A Hybrid Approach for Forecasting of Oil Prices Volatility

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Abstract

This study aims to introduce an ideal model for forecasting crude oil price volatility. For this purpose, the 'predictability' hypothesis was tested using the variance ratio test, BDS[1] test and the chaos analysis. Structural analyses were also carried out to identify possible nonlinear patterns in this series. On this basis, Lyapunov exponents confirmed that the return series of crude oil price is chaotic. Moreover, according to the findings, the rate of return series has the long memory property rejecting the efficient market hypothesis and affirming the fractal markets hypothesis. The results of GPH test verified that both the rate of return and volatility series of crude oil price have the long memory property. Besides, according to both MSE and RMSE criteria, wavelet-decomposed data improve the performance of the model significantly. Therefore, a hybrid model was introduced based on the long memory property which uses wavelet decomposed data as the most relevant model.

Key Words: Forecasting, Oil Price, Chaos, Wavelet Decomposition, Long Memory.

[1] Brock, Dechert and Scheinkman, (1987)

1. Introduction

Strong correlation between oil price and political as well as economic situation in the international scope, on the one hand, and the deep impacts of crude oil price on country-specific macroeconomic indicators, on the other, have made oil market and crude oil price fluctuations an ever-hot topic in the energy economics which has always received great attention from consumers, producers, governments, policy makers and scholars (Wang et al., 2011). Oil price, also, is identified as a key factor in financial markets because it significantly affects option pricing, portfolio management and risk measurement processes (Wei et al., 2010).

Oil importing and exporting countries pay great attention to oil price variations. However, this issue is of utmost importance for major oil exporting countries like Iran. For example, in

Iran, shaping 90 percents of country's export value, crude oil and gas exports constitute approximately 60 percents of government's income (Farzanegan and Mrakwart, 2011); Also, oil revenue is the major source of government spending and directly affects fiscal and monetary policies implemented by the government. Subsequently, oil price is recognized as the main source of macro-level fluctuations in the oil-based government-driven economy (komijani et al., 2013; Mehrara and Mohaghegh, 2012; Mehrara and Oskoui, 2007).

Considering its vital importance for decision makers, forecasting major indicators of oil price market is broadly accepted as both a scientific and applied goal. A brief review of the literature proves that almost all modeling and forecasting techniques _from theoretically designed structural models to thoroughly numerical models_ have been implemented in the oil market. Structural models are relatively more useful in explaining the status of a market; however, in forecasting studies most financial market analysts tend to use time series models. Such models which are relatively powerful in practice are based on the Efficient Market Hypothesis (Ozer and Ertokatli, 2010). As a modern competitive alternative, in recent years, a new approach has been suggested. This approach known as Fractal Market Hypothesis is based on Chaos Theory. This theory guides researchers towards detailed explanations of special events in the market. Technically speaking, implementing maximal Lyapunov exponent (to verify the predictability of time series by nonlinear models) and its inverse (to determine time span of forecasting) are two crucial prerequisites. This is a very important step because the results of linear estimation for those series recognized as chaotic are not valid.

The goals of this paper are to (a) investigate the predictability of the fluctuations of the time series of Iran's crude oil price on the basis of Fractal Market Hypothesis and Chaos Theory; (b) model these fluctuations by long memory models (in particular, ARFIMA-FIGARCH model); and (c) compare the performance of this model with a hybrid model consisted of ARFIMA-FIGARCH model and wavelet decomposition. For the aim of the study, the daily data from 1/2/2002 to 11/3/2011 were applied. From these 2536 observations, approximately 99 percent were used for estimations and the remaining 17 observations for out-of-sample forecasting.

1. Iran's Oil Price: a Brief Historical Review

For numerous political and economic reasons, oil market is always volatile; though they vary in size and durability. In sum, from 1973 to 2009, these factors had major impacts on oil price:

1. Arab Oil Embargo (1973-74) 2. Islamic Revolution in Iran (1978-79)
3. Iraq's Invasion of Iran (1979-80) 4. Saudi Arabia Excess Production[1] (1985-86)
5. Iraq's Invasion of Kuwait (1989-90) 6. Financial Crisis in South Asia (1997-98)
7. September 11 attacks in New York (2001) 8. US Invasion of Iraq (2003-04)
9. Some Geopolitical Issues (2006-07) 10. US financial crisis (2007-11)

In 2008, the extensive consequences of the financial crisis caused oil price to reduce from 150 US $ to nearly 35 US $ per barrel. Since then, in spite of some temporary decreases, crude oil price has consistently increased in the world market. Considering continuous increase in demand in the oil market, political instability in the Middle East, recent threats on the Islamic Republic of Iran, and the deep financial crisis in the Euro region, it is expected that this increasing trend will continue in the following years. Figure 1 depicts movements of Iran's heavy oil price in the past decade.

[1] Due to invention of a new method called Netback

Source: EIA[1] Reports

Figure 1: Iran's heavy oil price (2002-2011)

2. Literature Review: Chaos Theory in Oil Market

Chaos refers to a state of utter confusion or disorder. Philosophically, chaos is a total lack of organization in which accident determines the occurrence of events, but technically chaos is the order of seemingly disordered systems. Chaos theory studies such systems. Chaotic systems are deterministic, meaning that their future behavior is fully determined by their initial conditions, with no random elements involved. In other words, the deterministic nature of these systems does not make them predictable. But the analysts who are not aware of the nature of the system (or does not know it well enough), cannot distinguish between a chaotic and a random system. Unfortunately statistical tests cannot, also, distinguish between them. So, considering the measurement accuracy limit, the accuracy of forecasts based on usual statistical or econometric techniques, continuously _at an exponential rate_ decreases.

Chaotic systems are highly sensitive to initial conditions, an effect which is popularly referred to as the butterfly effect. Small differences in initial conditions (such as those due to rounding errors in numerical computation) yield widely

[1] Energy Information Administration

diverging outcomes for chaotic systems, rendering long-term prediction impossible in general. Chaotic systems are nonlinear dynamic systems which (1) are highly sensitive to initial conditions; (2) have unusual complicated absorbents; (3) sudden structural breaks in their trajectory are distinguishable (Prokhorov, 2008). However, it should be noted that:

1. The behavior of chaotic systems though seems random, in essence can be theoretically explained by deterministic rules and equations. Nevertheless, even though we accept the existence of equations explains the source of their chaotic behavior, proximity and inaccuracy (though very small) are inevitable due to measurement limitations.
2. Even very small inaccuracy in initial conditions, because chaotic system is highly sensitive to them, leads to huge differences between expected and realized values in the long term. In other words, as time passes, forecasted series and measured values of realized series totally diverge so much that previous forecasts are no longer reliable; a fact called unpredictability in the long run (Williams, 2005).

Lyapunov exponent tests and the inverse of maximal Lyapunov exponent allow us to recognize the dynamics of disturbing term in a chaotic process and distinguish between a chaotic error term and a stochastic process.

On the other hand, wavelet technique decomposes a non-stationary series into two (or more) approximation and detail series. Decomposed series can be modeled independently and separately (Lineesh and John, 2010). Therefore, wavelet analysis can be useful in describing the signals with discontinuous or fractal structures in the financial market. It also allows the removal of noise-dependent high frequencies while conserving the signal bearing high frequency terms (Homayouni and Amiri, 2011). This feature reduces the magnitude of error components and subsequently leads to more accurate forecasts.

Econometricians explain and forecast the dynamics of highly volatile price series by Generalized Autoregressive Conditional Heteroskedasticity (GARCH)-Type models. Such models perform well in tracking two critical features of the data; volatility clustering and Fat Tail series. These critical features can originate from such factors as sudden shocks, structural variations, domestic demands, global situations and political events can be included in these volatility models (Vo, 2011).

Oil market is one of the ever-volatile markets that make forecasting a challenging task. In addition, since oil is a strategic good, any price movement in oil market immediately affects other financial markets (Erbil, 2011). So, many scholars have modeled and forecasted oil price fluctuations with GARCH-Type models. But the crucial question is whether or not using such models is verified. To find appropriate answer, we need to know whether (i) oil price series predictable. If yes, (ii) whether oil price series is chaotic? And if yes (iii) whether oil price series has a long-memory property?. Many studies have been conducted on the predictability of oil price (e.g., Arouri et al., 2010a; Alvarez-Ramirez et al., 2010; Charles and Darné, 2009; Alvarez-Ramirez at al., 2008). Afees et al., 2013; Kang, 2011; vo, 2011; Wei, 2010; Mohammadi and Su, 2010; Arouri et al., 2010b; Cheong, 2009 also confirmed volatility of the oil price. Furthermore, the long memory feature was also confirmed to exist in oil markets in Mostafaei and Sakhabakhsh, 2011; Prado, 2011; Wang et al., 2011; Choi and Hammoudeh, 2009 study. The present study attempts to combine the findings of these studies and present a model that can make the most accurate forecasts about oil price volatility.

4. Methodology

4.1. Diagnosing Chaotic Processes

Several tests have been proposed in the chaos theory literature to distinguish between stochastic and chaotic processes. Some of these tests recognize stochastic processes while others are aimed at diagnosing chaotic processes; the former are called indirect and the latter direct tests. Indirect tests like BDS test almost always consider the residual series of a linear or nonlinear regression and test whether it is stochastic. Therefore, when the null hypothesis of randomness of residual is rejected, one cannot conclude that the series is necessarily chaotic (Brock et al., 1997; Takens, 1981).

Before the formulization of chaos theory, the concept of Lyapunov exponent was applied to verify the stability of linear or nonlinear systems. Its numerical value is calculated on the basis of our measurement from the kurtosis or curvature of the system. In fact, Lyapunov exponent is the average of the exponential divergence or convergence rate of chaotic trajectories in state space. Lyapunov exponent measures the sensitivity of the process to its initial values. Positive Lyapunov exponent indicates chaotic behavior in the series while its negative value indicates damping dynamic systems (Ellner et al., 1991). Besides, the inverse of maximal Lyapunov exponent points out the distance between fixed and randomness of the series. Therefore, based on its numerical value one can determine the predictability degree (the number of approved predictable periods) of the series (Wolf et al., 1985). This method was implemented for the purpose of this study.

4.2. Long Memory

Long memory property is a sign of strong correlation between far-distance observations in a given time series. Hurst (1951), first, noticed that some time series have this property. However, in mid 1980s, after suggestion of critical concepts like unit root and cointegration, econometricians realized some other types of nonstationarity and partial stationarities which are frequently found in economic and financial time series (Granger and Joyeux, 1980).

Generally, econometricians, used first-order differencing in their empirical analyses due to its ease of use (in order to avoid the problems of spurious regression in non-stationary data and the difficulty of fractional differencing). Undoubtedly, this replacement (of first-order differencing with fractional differencing) leads to over -or under- differencing and consequently loss of some of the information in the time series (Huang, 2010). On the other hand, considering the fact that majority of the financial and economic time series are non-stationary and of the Differencing Stationary Process (DSP[1]) kind, in order to eliminate the problems related to over differencing and to obtain stationary data and get rid of the problems related to spurious regression, we can use Fractional Integration.

4.2. Diagnosing Long Memory Process

Diagnosing the long memory process is the most important step. Auto Correlation Functions (ACF) as a graphical test and spectral density test or Geweke and Porter-

[1] And some are also trend stationary processes

Hudak (GPH) test as the frequently used numeric tests are two main groups of tests that diagnose the long memory feature.

4.3. Conditional Heteroskedasticity Models

These models were introduced by Engle (1982) and elaborated by Bollerslev (1986). After that, several other conditional heteroskedasticity models were suggested (see Arouri et al., 2010a). In this study, the Fractional Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) model was used. This model was first suggested by Baillie et al. (1996). FIGARCH, in general, specifies as $(1-L)^d \Phi(L)\varepsilon_t^2 = \omega + B(L)\upsilon_t$ in which $\Phi(L)$ and $B(L)$ are two lag functions with the optimal lag length of q and p, respectively; L is the lag operator and d is the fractional parameter. When $d = 0$ FIGARCH reduces to a usual GARCH model and when $d = 1$, FIGARCH is equal to IGARCH model (see Arouri et al., 2010a). In contrast with durable shocks in IGARCH or transient shocks in GARCH, in FIGARCH models it is assumed that imposed shocks have moderate durability and damp at a hyperbolic rate.

4.4. Wavelet Decomposition

Wavelet technique, using base functions, transmits time series to the frequency space and decomposes it in various scales. Contrary to fourier transformation which is only based on sine functions, wavelet decomposition includes various discrete and continuous base function, albeit all have finite energy (Reis et al., 2009). This property makes wavelet a pleasant tool for analyzing nonstationary and transient series. Such decompositions can be classified into two main categories; Continuous Wavelet Transform (CWT) and Discrete Wavelet Transform (DWT) (Karim et al., 2011). Crude oil price series is a discrete series. The most important DWT base functions include Haar, Daubechies, Symmelets, Coiflets, and Meyer functions. Among others, Daubechies function is widely used discrete base function (Al Wadi and Ismail, 2011); considering the similarity between the time series of Iran's crude oil price and a specific Daubechies base transformation function called db3, we used it for decomposing oil price series.

5. Empirical Results

5.1. Data

In this study, the daily data related to Iran's heavy crude oil price from 2/1/2002 to 11/3/2011 were used. Table 1 reports the main descriptive statistics for the series of natural logarithm of oil price (LOIL) as well as oil price return series (DLOIL).

Table 1: Descriptive Statistics

Stat.	Return Of Oil Prices Series	Tests	Return Of Oil Prices Series
Observations	2536	ADF[1]	-47.572(0.000)
Mean	0.000677	PP[2]	-47.659(0.000)
S.D	0.021500	ERS[3]	0.0355(3. 26)
Skewness	-0.291624	Box- Ljung Q(10)	23.083(0.010)
Kurtosis	6.187903	McLeod-Li $Q^2(10)$	477.43(0.000)
Jarque- Bra	1109.807(0.000)	ARCH 10)=F(10,2514)	25.209(0.000)

* All of numbers in parenthesis are probability of related test, but ERS test except that the critical value of the test is.

Source: Findings of Study

As seen in the table 1, the return series of crude oil price has the mean of 0.000667 and standard deviation of 0.0215 in the sample period suggesting that it has been highly volatile. Besides, Jarque-Bera and kurtosis statistics show that the series not only is not normally distributed but has wide tails. Based on the Ljung-Box statistics (10 lags), the null hypothesis of "No serial correlation" is rejected. Similarly, McLeod-Lee statistics reject the null hypothesis of "No serial correlation in squared series" and confirm Heteroskedasticity in return series suggesting that there exists some sort of nonlinear relationship in the squared series. This conclusion is also approved by Engle's ARCH test. Finally, according to unit root tests _ADF and PP tests_ the return series is stationary but ERS test unit root test shows this series is non-stationary. Thus, such conditions might have been caused by the long memory feature in this series. For this reason, tests for checking the existence of this feature will be focused upon in the next part.

[1] Augmented Dicky-fuller Test
[2] Phillips-Perron Test
[3] Elliott-Rothenberg-Stock Test

5.2. Predictability of Oil Price

i. Variance Ratio

Based on Lo & MacKinlay (1988), the variance ratio test investigates the Martingale hypothesis.

Table 2: Variance Ratio Test

Value	d.f	Prob.	Criterion
15.99	2535	0.000	Variance ratio test

Source: Findings of Study

As shown in Table 2, the martingale hypothesis –in the return series and its lag series- is strongly rejected. So, it can be concluded that the generating process of the data is not random walk; i.e. the series is predictable.

ii. BDS Test

This test was developed by Brock, Dechert and Scheinkman (1987). The main concept behind the BDS test is the correlation integral, which is a measure of the frequency with which temporal patterns are repeated in the data. BDS test makes it possible to distinguish between a nonlinear and a chaotic process. The results of BDS test are presented in Table 3.

Table 2: BDS Test

Dimension	BDS Stat.	Std. Error	z-Stat.	Prob.
2	0.012537	0.001635	7.667829	0.0000
3	0.027289	0.002591	10.53126	0.0000
4	0.037924	0.003077	12.32419	0.0000
5	0.043804	0.003198	13.69544	0.0000

Source: Findings of Study

As seen in Table 2, the null hypothesis of "the residual series is not random" is rejected. This result approves the existence of a nonlinear (may be a chaotic) process in the data. It should be noted that when BDS test in 2 (or higher) dimensions rejects the hypothesis that the series is random; existence of a nonlinear process is quite probable. This result points to the conclusion that BDS test also approves that the data generating process in this study is nonlinear.

iii. Maximal Lyapunov Exponent

Maximal Lyapunov Exponent measures the rate of convergence (divergence) of two initially close points along their trajectory over time. In Lyapunov method, this rate is measured by an exponential function. The value of these exponents can be used to investigate the Local Stability of linear or nonlinear systems. In this test, positive values of exponents indicate exponential divergence of the series, high sensitivity to initial conditions and therefore, chaotic process. On the other hand, negative values of exponents indicate exponential convergence. Finally, when Lyapunov exponents are zero, one may argue that there is no converging or diverging trajectory in the data; i.e. the series follows a fixed process. Therefore, in most of the cases, existence of only one Lyapunov exponent is enough to conclude that the system is chaotic.

There are two main approaches for computing Maximal Lyapunov Exponent; (1) direct method; and (2) Jacobian Method. In this study, we implemented both methods (using MATLAB software); the results are reported in Tables 4 and 5.

Table 4: Direct Approach to Maximal Lyapunov Exponent

Max Lag	1	2	3	4	5
Lyapanov Exponential	0.0743	0.0489	0.1075	0.1097	0.0956
Maximal Lyapunov Exponent			0.1097		

Source: Findings of Study

Table 5: Jacobian Approach to maximal Lyapunov exponent

Max Lag	1	2	3	4	5
Lyapanov Exponential	0	0	0. 0560	0	0.0537
Maximal Lyapunov Exponent			0.0560		

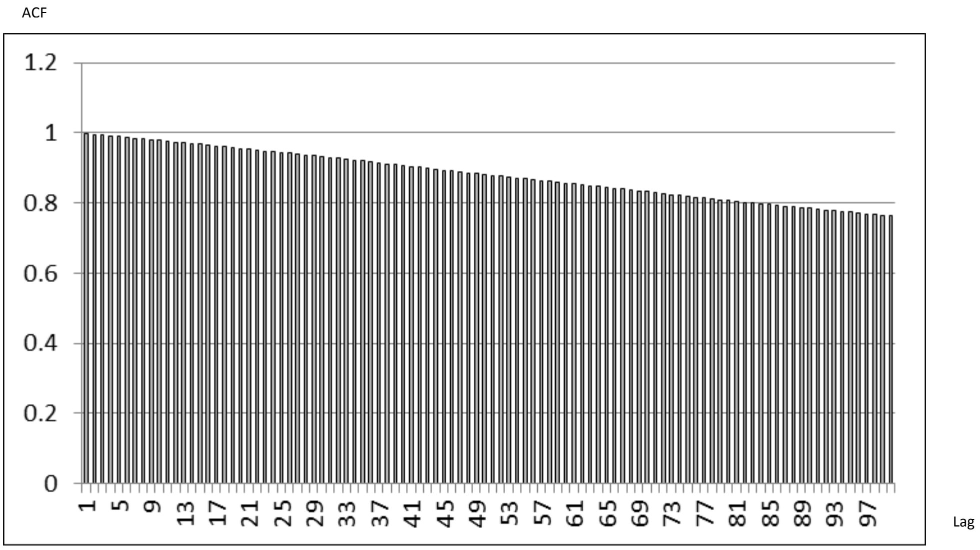
Source: Findings of Study

As seen in Tables 4 and 5, both methods lead to positive Lyapunov exponents. Besides, due to high sensitivity to initial conditions, for each combination of initial points, close trajectories diverge quickly in a way that there is no fixed point or cycle. Then, one may conclude that this process is chaotic. Furthermore, the maximal Lyapunov exponent in the direct method was 0.1097; so, the predictability limit (the maximum number of predictable periods) is approximately 10 days. In Jacobian method, this number was 18 days. To be prudent, we have selected the minimum value (10 days) for the rest of calculations and estimations. Therefore, achieving this result can not only increase the validity and strength of out-of-sampling forecasting but decrease the number of forecasting errors as well.

5.3. Quantitative analysis of the Long Memory Process

Estimating the long memory parameter (d) is the milestone of modeling long memory property. ACF and GPH are two commonly used methods for this purpose. Graph 1 depicts the ACF of the logarithm of the time series of crude oil price. As clearly shown, following an exponential trend, graph decreases very smoothly, a typical shape for time series that are non-stationary and have the long memory property.

Graph 1: ACF of LOIL



Source: Findings of Study

If such a series does not have the long memory property, it is expected that after first differencing, the series would become stationary. According to Table 6, although ADF and PP tests recognize the oil price series stationary after first differencing, the ERS and KPSS tests show some sort of non-stationarity in the data. This result further indicates the existence of the long memory property.

Table 6: Unit Root Tests

Tests	Accounting Value	Critical Value	Result
ADF	-47.572	-1.9409	Stationary
Phillips-Perron	-47.659	-1.9409	Stationary
ERS	0.0355	3. 26	Non-Stationary
KPSS	2.159	0.463	Non-Stationary

Source: Findings of Study

Models considering long memory property are very sensitive to the estimation of long memory parameter as well as the pattern of damping of auto-correlation functions. In this study, the long memory parameter was estimated using GPH approach. This method, invented by Gewek, Porter-Hudak (1987), is based on frequency domain analysis. GPH method applies a special regression technique called Log-Period Gram which allows us to distinguish between long-term and short-term trends. The slope of regression line calculated by this technique is exactly equal to long memory parameter.

Table 7 reports the estimated long memory parameter for both the logarithmic series and return series. To do so, we have used OX-Metrics software.

Table 7: Estimated Long Memory Parameters

Variable	d-Parameter	t-stat.	Prob.
LOIL	1.05207	65.8	0.000
dLOIL	0.440238	2.79	0.005

Source: Findings of Study

As table 7 shows, the estimated long memory parameter is statistically significant, i.e., it is not equal to zero suggesting that the series of (logarithm of) crude oil price in the level has the long memory property. However, the return series should be modeled after another differencing.

5.4. Modeling the Return Series of Crude Oil Price

Knowing that the crude oil price in the level has the long memory property, in this step, we fit an econometric model to our data. In this paper, the most famous and flexible long memory model, i.e., ARFIMA was applied to specify the mean equation:

$$\phi(L)(1-L)^d (y_t - \mu_t) = \theta(L)\varepsilon_t \qquad t = 1,2,3,...,T \tag{3}$$

$\phi(L)$ and $\theta(L)$ indicate AutoRegressive (AR) and Moving Average (MA) polynomials, respectively. L is the lag operator and μ_t represents the mean of the series. d is the differencing parameter and $(1-L)^d$ stands for fractional differencing operator. If d=1, this model reduces to ARIMA model. If, on the other hand, $d < 0.5$, the covariance is fixed and if $d > 0$, long memory property exists (Husking, 1981). When $0 < d < 0.5$, ACF has a hyperbolic decreasing pattern and when $-0.5 < d < 0$, medium-term (or short-term) memory exists; this property suggests that too many differencing have been made. In such cases, the invert of ACF has a hyperbolic decreasing pattern.

To estimate the ARFIMA model (and d parameter), three methods were implemented; Exact Maximum Likelihood (EML); Modified Profile Likelihood (MPL); and Non-Linear Least Square (NLS). Table 8 compares various estimated models on the basis of Akaike Information Criterion (AIC). As shown in this table, ARFIMA (1, 0.14, 2) has the best performance compared to other models.

Table 8: Estimated ARFIMA models

Model	AIC			ARCH-TEST
	MPL	NLS	EML	
ARFIMA(1,0.04,1)	-4.84134185	-4.85645431	-4.85611325	F(1,2519)=31.482(0.000)
ARFIMA(1,0.04,2)	-4.84141198	-4.86294341	-4.85659337	F(1,2518)= 34.235(0.000)
ARFIMA(2,0.04,1)	-4.8415002	-4.85703734	-4.85668659	F(1,2518)= 33.354(0.000)
ARFIMA(2,0.04,2)	-4.84418547	-4.85701168	-4.85928044	F(1,2517)=32.825(0.000)

Source: Findings of Study

Moreover, with respect to volatility equation, diagnostic ARCH tests approved the existence of ARCH effects in the residual series; to model this conditional heteroskedasticity, fractional (to track the long memory property) and non-fractional GARCH models were estimated. Table 9 compares them on the basis of AIC and Schwarz-Bayesian Criterion (SBC).

Table 9: ARFIMA-FIGARCH Models

Models	**ARFIMA(1,1)**		**ARFIMA(1,2)**		**ARFIMA(2,1)**		**ARFIMA(2,2)**	
	SBC	**AIC**	**SBC**	**AIC**	**SBC**	**AIC**	**SBC**	**AIC**
GARCH	-5.4439	-5.5492	-5.4589	-5.5477	-5.4571	-5.5459	-5.4535	-5.5486
EGARCH	-5.4165	-5.5116	-5.4172	-5.5186	-5.4168	-5.5182	-5.4164	-5.5242
GJR-GARCH	-5.4624	-5.5512	-5.4546	-5.5497	-5.453	-5.5481	-5.4495	-5.5509
APGARCH	-5.4575	-5.5391	-5.4491	-5.5505	-5.4472	-5.5487	-5.4429	-5.5507
IGARCH	-5.4637	-5.5398	-5.4558	-5.5383	-5.4538	-5.5363	-5.4504	-5.5391
FIGARCH(BBM)	-5.452	-5.5407	-5.4668	-5.5526	-5.442	-5.5371	-5.4385	-5.5399
FIGARCH (Chang)	-5.452	-5.5408	-5.444	-5.5391	-5.4421	-5.5371	-5.4385	-5.5399

Source: Findings of Study

Table 9 has three different parts: part 1 includes non-fractional heteroskedasticity models; part 2 is dedicated to some integrated non-fractional heteroskedasticity (IGARCH) models; and part 3 includes various fractional heteroskedasticity (FIGARCH) models. According to both information criteria, ARFIMA(1,2)-FIGARCH(BBM) proves to be the best specification. Table 10 reports the coefficients of this model as well as some diagnostic statistics in detail. As the table clearly shows, all the coefficients are significant (in 95% of confidence). Ljung-Box statistics shows no sign of serial correlation between residuals. Besides, according to both McLeod-Lee and ARCH statistics, there is no heteroskedasticity.

Table 10: Estimation of ARFIMA(1,2)-FIGARCH(BBM) Model

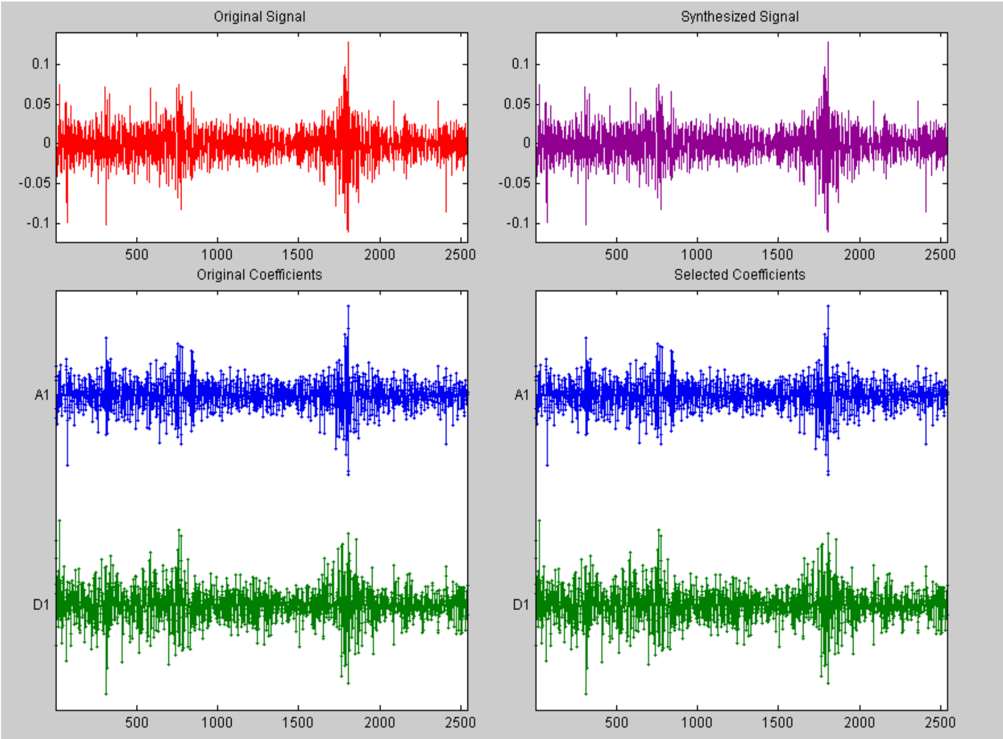
Variable	Coefficient	Standard Error	t-Stat.	Prob
Mean Equation				
C	0.001012	0.00036412	2.779	0.0055
d-ARFIMA	0.044040	0.0034389	12.81	0.0000
AR(1)	-0.887022	0.37596	-2.359	0.0184
MA(1)	0.952358	0.35649	2.671	0.0076
MA(2)	0.060525	0.023550	2.570	0.0082
Dum	0.141621	0.0071753	19.74	0.0000
Variance Equation				
C	0.061262	0.037146	1.649	0.0992
d-FIGARCH	0.421338	0.11706	3.599	0.0000
ARCH	0.288289	0.052668	5.474	0.0000
GARCH	0.724586	0.089571	8.089	0.0000
Log likelihood	6365.271	Box- Ljung Q(10)	10.3996(0.167)	
Akaike	-5.552605	McLeod-Li Q2(10)	5.80142(0..669)	
Schwarz	-5.466807	ARCH(10)=F(10,2503)	0.56651(0.842)	

Source: Findings of Study

5.5. Wavelet Decomposition

To determine the optimized decomposition level, the series was first decomposed up to 5 levels and then using graphical wavelet toolbox in MATLAB software, the best level was determined, which is one. It was also found that one of the Daubechies functions called db3 fits the data better than other base functions. Graph 2, depicts the approximation -A(1)- and detailed -d(1)- series resulting from decomposing oil price series with db3 up to 1 level.

Graph 2: Decomposed Series of Oil Price up to 1 Level Using db3



Source: Findings of Study

Modeling oil price fluctuations using approximation and detail series yielded valuable results. According to the results, the long memory property significantly increased in the "detail" series while not in the "approximation" series. Furthermore, comparing the accuracy of forecasts based on decomposed (DCM) and non-decomposed (NDCM) data are reported in Table 11.

Table 11: Forecasting Performance of Models Based on DCM and NDCM Data

Decomposition data		
Model	MSE	RMSE
ARFIMA-FIGARCH	0.0000276	0.00525
Non-Decomposition data		
Model	MSE	RMSE
ARFIMA-FIGARCH	0.0005281	0.02298

Source: Findings of Study

In line with previous studies mentioned in the literature (e.g., He et al., 2012; Haidar & Wolff, 2011, to name a few), DCM data have led to significantly more accurate forecasts as seen in the Table 11.

6. Conclusion

The strategic role of crude oil price and its deep wide effects on all countries in the world including Iran instigated the conduction of numerous studies in recent decades. Following this trend, we compared the performance of some various models in forecasting the crude oil price; these models include fractal chaos-based models, different GARCH-type volatility models and models based on DCM data using wavelet decomposition technique.

Our findings suggest that crude oil price series is not Martingale (i.e., it is predictable) and follows a nonlinear trend. So, the Efficient Market Hypothesis (EMH) is not valid in this case. Moreover, the results approved the Fractal Market Hypothesis (FMH) in the oil market; so, it can be concluded that this series is chaotic. Using -invert of- Maximal Lyapunov Exponent the predictability limit was determined as 10 days.

On the other hand, due to confirmed ARCH effects, GARCH-Type various models were used to track the variance fluctuations more accurately and reduce the forecasting error. Besides, the existence of long memory property in first and second momentums of the series (approved by ACF and GPH tests) led the researchers to fit an ARFIMA-FIGARCH specification model to crude oil price data. Compared with various fractal and non-fractal model, the nonlinear ARFIMA (1,2)-FIGARCH (BBM) model had the best forecasting performance on the basis of AIC and SBC information criteria. It is worth mentioning that, there are some complicated models which may improve forecasting performance in special cases even more than the model used in this study by including the long memory property in the econometric model which leads to systematic improvement.

Another goal of this paper was to compare the forecasting accuracy of models using the wavelet decomposed data with the models that use the original data. Our findings approve that the smoothed data are likely to provide more accurate forecasts not only in first momentum (mean) but also in second momentum (variance) of the crude oil price series.

In brief, it can be concluded that considering the long memory property and applying hybrid methods lead to more accurate forecasts. Besides, the best model in this study (ARFIMA-FIGARCH model using DCM data) can be used in forecasting other market indicators (e.g., Value at Risk (VaR)) in oil or other highly volatile financial markets.

References

Afees A., Salisu, I., Fasanya, O. (2013). Modelling oil price volatility with structural breaks. *Energy Policy* 52, 554-562.

Alvarez-Ramirez, J., Alvarez, J., Rodriguez, E. (2008). Short-term predictability of crude oil markets: A detrended fluctuation analysis approach. *Energy Economics* 30, 5, 2645-2656.

Alvarez-Ramirez, J., Alvarez, J., Solis, R. (2010). Crude oil market efficiency and modeling: Insights from the multiscaling autocorrelation pattern. *Energy Economics* 32, 5, 993-1000.

Al Wadia, S., Tahir Ismail, M. (2011). Selecting Wavelet Transforms Model in Forecasting Financial Time Series Data Based on ARIMA Model. *Applied Mathematical Sciences* 5, 7, 315 – 326.

Arouri, M. E. H., Dinh, Th. H., Nguyen, D. Kh. (2010b). Time-varying predictability in crude-oil markets: the case of GCC countries. *Energy Policy* 38, 8, 4371-4380.

Arouri, M. E. H., Lahiani, A., Nguyen, D.K. (2010a). Forecasting the Conditional Volatility of Oil Spot and Futures Prices with Structural Breaks and Long Memory Models. International Conference on Economic Modeling, (Istanbul, Turkey).

Baillie, R.T., King, M.L. (1996). Fractional Differencing and Long Memory Processes. *Journal of Econometrics* 73, 1, 1-3.

Bollerslev. T. (1986). Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics* 31, 3, 307-327.

Brock, W.A., Dechert, W.D., Sheinkman J.A. (1987). A Test of Independence Based on the Correlation Dimension. Working paper, University of Wisconsin at Madison, University of Houston, and University of Chicago, No. 8702, PP. 1-38.

Charles, A., Darné, O. (2009). The efficiency of the crude oil markets: Evidence from variance ratio tests. *Energy Policy* 37, 11, 4267-4272.

Cheong, C.W. (2009). Modeling and Forecasting Crude Oil Markets Using ARCH-Type Models. *Energy Policy* 37, 2346–2355.

Choi, K., Hammoudeh, S. (2009). Long Memory in Oil and Refined Products Markets. *the Energy Journal* 30, 2, 97-116.

Ellner, S., Gallant, A. R., McGaffrey, D. F., Nychka, D. (1991). Convergence Rates and Data Requirements for the Jacobian-Based Estimates of Lyapunov Exponents from Data. *Physics Letters* 15, 357–363.

Engle. R. F. (1982). Autoregressive Conditional Heteroskedadticity with Estimates of the Variance of UK Inflation. *Econometrica* 50, 4, 987-1008.

Erbil, N. (2011). Is Fiscal Policy Procyclical in Developing Oil-Producing Countries?. IMF Working Paper 171, 1-32.

Farzanegan, M., Markwardt, G. (2009). The Effect of Oil Price Shocks on Iranian Economy. *Energy Economics* 31, 134-151.

Geweke, J., Porter-Hudak, S. (1983). The Estimation and Application of Long-Memory Time Series Models. *Journal of Time Series Analysis* 4, 221–238.

Granger. C.W.J. and R. Joveux. 1980. An introduction to long-memory time series models and fractional differencing. *Journal of Time Series Analysis* 1, 15-39.

Haidar, I., Wolff, C. R. (2011). Forecasting crude oil price (revised). In the proceeding of the 30th USAEE Conference, Washington D.C. USA. 9-12 Octorber.

Homayouni, N., Amiri, A. (2011). Stock price prediction using a fusion model of wavelet, fuzzy logic and ANN. International Conference on E-business, Management and Economics, IPEDR Vol. 25, IACSIT Press, Singapore, 277-281.

Hosking, J.R.M. (1981). Fractional differencing. *Biometrika* 68, 165–176.

Huang, S.C. (2010). Return and Volatility Contagions of Financial Markets over Difference Time Scales. *International Research Journal of Finance and Economics* 42, 140-148.

Hurst, H.R. (1951). Long-Term Storage in Reservoirs. *Transactions of the American Society of Civil Engineers* 116, 770-799.

Kang, S.H., Cheong, C., Yoon, S.M. (2011). Structural Changes and Volatility Transmission in Crude Oil Markets. *Physica* A 390, 4317–4324.

Karim, S.A.A, Karim, B.A., Ismaeil, M.T., Hasan, M.K., Sulaiman, J. (2011). Application of Wavelet Method in Stock Exchange Problem. *Journal of Applied Science* 11, 8, 1131-1135.

Komijani, A., Gandali alikhani, N., Naderi, E. (2013). The Long-run and Short-run Effects of Crude Oil Price on Methanol Market in Iran. *International journal of Energy and Policy* 3, 1, 43-50.

Lineesh, M.C., John, C.J. (2010). Analysis of Non-Stationary Time Series using Wavelet Decomposition. *Nature and Science* 8, 1, 53-59.

Lo, A. W. and MacKinlay, A. C. (1988). Stock market prices do not follow random walks: evidence from a simple specification test. *Review of Financial Studies* 1, 1, 41-66.

Lorenz, H.W. (1989). Nonlinear Dynamical Economics and Chaotic Motion. New York: Springer- Verlag.

Mehrara, M., Mohaghegh, M. (2011). Macroeconomic Dynamics in the Oil Exporting Countries: A Panel VAR Study. *International Journal of Business and Social Science* 2, 21, 288-295.

Mehrara, M., Oskuei, K.N. (2007). The sources of macroeconomic fluctuations in oil exporting countries: A comparative study. *Economic Modeling* 24, 365–379.

Mohammadi, H., Su, L. (2010). International evidence on crude oil price dynamics: Applications of ARIMA-GARCH models. *Energy Economics* 32, 1001–1008.

Mostafaei, H., Sakhabakhsh, L. (2011). Modeling and Forecasting of OPEC Oil Prices with ARFIMA Model. *International Journal of Academic Research* 3, 1, Part Iii.

Ozer, G., Ertokatli, C. (2010). Chaotic processes of common stock index returns: An empirical examination on Istanbul Stock Exchange (ISE) market. *African Journal of Business Management* 4, 6, 1140-1148.

Prado. S, (2011). Free Lunch in the Oil Market: A Note on Long Memory. Working Paper, No.23.

Prokhorov, A.B. (2008). Nonlinear Dynamics and Chaos Theory in Economics: A Historical Perspective. *Quantile* 4, 1-27.

Reis, M.S., Saraiva, P.M., Bakshi, B.R. (2009). Denoising and Signal-to-Noise Ratio Enhancement: Wavelet Transform and Fourier Transform. Comprehensive Chemometrics, 25-55.

Takens, F. (1981). Detecting strange attractors in turbulence. Dynamical Systems and Turbulence. Lecture Notes in Mathematics Mathematics 898. Berlin: Springer, 366–381.

Vo, M. (2011). Oil and Stock Market Volatility: A Multivariate Stochastic Volatility Perspective. *Energy Economics* 33, 956–965.

Wang, y., Wu, c., Wei, y. (2011). Can GARCH-Class Models Capture Long Memory in WTI Crude Oil Markets?. *Economic Modeling* 28, 921–927.

Wei, Y., Wang, Y., Huang, D. (2010). Forecasting Crude Oil Market Volatility: Further Evidence Using GARCH-Class Models. *Energy Economics* 32, 1477–1484.

Williams, B. (2005). Trading Chaos: Applying Expert Techniques To Maximize Your Profits Press. John Wiley & Sons, 265 P.

Wolf, A., Swift, J., Swinney, H., Vastano, J. (1985). Determining Lyapunov Exponents from Time Series. *Physica* 16, 285-317.