometimes especially - if the damping is low. Hence, the classical viscous approach remains a rough approximation. It is enough for modal identification, frequency shift being negligible, but it leads to oversize of the structures in the bands of resonance. Our approach deals with dynamic conservative models with adequate truncations and corrections to meet the requirements for dimensioning, avoiding complex modes with low phases, and allowing reliable prediction for beam and shell structures. It is based on tests performed under vacuum, with use of aerospace technologies and focused on assembled carbon/epoxy thin-skin bending and twisting motions.

2. The Problem of Perturbed Linearity

2.1. Damping in an Elementary System

The behavior of the "One Degree of Freedom System" is the origin of any linear structural approach. Damping can usually be represented by Rayleigh's Viscous Model, leading to a First Order Complete Differential Equation, with constant coefficients when the system is considered as linear. In fact, this type of friction considers forces in proportion to speeds. In addition, Coulomb's Dry Friction Model constitutes a typical basic non-linearity in dissipation, with constant force in the module, the sign being obviously the

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opposite of speed. In terms of energies and powers, linear forces lead to energetic quadratic forms, and Heaviside Typical non-linearity to other forms. This induces specific formulations of Newton, Lagrange or Hamilton formalisms, with explicit separation of potentials.

2.1.1. Linear and non-linear system without excitation

Viscous and friction elementary vibrator admits an analytical solution when it is considered without excitation : sine shape with exponential decreasing for viscous dissipation, quarter-sinus sequences with linear decreasing for dry friction ([1] , [2]). In the latter, non-linear case, it must be observed that the final position is static but different from the original one (see FIG. 3). The reference pulsation here is the conservative one :

$$\omega_o \equiv \sqrt{\frac{k}{m}} \equiv 2\pi f_o \tag{1}$$

If the damping is low, the pseudo-frequency of response is close to f_0. In linear viscous approach, it is precisely :

$$\frac{f_{ov}}{f_o} = \sqrt{1-\zeta^2} \approx 1 - \frac{\zeta^2}{2} \approx 1 \qquad (\zeta \equiv \frac{c}{2\sqrt{km}} \ll 1) \tag{2}$$

where z is the adimensional damping ratio.

2.1.2. Solution with excitation

When we consider the excited system, the linearity leads naturally to classical Fourier's decomposition, which allows only a canonical sine applied load to be envisaged. In the frequency domain, the behavior of the system is then represented by dynamic stiffness - applied force inducing mass motion - or dynamic transmissibility - applied deflection inducing mass motion (see FIG. 5). The Principle of Equivalence allows inertial forces to be considered as equivalent applied ones, and makes the first configuration - dynamic stiffness - representative of general excitation, in relative motion. In addition, a static load does not change the dynamic approach, as it only slides the reference configuration of linearization, stiffnesses even being unchanged if the system is globally linear. The canonical applied acceleration is clearly also sinusoidal, with w^2 factor in relation with applied deflection. Thus, the elementary linear canonical equation is :

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = \cos(\omega_e t) \qquad (\omega_e \equiv 2\pi f_e) \tag{3}$$

The algebraic permanent general solution is well known as :

$$x(t) = \beta(\omega_e)\cos(\omega_e t + \varphi(\omega_e)) \qquad \beta(\omega_e) \in \mathbb{R}^+ \tag{4}$$

Or :

$$x(t) = Re[\beta exp(i\omega_e t)] \qquad \beta(\omega_e) \in \mathbb{C} \tag{5}$$

The amplitude is adimensionally expressed in relation to static stiffness :

$$\beta(\omega_e) \equiv \frac{\phi(\omega_e)}{k} \tag{6}$$

The dynamic factor - which represents the magnitude of the complex solution - is then :

$$|\phi_d(\gamma)| = \frac{1}{sqrt(1-\gamma^2)^2 + (2\zeta\gamma)^2} \tag{7}$$

With $\gamma \equiv \frac{f_e}{f_o} \in \mathbb{R}^+$ and $\gamma \in \mathbb{R}^+$
The maximal values are easily approximated as :

$$\gamma_{max} = sqrt1 - 2\zeta^2 \approx 1 - \frac{\zeta^2}{2} \approx 1 \qquad (\zeta \ll 1) \tag{8}$$

$$|\phi_d|_{max} = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \approx \frac{1}{2\zeta} \qquad (0 < \zeta \ll 1) \tag{9}$$

It is possible to propose approximations for the harmonic amplitude ratio $F_d(g)$. The first one is a single truncation in the resonance band at maximal value, the resonance here being pointed by saturation :

$$\begin{array}{l} |\phi_d(\gamma)| \approx \frac{1}{|\sqrt{1-\gamma^2}|} \\ \text{Truncated at } |\phi_d(\gamma)| \approx \frac{1}{2\zeta} \text{ For } \gamma \in [1-\zeta; 1+\zeta] \end{array} \tag{10}$$

The second proposed approximation consists in shifting g in a homothetic way from 0 to 1+z factor, in order to obtain the 1/2z maximal value at g=1. The resonance here is pointed by the discontinuity of the derivative, and the truncation is automatically operated by shifting, the discontinuity being contained in the proposed algebraic expression :

$$\begin{aligned} |\phi_d(\gamma)| &\approx \frac{1}{|1-[\gamma(1-\zeta sign(1-\gamma))]^2|} \\ &\approx \frac{1}{|1-[\gamma^2(1-2\zeta sign(1-\gamma))]|} \end{aligned} \tag{11}$$

The curves presented in Appendix A show clearly that, for z=0.02 (2%), which represents the typical significant damping in a structure, the two approximations allow convenient accuracy. On a logarithmic graph, the curves are practically superimposed. Concerning phase variation, the exact expression for viscous dissipation is given by :

$$\sin\varphi = -2\zeta\gamma|\phi_d(\gamma)| \tag{12}$$

Appendix A also shows the exact phase variation, with two single approximations : the linear interpolation in the band of resonance on the one hand, and the Heaviside function at resonance on the other. The phase clearly varies from 0 to -p, resonance being - at g=1 precisely - the changing point. It can be observed that the phases turn in opposition : +p or -p, which are here equivalent without any contradiction with the principle of causality, as the established solution is periodic (exact dual complement of general free solution specified above). In addition, this allows the phase to be represented schematically by the sign of F_d - modified from one hyperbolic branch to the other. Hence, if $|F_d|$ is represented, the p phase variation occurs at curve singularities - peaks or truncated peaks - as seen on curves.

Considering then non-viscous friction problem under excitation, the intrinsic nonlinearity restricts the use of Fourier's decomposition. Nevertheless, if damping is low, the study of canonical sine excitation presents some interest for comparison with viscous behavior, the basic equation remaining discontinuous and transcendant :

$$m\ddot{x}(t) + \chi\dot{x}(t) + kx(t) = \cos(\omega_e t) \quad (13)$$

In Appendix C figure several particular cases with explicit numerical integration step-by-step. This type of computation shows two major caracteristics of Constant Friction non-linearity :

- permanent solutions are not sine : quasi-periodic responses, combine frequencies
- amplitudes can grow infinitely, which is never the case if any viscosity exists.

Fig. 5 summarizes some results for pure friction, in comparison with exact solutions of linear reference systems : non-dissipative on the one hand, and 2% viscous on the other. The peak at resonance is widened, infinite and unsymmetrical.

The condition of low damping in friction is relative, considering the basic Eq. (13), and can be proposed on the basis of three forces originating from canonical excitation, elasticity and inertia :

$$\chi << 1; \quad \chi << k; \quad \chi << mw_e^2 \quad (14)$$

Thus, the phase is assumed to be schematically expressed as in a non-dissipative case :

$$|\varphi| = 0 \text{ for } \gamma \in [0;1[\text{ ; } |\varphi| = \pi \text{ for } \gamma \in]1;+\infty[\quad (15)$$

This system constitutes three independent conditions to be fulfilled separately to substantiate the perturbed proposed sine solution. Then, thanks to Heaviside's gap properties applied to a square wave, it is possible to propose a semi-analytical approach of permanent solution, assumed periodic with fundamental pulsation at w_e, by adapted Fourier's decomposition. In fact, the non-linearity expressed by sign can also be developed as a unitary square wave of pulsation w_e, defined as follows :

$$\left\{ \begin{array}{l} g_{\omega_e}(t) \equiv +1 \text{ for } t \in [o; \frac{\pi}{\omega_e}] \\ g_{\omega_e}(t) \equiv -1 \text{ for } t \in [\frac{\pi}{\omega_e}; \frac{2\pi}{\omega_e}] \end{array} \right. \quad g_{\omega_e} periodic \left[\frac{2\pi}{\omega_e}\right] \quad (16)$$

This function enables a development with an infinite number of known factors. This leads to the following shape of equation :

$$\begin{array}{l} -m\sum_{n=0}^{\infty} n\omega_e^2 a_n exp(in\omega_e t) \\ +\chi\sum_{n=0}^{\infty} n\omega_e^2 b_n exp(in\omega_e t) \\ +k\sum_{n=0}^{\infty} n\omega_e^2 a_n exp(in\omega_e t) = exp(i\omega_e t) \end{array} \quad (17)$$

Where a_n are the coefficients of the response decomposition, and b_n the coefficients of the unitary negative square wave $-g_{we}$ for $0 \leq \quad g < 1$, known as :

$$b_{2j+1} = i\frac{4}{\pi}\frac{1}{2j+1}; \quad b_{2j} = 0; \quad j \in \mathbb{N}^\star \quad (18)$$

By identification of the terms, it is then possible to restitute the coefficients of perturbed response. The static and fundamental terms are immediately given by :

$$ka_0 \approx b_0 = 0; \quad (k - m\omega_e^2)a_1 \approx 1 - b_1 = 1 - i\frac{4}{\pi} \quad (19)$$

And the following Harmonic Coefficients :

$$\begin{array}{l} (k - mn^2\omega_e^2)a_{2j+1} \approx -b_{2j+1} = -i\frac{4}{\pi}\frac{1}{2j+1} \\ a_{2j} \approx 0; \quad j \in \mathbb{N}^\star \end{array} \quad (20)$$

The signs of b_n coefficients must obviously be changed for $g > 1$. In addition, the general approach by Fourier's could be generalized to Laplace's transform, $\cos(wt)$ being replaced by $exp(st)$, but a structural problem induces vibrating cycling phenomena, for which the sine base is convenient.

2.2. Multibody and Continuous Systems

2.2.1. General Discrete System

For a natural discrete system, the quadratic shapes of kinetic energy, Rayleigh's viscous power and elastic potential induced by linearity allow the direct writing of the matrix equation with constant coefficients :

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{Q_e\} \quad (21)$$

where $[\mathbf{M}]\int[\mathrm{m}_{ij}]$, $[\mathbf{C}]\int[\mathrm{c}_{ij}]$ and $[\mathbf{K}]\int[\mathrm{k}_{ij}]$ are square nxn nonnegative defined symmetrical matrices, $\{q\}\int\{q_j\}$ the column-matrix of Lagrange's geometrical parameters of configuration (distances and angles), and $\{Q_e\}\int\{Q_{ej}\}$ the column-matrix of corresponding excitations - forces and moments. For a considered linear system, the general approach is conserved when considering only the p-th canonical harmonic excitation. In fact, those canonical excitations constitute a base - independent and generating system - for all possible excitations :

$$\{Q_e\} = \{Q_e\}_{p,\omega_e} \equiv d_{jp}\cos\omega_e t \equiv \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{Bmatrix}_{p-thline} \cos(\omega_e t) \tag{22}$$

For $j, p = 1; ...; n$.

2.2.2. Modal General Solution of Non-excited Homogeneous System

Fourier's transform leads to an eigenvalues problem, with a characteristic homogeneous equation allowing the existence of a base of decoupled and general sine solutions. The characteristic non-dissipative equation is defined by harmonic impedance :

$$|[Z(w)]| \equiv |-\omega^2[M] + [K]| \equiv 0 \text{ for } \omega = \omega_k \tag{23}$$

The non-dissipative system allows rigorous diagonalization in the common base, with real eigenvectors $\{v\}_k$ associated to eigenfrequencies $f_k \int w_k/2/p$:

$$[Z(w)]\{v\}_k \equiv 0 \quad (k = 1; ...; n) \tag{24}$$

When the vectors are normalized (with Euclidian norm), they consitute an orthonormal system directly defining the principal parameters for diagonalization

$$\begin{array}{l} [M]_{diag} =^t [P][M][P]; \;\; [K]_{diag} =^t [K][M][K] \\ \{q\} = [P]\{p\} \text{ With } [P] = [\{p\}_1\{p\}_2 \dots \{p\}_n] \end{array} \tag{25}$$

Consequently, in this case, the problem is strictly equivalent to n problems with one DoF. In addition, for each eigenfrequency, the m_k terms of diagonalized $[\mathbf{M}]_{diag} \int [m_k]_{diag}$ matrix directly define the modal effective inertias. This notion can be enlarged to rigid ($m_0 = \mathrm{M}_{system}$) and circular modes ($m_j = m_l$). In addition, symmetries or pseudo-symmetries can be used to define active effective inertias of a part of the structure.

This modal approach can be generalized to viscous linear damping, with modes built on the field of complex numbers. Hasselman's conditions are here necessary :

$$\frac{2\zeta_k\omega_k}{|\omega_k - \omega_i} \ll 1 \text{ with } j, k = 1, ..., n \text{ and } j \neq k \tag{26}$$

These conditions - low modal damping and decoupled frequencies, implying a special mention for circular modes - are mandatory for applying Basile's assumption of common diagonalizing base. Then, the characteristic equation obviously becomes :

$$|[Z(w)]| \equiv |-\omega^2[M] + i\omega[C] + [K]| \equiv 0 \tag{27}$$

In any case, with the modal approach, the system can be represented by n independent unidimensional eigensystems, the n modes equivalent to n^2 numerical data ([3]). Thanks to lower frequential truncation (quasi rigid modes) and higher frequential truncation (low effective masses and effect of damping at high frequency), it is possible to restrict the number of usual modes to m¡n (and even m¡¡n). Finally, the vibratory behavior of the system is represented with limited mxn numerical data (see FIG. 6).

This approach can be generalized to friction or other dissipation under the assumption of low power of dissipation (in relation with kinetik and potential powers). But the equivalent assumption of common diagonalization of matrices is basically approximate, and can remain valid only for very low dissipation.

2.2.3. Harmonic Particular Solution of Excited System

For a non-dissipative system, the excited equation and solution to consider is, in coherence with EQ. (22) :

$$\begin{array}{l} [M]\{\ddot{q}\} + [K]\{q\} = \{Q_e\}_{p,\omega_e} \\ \text{So } \begin{cases} [Z(\omega_e)]\{\beta\} \equiv \{\delta_{jp}\} \\ \{q\}_{permanent} \equiv \{\beta\}\cos(\omega_e t) \end{cases} \end{array} \tag{28}$$

The real established solution in amplitude shows the resonance with a characteristic determinant at the denominator of Kramer's unique solution of non-homogeneous system, existing precisely out of eigenvalues. Under Basile's assumption, such a result is generalized to viscosity, with b_j amplitudes in the field of complex numbers. The generalization is not explicit for solid friction because the modes are not defined rigorously and also because the divergence without phase exists at resonance even with dissipation, as it was observed for one DoF system. Nevertheless, this elementary reference is indicative for the qualitative modal modification when a very low solid friction occurs.

2.2.4. Continuous System

The discretization - necessary for digital processing - can be made by method of equivalence or by discrete Lagrange's observative variables. Applying Absi's method of equivalence consists in replacing the actual structure by an equivalent "rational" one, with natural finite number degrees of freedom ([4]). This method then simplifies the computation, but the difficulty is to establish a rigorous equivalence with a discrete system. Lagrange's observative variables consist in applying observation points (knots) on the continuous system, and defining the configuration by their position, as it is usually done in static or dynamic tests. The matrices of elements - between adjacent knots - are restituted by the Rayleigh-Ritz method with interpolation of displacements, with virtual principle application, the elementary matrices being then assembled to restitute the global ones in a common base of parameters. In addition, the definition of interpolation can involve thin element theories (shells, plates, linear or curved beams) which simplify the formulation ([5]), and special techniques of condensation or boundary elements allow only chosen degrees of freedom to be observed. In any case, the dynamic system with classical resolutions leads - once again - to the elementary one DoF system (see FIG. 8).

3. Vacuum Tests and Structural behavior

3.1. Description of Test Facility

The 0.3 m^3 chamber restitutes statically thermo-mechanical conditions equivalent to low orbit : 10^{-9} bar pressure with temperatures between -80 and +100 ° C obtained by liquid nitrogen and electrical resistances. But for this particular use, it was necessary to allow dynamic tests with excitation crossing the wall, and with the induced airflow, the group of pumps allowed only a 10^{-3} level vacuum, which is nevertheless sufficient for the mechanical removal of atmospheric viscosity and inertia, in relation to the dynamic behavior of thin structures. FIG. 9 shows this chamber with an external system for generation of the thermal vacuum, the electrodynamic actuator and the system for data processing. In the chamber itself, the structural specimen is excited here by imposed linear motion of the base chariot.

The transmissibility is then obtained by simultaneous measurment of the base accelerations and free parts of the specimen, by accelerometers or camera stroboscopic views. Thus, the restitution of the transfer function for identification of the parameters is obtained by analytical models of transmissibility in the reference case of simple cantilever beams, or by Rayleigh-Ritz analogy for plates with accelerometers.

3.2. Free Vibrating Tests and Results

In the tests, free vibrating is induced by abrupt interruption of the initial established excitation of the fundamental mode. Then, the natural motion of the system is observed in time, and shows a combination of exponential and linear amplitude decrease. For the composite specimen, the free motion also shows an initial beating, suggesting an interference with another eigenfrequency, as shown in FIG. 7, where a temporal curve combining two frequencies with linear and solid damping has been presented for comparison.

3.3. Excited Vibrating Tests and Results

The reference test is a sine excitation with logarithmic scanning in frequency, to obtain the amplification ratio, in relation to eigenshapes in deflection transmissibility ([3]). FIG. 10 presents the results for carbon plate (dots), under vacuum and at 20 ° C specimen temperature, showing global behavior and dispersion. This modal appropriate approach allows direct comparison with linear modal resonance, which has been represented in the graphs by black lines.

3.3.1. Interpretation of Harmonic Results

The shape of the unsymmetrical peak shows clearly that a non-linear component is present in vibration. Consequently, the height of the peak is more pertinent (precise and robust) to define the equivalent adimensional damping, rather than the analysis of peak width and accuracy. This height is used to determine the damping ratio at resonance, either by direct analytical method (beam transmissibility) and numerically (Finite Elements with modal defined damping).

For identical technology (T300/914 Bolted Carbon Plate in Bending), the variation of length and the addition of punctual masses allow variations of eigenfrequencies, and consequently a precise analysis of variation of intrinsic damping z in relation to frequency f, as shown in FIG. 11 (dots).

Two major characteristics appear immediately :

- the dispersion is high, but is applied to very low values
- the constant value of z constitutes a very rough approximation, as it is clear that the variation z(f) is non-monotonous.

Thus, it is possible to propose a simple algebraic representation of the equivalent damping z , taking into account different origins, with the rational curve presented in FIG. 11 (a, b, c constants being determined by quadratic minimal difference criterion). This kind of representation induces a relative error on damping in the range of 25%, to be compared with more than 50% for constant or any monotonous representation. Obviously, this sort of approach is valid in the defined band of frequency, and for a precise technology. In a coherent way, the equivalent viscous maximal amplification ratio - represented by logarithmic symmetrical inverted non-monotonous curve - is as follows :

$$|\phi_d|_{maxequiv} = Q_{equiv}(f) \equiv \frac{1}{2\zeta_{equiv}(f)} \tag{29}$$

3.4. Recommendation on Intrinsic Truncation for Real Models

Considering the nature of results presented above, it is now possible to propose a Low Approach dealing only with non-damped conservative structures, inducing real computations with inertias and rigidities, and to truncate the infinite peaks of resonance at Q_{equiv}(f) levels defined by intrinsic curve associated with the type of structure concerned. This curve obviously requires a specific definition for each technology, and can lead to more intrinsic variations than the ones presented previously for a basic non-linear structural element It is also necessary to mention that possible anti-resonances must be under-truncated with 1/Q factor.

This methodology is schematically represented in Fig. 12, which illustrates simply the classical transmissibility in frequency without damping and the limitations to intrinsic curve in logarithmic scale. Thus, the conservative model is simply truncated to represent the actual damping with a convenient approximation based on the empirical intrinsic Q-curve typical of structural technology and assembly. As mentioned previously, it is obviously possible to define a shifted non-damped transfer curve with intrinsic Q values locally at each singularity - pole or zero - of the transfer function.

Conclusion

The proposed methodology is based here on a non-dissipative inertial-flexible approach, developed with real numbers. Even in usual and well conditioned thin aerospace structures, the intrinsic damping is basically non-linear, but remains low in an adimensional value, with a modal-type approach. Consequently, it is possible to truncate the resonance and anti-resonance singularities through a non-damped approach thanks to an empirical approach of the intrinsic modal damping defining the damping variations in relation with the frequency for one determined structural technology. The accuracy is then better than with a classical viscous approach, in terms of amplitude, and the frequency shift is compatible with this kind of precision. This approach allows a cost-efficient simplification of the dynamics applied to thin shells and beams with static linear structures.

Acknowledgements

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APPENDIX A : Representations of proposed approximations for damped elementary system

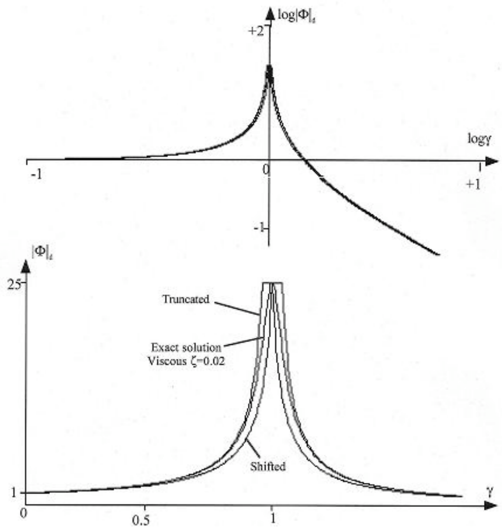


Fig. 1. Amplitude ratio algorithmic adimensional frequency graph and cartesian zooming

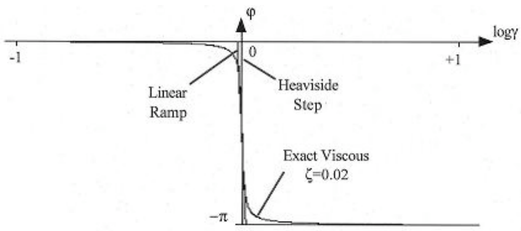
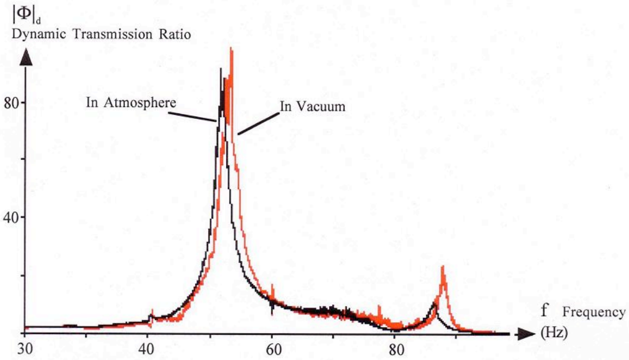


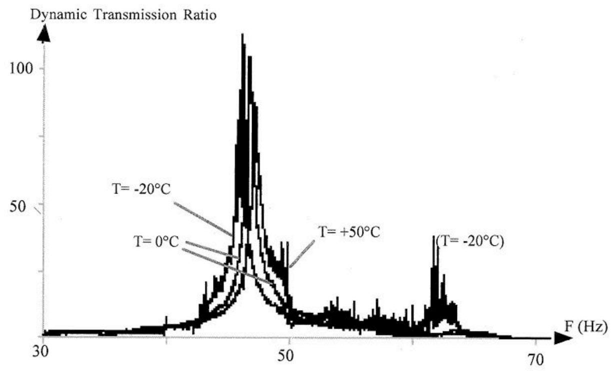
Fig. 2. Phase logarithmic adimensional frequency graph

APPENDIX B : Effect of Vacuum on Sine Transmissibility

The figure hereunder shows the influence of vacuum on fundamental modes in the bending and twisting of a 2024-T4 aluminum plate dynamically excited. The amplitude of acceleration measured at the free end of the plate clearly shows a diminution of dynamic ratio and frequency shift, due partially to inertial and viscous effect of the air :



The figure hereunder shows, again on an aluminum plate under vacuum, the effect of temperature, which has a certain influence mainly on the second mode (twisting), and which also adds noise on measurement :



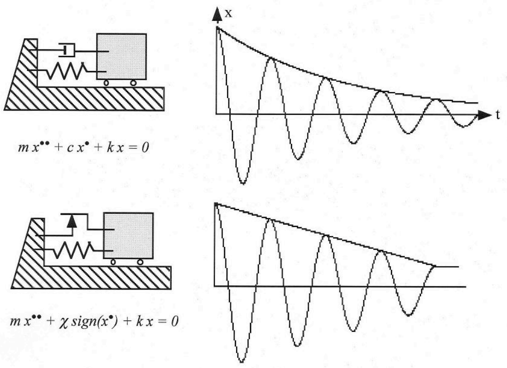


Fig. 3. Linear Viscous and Non-Linear Friction free motion elementary solutions

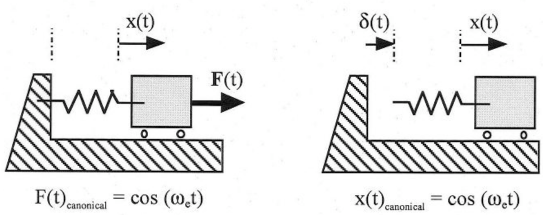


Fig. 4. Excitation in Force or in Deflection, presented on linear conservative system

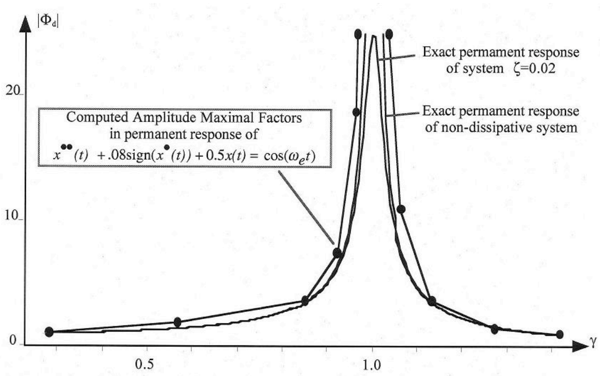


Fig. 5. Dynamic Amplification Ratios in Amplitude computed with pure friction

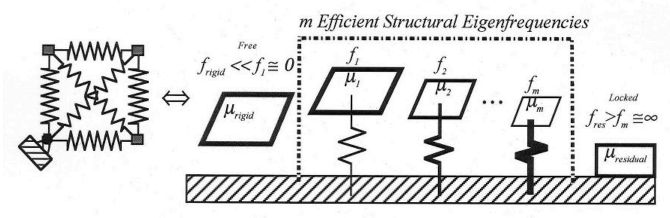


Fig. 6. Effective Modal Elementary Systems and Truncations

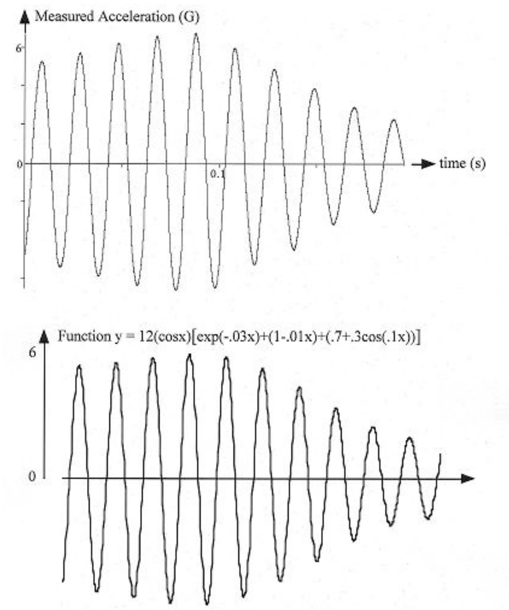


Fig. 7. Measured Temporal free response of Carbon Plate - Analogy with mixed system

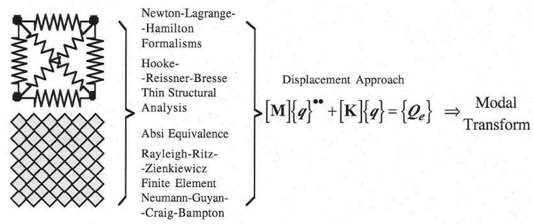


Fig. 8. Schematic process for structural discretization

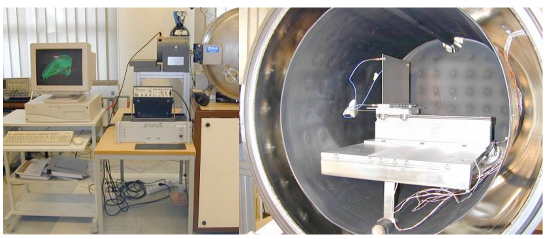


Fig. 9. Vacuum Facility and Mechanical Fixture in Chamber (photo : ENSICA)

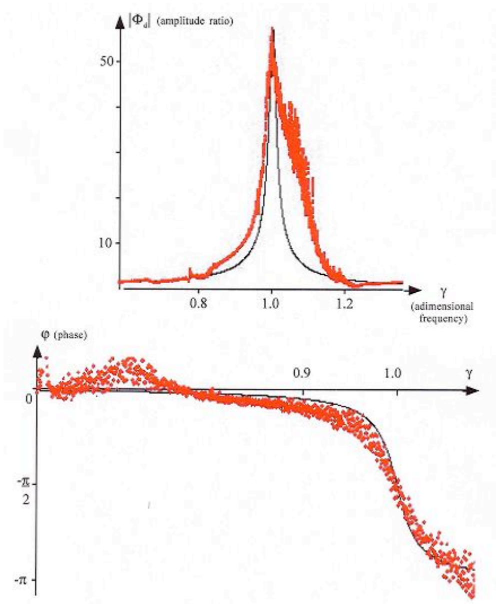


Fig. 10. Measured Frequential Excited response of Carbon Plate compared with viscosity

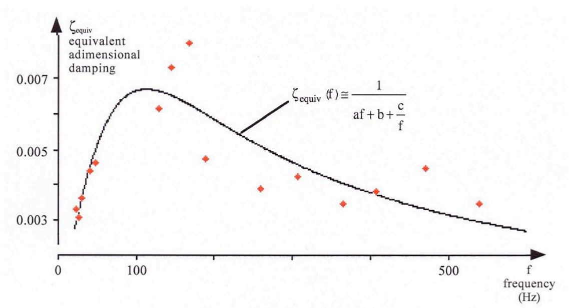


Fig. 11. Measured Damping in Composite Bolted Plate - Proposed analytic curve

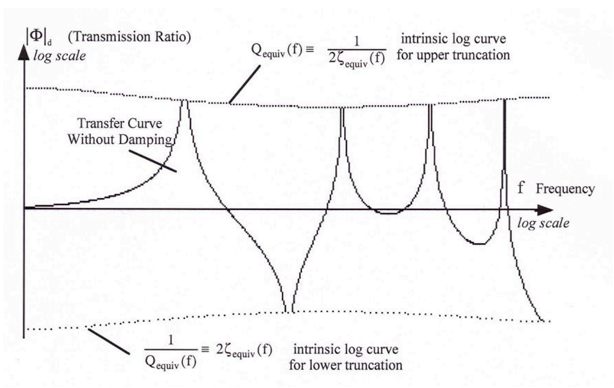


Fig. 12. Principle of truncations in Amplitude with Empirical Intrinsic Curve

APPENDIX C : Explicit integration of viscous and friction excited system