Discounting, Distribution and Disaggregation

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Abstract

To consider the implication of disaggregated consumption and discounting, we study discounting in a world composed of the rich and the poor, a standard setting in the literature of cost-benefit analysis with distributional considerations. We derive several discount rates for different numéraires, which would enable us to discuss intergenerational and intragenerational equity in common terms. In the example of CES-CRRA utility, we also show that disaggregated discount rates may vary owing to several factors. One important parameter of such –inequality aversion– can be determined in unknown weighting of intergenerational and intragenerational concerns.

Key Words: Discounting; income distribution; climate change; environmental stock

1 Introduction

In the series of reports published by IPCC over the past decade, one of the most controversial socioeconomic factor has been the choice of discount factor for measuring costs and benefits of future climate change, which was especially evident in Second Assessment Report of 1995. Similarly, right after the launch of the *Stern Review*, most criticisms (and plaudits) referred to its choice of low utility discount rate that could have led to the opposite conclusion were it higher.

Standard analysis suggests that the consumption discount rate is made up of pure rate of time preference and the product of consumption growth rate and the elasticity of marginal utility of consumption. But the rate of "consumption growth" seems too aggregate to discuss such a global and durable issue, so some authors scrutinize the prospect of consumption, along the line of the limited substitutability argument of Gerlagh and van der Zwaan (2002). In such a move, Hoel and Sterner (2007) and Traeger (2011) assume consumption of usual commodities and environmental quality as arguments of utility. Assigning relative prices to both consumption and the environment, they show that when discounting the environment the possible increase of the relative price of the environment should be deducted from the usual consumption discount rate. Applying this formulation to the DICE model of Nordhaus (2008) would propose the optimal emission of carbon dioxide become zero within a century (Sterner and Persson 2008).

Aside from the relatively oft-debated issue of uncertain consumption, another route that can be taken to elaborate the consumption prospect is to disaggregate consumption into the rich and the poor. Equity consideration in cost-benefit analysis dates back at least to Dasgupta et al.'s (1972) exposition which proposed some unknown redistributional weight be placed on the poor's consumption. Since then, however, equity weighting has been almost evicted from standard analysis under the banner of Kaldor-Hicks compensation criterion, but it has recently been making a comeback in the context of climate change. An obvious reason is that it is very costly or unrealistic to compensate those who suffer positive net damage from climate change in the present and in the future. A pioneering study in this regard is Azar and Sterner (1996) again and Fankhauser et al. (1997), who have considered weighting costs and benefits by the elasticity of marginal utility of income[1]. Kaplow and Weisbach (2011) clarify that the elasticity of marginal utility is essentially a reduced form derived from the composite of the utility function

[1]Their numéraires were marginal utility of the rich and the average, respectively, but they were shown to be equivalence process by Azar (1999).

and the social welfare function.

In debates about climate change, unarguably one of the most contentious practical problems of equity consideration, it is sometimes suggested that enhancing adaptive behavior of the vulnerable should be prioritized over costly, uncertain mitigative policies by the rich countries. For example, referring to the relation between the two policies as "trade-offs," Tol (2005) suggests that reducing impacts might be more efficient than reducing emissions in Africa and Latin America[2]. In this context also, discounting based on disaggregated consumption could be of more significance if we pose the problem as mitigation versus adaptation, each affecting the rich and the poor in a different manner.

All these explain the motivation of the current paper. Quite a few papers have accumulated regarding the substitutability and the equity consideration, but few studies focus on their combination, albeit its apparent relevance in the policy debate. If we disaggregate consumption into the rich and the poor, while keeping the dual consumption of the composite good and the environment, we get a more precise idea of what is going on in terms of the discount rate. In the course of doing so, we start with a general social welfare function (SWF), and gradually proceed while clarifying several formulations of social discount rates. By introducing income groups and the environment in a social welfare function, we will find how population change, time preference rates in each group, utility growth, the self-elasticity of consumption and cross elasticity of consumption and the environment all help to determine the discount rates. Specifically, the combination of inequality aversion, relative risk aversion, and the cross elasticity of marginal utility is shown to yield interesting diversion between the income groups in the simulation results. This two-good, two-country model highlights the problem of applying "one-size-fits-all" discount rate in the global warming discussion in the current world[3]. Also, the discount rate for the environment numéraire now includes the "welfare value share" of each income group. Furthermore, it would clarify the oft-debated balancing act of intergenerational equity and intragenerational equity (Schelling 1995; Portney and Weyant 1999; López 2005), by directly comparing the welfare of people in a different land, living in different times.

In the next section, we study a general SWF and state some remarks on several discount rates. Section 3 introduces a CES-CRRA utility function which is dependent on both consumption and the environmental stock. In Section 4, we

[2]See also Lomborg (2008) for a similar non-technical argument.

[3]In a related vein, a recent paper by Anthoff and Tol (2010) calculates different social costs of carbon according to different social welfare functions.

explore an intertemporal version of unknown equity weighting, which can be applied practically to study the significance of inequality aversion discussed in the preceding sections. Section 5 concludes.

2 General Formulation

We would like to consider a general SWF, which go beyond the dichotomy of either a utilitarian or Rawlsian. Indeed, as Pearce (2003) put it, distributional weights raise the problem that "any number of social welfare functions (SWFs) can be postulated, each producing different weightings", so we start with a formulation as general as possible, following Azar (1999) and Johansson-Stenman (2000, 2005). We assume that the world is divided between the rich and the poor. Our "welfaristic" social welfare function is inherently an intertemporal one, and differs from a representative agent utility:

$$W = \int_0^\infty w(u_r(t),\ u_p(t), t)dt, \tag{1}$$

where w is an instantaneous Bergson-Samuelson welfare valued at the present term, $u_r(t)$ and $u_p(t)$ represent utility for an individual in the rich and the poor world, respectively. We extend this expression by defining the instantaneous welfare (Fankhauser et al. 1997):

$$W = \int_0^\infty \frac{1}{1-\alpha}[L_r u_r^{(1-\alpha)} e^{-\delta_r t} + L_p u_p^{(1-\alpha)} e^{-\delta_p t}]\, dt, \quad \alpha \neq 1, \tag{2}$$

where L_r and L_p stand for the populations in the rich and the poor, assumed to be exogenously determined. This type is sometimes called constant inequality aversion, or constant equality aversion, depending on the value of α (i.e. $-w_{uu}u/w_u = \alpha$, where the subscript i is omitted).

Note also that the time preference rates, δ_r of the rich and δ_p of the poor, can differ from each other in the above setting. Utility discount rate, or social rate of time preference, should be firmly founded from an ethical point of view. Then discounting utility at a different rate might appear an erroneous move. That said, different rates of utility discount can not be so much of a cardinal sin for some reasons. The rate, also reflecting people's rate of impatience, can differ in each income group. If those in each group reach a consensus on the group's discount rate, which then turn out to be different from the other group, different rates of utility discount should be accepted. Indeed, this is loosely similar to the formulation of

Li and Löfgren (2000), who considered a world of two persons, one conservationist and the other utilitarian. In such a setting, the conservationist's view is more and more weighted as time passes. Also, while human being as a whole is on the same boat called the Earth, one may consider one group's possibility of extinction is higher than the other. After all, if we heuristically find that this cannot be permissible, then we can simply give it up and set $\delta_r = \delta_p$.

As for the utility, we briefly note here that it is dependent on per capita consumption, C_i/L_i, and the environment, namely,

$$u_i = u(C_i/L_i,\ E),\quad for\ \ i = p, r, \tag{3}$$

where $\partial u_i/\partial c_i > 0$, $\partial u_i/\partial E > 0$, $\partial^2 u_i/\partial c_i^2 < 0$, and $\partial^2 u_i/\partial E^2 < 0$, but the sign of $\partial^2 u_i/\partial c_i \partial E$ is ambiguous. Here E stands for an aggregate proxy for the environment, typically a global ecosystem or a global sink for atmospheric concentration of carbon dioxide. E does not come with a suffix for the income category as it stands for a global public good, but the consequences of a perturbation in it can be widely different, so that it may be that those in each group have different preferences toward E. It is also important to note that we abstract away from inequality *within* each group.

Now let us define a discount rate as the rate of change of the shadow price of the object in question. For example, the shadow price of per capita utility at t is $\partial w/\partial u_i = L_i u_i^{-\alpha} e^{-\delta_i t}$ for $i = p, r$, so that i's per capita utility discount rate ρ_i^{pcu} becomes

$$\rho_i^{pcu} = -\frac{d}{dt}\frac{\partial w}{\partial u_i(t)} = \frac{-\partial^2 w/\partial u_i \partial t}{\partial w/\partial u_i} + \frac{-u_i \partial^2 w/\partial u_i^2}{\partial w/\partial u_i}\frac{\dot{u}_i}{u_i} = \delta_i - \frac{\dot{L}_i}{L_i} + \alpha\frac{\dot{u}_i}{u_i},\quad for\ \ i = p, r. \tag{4}$$

In the literature it is common to refer to the pure rate of time preference δ_i as the utility discount rate, but it is clear from the above expression that it holds only under utilitarianism. When the numéraire is per capita utility, the latter is equal to the former, net of the rate of population growth, plus the rate of utility growth multiplied by the inequality aversion parameter, α. The message is that, for a developing world where population is increasing at a rate higher than the developed world, per capita utility discount rate should be smaller. The second term expresses decreasing marginal welfare of utility, also appearing in the discount rate in the utility term. For the case of classical utilitarianism, however, one unit of increase in utility in any region would amount to one unit of social welfare at that moment, i.e. $\alpha = 0$, so that the third term disappears. Also, a representative agent model assumes constant population by definition, which would nullify the second term as well.

Likewise, the shadow price of per capita consumption at t is $\partial w/\partial c_i = L_i u_i^{-\alpha} e^{-\delta_i t} \partial u_i/\partial c_i$ for $i = p, r$, so we have

$$\rho_i^{pcc} = \delta_i - \frac{\dot{L}_i}{L_i} + \alpha\frac{\dot{u}_i}{u_i} - \frac{\frac{d}{dt}\frac{\partial u_i}{\partial c_i}}{\partial u_i/\partial c_i}, \quad for \;\; i = p, r. \tag{5}$$

That is, in addition to the per capita utility discount rate ρ^{pcu}, we should discount further for the growth of the derivative of utility with regard to its arguments. If we consider *aggregate* consumption, rather than *per capita* consumption, in a given group, the second term of the rate of the population change is canceled out.

Now, as we assume that people derive utility from the environment as well as from consumption, care is taken in developing the last term of the RHS in (5), as

$$-\frac{\frac{d}{dt}\frac{\partial u_i}{\partial c_i}}{\partial u_i/\partial c_i} = -\frac{\frac{\partial^2 u_i}{\partial c_i^2}\dot{c}_i + \frac{\partial^2 u_i}{\partial c_i \partial E}\dot{E}}{\partial u_i/\partial c_i} = \eta_{cc}^i \frac{\dot{c}_i}{c_i} + \eta_{cE}^i \frac{\dot{E}}{E}, \quad for \;\; i = p, r, \tag{6}$$

where $\eta_{cc}^i \equiv -u_{icc}c_i/u_{ic}$ and $\eta_{cE}^i \equiv -u_{icE}E/u_{ic}$ In a standard Ramsey model, where consumption is the only argument of utility, (6) corresponds to the elasticity of marginal utility of consumption multiplied by the consumption growth rate. When the environment also affects utility, what should be added to this is the cross elasticity of marginal utility of consumption and the environment, multiplied by the growth of the environmental stock (e.g. Krautkraemer 1985).

Plugging (6) into the equation (5) for $i = p, r$, we obtain the difference between the per capita consumption rate of the rich and that of the poor:

$$\rho_r^{pcc} - \rho_p^{pcc} = (\delta_r - \delta_p) - \Big[\frac{\dot{L}_r}{L_r} - \frac{\dot{L}_p}{L_p}\Big] + \alpha\Big[\frac{\dot{u}_r}{u_r} - \frac{\dot{u}_p}{u_p}\Big] + \Big[\eta_{cc}^r \frac{\dot{c}_r}{c_r} - \eta_{cc}^p \frac{\dot{c}_p}{c_p}\Big] + (\eta_{cE}^r - \eta_{cE}^p)\frac{\dot{E}}{E}. \tag{7}$$

In other words, per capita consumption discount rates would be the same value only when the RHS of (7) becomes zero.

For the cross elasticity of marginal utility of consumption, little is known yet. But when consumption and the environment are substitutes, $\eta_{cE} > 0$, when complements, $\eta_{cE} < 0$. Heal (2009) argues that it is likely to be negative, and so assume Weikard and Zhu (2005), but things may vary according to income groups. When people enjoy a high level of average consumption, the environment can be perceived as a substitute, so the growth in the environmental good can actually decrease the marginal utility of consumption. On the contrary, when the economy is based on subsistence, or physically and economically vulnerable to environmental damage, it seems safe to assume consumption and the environment are complements. The feasibility of formulating such functions is quite another matter, which is to be explored in the next section.

3 An Example of CES-CRRA Utility

In this section, we further specify the utility function and elaborate on the characteristics of the formulation shown in the previous section. We follow Hoel and Sterner (2007) in assuming a constant elasticity of substitution between consumption c and the environment E. Also, relative risk aversion is set constant at $\beta(>0, \neq 1)$. Formally, we have

$$u_i = u(c_i, E) = \frac{1}{1-\beta}[(1-\gamma)c_i^{1-1/\sigma} + \gamma E^{1-1/\sigma}]^{(1-\beta)\frac{\sigma}{\sigma-1}} \quad for \;\; i = p, r, \tag{8}$$

where $\sigma > 0$ is the elasticity of substitution between consumption and the environment, and γ is a weight-like constant which is in the range of (0,1). Now we will have a look at each term in the RHS of (7) by turn.

3.1 Utility discount rate

The utility discount rate is sometimes called the rate of pure time preference, or the rate of impatience in others. It is considered to be a parameter to be ethically determined, which is why so controversial. Some authors including Ramsey and Harrod famously state that it should be set at nil since future generations cannot be discriminated just because they live in the future, resorting to analogy with the legitimacy of modern spatial discrimination. Others point out, however, that with our possibility of our extinction being positive, it should be above zero by nature, however abysmally low it may be. Also, zero impatience would imply an absurdly high saving rate in a productive economy (Dasgupta 2008).

The question of relevance to the current debate is: do we have a different utility discount rate depending on the income level[4]? Three rationales for differentiation come up immediately. First, when the utility discount rate is seen as

[4]A survey by Frederick et al. (2002) shows that utility discounting varies among individuals even within one empirical research.

the survival probability of individuals, the shorter average lifetime they have, the higher discount rate they are taken to have. Second, similarly, if the discount rate is interpreted as the survival probability of rich and poor dynasties, the poorer group might face larger extinction risk because of their harsh environment. Third, and most importantly, there is ethical intuition that rationale for utility discounting is stronger in the case of the rich than for the poor.

For all this, just because different income groups may have different patience level doesn't mean different utility discount rates *should* be applied when we assume a social (world) planner maximizing (2)[5]. Although we cannot discuss this issue furthermore in this paper, it seems safer to keep assuming that $\delta_r - \delta_p$ is at most negligible until we come up with rationale for discrimination.

3.2 Population growth rate

As described above, this term appears only when we are talking about per capita consumption as numéraire, while it disappears for the case of the conventional aggregate consumption discount rate. We simply take it that population growth rates are exogenously given[6]. So the message here is simply that there might arise a wide diversion of per capita consumption discount rates due to the different prospects of population growth.

3.3 Elasticity of marginal utility of consumption and consumption growth

The product of elasticity of marginal utility of consumption and consumption growth rate is also revealed in the conventional Ramsey discounting. There is already a huge discussion on this, although apparently there is no consensus up to now. The ethical part is the elasticity η^i_{cc}, which is usually assumed to fall somewhere in the range of 1 and 4. In the current formulation of utility, it becomes

$$\eta^i_{cc} = -u_{icc}c_i/u_{ic} = (\beta - \frac{1}{\sigma})(1-\phi^i) + \frac{1}{\sigma} = \beta(1-\phi^i) + \frac{1}{\sigma}\phi^i > 0, \quad (9)$$

$$where \quad \phi^i \equiv \frac{\gamma E^{1-1/\sigma}}{(1-\gamma)c_i^{1-1/\sigma} + \gamma E^{1-1/\sigma}}.$$

[5]Furthermore, as time goes to infinity, different pure rates of time difference exhibit problematic consumption paths.

[6]Endogenous population would raise another deep policy implication, as is noted in Dasgupta (2008).

The newly denoted ϕ^i can be rewritten as $\frac{u_E E}{u_c c + u_E E}$, implying this variable is the value share of the environment (Hoel and Sterner 2007). Again, our interest lies in the difference of the elasticity between the rich and the poor. The partial derivative of the value share of the environment w.r.t. consumption reveals itself as

$$\frac{\partial \phi^i}{\partial c_i} = -\frac{\gamma(1-\gamma)(1-1/\sigma)(c_i/E)^{-1/\sigma}/E}{[(1-\gamma)(c_i/E)^{1-1/\sigma}+\gamma]^2} = \begin{cases} < 0 & \text{if } \sigma > 1 \\ > 0 & \text{if } 0 < \sigma < 1, \end{cases}$$

which is to say, when the consumer cuts back on "consumption" of the environment by $\sigma(> 1)$% in responding to a relative price hike of the environment by 1%, the value share of the environment decreases as consumption rises. On the contrary, if the elasticity of substitution is less than 1%, it decreases. So we have

$$\eta^r_{cc} - \eta^p_{cc} = -(\beta - \frac{1}{\sigma})(\phi^r - \phi^p),$$

whose sign is dependent upon the sign of $\phi^r - \phi^p$, as well as upon whether $\beta\sigma > 1$. When $0 < \sigma < 1$, which is an interesting case for climate change debate, it is likely that $\phi^r > \phi^p$, in which case the elasticity of the rich, η^r_{cc}, is smaller (larger) than that of the poor, η^p_{cc}, if and only if $\beta\sigma > (<)1$.

The portion of consumption growth includes nothing ethical, and it is governed by consumption prospects written on the wall of scenarios. It is usually considered to be positive at least for the short- and mid-term, except for some anomalies. Azar and Sterner (1996) and Dasgupta (2008), among others, consider the possibility of declining consumption when deriving consumption discount rate, particularly accounting for the externality of climate change.

3.4 Inequality aversion and utility growth

Our assumption of global social welfare function generates the difference of utility growth rates weighted by inequality aversion parameter. The case of $\alpha = 0$ collapses to the usual representative agent utility; in contrast, a Rawlsian would set the parameter at $\alpha = -\infty$. α is an ethical parameter to address intragenerational equity between the rich and the poor[7]. It is hard to allocate a priori a specific number to this, so it would be safest to conduct sensitivity analysis, or even resort to unknown weighting (see Section 4).

[7] As mentioned above, we skip the problem of inequality *within* each group.

The utility growth rate for i can be obtained by taking the time derivative of (8):

$$\begin{aligned}\dot{u}/u &= (1-\beta)\frac{\sigma}{\sigma-1}\frac{d}{dt}\log[(1-\gamma)c_i^{1-1/\sigma}+\gamma E^{1-1/\sigma}] \\ &= (1-\beta)[(1-\phi^i)\dot{c}_i/c_i+\phi^i\dot{E}/E].\end{aligned} \tag{10}$$

3.5 Environment growth and cross elasticity of marginal utility of consumption

Whether the environment is complement or substitute of consumption is both theoretically and empirically a challenge. In our formulation, however, a straightforward calculation shown in Hoel and Sterner (2007) leads to the cross elasticity of marginal utility of consumption

$$\eta_{cE}^i \equiv -u_{icE}E/u_{ic} = (\beta - \frac{1}{\sigma})\phi^i, \tag{11}$$

which is positive (negative) if and only if $\beta\sigma > (<)1$. The last term of the RHS in (7) is then

$$(\beta-\frac{1}{\sigma})(\phi^r-\phi^p)\dot{E}/E.$$

3.6 Summary

A numerical example will help us understand the current exercise. Before that, to recap, the per capita consumption discount rates for the current specific utility function is, by substituting (10), (9) and (11),

$$\rho_i^{pcc} = \delta_i - \frac{\dot{L}_i}{L_i} + (\alpha(1-\beta)+\beta-\frac{1}{\sigma})\Big[(1-\phi^i)\frac{\dot{c}_i}{c_i}+\phi^i\frac{\dot{E}}{E}\Big] + \frac{1}{\sigma}\frac{\dot{c}_i}{c_i} \tag{12}$$

in contrast to the benchmark conventional discount rate, $\rho = \delta + \beta\dot{c}/c$, or another benchmark for aggregate consumption discount rate (Hoel and Sterner 2007),

$$\rho = \delta + \Big[(\beta-\frac{1}{\sigma})(1-\phi)+\frac{1}{\sigma}\Big]\frac{\dot{c}}{c} + (\beta-\frac{1}{\sigma})\phi\frac{\dot{E}}{E}.$$

If the numéraire is the environment, its shadow price $\sum_{i=r,p}[L_iu_i^{-\alpha}e^{-\delta_i t}\partial u_i/\partial E]$ can be shown to change according to

$$\rho^E = \sum_{i=p,r}\psi_i\Bigg[\delta_i - \frac{\dot{L}_i}{L_i} + (\alpha(1-\beta)+\beta-\frac{1}{\sigma})\Big[(1-\phi^i)\frac{\dot{c}_i}{c_i}+\phi^i\frac{\dot{E}}{E}\Big]+\frac{1}{\sigma}\frac{\dot{E}}{E}\Bigg],$$

where $\psi_i = \frac{L_i u_i^{-\alpha} e^{-\delta_i t} \partial u_i/\partial E}{\sum[L_i u_i^{-\alpha} e^{-\delta_i t} \partial u_i/\partial E]}$ is, by analogy with the value share of the environment ϕ, the "marginal welfare share" of the income group $i = p, r$. Using the per capita consumption rate above, the environment discount rate is equivalent to

$$\rho^E = \sum_{i=p,r} \psi_i \Big[\rho_i^{pcc} - \frac{1}{\sigma}(\frac{\dot{c}_i}{c_i} - \frac{\dot{E}}{E})\Big].$$

This shows that the scarcity of the environment relative to consumption should be deducted from the usual consumption discount rate, and that the weighted sum of the residual rates is the correct environment discount rate.

To formalize the impacts of parameters on the disaggregated discount rates, below are the derivatives of the discount rates with regard to selected parameters.

$$\partial\rho_i^{pcc}/\partial\alpha = (1-\beta)[(1-\phi^i)\frac{\dot{c}_i}{c_i} + \phi^i\frac{\dot{E}}{E}], \quad (13)$$

$$\partial\rho_i^{pcc}/\partial\beta = (1-\alpha)[(1-\phi^i)\frac{\dot{c}_i}{c_i} + \phi^i\frac{\dot{E}}{E}], \quad (14)$$

$$\partial\rho_i^{pcc}/\partial\sigma = \frac{\phi^i}{\sigma^2}(\frac{\dot{E}}{E} - \frac{\dot{c}_i}{c_i}). \quad (15)$$

The partial derivative with regard to α is just $\dot{u}/u$, since the parameter does not appear in the other parts than the utility growth rate in (5). Note also that the effect of α and β on the discount rates of i is symmetric if $0 < \alpha, \beta < 1$.

Now, let us rewrite the difference in ρ_i^{pcc}, (7):

$$\begin{aligned}\rho_r^{pcc} - \rho_p^{pcc} =& (\delta_r - \delta_p) - \Big[\frac{\dot{L}_r}{L_r} - \frac{\dot{L}_p}{L_p}\Big] \\ &+ \Big[\alpha(1-\beta) + \beta - \frac{1}{\sigma}\Big]\Big[(1-\phi^r)\frac{\dot{c}_r}{c_r} - (1-\phi^p)\frac{\dot{c}_p}{c_p} + (\phi^r - \phi^p)\frac{\dot{E}}{E}\Big] \\ &+ \frac{1}{\sigma}[\frac{\dot{c}_r}{c_r} - \frac{\dot{c}_p}{c_p}]\end{aligned} \quad (16)$$

It is straightforward to interpret the third term on the RHS of (16), the first bracket of which can be re-arranged as $[\alpha(1-\beta) + \beta - \frac{1}{\sigma}] = (1 - \frac{1}{\sigma}) - (1-\alpha)(1-\beta)$, and becomes negative if, say, $\alpha, \beta < 1$ for the poor substitutability case $(0 < \sigma < 1)$. Whereas its sign is ambiguous when either α or β is more than unity.

3.7 Examples

For simplicity, throughout the examples below, the pure rate of time preference and the population growth rate are set equal between the two groups ($\delta_r = \delta_p =$ 0.1% and $\dot{L}_r/L_r = \dot{L}_p/L_p = 0.5\%$).

We first look at the case of different consumption growth rates (Figure 1). We let α to be varied at 4, 2, 1, and -1. The other assumptions are: $\beta = 2$, $\sigma = 0.7$, $\gamma = 0.1$, $\dot{c}_r/c_r = 1.3\%$ (as is the average employed in Stern (2006]), $\dot{c}_p/c_p = 0.65\%$ and $\dot{E}/E = 0.5\%$ (i.e. environmental quality is growing at an even slower pace than consumption of the poor), so that the cross elasticity η_{cE} is positive. The initial level of consumption in the rich is 100 times higher than that in the poor. The conventional Ramsey discount rate would read $r = 2.7\%$ in the rich and $r = 1.4\%$ in the poor. The wedge of discount rates between the rich and poor widens as time goes by, since the poor consumption growth rate is so low to settle to the long-run discount rate. When inequality aversion is so strong as $\alpha = 4$, discounting of the rich, poor, and the environment are all negative for the first century. The environment discount rate, which is actually negative, starts with -0.5%, then gradually increases, and almost reaches 0% within the same period.

To focus on how the value share of the environment affects the divergence, let us assume away the difference in the consumption growth, $\dot{c}_r/c_r = \dot{c}_p/c_p = \dot{c}/c$, corresponding to the Figures 2 and 3. In this case the discount rate wedge, (16), is simplified as

$$\rho_r^{pcc} - \rho_p^{pcc} = [(1 - \frac{1}{\sigma}) - (1 - \alpha)(1 - \beta)](\phi^r - \phi^p)(\frac{\dot{E}}{E} - \frac{\dot{c}}{c}) \tag{17}$$

Let the consumption growth rates for the rich and the poor equal to 1.3% and the environmental growth 0.5%. And even so, the divergence of the two consumption discount rates is not negligible especially from 20 years on, as is depicted in Figures 2 and 3 (but eventually it disappears since the value share of the lower income group catches up with the rich under equal consumption growth prospects). Since all the other parameters are set equal between the two groups, this is traceable to the third term in the RHS of (16); and this term is amplified by previously dismissed parameter of inequality aversion. Figure 2 differs from Figure 1 only in that consumption growth rates are identical for the rich and poor, but the gap of disaggregated discount rates is as huge as 1% for the case of strong inequality aversion, $\alpha = 4$. Figure 3 preserves most of the assumptions of Figure 2, but the substitutability of consumption goods and the environment is as high as 1.3.

To sum up these examples, we note, among others, that different consumption

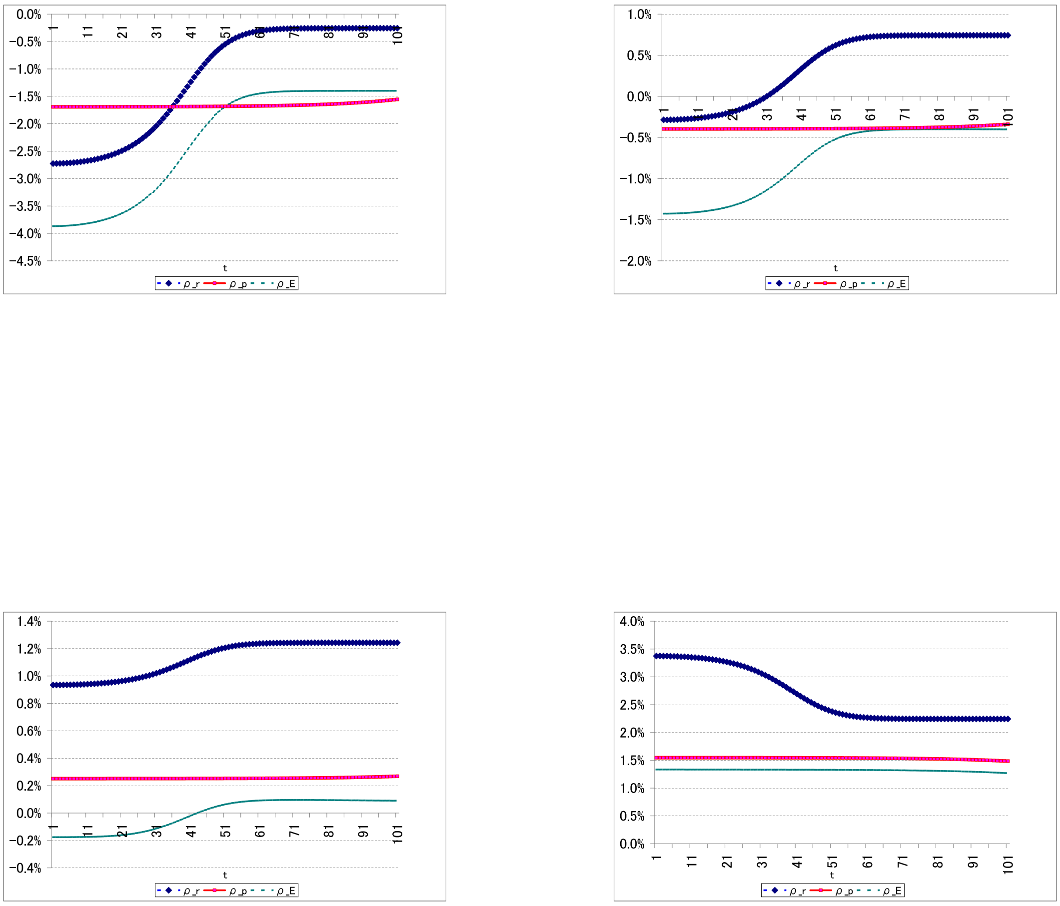

Figure 1: Discounting when the initial levels of per capita consumption are different ($c_r(0) = 100c_p(0)$) and per capita consumption prospect for the rich is higher ($\dot{c}_r/c_r = 1.3\%$, $\dot{c}_p/c_p = 0.65\%$). The other parameters are: $\alpha = 4$, $\beta = 2$, $\sigma = 0.7$, $\gamma = 0.1$ and $\dot{E}/E = 0.5\%$) for the top left, $\alpha = 2$ for the top right, $\alpha = 1$ for the bottom left, and $\alpha = -1$ for the bottom right.

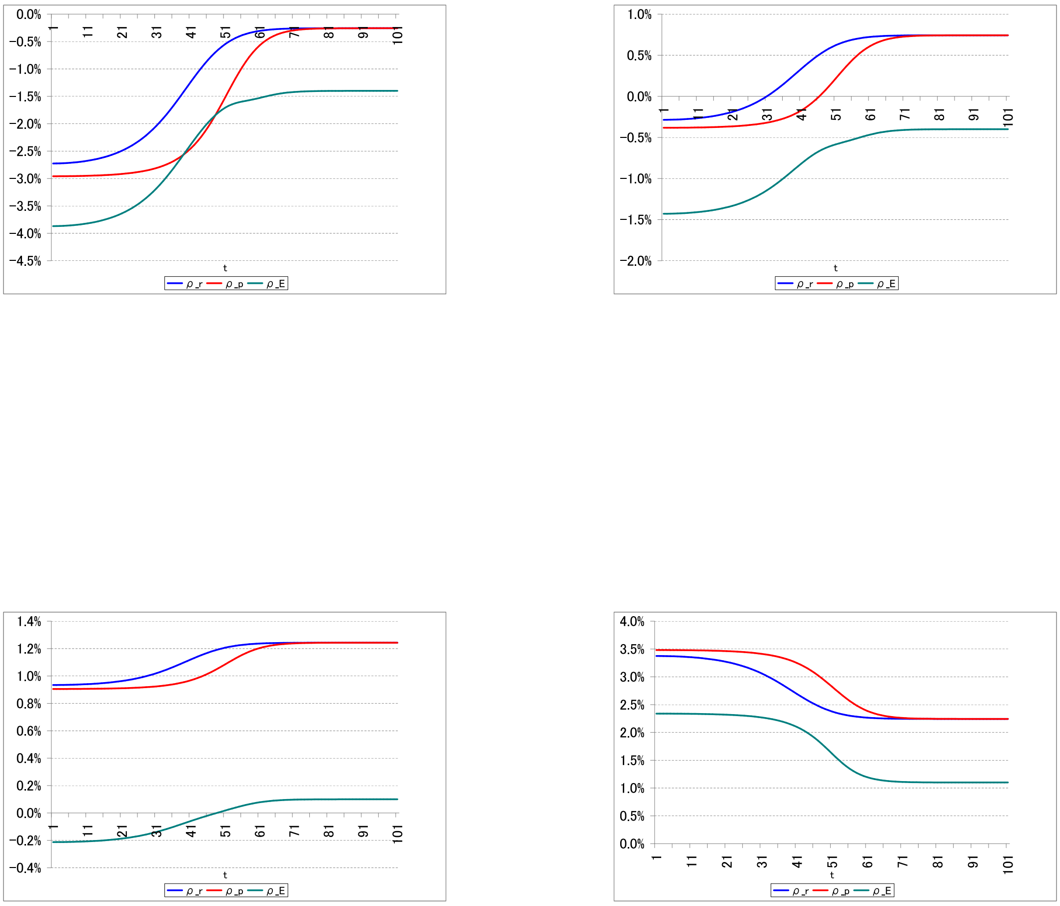

Figure 2: Discounting when the initial levels of per capita consumption are different ($c_r(0) = 100c_p(0)$) but the prospects of consumption growth are the same($\dot{c}_r/c_r = \dot{c}_p/c_p = 1.3\%$). All the other assumptions in Figure 1 hold: $\beta = 2$, $\sigma = 0.7$, $\gamma = 0.1$, $\dot{E}/E = 0.5\%$ and $\alpha = 4$ for the top left, $\alpha = 2$ for the top right, $\alpha = 1$ for the bottom left, and $\alpha = -1$ for the bottom right.

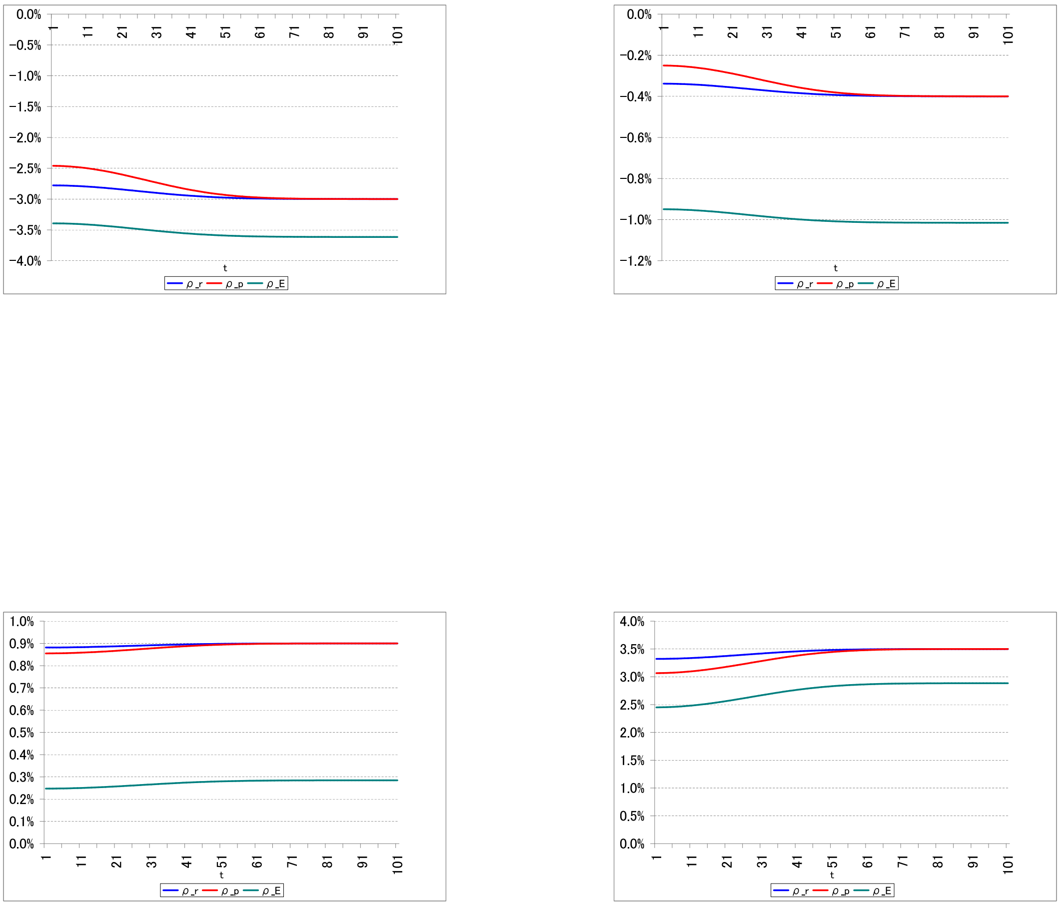

Figure 3: Discounting with high substitutability between consumption and the environment. As in Fig.2, the initial levels of per capita consumption are different ($c_r(0) = 100c_p(0)$) but the prospects of consumption growth are the same($\dot{c}_r/c_r = \dot{c}_p/c_p = 1.3\%$). The other parameters are: $\beta = 2$, $\sigma = 1.3$, $\gamma = 0.1$, $\dot{E}/E = 0.5\%$ and $\alpha = 4$ for the top left, $\alpha = 2$ for the top right, $\alpha = 1$ for the bottom left, and $\alpha = -1$ for the bottom right.

growth rates widen the disaggregated discount rates, but the value share of the environment, teaming up with inequality aversion and elasticity of marginal utility, drives the wedge even if the consumption growth rates are the same. In the latter case, the gap is to be warranted for the transition period until the eventual convergence. Much has been discussed on elasticity of marginal utility of consumption, but how inequality aversion can be determined is not all that clear, which we turn to in the next section.

4 Unknown Weighting to Determine Inequality Aversion

Thus far, we have been proceeding the discussion as if all the parameter values are to be given once we face a problem in practice. In practice, we can move forward in the opposite direction with some parameters being left unknown, and see an implication of an hypothetical decision to be revealed in the unknown parameters. This is especially relevant in the current context since, in delineating (16), we have seen that (in)equality aversion in our SWF boosts the difference in the discount rates between the rich and the poor. However, we know nothing beforehand about descriptively or normatively appropriate value of α, let alone the other ethical parameters. In a classic literature in the theoretical and practical basis for project evaluation, Dasgupta et al. (1972) proposed distributional weights be treated as unknown. The role of the central planning officer, they argue, is to calculate the switching value of the weight, placed on distributional considerations, and let them see the implications of policy options they have to allocate to consumption in the aggregate and that of the disadvantaged. This approach, by nature, has its own limitation: it's hard to identify what governs allocative decisions in reality. In this section, we simply assume that all the parameter values but the parameter in question are known.

In this section, we modify their example and reinterpret it in a dynamic context of climate change policy to see how disaggregated discounting and inequality aversion is applied. Their original problem is summarized in Table I. Assume there are two agricultural projects for irrigation. Project A is to provide irrigation for commercial agriculture, in which case society as a whole receives net benefit of 5 and poor peasants obtain net benefit of 2. Project B is an irrigation for subsistence agriculture, which gives society and peasants net benefits of 1 and 5, respectively. The society as a whole wants to maximize the social profit $V = B_1 + \omega B_2$,

	net aggregate consumption	peasants' consumption
Project A	5	2
Project B	1	5

Table I. The original problem of unknown weighting in a hypothetical cost-benefit analysis in Dasgupta et al. (1972).

where B_1 and B_2 stand for net benefit for aggregate and for peasants and ω is a weight on the consumption of peasants, which is presumably taken as unknown.

It will prove useful if we consider the enterprise as a mix of Project A and Project B, each being allocated shares of a and $(1-a)$. So the social profit becomes $(4-3\omega)a+(1+5\omega)$, which then leads to conditional, corner solution of $a = 1$ for $\omega \leq 4/3$ and $a = 0$ for $\omega \geq 4/3$. In sum, if the social weight given to the poor is roughly more than 1.33, the subsistence agriculture project should be employed. We can depict iso-profit curves in a (B_1, B_2) plane for a given value of ω, on which a new set of curves corresponding to this switching value of the weight, 4/3, can be overdrawn. According to this sensible frame of discussion, rather than laying down the law of a certain number as a normative weight, the planner should show the political leadership implied weighting values for each project to be opted for. The leadership would then know rather objectively how much significance she would juxtapose if a project or the other is to be adopted.

Our natural extension here is to assume a modern central planning officer who faces climate change, an intergenerational problem. Leave the numerical examples intact from the original problem, but net *per capita* social profit from Project A is composed of 5 and 2, the former reaped by the (richer) future generations and the latter by the poor in the contemporary (Table II)[8]. Likewise, net *per capita* social profit from Project B generates only 1 unit for future generation and 5 units for the poor group at the current time. To simplify the two pillar strategies for climate change, label the two recipient sectors as "mitigation" and "adaptation" for convenience. The effect of mitigation is revealed in t years, whereas adaptation is an approach that benefits the contemporary poor who live in the lowland areas, deemed most vulnerable to climate change. The objective function of the social planner is $V = B_1 + \omega B_2$, where B_1 is the net per capita social profit that will arise to the rich in t years by the mitigation policy, and B_2 is that to the poor in the present generation by the adaptation policy. Note that both are expressed in per capita consumption terms, as is the case for many social cost-benefit analyses.

[8] In this section we use net per capita social profit and per capita consumption interchangeably.

	mitigation (for the future rich)	adaptation (for the current poor)
Project A	5	2
Project B	1	5

Table II. A problem of unknown weighting for climate change.

Finally, ω is an ex-ante unknown parameter.

By construction, per capita consumption of the mitigation sector, rather than social welfare or utility, is the numéraire ($\partial V/\partial B_1 = 1$). In this problem we are asked to compare the benefits of the two groups who live in different time *and* space. In other words, this is a joint problem of intergenerational *and* intragenerational equity. To consider the benefits of the mitigation (i.e. rich) sector in the future, we can simply use the discount rate of (5) for the rich. We turn to the expression of the shadow price of the adaptation (i.e. poor) sector of the current time. Since the shadow price of the per capita social profit to the mitigation sector (the rich) in t years is normalized to unity, referring to the shadow price for deriving equation (5), the weight factor in $V = B_1 + \omega B_2$ for this dynamic problem becomes

$$\omega = \frac{L_p(0)}{L_r(t)}\left[\frac{u^p(0)}{u^r(t)}\right]^{-\alpha}\frac{u_c^p(0)}{u_c^r(t)/(1+\delta_r)^t}, \tag{18}$$

which is the marginal rate of substitution (minus 1) of the consumption of the future rich by the present poor. As we have seen above, opting for Project B (adaptation) with $a = 0$ would mean that the planner must have set the weight parameter ω as more than 4/3.

Taking the natural logarithm of the above equation for ω, which is set at the switching value of 4/3, we have

$$\ln(4/3) = (\ln L_p(0) - \ln L_r(t)) - \alpha(\ln u^p(0) - \ln u^r(t)) + (\ln u_c^p(0) - \ln u_c^r(t)) + t\ln(1+\delta_r),$$

which reads as $\ln(4/3) \cong 0.0025$.

The other parameters and assumptions continue to hold from the previous section: the population growth rate is 0.5% and the forecast of the per capita consumption growth rate is 1.3% for the both regions. Regarding our ethical or normative parameters, let us assume the relative risk aversion $\beta = .5$ and the pure rate of time preference for the rich, δ_r, equals .001 (0.1%) per annum. The parameter attached to the environment, γ, is .1, so that the value share of the environment also becomes .1 in the year 0. Then the switching value of inequality aversion

α is computed as 3.14. Thus, for this specific numerical sample, α turns out to represent *equality* aversion. It is difficult to determine an appropriate value for inequality aversion ex-ante, but we can tell the political leadership governing both worlds that the implied inequality aversion is such and such in this way.

5 Conclusion

In the literature of discounting and climate change, it is customary to discuss discounting in terms of aggregate consumption. Consumption prospects in the future, however, should be wide apart between, say, the wealthiest nations and the bottom billion. Care for such a global distribution of income would lead to a move to disaggregate consumption discount rates. In this paper, we go back to the drawing board of social welfare function comprising the well-being of the rich and that of the poor, and have derived several discount rates for different numéraire. The most refined of such may be per capita consumption discount rate, which is comprised of utility discount rate, population growth, utility growth rate weighted by inequality aversion parameter, consumption growth rate weighted by elasticity of marginal utility, and environment growth rate multiplied by cross elasticity of marginal utility. All these might contribute to a wide difference in the rates of discount for the rich and the poor, from a global social planner perspective. Of particular significance is the mixed effect of relative risk aversion, elasticity of substitution, and inequality aversion. Furthermore, in this setting, one can treat intergenerational and intragenerational equity in a seamless manner. We carried on to assume a constant-elasticity-of-substitution and constant relative risk aversion utility for each group to specify characteristics of disaggregated discounting. Within the given framework, each of the above does make a difference between the two rates in numerical examples.

As is the case for all the discussion going on around discounting, ethical parameters are hard to be determined, so they should be discussed with sensitivity analysis in a reflective equilibrium manner. One potential way to do this was shown in the last section with a practice of unknown weighting.

There remain some important research items on the plate. We have assumed that the world can be divided simply into the rich and the poor, but in practical application they can vary as time goes by. Furthermore, climate change policy inherently involves income transfer among nations, which is not addressed in the current paper. It is not clear in such circumstances how, and according to what, we should disaggregate consumption and discounting. Another interesting issue that arises from the current SWF is the order of aggregation and its effect on discount rates.

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