

Capital market of Bulgaria: testing different CAPM corrections

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The present study makes comparison in the usage and the level of accuracy of different methods for calculation of the expected return. The presented methods are based on CAPM, but with different corrections. We are going to test the traditional CAPM of Sharpe (1963) and Lintner (1964), the downside D-CAPM proposed by Estrada (2002), and three methods presented by the authors of this study. The first method uses combination of downside and upside beta to compute the risk in CAPM; the second uses the absolute deviation as a measure of risk; the third method integrates skewness in CAPM, but makes it by using different approach than familiar downside methods. The skewness is added as additional multiplier in the CAPM.

In traditional CAPM the risk of a single asset is measured by the standard deviation of its return.

$$(1) \sigma_i = \sqrt{\frac{1}{n} \sum_{t=1}^n (R_{it} - \mu_i)^2}$$

Where:

R_i - return of asset i;

μ_i - average return of asset i;

n - number of periods;

When the asset i is in a diversified portfolio the risk can be measured by its covariance with the market portfolio:

$$(2) \text{cov}_{iM} = \frac{1}{n} \sum_{it=1}^n (R_{it} - \mu_i)(R_{Mt} - \mu_M)$$

Where:

cov_{iM} is the covariance of asset i with the market portfolio M;

R_M is the return of the market portfolio M;

μ_M is the average return of market portfolio M.

If we divide the covariance with the variance of the market return we receive the traditional beta used in CAPM.

$$(3) \beta_i = \frac{\frac{1}{n} \sum_{t=1}^n (R_{it} - \mu_i)(R_{Mt} - \mu_M)}{\frac{1}{n} \sum_{t=1}^n (R_{Mt} - \mu_M)^2}$$

It is possible to calculate the risk of the asset using an alternative approach. This approach is originally proposed by Markowitz (1959). This is well known method that uses downside standard deviation and downside variance instead of traditionally used standard deviation and variance to calculate the risk. Downside standard deviation can be expressed by:

$$(4) \sigma_i^- = \frac{1}{n} \sqrt{\sum_{t=1}^n [(\min(R_{it} - \mu_i, 0))^2]},$$

Where:

σ_i^- is the downside standard deviation ,

$\min(R_i - \mu_i, 0)$ means that the expression $R_i - \mu_i$ takes only the value that are less than 0.

If we use downside deviation we can calculate downside covariance and downside beta. This approach is used by Estrada (2000, 2002) to calculate beta, that he call D-beta (downside beta). The downside covariance is presented in (5):

$$(5) \text{cov}_{iM}^- = \frac{1}{n} \sum_{t=1}^n [\min(R_{it} - \mu_i, 0) \cdot \min(R_{Mt} - \mu_M, 0)]$$

Respectively we can write the D-beta:

$$(6) \beta_i^D = \frac{\text{cov}_{iM}^-}{\sigma_M^{2-}} = \frac{\frac{1}{n} \sum_{t=1}^n [\min(R_{it} - \mu_i, 0) \cdot \min(R_{Mt} - \mu_M, 0)]}{\frac{1}{n} \sqrt{\sum_{t=1}^n [(\min(R_{Mt} - \mu_M, 0))^2]}}$$

The above methods compute CAPM by using well known models for risk calculations that are based on mean – variance and mean - semideviation. The economic logic for the use of downside standard deviation, downside covariance and D-beta we can search in two main directions: first it can calculate mainly negative values of the risk or the possibility of losses. This logic corresponds to the psychological understanding of the risk (the risk of losses - Markowitz (1959)). From the other hand this logic is confirmed from several researches made by interviews of managers by Mao (1970), March and Shapira (1992); second direction is that this approach can overcome the problem with the skewness of return of the shares. This problem is pronounced on the emerging markets and is shown empirically on the Russian capital market by Teplova, Selivanova (2007)

When we calculate risk using downside deviation, in fact we neglect all positive changes in the distribution of the return. These changes bear a risk and their complete neglecting is not justifiable. That's why we try to introduce a new formula that can use upside and downside changes in the return, but using a different, more flexible approach. In order to achieve our goal let's write the models for calculation of the upside standard deviation, upside covariance and upside beta. The upside standard deviation is:

$$(7) \sigma_i^- = \frac{1}{n} \sqrt{\sum_{t=1}^n [(\max(R_{it} - \mu_i, 0))^2]}$$

Respectively we can write upside covariance:

$$(8) \text{cov}_{iM}^- = \frac{1}{n} \sum_{t=1}^n [\max(R_{it} - \mu_i, 0) \cdot \max(R_{Mt} - \mu_M, 0)]$$

If we divide the upside covariance to the upside variance of the market we can receive the upside beta or U-beta:

$$(9) \beta_i^U = \frac{\text{COV}_{iM}^-}{\sigma_M^2} = \frac{\frac{1}{n} \sum_{t=1}^n [\max(R_{it} - \mu_i, 0) \cdot \max(R_{Mt} - \mu_M, 0)]}{\frac{1}{n} \sqrt{\sum_{t=1}^n [\max(R_{Mt} - \mu_M, 0)]^2}}$$

The main idea is to construct beta coefficient which can combine the downside and the upside beta. We can do it by (10):

$$(10) \beta_i^{DU} = w\beta_i^D + (1-w)\beta_i^U ;$$

Where:

w is the weight of downside beta and the weights of both upside and downside betas, should be equal to 1.

(11) and (12) shows the way CAPM looks if we use combined upside and downside beta:

$$(11) E(R_i) = R_f + MRP \cdot \beta_i^{DU} ,$$

or

$$(12) E(R_i) = R_f + MRP [w\beta_i^D + (1-w)\beta_i^U]$$

The advantages of this approach in the calculation of beta are that it can overcome the theoretical difficulties with the skewness of the return. On the other hand the method may reflect the managers' perception concerning the risk, as defined as danger of loses. This is achieved in a more moderate way. When downside deviation is used we assume that 100% of negative changes are regarded as risk and 0% of positive changes are included in risk calculation. By using combined method weights can be implied and the risk can be calculated considering both positive and negative changes. For example we can construct beta, which imply 70% negative (the downside) and 30% positive (the upside) beta.

A different method for risk calculation can be used instead, giving us some additional methods for beta calculation. Another method for risk calculation is the method of absolute deviation, which we mark as AD.).

$$(13) AD = \frac{\sum_{t=1}^n |R_{it} - \mu_i|}{n}$$

What characterizes this method is that it uses the modulation of difference of the expected and average values.

If we use the same logic that we have used to construct the downside and upside covariance we can write the covariance using absolute deviation as:

$$(14) Acov_{iM} = \frac{1}{n} \sum_{t=1}^n |R_{it} - \mu_i| |R_{Mt} - \mu_M|$$

Thus beta coefficient when absolute deviation and absolute covariance are used looks like:

$$(15) \beta^A = \frac{\frac{1}{n} \sum_{t=1}^n |R_{it} - \mu_i| |R_{Mt} - \mu_M|}{\frac{1}{n} \sum_{t=1}^n |R_{Mt} - \mu_M|^2}$$

So the CAPM we can write in (16):

$$(16) E(R_i) = R_f + MRP \cdot \beta_i^A,$$

The next model, we would like to present is CAPM model, where an additional premium is added to it, which on the other hand depends from skewness of the return. We can state that the return of shares are stochastic and in year t it can be described by:

$$(17) E(r_i) = R_f + \beta_i (R_m - R_f) + \varepsilon$$

Where:

$R_f + \beta_i (R_m - R_f)$ gives the description of the expected return of the shares that depends from risk free return, beta and the market risk premium;

ε is the error or stochastic element in the formula, which is not subject of prediction. For the model to be correct two conditions related to the ε should be met. First, the sum of ε for all shares should equals 0. When the real empirical data is used this

condition is almost never met. There is skewness of the distribution of the shares in almost every case. Assuming that the following two statements as being true:

- Presence of skewness of assets return and market portfolio return;
- ϵ is different than 0

we can make the conclusion that ϵ can be expressed by an equation. That's why we can write (17) as:

$$(18) E(r_i) = R_f + \beta_i(R_m - R_f) + AP$$

Where:

AP is the additional premium.

The questions that arise at this point are two. The first is how to find the equation of the additional premium. The second is which is the independent variable or variables in the model.

Our basic idea is to find relation that is simple and thus easy applicable in the practice of the companies. That's why the mathematical equation should not be complicated and the data it needs should be easy to find. As it was mentioned above the skewness of the return is one of the arguments for the existence of the additional premium (AP). That's why we are going to try to find the relation between additional premium as a function of the skewness of the return of the shares and the skewness of the market return. We can write it as:

$$AP = f(A_{R_i})$$

$$AP = f(A_{R_i}, A_{R_m})$$

Where A_{R_i} is the skewness of the return of the shares; A_{R_m} is the skewness of the market return.

That means that we are going to try to find the additional premium as a function of the skewness of the return of the shares and from other hand we are going to try to find the additional premium as a function of the skewness of the return of the shares and the skewness of the market return.

The next question that we have to solve is how to find the functional dependence. For that purpose we can create a database with historical returns of the

shares, and to calculate additional premium and skewness for every share. After we have AP and skewness it is easy to run a regression and to estimate the equation.

Theoretically we can use two types of regressions. The first is time series regression. But when we work with emerging markets (like Bulgarian) we can gather data for a short period. That means that we can calculate beta for up to 10 years. That's why if we use time series regression it will give us a large mistake and the results would be doubtful.

The second type of regression that we can use is cross-sectional regression. When we use it we do not need large time period in order to receive statistically significant results. This time the relation between AP and the skewness is searched by testing a large number of shares. The sequences of the actions that we have to do in order to run cross-sectional regression are:

1. Calculation of AP for every share as a difference between historical and expected return for period t:

$$AP_{i,t} = R_{i,t} - E(R_{i,t}),$$

Where:

$$i = 1 \dots n$$

n is the number of shares of the market..

2. Calculation of the skewness of the return of the shares and of the market return for the period t.

Thus we receive two rows of data that we have to regress. This time we can perform regression using the data for only one year. When we use cross-sectional regression it is important the number of the companies to be as high as possible.

At this point we have to determine the shape of the function. In order to estimate it we can draw a chart of the relation between AP and skewness. Thus we can make visual estimation of the shape of the relation (appendix 2).

Basically we can search the following relations between additional premium and the skewness:

$$(19) AP_{i,t} = y_0 + y_1 A_{Ri,t} + \delta$$

Equation (19) is the simplest form of the regression that we can run. We can also construct more complicated regressions:

$$(20) AP_{i,t} = y_0 + y_1 A_{Ri,t} + y_2 A_{Ri,t}^2 + \delta$$

Where:

y_0 is the intercept;

y_1 and y_2 are the coefficients of the regression;

δ is the error that should meet the condition:

$$E(\delta) = 0$$

The regression should estimate y_0 , y_1 and y_2 . If some of the coefficients (y_1 and y_2) equals or is very close to 0, than the corresponding multiple (for example $A_{Ri,t}^2$) has no influence over AP.

The cross-section regression does not give us the opportunity to test directly the relation between skewness of the market return and additional premium. In order to include skewness of the market in research we can try to include it into the model (20).

We can include it using two models. The first looks exactly as (20) but the definition of $A_{Ri,t}^2$ is changed:

$$(21) AP_{i,t} = y_0 + y_1 A_{Ri,t} + y_2 A_{Ri,t}^2 + \delta$$

where:

$A_{Ri,t}^2$ is the skewness of the return of all shares in the research and the skewness of the market return.

The second approach that we can use is to include the skewness of the market return as additional multiple:

$$(22) AP_{i,t} = y_0 + y_1 A_{Ri,t} + y_2 A_{Ri,t}^2 + y_3 A_{m,t} + \delta$$

In fact we can try to search for much more complicated model where AP is expressed as a functional dependence from $A_{Ri,t}^2$ and $A_{m,t}$. The regression that we can run is:

$$(23) \quad AP_{i,t} = y_0 + y_1 A_{Ri,t} + y_2 A_{Ri,t}^2 + y_3 A_{m,t} + y_4 A_{Rm,t}^2 + y_5 A_{Ri,t} A_{m,t} + y_6 A_{Ri,t}^2 A_{m,t}^2 + y_7 A_{Ri,t}^2 A_{m,t} + y_8 A_{Ri,t} A_{m,t}^2 + \delta$$

The basic idea beside the complicated model is that it can test all combination between $A_{Ri,t}^2$ and $A_{m,t}$. The regression will give us the values of the coefficient from y_1 to y_8 . If they are equal or close to 0 they are insignificant.

Methods for assessing accuracy of the models.

The next issue to discuss is about how to evaluate the accuracy of models. For the purposes of the study we will apply two methods to evaluate the results. The first method is well-known determination coefficient which assesses the degree of accuracy in the regression model. We cannot apply the method of coefficient of determination to assess all corrections and we propose another method, by which we can compare all CAPM corrections. We are going to use the coefficient of determination to assess the relation between the additional premium and skewness.

We can apply the backtesting method in order to assess all CAPM corrections. This method will give an assessment of the average mistake that makes every CAPM correction for all years of the study. The method which gives least mistake can be accepted as the most accurate method for expected return calculation. The methodology of the backtesting is:

First we will find the difference between expected return and historical return for each share for all periods by the model:

$$B_{i,t} = E(R_{i,t}) - R_{i,t}$$

where:

$B_{i,t}$ is the difference between expected return and historical return of the share i in moment t . It is the error in prediction that share i makes in year t .

$E(R_{i,t})$ is the expected return of the share i in moment t ;

$R_{i,t}$ is the historical return of the share i in moment t ;

We can find the average error for all shares that take part in the study and for all periods. This gives us an idea for the overall performance of the CAPM corrections. The

average error for the entire period and for all shares that take part in it can be computed by the model:

$$(24) B = \frac{\sum_{t=1}^m \sum_{i=1}^n \sqrt{(E(R_{i,t}) - R_{i,t})^2}}{\sum_{t=1}^m n_t}$$

where:

m is the number of periods in the research;

n is the number of shares included in the study in year t;

Empirical results

For the empirical analysis we are using data about prices of the shares listed on Bulgarian stock exchange (BSE) during the period from 1998 to 2009. Data are downloaded from the graphic operator at the BSE website. The main idea is to calculate and test beta coefficients and CAPM corrections based on monthly data about share prices and monthly return of the securities. Due to lack of liquidity on the BSE, some of the shares are not traded regularly and we exclude them from the research. Each year the number of companies, fulfilling the requirements and included in the research is different. The number of included companies by years are shown in table 1.

Table 1 : number of companies which are included in the research by years.

2004	2005	2006	2007	2008	2009
29	42	71	90	126	141

In table 2 it is shown the calculations for the average expected return for all companies for different CAPM corrections. Column CAPM shows us the average value of the expected return calculated through traditional CAPM. Columns D-CAPM and A-CAPM represents the expected return calculated by the downside model in (6) and the model that uses the absolute deviation from (16). The models marked with CAPM (AP1), CAPM (AP2) and CAPM (AP3) calculate the expected return by respectively (20), (22) and (23). The columns DU-CAPM use the model shown in (11) for the combined downside and upside beta. The figures under the head row show the weights of the

downside and upside beta. For example 20/80 shows that it is used 20 % downside beta and 80% upside beta.

Table 2: Average expected return of all CAPM corrections by years.

	CAPM	D-CAPM	A-CAPM	CAPM(API)	CAPM(AP2)	CAPM(AP3)
2004	33.5%	40.8%	62.2%	3.8%	12.4%	-8.6%
2005	42.3%	100.9%	125.8%	-4.5%	1.3%	-2.7%
2006	52.8%	82.7%	101.2%	17.4%	18.3%	10.7%
2007	47.7%	63.1%	72.9%	11.2%	5.0%	26.8%
2008	49.3%	59.8%	74.8%	12.4%	13.0%	8.8%
2009	-16.4%	-15.7%	-23.7%	-35.3%	-9.7%	-5.0%
	DU-CAPM	DU-CAPM	DU-CAPM	DU-CAPM	DU-CAPM	DU-CAPM
	0/100	20/80	40/60	60/40	80/20	100/0
2004	54.4%	51.7%	48.9%	46.2%	43.5%	40.8%
2005	75.2%	80.4%	85.5%	90.6%	95.8%	100.9%
2006	73.3%	75.2%	77.1%	79.0%	80.8%	82.7%
2007	58.6%	59.5%	60.4%	61.3%	62.2%	63.1%
2008	63.6%	62.9%	62.1%	61.3%	60.5%	59.8%
2009	-30.2%	-27.3%	-24.4%	-21.5%	-18.6%	-15.7%

In table 3 we can follow the size of the additional premium by years from (18). As we can see the size of additional premium is considerable (theoretically it should be equal to 0). For 2004 the additional premium is 127.6%, which is the highest value. The lowest is 35.4% during the last, 2009 year from the research. The high average size of the additional premium is the main reason for us to try to express it as dependable variable.

Table 3: Average size of the additional premium.

	2004	2005	2006	2007	2008	2009
Additional premium	127.6%	71.9%	83.2%	126.9%	107.8%	35.4%

The regression is performed by the software for statistical analysis SPSS 16.0. In tables 4a, 4b, 4c, 4d are introduced the results for the coefficient of determination for all regressions. In table 4a we are looking for linear relation shown in (19). The coefficient of determination is 0.05, which means that the skewness of the return of the shares is capable to explain 5% of the changes in additional premium. In table 4b we are looking the relation shown in model (20). Although the more complicated model the relation remains very low – 5.2%. Table 4c represents the model that integrates the skewness of the market return from equation (22). The coefficient of determination is 8.5 %. In the last and most complicated model in (23) the coefficient of determination has

value of 11.4%. Although the coefficient of determination rises with the more complicated models its value remains too low. So the main conclusion is that the skewness of return of the shares and the skewness of market return cannot explain the main changes in the additional premium.

Table 4a: Coefficient of determination from the regression model:

$$AP_{i,t} = y_0 + y_1 A_{Ri,t} + \delta$$

Model Summary				
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
19	0,223	0,050	0,048	0,78993

Table 4b Coefficient of determination from the regression model:

$$AP_{i,t} = y_0 + y_1 A_{Ri,t} + y_2 A_{Ri,t}^2 + \delta$$

Model Summary				
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
20	0,229	0,052	0,048	0,78969

Table 4c: Coefficient of determination from the regression model:

$$AP_{i,t} = y_0 + y_1 A_{Ri,t} + y_2 A_{Ri,t}^2 + y_3 A_{m,t} + \delta$$

Model Summary				
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
22	0,291	0,085	0,079	0,77689

Table 4d: Coefficient of determination from the regression model:

$$AP_{i,t} = y_0 + y_1 A_{Ri,t} + y_2 A_{Ri,t}^2 + y_3 A_{m,t} + y_4 A_{Rm,t}^2 + y_5 A_{Ri,t} A_{m,t} + y_6 A_{Ri,t}^2 A_{m,t}^2 + y_7 A_{Ri,t}^2 A_{m,t} + y_8 A_{Ri,t} A_{m,t}^2 + \delta$$

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
23	0,338	0,114	0,099	0,76856

The unsatisfactory results from regression can be confirmed by the results from the backtesting. Table 5 gives information about the average mistake that makes every method for CAPM correction for the entire researched period. According to the backtesting method the average mistake of the traditional CAPM is 92.1%. All CAPM corrections give worse results. For example the Estrada's D-CAPM makes 94% average mistake, which is very close to the result of CAPM but worse. Using combined downside and upside beta (DU-CAPM) makes mistake from 92.8% to 94.1%. A-CAPM makes the highest mistake – 99.1%. When we try to correct CAPM with additional premium the average error is between 94.8 to 97.9%.

Table 5: The average error for the entire period for all methods based on backtesting.

Backtesting					
CAPM	D-CAPM	DU-CAPM	DU-CAPM	DU-CAPM	DU-CAPM
		0/100	20/80	40/60	60/40
92.1%	94.0%	94.1%	93.2%	92.8%	92.8%
DU-CAPM	DU-CAPM	A-CAPM	CAPM(DP1)	CAPM(DP2)	CAPM(DP3)
80/20	100/0				
93.2%	94.0%	99.1%	97.9%	94.8%	95.3%

We can summarize the following conclusions about the results of the research:

- All CAPM based models performs badly on Bulgarian market.
- It seems that skewness is not major factor for determining the expected return.
- When overall return of the shares switches over from positive to negative return and from negative to positive return the difference between expected and historical return is highest.
- In a market with high volatility the error in estimation is very high (average about 90% points)

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