

The two sides of envy

Boris Gershman

Brown University

17. October 2011

Online at <http://mpra.ub.uni-muenchen.de/34168/> MPRA Paper No. 34168, posted 17. October 2011 16:42 UTC

The Two Sides of Envy[∗]

Boris Gershman† Brown University

First Draft: September 2010 This Draft: October 2011

Abstract

The two sides of envy, destructive and competitive, give rise to qualitatively different equilibria, depending on the economic, institutional, and cultural environment. If inequality is high, property rights are poorly protected, and social comparisons are strong, society is likely to be in the "fear equilibrium," in which better endowed agents restrain their efforts to prevent destructive envy of the relatively poor. In the opposite case, the standard "keeping up with the Joneses" competition arises, and individuals satisfy their positional concerns through suboptimally high efforts. The different nature of these equilibria leads to strikingly contrasting effects of envy on economic performance. From a welfare perspective, the adoption of better institutions that move the economy away from the low-output fear equilibrium may not be Pareto improving since it aggravates the negative consumption externality. In a dynamic setting, envy-avoidance behavior, dictated by the destructive side of envy, reduces inequality and paves the way to emulation, driven by its competitive side. The paper provides a foundation for understanding the evolving role of envy in the process of development.

Keywords: Envy, Fear of envy, Inequality, Property rights, Relative standing

JEL Classification Numbers: D31, D62, D74, O10, O43, Z13

[∗] I am grateful to Oded Galor, Peter Howitt, and David Weil for their guidance and advice. Quamrul Ashraf, Pedro Dal Bó, Geoffroy de Clippel, Mana Komai, Mark Koyama, Nippe Lagerlöf, Ross Levine, Glenn Loury, Michael Ostrovsky, Louis Putterman, Eytan Sheshinski, Ilya Strebulaev, Holger Strulik, and Anand Swamy provided valuable comments. I also would like to thank seminar participants at Brown University, New Economic School, SEA 80th annual conference in Atlanta, Moscow State University, and Williams College.

[†]Department of Economics, Brown University, Box B, 64 Waterman St, Providence, RI 02912 (e-mail: Boris Gershman@brown.edu).

1 Introduction

In recent years it has become increasingly apparent that variations in cultural characteristics contribute to differential development paths across the globe. Trust, religion, family ties, and risk attitudes are among the attributes shown to have profound effects on eco-nomic outcomes.^{[1](#page-2-0)} This paper takes a new look at the role of envy, another prominent cultural force guiding human behavior.

Although concern for relative standing is widely recognized as a salient feature of individuals interacting in a society, there is no agreement on how it affects the economy, either directly, through its impact on incentives to work and invest, or indirectly, through its connections to institutions and culture.^{[2](#page-2-1)} In some parts of the world people engage in conspicuous consumption and overworking, driven by competition for status, while in others they hide their wealth and underinvest, constrained by the fear of malicious envy.

This paper develops a unified framework capturing the qualitatively different equilibria that can emerge in the presence of concern for relative standing, depending on the economic, institutional, and cultural environment. It sheds new light on the implications of positional concerns for economic performance and social welfare, and adds a new perspective to the analysis of redistributive mechanisms and the dynamics of wealth inequality.

Throughout the paper envy is taken to be a characteristic of preferences that makes people care about how their own consumption level compares to that of their reference group. This operational definition implies that envy can be satisfied in two major ways: by increasing own outcome (competitive envy) and by decreasing the outcome of the reference group (destructive envy).^{[3](#page-2-2)} Given the two sides of envy, individuals face a fundamental trade-off. On the one hand, they strive for higher relative standing. On the other hand, they want to avoid the destructive envy of those falling behind. The way this trade-off is resolved shapes the role of envy in society.

¹For a state-of-the-art overview see Fernández (2011) and other chapters in Benhabib et al. (2011).

²A number of evolutionary theoretic explanations have been proposed for why people care about relative standing, see Hopkins (2008, section 3) and Robson and Samuelson (2011, section 4.2) for an overview. Evidence in support for positional concerns is abundant and comes from empirical happiness research (Luttmer, 2005), experimental economics (Zizzo, 2003; Rustichini, 2008), neuroscience (Fliessbach et al., 2007), and surveys (Solnick and Hemenway, 2005; Clark and Senik, 2010); see Clark et al. (2008, section 3) and Frank and Heffetz (2011, section 3) for an overview.

³An alternative option is to drop out of competition for status (Banerjee, 1990; Barnett et al., 2010). Yet another possibility is to redefine the reference group, see Falk and Knell (2004) for a model with endogenous formation of reference standards.

The basic model is set up as a simple two-stage dynamic game between two individuals who are each other's reference points. In the first stage of the game each individual exerts effort that, combined with endowment, yields an investment outcome, or potential output. In the second stage, the available time may be spent on the realization of own potential output and disruption of the other individual's production process. The optimal allocation of time between productive and destructive activities depends on the disparity of investment outcomes and the tolerance for inequality, the latter being determined endogenously by the level of property rights protection (effectiveness of destruction technology) and the strength of social comparisons. The unique equilibrium of the model can be of three different types that are broadly consistent with the conflicting evidence on the role of envy from around the world.

If the initial inequality is low and/or the tolerance for inequality is high, the familiar "keeping up with the Joneses" (KUJ) equilibrium arises. In this case individuals compete peacefully for their relative standing, and the consumption externality leads to high effort and output. These are typical characteristics of a consumer society prominently documented for the U.S. by Schor (1991) and Matt (2003).

Higher inequality and/or lower tolerance for inequality may lead to the "fear equilibrium," in which the better endowed individual anticipates destructive envy and prevents it by restricting effort at the investment stage. Such envy-avoidance behavior is standard for traditional agricultural communities in developing economies, where the fear of inciting envy discourages production (Foster, 1979; Dow, 1981). It is also typical for the emerging markets like China and Russia, where potential entrepreneurs are reluctant to start new business that can provoke envious retaliation of others (Mui, 1995).

Finally, if the tolerance for inequality is very low and/or the distribution of endowments is highly unequal, a "destructive equilibrium" arises, in which actual conflict takes place and time is wasted unproductively to satisfy envy. Allowing for transfers replaces destructive activities of the poor with "voluntary" sharing of the rich, reflecting the evidence on the fear-of-envy motivated redistribution in peasant societies of Latin America (Cancian, 1965), Southeast Asia (Scott, 1976), and Africa (Platteau, 2000).

The qualitatively different nature of these equilibria leads to opposite effects of positional concerns on aggregate economic performance. In the KUJ equilibrium envy increases effort and output by intensifying status competition, while in the fear equilibrium it aggravates the already binding "fear of envy constraint" further discouraging investment.

Another striking difference between the alternative equilibria becomes clear in a comparative welfare analysis. While better institutions can move the society from the lowoutput fear equilibrium to the high-output KUJ equilibrium, such change need not be welfare enhancing if it triggers the "rat race" competition. That is, individuals may be happier in the fear equilibrium, since the threat of destructive envy constrains the suboptimally high efforts induced in the KUJ equilibrium.

A dynamic version of the model explores the transitions between equilibria. It shows how the implications of envy change endogenously as the society evolves from destructive equilibrium to the long-run KUJ steady state, following the dynamics of inequality. In this setting, institutional and cultural parameters not only affect economic outcomes at a given point in time but also determine the division of the transition process into three "envy regimes" qualitatively corresponding to the equilibria of the static model. Initially, destructive envy and the fear of it reduce inequality by discouraging productive effort of the better endowed individual or destroying part of his output. This leads to more equal investment opportunities for next generations, up to the point when it becomes optimal to compete productively. Thus, envy-avoidance behavior, dictated by the destructive side of envy, eventually paves the way to emulation, driven by its competitive side. This transition can be delayed or accelerated by factors, such as religion, political ideology, and legal institutions, that affect the strength of social comparisons and the relative attractiveness of productive and destructive effort.

The model is designed to be "scale-free," that is, inequality, but not the scale of production, is the key variable of the analysis. Nevertheless, it provides a foundation for understanding the evolving role of envy in the process of development. Specifically, Gershman (2011b) incorporates the basic model of this paper in a simple endogenous growth framework, in which resources are limited and productivity is driven by learning-by-doing and knowledge spillovers. Rising productivity expands the production possibilities frontier, and the economy experiences an endogenous transition from the fear equilibrium to the KUJ equilibrium. At the same time, the type of equilibrium at each point feeds back into the growth rate of productivity by constraining or spurring investment. Such analysis explicitly connects the alternative types of equilibria to different stages of economic development, consistent with the evidence of the following section, and qualifies some results of the static model by accounting for the long-run dynamics.

The rest of the paper is organized as follows. Section [2](#page-5-0) summarizes the evidence on the two sides of envy. Section [3](#page-7-0) lays out the basic model and examines comparative statics. Section [4](#page-18-0) conducts the welfare analysis. Section [5](#page-21-0) considers the implications of allowing voluntary transfers. Section [6](#page-26-0) presents the dynamic version of the model. Section [7](#page-31-0) concludes. All proofs are collected in the Appendix.

2 Evidence on the Two Sides of Envy

As wealth increases, the continued stimulus of emulation would make each man strive to surpass, or at least not fall below, his neighbours, in this.

Richard Whately (1831) Introductory Lectures on Political Economy The affluence of the rich excites the indignation of the poor, who are often both driven by want, and prompted by envy, to invade his possessions.

> Adam Smith (1776) The Wealth of Nations

As exemplified by the quotations above, the distinction between competitive and destructive sides of envy in economics goes back to the classicists. Since then, this dichotomy has been discussed extensively by anthropologists (Foster, 1972), philosophers (D'Arms and Kerr, 2008), political scientists (Fernández de la Mora, 1987), psychologists^{[4](#page-5-1)} (Smith and Kim, 2007), sociologists (Schoeck, 1969; Clanton, 2006), and recently got a renewed interest from economists (Elster, 1991; Zizzo, 2008; Mitsopoulos, 2009).

Evidence on the destructive potential of envy comes primarily from the developing world. For instance, recent data from the Afrobarometer surveys^{[5](#page-5-2)} indicate that envy is perceived as an important source of conflict. Respondents were asked the following question: "Over what sort of problems do violent conflicts most often arise between different groups in this country?" They were offered to choose the three most important problems from a list of several dozens of possible answers (such as religion, ethnic differences, economic issues). Remarkably, in nine countries, where the list featured "envy/gossip" as a potential source of violent conflict, an average of over 9% of respondents have picked it as one of the three most important reasons.

The threat of destructive envy naturally leads to a rational fear of it that motivates envy-avoidance behavior. Numerous examples of such behavior come from anthropological research on small-scale agricultural communities around the world. According to Foster (1972), in peasant societies "a man fears being envied for what he has, and wishes to protect himself from the consequences of the envy of others" (p. 166). People are reluctant to exert effort or innovate since they expect sanctions in the form of plain destruction, forced redistribution, or envy-driven supernatural punishment.^{[6](#page-5-3)}

⁴Certain psychological approaches treat benign and malicious envy as two separate emotions (van de Ven et al., 2009). The present theory is crucially different in that the motive, envy, is the same, but its manifestation is an equilibrium outcome.

⁵Second round, 2002–2003; raw data available at <http://www.afrobarometer.org>.

⁶The rational fear of envy becomes curiously embedded in culture. The term "institutionalized envy" coined by Wolf (1955) summarizes the set of cultural control mechanisms related to the fear of envy including gossip, fear of witchcraft, and the evil eye belief. See Gershman (2011a) for more details.

In Tzintzuntzan, a Mexican village, people do not go beyond minimum conveniences even if they can afford it (Foster, 1979). A relatively wealthy peasant refuses to lay a cement or tile floor, or cut windows in his rooms since he is "frankly afraid people will envy him" (p. 154). Similarly, the "envious hostility of neighbors" discourages the villagers of Northern Sierra de Puebla, also in Mexico, to produce food beyond subsistence (Dow, 1981). In peasant communities of Southeast Asia people work "in large measure through the abrasive force of gossip and envy and the knowledge that the abandoned poor are likely to be a real and present danger to better-off villagers" (Scott, 1976, p. 5).

The fear of destructive envy is not an exclusive feature of simple agricultural societies. Mui (1995) focuses on the large industrial economies, Russia and China, in the process of transition to the free market. He brings up evidence on emerging cooperative restaurants and shops in the Soviet Union being regularly attacked by people resenting the success of their owners. Mui then tells a similar story of a Chinese peasant whose successful entrepreneurship provoked the envious neighbors to steal timber for his new house and kill his farm animals. "I dare not work too hard to get rich again" was his comment. The fear of envy may be a particularly serious issue in societies with socialist experience characterized by the ideology of material egalitarianism and neglect of private property rights, both reducing the tolerance for inequality.

A very different strand of evidence comes from the modern consumer societies, in which the competitive side of envy is predominant and the fear of destructive envy vanishes. As observed with dismay by the Christian Advocate newspaper (1926), consumer society offers a new version of the tenth commandment: "Thou shalt not be outdone by thy neighbor's house, thou shalt not be outdone by thy neighbor's wife, nor his manservant, nor his car, nor anything – irrespective of its price or thine own ability – anything that is thy neighbor's" (cited in Matt, 2003, p. 4). Benign consumer envy becomes the target of producers and salesmen (Belk, 2008).

Under "keeping up with the Joneses" envy acts a stimulus to work harder to be able to match the spending patterns of the reference group.^{[7](#page-6-0)} According to Schor (1991), the steady increase in work hours in the U.S. since the early 1970s is primarily due to the competitive effects of social comparisons. The positive impact of relative standing concerns on labor supply in developed economies is supported by rigorous empirical research. Neumark and Postlewaite (1998) study the employment decisions of women using data from the U.S.

⁷As argued by Frank (2007), hard work is not the only way to keep up with the Joneses in the consumption domain, the most worrisome alternative being reduced savings.

National Longitudinal Survey of Youth and find evidence that those are partly driven by relative income. Bowles and Park (2005) use the aggregate data from ten OECD countries over the period 1963–1998 to show that greater earnings inequality is associated with longer work hours. They attribute this finding to the Veblen effect of the consumption of the rich on the less wealthy, that is, emulation.

Overworking caused by competitive envy and the welfare consequences of the KUJ-type competition have become the subject of recent research in the field of happiness economics (Graham, 2010). In particular, accounting for the relative standing concern is one of the keys to understanding why happiness and material well-being might not always go together. In the words of Schor (1991, p. 124), those caught in a race for relative standing "would be better off with more free time; but without cooperation, they will stick to the long hours, high consumption choice."

The controversial evidence on the role of envy in society poses a number of questions. How does the same feature, concern for relative standing, give rise to these qualitatively different cases? Under what conditions do the fear of envy or the KUJ competition emerge? What are the implications of social comparisons for economic performance and welfare? The first step towards thinking about these issues is to construct a unified framework reconciling the above evidence.

3 The Basic Model

3.1 Environment

Consider two agents that may be thought of as representatives of equal-sized homogeneous groups of people. They differ only in the amount of broadly defined initial endowments, K_i , $i = 1, 2$. In particular,

$$
K_1 = \lambda K, \qquad K_2 = (1 - \lambda)K,\tag{1}
$$

where K is the total endowment in the economy and parameter $0 < \lambda < 1/2$ captures the degree of initial inequality: Agent 1 is "poor" and Agent 2 is "rich." They interact in the following two-stage game (see Figure [1\)](#page-8-0).

In the first stage of the game both agents invest, for instance, in education or innovation. Specifically, each agent exerts effort, $L_i \geq 0$, to produce an investment outcome, Y_i , according to

$$
Y_i = F(K_i, L_i) = K_i L_i, \quad i = 1, 2.
$$
\n(2)

Figure 1: Timing of events in the envy game.

This outcome may be thought of as a factor of production like human or physical capital, intermediate product, or potential output in general, to be realized in the second stage.

In the second stage, each agent has a unit of time that has to be allocated between realization of own potential output (production) and disruption of the other agent's production process (destruction). Clearly, in this setup the only reason for spending time on destruction is envy.^{[8](#page-8-1)} If Agent i allocates a fraction $d_i \in [0,1]$ of his time to disrupt the productive activity of Agent j, the latter retains only a fraction p_j of his final output, where

$$
p_j = p(d_i) = \frac{1}{1 + \tau d_i}, \quad i, j = 1, 2, \ i \neq j.
$$
 (3)

As will become clear, the formulation with time allocation makes the model scale-free: optimal destruction intensity will be a function of the inequality (but not the scale) of first-stage outcomes, which captures the essence of destructive envy.^{[9](#page-8-2)} The function $p(d_i)$ has standard properties: it is bounded, with $p(0) = 1$, decreasing, and convex (Grossman and Kim, 1995). Parameter $\tau > 0$ measures the effectiveness of destruction technology and may capture the overall level of private property rights protection. In particular, property rights are secure if τ is low.

Time $1 - d_i$ is spent on the realization of potential output Y_i yielding the final output $(1-d_i)Y_i$. Since only a fraction $p(d_j)$ of this output is retained, the consumption level of Agent i is given by

$$
C_i = (1 - d_i)p(d_j)Y_i, \quad i, j = 1, 2, i \neq j.
$$
\n(4)

Finally, payoffs are generated. The utility function is of the following form:

$$
U^i = U(C_i, C_j, L_i) = v(C_i - \theta C_j) - e(L_i) = \frac{(C_i - \theta C_j)^{1-\sigma}}{1-\sigma} - L_i,
$$
\n(5)

⁸The model can be generalized to incorporate protection and theft, instead of pure destruction, without affecting the qualitative results. The implications of defense in a model of appropriation were examined by Grossman and Kim (1995; 1996).

⁹If, apart from time, material resources or extra effort are needed to destroy, the scale effect may arise, which does not change the qualitative results of the basic model, but undermines analytical tractability.

where $i, j = 1, 2, i \neq j, \sigma > 1$, and $0 < \theta < 1$.^{[10](#page-9-0)} It is increasing in own consumption while decreasing in the other agent's consumption and own effort. Agents are each other's reference points, which is natural in the setup with two individuals, and parameter θ captures the strength of concern for relative standing. This utility function features additive comparison which is one of the two most popular ways to model envy, the other being ratio comparison.^{[11](#page-9-1)} Overall, the form of the utility function is identical to that in Ljungqvist and Uhlig (2000), but in their model the reference point for each agent is the average consumption in the population.

A crucial property of this utility function is that the cross-partial derivative U_{12} is positive, leading to the "keeping up with the Joneses" kind of behavior, or emulation, due to complementarity between own and reference consumption. Clark and Oswald (1998) call this a "comparison-concave" utility (since v is concave). Intuitively, individuals are willing to match an increase in consumption of their reference group. The reason is that an increase in C_i reduces the consumption rank of individual i and, under concave comparisons, raises the marginal utility of his own consumption.

Finally, the payoff function implies that consumption is a positional good, while leisure (disutility of effort) is not. This hypothesis has been consistently advocated by Frank (1985; 2007) and finds support in the data (Solnick and Hemenway, 2005).

3.2 Equilibria

The dynamic structure of the model makes subgame perfect equilibrium a natural solution concept. The model is solved backwards, starting at stage 2, when effort is sunk. Given the outcomes of the investment stage, Y_1 and Y_2 , Agent i chooses the intensity of destruction, d_i , to maximize his payoff:

$$
v((1-d_i)p(d_j)Y_i - \theta(1-d_j)p(d_i)Y_j) \longrightarrow \max_{d_i} \qquad s.t. \quad 0 \leq d_i \leq 1,
$$
 (6)

where $i, j = 1, 2, i \neq j$.

¹⁰Assume for simplicity that $U^i = -\infty$ whenever $C_i \leq \theta C_j$. Since effort is unbounded, this will never be the case in equilibrium. The assumption on the elasticity of marginal utility with respect to relative consumption, σ , is a convenient regularity condition that is not crucial for the main results; see also the discussion in section [3.3.](#page-15-0) Linearity in effort is assumed for analytical tractability.

¹¹Additive comparison was used, among others, by Knell (1999), Ljungqvist and Uhlig (2000) and Mitsopoulos (2009). Boskin and Sheshinski (1978) and Carroll et al. (1997) are examples of models with ratio comparison. Clark and Oswald (1998) examine the properties of both formulations. Gershman (2011a) shows that the qualitative results of section [3](#page-7-0) can be obtained in a model with ratio comparison.

This yields the optimal second-stage $respose¹²$ $respose¹²$ $respose¹²$:

$$
d_i^* = \begin{cases} 0, & \text{if } Y_i/Y_j \ge \tau \theta (1 - d_j)(1 + \tau d_j); \\ \frac{1}{\tau} \cdot \left(\sqrt{\tau \theta \frac{Y_j}{Y_i} (1 - d_j)(1 + \tau d_j)} - 1 \right), & \text{if } Y_i/Y_j < \tau \theta (1 - d_j)(1 + \tau d_j). \end{cases}
$$
(7)

Several features are to be noticed about this expression. First, without envy $(\theta = 0)$ there is no destruction. If envy is present, the decision about engaging in destruction depends on the inequality of investment outcomes, Y_i/Y_j . If Y_i is high enough compared to Y_j , Agent *i* finds it optimal to refrain from destruction. Otherwise, it is optimal to engage in destruction and its intensity is increasing in inequality, effectiveness of destructive technology, and the strength of positional concerns.

The product $\tau\theta$ is an endogenous measure of tolerance for inequality. Given the ratio of investment outcomes, destructive envy is more likely to occur if positional concerns are strong (large θ) and property rights are poorly protected (large τ). Hence, large $\tau\theta$ means low tolerance for inequality. If $d_j = 0$, the product $\tau \theta$ provides a critical inequality level, beyond which Agent i chooses to engage in destruction. Assume from now on that $\tau\theta < 1$, that is, in the case when $d_j = 0$, there is some tolerance for inequality and $d_i^* = 0$ if $Y_i \geq Y_j$. Given this assumption, the agent with higher investment outcome will never attack in the second-stage equilibrium, as established in the following lemma.^{[13](#page-10-1)}

Lemma 1. (Second-stage equilibrium). Let $Y_2 > Y_1$. Then, in the second-stage equilibrium $d_2^* = 0$ and d_1^* is given by

$$
d_1^* = \begin{cases} 0, & \text{if } Y_1/Y_2 \geqslant \tau \theta; \\ \frac{1}{\tau} \cdot \left(\sqrt{\tau \theta \frac{Y_2}{Y_1}} - 1 \right), & \text{if } Y_1/Y_2 < \tau \theta. \end{cases}
$$
 (8)

Hence, Agent 2 retains a fraction p_2^* of his final output, where

$$
p_2^* = p(d_1^*) = \min\left\{\sqrt{\frac{Y_1}{Y_2} \cdot \frac{1}{\tau \theta}}, 1\right\} \tag{9}
$$

and is strictly decreasing in τ , θ , and Y_2/Y_1 , if destruction takes place.

¹²Clearly, $d_i^* = 1$ is never optimal, see footnote [10.](#page-9-0) Since the first-stage efforts guarantee that $C_i - \theta C_j$ 0 in equilibrium, $d_i^* < 1$ is the only relevant case.

¹³This assumption rules out the case in which individuals with higher relative standing engage in destructive activities to push their relative position even further up. While such behavior is a theoretical possibility, the case considered in this paper is more intuitive and consistent with the anecdotal evidence on destructive envy from section [2.](#page-5-0)

It can be shown that in the subgame perfect equilibrium of the game the initially better endowed Agent 2 will always have a higher investment outcome, that is, $Y_2 > Y_1$, and so, given the result above, he will never spend time on destructive activities. For expositional simplicity we proceed as if there is a "predator and prey" type relationship (Grossman and Kim, 1996) between Agent 1 and Agent 2 from the outset, so that only the initially poor Agent 1 is allowed to engage in destruction at stage 2. Such assumption is made without loss of generality and underlines the asymmetric equilibrium roles played by the agents in the general formulation of the model. 14 14 14

Agents are forward-looking and anticipate the optimal second-stage actions when making their first-stage decisions. Although Agent 2 is passive at stage 2, he is perfectly aware of how his investment outcome affects d_1^* and takes this into account at stage 1:

$$
U^{2} = v(p(d_{1}^{*})Y_{2} - \theta(1 - d_{1}^{*})Y_{1}) - Y_{2}/K_{2} \longrightarrow \max_{Y_{2}} \tag{10}
$$

For technical reasons, it is easier to analyze the best responses of both agents in terms of their consumption levels, C_i , rather than the first-stage outcomes Y_i . Note that these are different only if destruction actually takes place. In that case there is a one-to-one mapping between Y_i and C_i .

Lemma 2. (BR of Agent 2). The best-response function of Agent 2, $BR_2 \equiv C_2^*(C_1)$, has the following form:

$$
C_2^*(C_1) = \begin{cases} K_2^{1/\sigma} + \theta C_1, & \text{if } C_1 \ge \tilde{C}_1; \\ C_1 \cdot \frac{1}{\tau \theta}, & \text{if } \tilde{C}_1 \le C_1 < \tilde{C}_1; \\ C_2^d(C_1), & \text{if } C_1 < \hat{C}_1, \end{cases}
$$
(11)

where

$$
\widetilde{C}_1 \equiv \frac{\tau \theta}{1 - \tau \theta^2} K_2^{1/\sigma}, \quad \widehat{C}_1 \equiv \frac{\tau \theta}{1 - \tau \theta^2} \left(\frac{1 + \theta^2}{2} K_2\right)^{1/\sigma},
$$

and $C_2^d(C_1)$ is implicitly given by

$$
C_2 - \theta C_1 = \phi \cdot \left(\frac{C_1 + \theta C_2}{C_2}\right)^{1/\sigma}, \quad \phi \equiv \left(\frac{1 + \theta^2}{2\theta(1+\tau)} K_2\right)^{1/\sigma}.
$$
 (12)

The function $C_2^d(C_1)$ is strictly increasing and concave.

The best response function of Agent 2, depicted in the left panel of Figure [2,](#page-12-0) consists of three segments that correspond to the following cases. If the output of Agent 1 is high

¹⁴For further analysis of the general case see the Appendix.

enough, Agent 2 is not constrained by destructive envy and displays the standard KUJ behavior. If the output of Agent 1 is in the intermediate range, the best response of Agent 2 is to avoid destructive envy by constraining his own effort at stage 1. This is the "fear segment," in which Agent 2 exerts the maximum possible effort that prevents destruction at stage 2. Finally, if the output of Agent 1 is low, it becomes too costly for Agent 2 to avoid destruction by self-restraint and the optimal action is to allow some of it.

Figure 2: Best responses of Agent 2 (left) and Agent 1 (right).

Agent 1 is also forward-looking and knows his own optimal second-stage behavior when optimizing at the investment stage:

$$
U^{1} = v((1 - d_{1}^{*})Y_{1} - \theta p(d_{1}^{*})Y_{2}) - Y_{1}/K_{1} \longrightarrow \max_{Y_{1}} \tag{13}
$$

Lemma 3. (BR of Agent 1).^{[15](#page-12-1)} The best-response function of Agent 1, BR₁ $\equiv C_1^*(C_2)$, has the following form:

$$
C_1^*(C_2) = \begin{cases} K_1^{1/\sigma} + \theta C_2, & \text{if } C_2 \leq \widehat{C}_2; \\ C_1^d(C_2), & \text{if } C_2 > \widehat{C}_2, \end{cases}
$$
(14)

where

$$
\widehat{C}_2 \equiv \frac{K_1^{1/\sigma}}{\theta(\tau - 1)}
$$

and $C_1^d(C_2)$ is implicitly given by

$$
C_1 - \theta C_2 = \psi \cdot \left(\frac{C_1}{C_1 + \theta C_2}\right)^{1/\sigma}, \quad \psi \equiv \left(\frac{1+\tau}{\tau} K_1\right)^{1/\sigma}.
$$
 (15)

The function $C_1^d(C_2)$ is strictly increasing and convex.

¹⁵As follows from the proof of Lemma [3,](#page-12-2) if $\tau < 1$, it is never optimal for Agent 1 to engage in destruction, and catching up is always the best response. To focus on the largest set of possible equilibria, the assumption $\tau > 1$ is maintained throughout the paper.

The best response function of Agent 1, depicted in the right panel of Figure [2,](#page-12-0) consists of two segments. If the output of Agent 2 is low enough, Agent 1 prefers to catch up peacefully without causing destruction. This is the KUJ case. If, however, Y_2 is too high, second-stage destruction substitutes for some of the first-stage investment.

Depending on parameter values, the envy game may end up in three different types of equilibria which we consider in turn.

1. Keeping up with the Joneses equilibrium (KUJE). This equilibrium would always emerge if destruction were not possible (right panel of Figure [3\)](#page-13-0). Its features are well-known in economics literature and have been formally analyzed by Frank (1985) and Hopkins and Kornienko (2004), among others. In the present model it arises when the "competitive segments" of the best response functions intersect (left panel of Figure [3\)](#page-13-0) and is given by

$$
C_i^{\text{KUJ}} = Y_i^{\text{KUJ}} = \frac{K_i^{1/\sigma} + \theta K_j^{1/\sigma}}{1 - \theta^2}, \quad i, j = 1, 2, \quad i \neq j. \tag{16}
$$

Figure 3: KUJE in the envy game with (left) and without (right) the second stage.

2. Fear equilibrium (FE). In this case the richer agent constrains his first-stage investment to avoid destructive envy in the second stage of the game. He consumes right at the point that makes it not optimal for the poorer agent to engage in destruction. This equilibrium resembles the fear of envy documented in many developing societies, as discussed in section [2.](#page-5-0)^{[16](#page-13-1)} It arises when the "fear segment" of $BR₂$ intersects $BR₁$ at the kink point (left panel of Figure [4\)](#page-14-0) and is given by

$$
C_1^{\mathcal{F}} = Y_1^{\mathcal{F}} = \frac{\tau K_1^{1/\sigma}}{\tau - 1}, \quad C_2^{\mathcal{F}} = Y_2^{\mathcal{F}} = \frac{K_1^{1/\sigma}}{\theta(\tau - 1)}.
$$
 (17)

¹⁶Mui (1995) constructs a theoretical framework in which technological innovation may not be adopted in anticipation of destructive envy. The intuition of the fear equilibrium is similar, except that here the fear of envy affects the intensive margin by discouraging effort.

3. Destructive equilibrium (DE). In this equilibrium it is not optimal for Agent 2 to reduce effort to completely avoid destructive envy and he accommodates some of it. Equilibrium consumption levels are implicitly given by the system of equations

Figure 4: Fear (left) and destructive (right) equilibria.

The right panel of Figure [4](#page-14-0) shows a generic destructive equilibrium, which occurs at the intersection of "destructive segments." It follows from the properties of $C_2^d(C_1)$ and $C_1^d(C_2)$ that they can only intersect once (see the proof of Proposition [1\)](#page-14-1).

The following proposition provides a full taxonomy of equilibria in terms of the three main parameters of the model: λ , the initial level of inequality (economic environment), τ , the degree of property rights protection (institutional environment), and θ , the strength of social comparisons (cultural environment).

Proposition 1. (Equilibrium of the envy game). There exists a unique subgame perfect equilibrium of the envy game. Denote $k \equiv K_1/K_2$. Then, if $k \geq \tilde{k}$, it is the KUJ equilibrium [\(16\)](#page-13-2). If $\hat{k} \leq k < \tilde{k}$, it is the fear equilibrium [\(17\)](#page-13-3). Finally, if $k < \hat{k}$, it is the destructive equilibrium (18) . The threshold values for k are given by

$$
\tilde{k} \equiv \left[\frac{\theta(\tau - 1)}{1 - \tau \theta^2}\right]^\sigma, \quad \hat{k} \equiv \frac{(1 + \theta^2)}{2} \cdot \left[\frac{\theta(\tau - 1)}{1 - \tau \theta^2}\right]^\sigma < \tilde{k} < 1. \tag{19}
$$

This result can be expressed in terms of regions for λ , τ , and θ , with the relevant thresholds uniquely determined by [\(19\)](#page-14-3).

Given the setup of the game, this result is very intuitive. Other things equal, the standard KUJ equilibrium is more likely to emerge if the level of initial inequality is low

and/or the tolerance for inequality is high, that is, property rights are well-protected and social comparisons are not strong. In the opposite case, destructive envy is in action and either fear or destructive equilibrium arises. It is important to emphasize that the three parameters of interest jointly determine the type of equilibrium. For instance, just having low inequality is not enough to obtain the KUJE. If, at the same time, institutions are very weak (destructive technology is efficient) and/or relative standing concerns are very strong, the economy may still end up in the fear or destructive equilibrium.

3.3 Comparative statics

The next issue to address is how the parameters of interest affect economic performance (measured by final outputs) across different equilibria.

As follows from [\(16\)](#page-13-2), in the KUJ equilibrium outputs do not depend on τ since destructive envy is not binding. The effect of θ is straightforward: increasing the strength of relative concerns in the KUJ equilibrium acts as additional incentive to work, which leads to higher levels of effort and output.

The effect of raising λ (increasing equality) on aggregate output depends crucially on σ . Equations [\(1\)](#page-7-1) and [\(16\)](#page-13-2) imply that the total output is

$$
Y^{KUJ} = Y_1^{KUJ} + Y_2^{KUJ} = \frac{\lambda^{1/\sigma} + (1 - \lambda)^{1/\sigma}}{1 - \theta} K^{\frac{1}{\sigma}}.
$$
 (20)

Under baseline assumption $\sigma > 1$, it is a strictly concave and increasing function of λ . Concavity of output in endowments seems more natural than a kind of nondecreasing returns to scale that would emerge under $\sigma \leq 1$.

The total effect of redistribution on private outputs consists of two parts: wealth effects and comparison effects. Wealth effects are just the direct effects of making one agent poorer and the other richer. In case of increasing λ the wealth effect is positive for Agent 1 and negative for Agent 2. The total wealth effect on output is positive since under concave outputs the poor agent is more productive on the margin. Comparison effects reflect the fact that the reference group becomes poorer for Agent 1 and richer for Agent 2. Consequently, comparison effect is negative for Agent 1 and positive for Agent 2. The total comparison effect has the same sign as the total wealth effect. In particular, under $\sigma > 1$, the negative comparison effect on the output of the poor is outweighed by the positive comparison effect on the output of the rich.

Next, consider the fear equilibrium [\(17\)](#page-13-3). In this case the outcomes depend on τ , the measure of property rights protection. Raising τ (reducing the quality of institutions)

decreases the tolerance of Agent 1 for inequality and aggravates the "fear constraint" of Agent 2. This means that with higher τ Agent 2 has to produce less to avoid destructive envy which leads to lower total output given by

$$
Y^{\mathcal{F}} = Y_1^{\mathcal{F}} + Y_2^{\mathcal{F}} = \frac{1 + \tau \theta}{\theta(\tau - 1)} (\lambda K)^{\frac{1}{\sigma}}.
$$
 (21)

The effect of raising θ is similar since it, too, decreases the tolerance for inequality. This is in stark contrast with the role of positional concerns in the KUJ equilibrium. In the latter case it acts as additional incentive to work, while in the fear equilibrium it constrains productive effort by increasing the hazard of destructive envy. On the other hand, the effect of raising equality in the fear equilibrium is unambiguously positive, as it is in the KUJE. Increasing λ means a positive wealth effect for Agent 1 which translates into higher output of Agent 2. That is, redistribution from the rich to the poor increases investment and final output by alleviating the fear of envy constraint.

Destructive equilibrium is harder to analyze analytically. Multiple effects are at work which makes the aggregate comparative statics with respect to θ ambiguous. If inequality is high or tolerance for inequality is low, stronger envy leads to substantial destruction which may lower the consumption of Agent 2, as well as total final output, C (left panel of Figure [6\)](#page-17-0). In contrast, if the destructive environment is not severe (or σ , the catching-up propensity, is high), the stimulating effect of envy dominates (right panel of Figure [6\)](#page-17-0). Thus, comparative statics in the DE combines the features of the FE and the KUJE. Moreover, it can be shown that higher τ and lower λ unambiguously decrease total consumption. The following proposition summarizes the comparative statics results.

Proposition 2. (Comparative statics of the envy game).

1. KUJE:

- (a) $\partial Y/\partial \lambda > 0$; $\partial Y_1/\partial \lambda > 0$; $\partial Y_2/\partial \lambda > 0$ iff $\theta > [(1 \lambda)/\lambda]^{\frac{1 \sigma}{\sigma}}$.
- (b) $\partial Y/\partial \theta > 0$; $\partial Y_i/\partial \theta > 0$.
- (c) $\partial Y/\partial \tau = \partial Y_i/\partial \tau = 0.$

2. FE:

3. DE:

- (a) $\partial Y/\partial \lambda > 0$; $\partial Y_i/\partial \lambda > 0$;
- (b) $\partial Y/\partial \theta < 0$; $\partial Y_1/\partial \theta = 0$; $\partial Y_2/\partial \theta < 0$;
- (c) $\partial Y/\partial \tau < 0$; $\partial Y_i/\partial \tau < 0$.
- (a) $\partial C/\partial \lambda > 0$; $\partial C_1/\partial \lambda > 0$; $\partial C_2/\partial \lambda \leq 0$; (b) $\partial C/\partial \theta \leq 0$; $\partial C_1/\partial \theta > 0$; $\partial C_2/\partial \theta \leq 0$;
	- (c) $\partial C/\partial \tau < 0$; $\partial C_i/\partial \tau < 0$.

Figure 5: Inequality (left), property rights (right), and economic performance.

Figure 6: Concern for relative standing and economic performance.

Proposition [2](#page-16-0) makes clear that the mechanics of the qualitatively different equilibria of the envy game may imply contrasting comparative statics. Stronger envy is detrimental for economic performance in the fear equilibrium, increases total output under the KUJtype competition, and has an ambiguous effect in the DE. Higher inequality has a negative effect across all equilibria types. Well-protected private property rights are conducive to economic performance in both the FE and the DE and play no role beyond the KUJE threshold. Overall, Proposition [2](#page-16-0) establishes the relationship between aggregate economic performance and the parameters of the model, reflecting the economic, institutional, and cultural environment (see Figures [5](#page-17-1) and [6\)](#page-17-0).

The following section shows that consumption and happiness need not go together when relative standing is what matters, and hence, the established comparative statics results need to be challenged in a welfare analysis.

4 Welfare, property rights, and inequality

In this section we focus on the KUJE and the FE to conduct a comparative welfare analysis. We start by analyzing how an institutional change affects the welfare of individuals. Then we show how it is affected by ex-ante redistribution of endowments.

The connection between the emergence of property rights and externalities was famously considered by Demsetz (1967). He argued that an important function of property rights is to internalize externalities, and institutional change in this domain is intended to cope with new externality problems. In the context of the present theory it is the consumption externality that makes institutions matter. As shown below, it is not the emergence of property rights protection but its demise that may internalize this externality.

Consider the following thought experiment. Assume that the agents start in the fear equilibrium. Would they want to accept a lower value of τ that would move them to the KUJ equilibrium? The following proposition provides conditions under which the agents will (not) want to adopt such institutional change.

Proposition 3. Let $\hat{k} \leq k < \tilde{k}$, that is, agents are in the FE. Then, $\exists! \ \bar{\theta} \in (0,1)$ such that: 1) If $\theta > \overline{\theta}$, adopting a (lower) τ that would bring up the KUJ equilibrium is a Pareto worsening; 2) If $\theta \leq \bar{\theta}$, \exists ! $\bar{k}(\tau) \in [\hat{k}, \tilde{k})$, such that adopting such τ is a Pareto worsening if and only if $k > \bar{k}$. Furthermore, private outputs in the initial FE are always lower than those that would have been achieved by improving the property rights protection.

The intuition for this result is simple and revolves around the negative effect of positional externality on social welfare. As shown in the proof of the above proposition, Agent 1 always prefers the FE. In particular, he maintains the same level of relative consumption in the two equilibria, but the amount of effort he has to exert to do that is always lower in the FE. The rich agent prefers to stay in the FE if only if in the alternative KUJE he would have to work too hard to support his relative standing. That would be the case if either social comparisons are strong $(\theta \text{ is high})$ or inequality is low $(k \text{ is large})$. Under these conditions the FE is Pareto dominant since the fear of destructive envy restrains effort and curbs the consumption externality that otherwise leads to overworking in the KUJ equilibrium. Thus, worse property rights protection corrects the distortion caused by positional concerns.[17](#page-18-1)

¹⁷Curiously, this is akin to the effects of Pareto improving redistributive taxation schemes suggested in similar contexts by Boskin and Sheshinski (1978), Oswald (1983), and Frank (1985).

Figure 7: From FE to KUJE: Improving the property rights protection.

Figure [7](#page-19-0) illustrates the case, in which moving from FE to KUJE is a Pareto worsening (left panel) in spite of increased private outputs (right panel).^{[18](#page-19-1)} The result that both agents can be better off in an equilibrium with lower consumption is reminiscent of what Graham (2010) calls the "paradox of happy peasants and miserable millionaires" and the related research in happiness economics.^{[19](#page-19-2)} It is instructive to read Proposition [3](#page-18-2) in the "reverse" order, assuming that initially agents are in the KUJ equilibrium.

Proposition [3](#page-18-2)'. Let $k \geq \tilde{k}$, that is, agents are in the KUJE. Then, $\exists! \bar{\theta} \in (0,1)$ such that: 1) If $\theta > \bar{\theta}$, adopting a (higher) τ that would bring up the FE is a Pareto improvement; 2) If $\theta \leq \bar{\theta}$, \exists ! $\bar{k}(\tau) \in [\hat{k}, \tilde{k})$, such that adopting such τ is a Pareto improvement if and only if $k > \bar{k}$. Furthermore, private outputs in the initial KUJE are always higher than those that would have been achieved by distorting the property rights protection.

Taken literally, Proposition [3](#page-18-2)' may be seen as a formalization of a more than a centuryold argument raised by Veblen (1891) in an attempt to explain the support for socialist movement, specifically, for the abolition of private property rights. Veblen argued that such support was driven by the competitive envy of a consumer society (p. 65–66): "The ground of the unrest with which we are concerned is, very largely, jealousy, – envy, if you choose: and the ground of this particular form of jealousy that makes for socialism, is to be found in the institution of private property."

¹⁸It can be shown that a result similar to Proposition [3](#page-18-2) holds in the region corresponding to the fear equilibrium. In particular, if the strength of positional externality, θ , is high enough, both agents would be better off with higher τ . Yet this does not hold in the destructive equilibrium.

¹⁹ While intuitive in a static model, this result need not hold in a dynamic framework. Gershman (2011b) shows in a simple endogenous growth model that the long-run KUJ steady state may well Pareto dominate the FE due to labor-saving productivity growth supported by higher effort levels.

Veblen goes on to describe what in the model can be called a transition from the KUJE ("keep up appearances") to the FE ("socialism"): "The ultimate ground of this struggle to keep up appearances by otherwise unnecessary expenditure, is the institution of private property. . .With the abolition of private property, the characteristic of human nature which now finds its exercise in this form of emulation, should logically find exercise in other, perhaps nobler and socially more serviceable, activities." He then emphasizes that the abolition of the KUJ competition would lessen the amount of labor and output required to support the economy. This is exactly what happens after transition to the fear equilibrium: output and labor supply fall, individuals enjoy more leisure and are, at least initially, happier with less output (see, however, footnote [19\)](#page-19-2). As discussed above, the rich may prefer well-protected property rights if inequality is high enough, since moving to the FE would mean losing too much in terms of relative standing. Thus, in a KUJE with relatively high inequality there is a conflict of interests, or, in Veblen's terms, opposite sentiments towards the socialist movement.

If a Pareto improvement can be achieved by increasing τ , can it be accomplished through ex-ante redistribution of endowments from the tich to the poor (increasing k)?

Proposition 4. Let $\hat{k} \leq k < \tilde{k}$, that is, agents are in the FE. Then: 1) U_{F}^1 is strictly increasing in k ; 2) U_{F}^2 is either strictly decreasing in k or has a global maximum at

$$
k^* = \frac{\sqrt{1+4\sigma\delta}-1}{2\sigma}, \quad \delta \equiv \theta(\tau-1) \left[\frac{1-\tau\theta^2}{\theta(\tau-1)} \right]^{1-\sigma},
$$

where $\hat{k} < k^* < \tilde{k}$. Furthermore, if $k \geqslant \tilde{k}$, U_{KUJ}^2 is strictly decreasing in k, while U_{KUJ}^1 is strictly increasing in k.

Proposition [4](#page-20-0) implies that, while redistribution is always desirable for the poor agent, from the point of view of the rich agent it can only be beneficial under certain conditions and only to a certain extent. In particular, switching to the KUJE via a decrease in initial inequality cannot result in a Pareto worsening or a Pareto improvement, since the poor always win while the rich always lose. The big difference between an institutional change analyzed above and an ex-ante redistribution is that in the latter case the rich are actually giving away part of their endowment, lowering their own productivity and increasing the productivity of the poor.

If endowments represent ability or other indivisible resource, ex-ante redistribution is not feasible. In that case agents can potentially share the fruits of their first-stage investment. The following section modifies the basic model to allow for voluntary transfers of this sort and examines its implications.

5 Envy and Redistribution

It is not surprising that a common way to deal with destructive envy is through redistribution of various forms. Cancian (1965) and Foster (1979) examine redistributive practices in Latin American peasant societies, such as ceremonial expenditures and sponsorship of religious fiestas by the rich (cargo system). In particular, Cancian (1965, p. 140) suggests that "service in the cargo system legitimizes the wealth differences that do exist and thus prevents disruptive envy." Platteau (2000, chapter 5) examines similar arrangements in Sub-Saharan Africa and Asian village communities, where "in seeking neighbourer's goodwill or in fearing their envy, incentives operate for the rich to redistribute income and wealth to the poor" (p. 235).

Mui (1995, section 4) discusses sharing in the emerging market economies of the Soviet Union and China manifested in contributions of the nascent entrepreneurial class to charity or local public goods, which he sees as an attempt to alleviate destructive envy. Mui then goes on to formally show how sharing can support the adoption of innovation in equilibrium. Some authors (Schoeck, 1969; Fern´andez de la Mora, 1987) even see the modern progressive taxation system as a remnant of "egalitarian envy." Incidentally, using recent survey data from 18 European countries, Clark and Senik (2010) showed that the strength of income comparisons is associated with higher demand for redistribution.

Within the framework of section [3,](#page-7-0) a natural question is whether the rich would be willing to share the fruits of their effort to avoid destructive envy and how this affects the possible equilibrium outcomes of this modified envy game. Specifically, assume that Agent 2 can make a transfer to Agent 1 before stage 2 begins, as shown in Figure [8.](#page-22-0)

Consider the node of the game where the rich agent decides on transfer having seen the outcomes of the investment stage. Obviously, non-zero transfer may only be optimal if $Y_1 < \tau \theta Y_2$, that is, destructive envy is binding. In this case Agent 2 may want to make a positive transfer T to lower the intensity of destruction, d_1^* .

Given that effort is sunk at this stage, Agent 2 maximizes

$$
v(p_2^*(T)(Y_2 - T) - \theta(1 - d_1^*(T))(Y_1 + T)) \longrightarrow \max_T \qquad s.t. \quad 0 \le T \le \overline{T}, \tag{22}
$$

where $\overline{T} \equiv (\tau \theta Y_2 - Y_1)/(1 + \tau \theta)$ is the minimum transfer sufficient to completely avoid destruction.[20](#page-21-1) Lemma [4](#page-22-1) characterizes the optimal transfer of Agent 2.

²⁰Formally, $(1-d_1^*)Y_1 - \theta p_2^*Y_2$ is positive if and only if $Y_2/Y_1 < (1+\tau)^2/4\tau\theta$. Otherwise, $U^1 = -\infty$ and Agent 1 is indifferent between any feasible destruction intensities. For concreteness, focus on d_1^* as the best response of Agent 1 in this case.

Stage 1 (Investment, Transfer) Stage 2 (Production, Destruction, Consumption)

Effort L_i Outcomes Y_i Transfer T Production/Destruction d_i Consumption C_i Payoffs U^i

Figure 8: Timing of events in the envy game with transfers.

Lemma 4. (Optimal transfer). For $Y_1/Y_2 \leq \tau\theta$, the optimal transfer is $T = \overline{T}$ if and only if $\tau \theta \leq (1 - \theta)/(1 + \theta)$. Otherwise,

$$
T = \begin{cases} T^*, & \text{if } Y_1/Y_2 < \mu; \\ 0, & \text{if } Y_1/Y_2 \geqslant \mu, \end{cases}
$$

where T^* is given by the first-order condition of [\(22\)](#page-21-2) for the interior solution and $\mu \in (0, \tau \theta]$ is a constant depending on τ and θ (see the Appendix).

In what follows consider the simple case $\tau \theta \leq (1 - \theta)/(1 + \theta)$. Then, as established in Lemma [4,](#page-22-1) it is always optimal for Agent 2 to make the full transfer \overline{T} . Intuitively, the rich individual will be willing to do so if \overline{T} is low. The next lemma establishes what the best response of Agent 2 looks like under full transfer.

Lemma 5. (BR of Agent 2 with full transfer). The best response function of Agent 2, $BR_2^T \equiv Y_2^*(Y_1)$, has the following form:

$$
Y_2^*(Y_1) = \begin{cases} K_2^{1/\sigma} + \theta Y_1, & \text{if } Y_1 \ge \tilde{Y}_1; \\ Y_1 \cdot \frac{1}{\tau \theta}, & \text{if } \check{Y}_1 < Y_1 < \tilde{Y}_1; \\ (\gamma_2 K_2)^{1/\sigma} - Y_1, & \text{if } Y_1 \le \check{Y}_1, \end{cases} \tag{23}
$$

.

where

$$
\widetilde{Y}_1 \equiv \frac{\tau \theta}{1 - \tau \theta^2} K_2^{1/\sigma}, \quad \breve{Y}_1 \equiv \frac{\tau \theta}{1 + \tau \theta} (\gamma_2 K_2)^{1/\sigma}, \quad \gamma_2 \equiv \left[\frac{1 - \tau \theta^2}{1 + \tau \theta} \right]^{1 - \sigma}
$$

Now, instead of "destructive region" the best response of Agent 2 has a "transfer region" (left panel of Figure [9\)](#page-23-0). For low levels of Y_1 it is optimal to prevent destruction through transfers rather than by producing less in the first stage.

Agent 1 anticipates to get the full transfer in case of high inequality, because he knows that the rich agent will be afraid of destructive envy. He takes this into account when choosing his first period effort. Intuitively, given the high potential output of the rich, Agent 1 has an incentive to invest as little effort as possible and cause a threat of destructive envy thereby provoking the rich to make a transfer. Lemma [5](#page-22-2) gives a characterization of the best response function of Agent 1 for a special case that illustrates all possible kinds of equilibria that can emerge in a modified envy game.

Figure 9: Best responses in the envy game with transfers.

Lemma 6. (BR of Agent 1 with full transfer). Under parametric conditions [\(A12\)](#page-37-0) and [\(A13\)](#page-37-1) provided in the Appendix the best response function of Agent 1, $BR_1^T \equiv Y_1^*(Y_2)$, is given by

$$
Y_1^*(Y_2) = \begin{cases} K_1^{1/\sigma} + \theta Y_2, & \text{if } Y_2 \le \tilde{Y}_2; \\ (\gamma_1 K_1)^{1/\sigma} - Y_2, & \text{if } \tilde{Y}_2 < Y_2 < \tilde{Y}_2; \\ 0, & \text{if } Y_2 \ge \tilde{Y}_2, \end{cases}
$$
(24)

.

where

$$
\widetilde{Y}_2 \equiv \frac{\sigma(\gamma_1^{1/\sigma} - 1)}{(\sigma - 1)(1 + \theta)} K_1^{1/\sigma}, \quad \widetilde{Y}_2 \equiv (\gamma_1 K_1)^{1/\sigma}, \quad \gamma_1 \equiv \left[\frac{\theta(\tau - 1)}{1 + \tau \theta}\right]^{1 - \sigma}
$$

The right panel of Figure [9](#page-23-0) depicts this function. Note, in particular, that there is a discontinuity at point \widetilde{Y}_2 , as it becomes optimal for Agent 1 to switch to the KUJ-type best response when Y_2 is low enough.^{[21](#page-23-1)}

Given the best responses of the agents, two qualitatively different types of equilibria may arise: fear equilibrium with transfers (FT) and KUJ equilibrium. Moreover, as shown in the bottom panels of Figure [10,](#page-24-0) multiplicity of equilibria cannot be ruled out. Proposition [5](#page-23-2) characterizes the equilibria under conditions of Lemma [6](#page-23-3) and an additional restriction that guarantees existence of at least one equilibrium for all possible values of k .

Proposition 5. (Equilibria of the envy game with transfers). Assume that $\omega < \gamma_2/\gamma_1$, where $\omega \equiv [\sigma(1-\theta)(\gamma_1^{1/\sigma}-1)/(\sigma-1)-\theta]^{-\sigma}$. Then the envy game with transfers has: 1) a unique KUJ equilibrium [\(16\)](#page-13-2) if $k > \gamma_2/\gamma_1$; 2) a unique fear equilibrium with full transfer

²¹Under alternative parametric restrictions the best response of Agent 1 consists of only two regions, KUJ and "zero output," and a statement analogous to Proposition [5](#page-23-2) holds.

of the form

$$
\{Y_1^{\text{FT}}, Y_2^{\text{FT}}\} = \{0, (\gamma_2 K_2)^{1/\sigma}\};
$$

$$
\{C_1^{\text{FT}}, C_2^{\text{FT}}\} = \{\varepsilon(\gamma_2 K_2)^{1/\sigma}, (1-\varepsilon)(\gamma_2 K_2)^{1/\sigma}\},
$$
 (25)

if $k < \omega$, where $\varepsilon \equiv [\tau \theta/(1 + \tau \theta)];$ 3) two equilibria [\(16\)](#page-13-2) and [\(25\)](#page-24-1) if $\omega \leq k < \gamma_2/\gamma_1;$ 4) multiple equilibria with full transfer of the form

$$
\{Y_1^{\text{FT}}, Y_2^{\text{FT}}\} = \{Y_1^{\text{FT}}, (\gamma_2 K_2)^{1/\sigma} - Y_1^{\text{FT}}\}, \quad 0 \leq Y_1^{\text{FT}} \leq \widetilde{Y}_2; \{C_1^{\text{FT}}, C_2^{\text{FT}}\} = \{Y_1^{\text{FT}} + \bar{T}, Y_2^{\text{FT}} - \bar{T}\},
$$
\n(26)

if $k = \gamma_2/\gamma_1$, along with a KUJ equilibrium.

Figure 10: Equilibria in the envy game with transfers.

Destructive and fear equilibria of the basic model are replaced by "fear equilibrium with transfers," in which the rich agent redistributes part of his investment outcome to avoid destructive envy of the poor agent. Since inequality is what matters for the amount of destruction, by investing nothing in the first stage the poor agent creates the fear of envy forcing the rich to share. In turn, when the poor agent expects to receive a transfer, he does not need to work hard and chooses to produce the minimum amount needed to create a credible threat to destroy. This type of equilibrium arises when initial inequality is high. In contrast, if it is low, the only possible outcome of the game is the standard KUJ equilibrium. It is easy to see that both γ_2/γ_1 and ω are increasing in τ and θ , that is, reducing tolerance for inequality increases the likelihood of the redistributive equilibrium. This is intuitive and parallels the taxonomy of equilibria in Proposition [1.](#page-14-1)

Interestingly, the possibility of transfers gives rise to (stable) multiple equilibria for intermediate levels of inequality (bottom panels of Figure [10\)](#page-24-0). This implies that societies with similar characteristics and moderate wealth differences may end up in one of the two alternative equilibria: one with redistribution and fear of destructive envy and the other with KUJ-type competition. The following proposition highlights the contrast between the two equilibria.

Proposition 6. Assume that $\omega < k < \gamma_2/\gamma_1$, that is, the envy game with transfers has an FT equilibrium [\(25\)](#page-24-1) and a KUJ equilibrium [\(16\)](#page-13-2). Then, the FT equilibrium is characterized by: 1) higher ex-post inequality, that is, $C_1^{\text{FT}}/C_2^{\text{FT}} < C_1^{\text{KUJ}}/C_2^{\text{KUJ}}$; 2) lower total output, that is, $Y_2^{\text{FT}} < Y^{\text{KUJ}}$. Moreover, 3) $U_{\text{KUJ}}^2 > U_{\text{FT}}^2$, while $U_{\text{KUJ}}^1 > U_{\text{FT}}^1$ if and only if $(1 - \theta^2) \gamma_1 \gamma_2^{(1 - \sigma)/\sigma} k > (\sigma - \theta^2) k^{1/\sigma} + \theta(\sigma - 1)$.

Curiously, the FT equilibrium is more unequal ex post than the KUJE. That is, competitive envy turns out to be more effective in narrowing the gap between the rich and the poor than redistribution under the threat of destructive envy. The second part of Proposition [6](#page-25-0) is intuitive. In the KUJE, positional externality makes both agents work hard and leads to high total output. In FT equilibrium, the poor agent does not work at all to create a threat of destructive envy, while the rich provides consumption for both.

Parts 1 and 2 help to understand the third statement. In the FT equilibrium, the poor do not work and receive a transfer, just necessary to prevent destructive envy, while in the KUJE they have to exert substantial effort in order to keep up with the rich. They prefer the latter if the initial inequality is low enough, that is, k is large enough. In that case, expost relative standing in the KUJE is good enough to justify extra effort. In the opposite case, it is too hard to maintain the relative standing peacefully and the poor prefer to live on transfers at the cost of lower social status. The rich, however, are always better off in the KUJE: overworking due to positional externality in the KUJE dominates overworking (and then sharing) under the threat of destructive envy in the FT equilibrium. Overall, this result means that, if the two equilibria are in fact Pareto rankable, KUJ equilibrium is dominant. Otherwise, there is a conflict of interests with the poor favoring the FT equilibrium and the rich opting for the KUJE.

6 Envy and the Dynamics of Inequality

So far, the analysis has focused on the equilibria of the static model in which inequality of endowments was taken as fixed. In this section intergenerational links are introduced in the basic model of section [3](#page-7-0) to explore the dynamics of inequality driving the endogenous evolution of the economy through different envy regimes.

Environment. The economy is populated by a sequence of non-overlapping generations, indexed by $t \geq 0$. Time is discrete, and each generation lives for one period. The initial population consists of 2 homogeneous groups of people (representative agents), the poor and the rich, with initial endowments K_{10} and $K_{20} > K_{10}$. Each person has 1 child, so that in each time period two groups of people are descendants of the two original groups. Parents care about their children and leave bequests, b_{it} , $i = 1, 2$, at the end of each period $t.^{22}$ $t.^{22}$ $t.^{22}$ In particular, they derive utility not just from relative consumption but the relative Cobb-Douglas aggregator of consumption, now denoted c_{it} , and bequest, b_{it} :

$$
U_t^i = \frac{\pi (c_{it}^{1-\alpha} b_{it}^{\alpha} - \theta c_{jt}^{1-\alpha} b_{jt}^{\alpha})^{1-\sigma}}{1-\sigma} - L_{it},
$$
\n(27)

where $i, j = 1, 2, i \neq j, 0 < \alpha < 1$ parameterizes the fraction of final output allocated to bequest, and $\pi \equiv [(1-\alpha)^{1-\alpha} \alpha^{\alpha}]^{\sigma-1}$ is a normalization constant. Consumption and bequest are made out of the final output, C_{it} , so that $b_{it} + c_{it} = C_{it}$. This formulation leaves the workings of the basic model from section [3](#page-7-0) intact while introducing dynamic linkages.

Dynamical system. The initial endowment of generation $t + 1$, K_{it+1} , is assumed to depend on the endowment of their parents and parental investment in children:

$$
K_{it+1} = K_{it}^{\beta} b_{it} = K_{it}^{\beta} \alpha C_{it},\tag{28}
$$

where $i = 1, 2$, and $0 < 1 - \beta < 1$ is the rate of geometric depreciation of parental endowment.^{[23](#page-26-2)} Note that Proposition [1](#page-14-1) holds each period t , and the level of initial inequality in period $t + 1$ is determined endogenously:

$$
k_{t+1} \equiv \frac{K_{1t+1}}{K_{2t+1}} = k_t^{\beta} \cdot \frac{C_{1t}}{C_{2t}},
$$
\n(29)

given the initial condition $0 < k_0 < 1$. The joint dynamics of K_{1t} and K_{2t} depends on the type of equilibrium, in which the economy resides, and the equilibrium next period is, in

²²These bequests may represent any kind of investment in children that increases the productivity of their effort, for example, expenditure on human capital.

²³Persistence of endowments is introduced to make the dynamics more realistic. If $\beta = 0$, the qualitative results remain unchanged, but the economy will not stay in the fear equilibrium for more than 1 period.

turn, determined by the economic outcomes of the current period. Lemma [7](#page-27-0) characterizes this dynamical system and follows directly from Proposition [1](#page-14-1) and the law of motion [\(28\)](#page-26-3).

Lemma 7. (Dynamics of endowments). The two-dimensional dynamical system for K_{1t} and K_{2t} is given by

$$
\begin{bmatrix}\nK_{1t+1} \\
K_{2t+1}\n\end{bmatrix} = \begin{cases}\n\frac{\alpha}{1-\theta^2} [K_{1t}^{\beta} (K_{1t}^{1/\sigma} + \theta K_{2t}^{1/\sigma}), K_{2t}^{\beta} (K_{2t}^{1/\sigma} + \theta K_{1t}^{1/\sigma})], & \text{if } k_t \geq \tilde{k}; \\
\frac{\alpha}{\theta(\tau-1)} [\tau \theta K_{1t}^{1/\sigma+\beta}, K_{2t}^{\beta} K_{1t}^{1/\sigma}], & \text{if } \hat{k} \leq k_t < \tilde{k}; \\
\alpha [K_{1t}^{\beta} \cdot C_1^D (K_{1t}, K_{2t}), K_{2t}^{\beta} \cdot C_2^D (K_{1t}, K_{2t})], & \text{if } k_t < \hat{k},\n\end{cases}
$$
\n(30)

where $C_1^D(K_{1t}, K_{2t})$ and $C_2^D(K_{1t}, K_{2t})$ are the final outputs in the DE given by [\(18\)](#page-14-2).

The thresholds \tilde{k} and \hat{k} divide the (K_{1t}, K_{2t}) phase plane into three regions according to the types of equilibria (see Figures [11](#page-28-0) and [12\)](#page-29-0): KUJ, F (fear), and D (destruction). In each of these regions the motion is governed by the corresponding part of the dynamical system [\(30\)](#page-27-1). To rule out explosive dynamics it is assumed throughout this section that $\sigma(1-\beta) > 1$ which also implies that the results of all previous sections hold, since $\sigma > 1$.

It is convenient to analyze a companion one-dimensional difference equation driving the dynamics of inequality. Some of its properties are established in the following lemma.

Lemma 8. (Dynamics of inequality). The dynamics of k_t is given by

$$
k_{t+1} = \begin{cases} g_1(k_t) \equiv [k_t^{1/\sigma + \beta} + \theta k_t^{\beta}]/[1 + \theta k_t^{1/\sigma}], & \text{if } k_t \geq \tilde{k};\\ g_2(k_t) \equiv \tau \theta k_t^{\beta}, & \text{if } \hat{k} \leq k_t < \tilde{k};\\ g_3(k_t), & \text{if } k_t < \hat{k}, \end{cases}
$$
(31)

where $g_3(k_t)$ is implicitly given by

$$
k_t = \frac{\tau (1 + \theta^2)}{2\theta (1 + \tau)^2} \cdot \frac{(z_{t+1} + \theta)^2}{z_{t+1}} \cdot \left(\frac{z_{t+1} - \theta}{1 - \theta z_{t+1}}\right)^{\sigma}, \quad z_{t+1} \equiv k_{t+1}/k_t^{\beta}.
$$
 (32)

Moreover, $k_{t+1} > k_t$ for all $0 < k_t < 1$.

Further analysis focuses on the dynamics of the system in the fear region and the KUJ region. The two lemmas below provide a characterization.

Lemma 9. (KUJ region dynamics). The system converges to a unique stable "equal" long-run steady state $\bar{K}_1^{\text{KUJ}} = \bar{K}_2^{\text{KUJ}} = \bar{K} = [\alpha/(1-\theta)]^{\frac{\sigma}{\sigma(1-\beta)-1}}$. The steady-state levels of output are equal to $\bar{Y}_1^{\text{KUJ}} = \bar{Y}_2^{\text{KUJ}} = \bar{Y} = \bar{K}^{1/\sigma}/(1-\theta)$. The evolution of endowments is determined by the loci

$$
\Delta K_i \equiv K_{it+1} - K_{it} = 0: \quad K_{it}^{1/\sigma} + \theta K_{jt}^{1/\sigma} = K_{it}^{1-\beta} \cdot (1 - \theta^2) / \alpha, \quad i, j = 1, 2, \quad i \neq j,
$$

such that $dK_{it}/dK_{jt} > 0$ and $d^2K_{it}/dK_{jt}^2 < 0$.

Figure 11: Dynamics and steady states in the KUJ (left) and fear (right) regions.

Figure [11](#page-28-0) (left panel) depicts schematically the dynamics in the KUJ region. The descendants of the initially poor eventually catch up with the rich dynasty, and in the steady state both have the same endowments.

Lemma 10. (Fear region dynamics). In the fear region the system moves towards a unique stable "unequal" long-run steady state $\bar{K}_1 = [\alpha \tau/(\tau - 1)]^{\frac{\sigma}{\sigma(1-\beta)-1}}$, $\bar{K}_2 = \bar{K}_1 \cdot (\tau \theta)^{\frac{1}{\beta-1}}$. The corresponding levels of output are $\bar{Y}_1 = [\tau/(\tau - 1)] \bar{K}_1^{1/\sigma}$ $Y_1^{1/\sigma}$ and $\bar{Y}_2 = \bar{K}_1^{1/\sigma}$ $\int_1^{1/\sigma}/[\theta(\tau-1)]$. This steady state is, however, unattainable, since it is located in the KUJ region, that is, the system moves to the KUJ region before reaching the fear steady state. The evolution of endowments is determined by the loci

$$
\Delta K_1 = 0: \quad K_{1t} = \bar{K}_1 = [\alpha \tau/(\tau - 1)]^{\frac{\sigma}{\sigma(1-\beta)-1}};
$$

$$
\Delta K_2 = 0: \quad K_{1t} = K_{2t}^{\sigma(1-\beta)} \cdot [\theta(\tau - 1)/\alpha]^{\sigma}.
$$

Figure [11](#page-28-0) (right panel) depicts schematically the dynamics in the fear region. It is instructive to look at the comparative statics of the long-run levels of output with respect to τ and θ . They resemble the results of the static model: in the KUJ steady state, outputs are increasing in θ and independent of τ ; in the (unattainable) fear steady state, the output of group 1 is independent of θ and decreasing in τ while the output of group 2 is decreasing in θ and τ . Note also that, despite their qualitatively different nature, as $\tau\theta \to 1$, the two long-run equilibria get closer and coincide in the limit.

Given Lemmas [8](#page-27-2)[–10,](#page-28-1) it is easy to establish how the possible development trajectories look. The long-run convergence result is stated in Proposition [7](#page-29-1) and depicted in Figure [12.](#page-29-0)

Proposition 7. (Long-run convergence). Starting with any initial conditions $\{K_{10}, K_{20}\}\$ such that $0 < k_0 < 1$, the endowments converge to a unique stable long-run "equal" steady state of the KUJ region, \bar{K} . Inequality decreases monotonically along the transition path.

Figure 12: Evolution through envy regimes.

Thus, if the economy starts off, say, in the destructive region, it experiences a transition to the KUJ steady state, possibly passing through the fear region and staying there for a while. Initially, destructive envy and the fear of it reduce original inequality of endowments by discouraging productive effort of the rich or destroying part of their output. This leads to more equal investment opportunities for future generations who eventually find it optimal to compete productively. Envy-avoidance behavior, dictated by the destructive side of envy, paves the way to emulation, driven by its competitive side.

The transitions from one envy region to another can be delayed or accelerated by factors that affect the intensity of social comparisons, such as religion and ideology in general.^{[24](#page-29-2)} In the context of the model, religious and moral teachings condemning envy may be thought of as causing downward pressure on θ . Assume that the economy is in the fear region and θ falls. Then, as follows from Proposition [2,](#page-16-0) outputs will rise because the fear constraint of

²⁴All major world religions denounce envy. In Judeo-Christian tradition envy is one of the deadly sins and features prominently in the tenth commandment. Schoeck (1969, p. 160) goes as far as to say that "a society from which all cause of envy had disappeared would not need the moral message of Christianity."

Figure 13: Dynamics of inequality under low (left) and high (right) τ .

the rich is alleviated permitting higher effort without fear of destructive envy. Moreover, as follows directly from [\(19\)](#page-14-3), a fall in θ lowers the inequality threshold k contributing to a faster transition from FE to KUJE. As the economy enters the KUJ region, destructive envy turns into emulation, and changes in θ have an opposite impact on economic performance. In the KUJ region the same factors that drive the society out of the fear equilibrium by reducing the strength of envy have a negative effect on output.

An example of an ideology affecting θ is that of material egalitarianism. The concept of everyone being equal and the neglect of private property rights are effective in fostering social comparisons and lowering tolerance for inequality. Hence, this ideology operates in favor of the destructive side of envy and delays the transition to the KUJE.

Similarly, one could analyze the dynamic effects of a shock in τ that could be due to a variety of reasons, from changes in technologies of destruction and protection to reforms of legal institutions. If τ increases, it can be shown that: 1) the threshold \tilde{k} increases; 2) the scope of the fear region, $(\tilde{k} - \hat{k})$, extends; 3) the long-run fear steady state moves closer to the long-run KUJ steady state. So, an increase in τ would endogenously prolong the presence of the economy in the fear region (see Figure [13\)](#page-30-0), and at the same time would make the FE more egalitarian since the erosion of institutions decreases tolerance for inequality and exacerbates the fear constraint. This may explain the persistence of the fear equilibrium, along with such characteristics as poorly protected private property rights, fear of envy, and relatively low inequality.

7 Concluding remarks

This paper develops a unified framework for the economic analysis of envy by capturing its two main forces, destructive and competitive. The role of envy in society is determined in equilibrium by the level of fundamental inequality and the endogenous tolerance threshold shaped by the quality of institutions and the strength of positional concerns.

The qualitatively different equilibria that arise in this framework are broadly consistent with the available evidence on the implications of envy for economic incentives. The "keeping up with the Joneses" equilibrium roughly corresponds to modern economies, in which emulation is the main driver of consumer demand. The "fear equilibrium" resembles the role of envy in developing countries, where the fear of envious retaliation prevents productive investment and retards progress. The different nature of these equilibria yields contrasting comparative statics with respect to the strength of positional concerns. In the KUJ equilibrium, envy enhances production by intensifying emulation, while in the fear equilibrium it reduces output by aggravating the fear constraint.

The model sheds new light on the interplay between institutions, welfare, and economic performance. First, while better institutions can move the society from the low-output fear equilibrium to the high-output KUJ equilibrium, such change need not be welfare enhancing if it triggers the "rat race" competition. Second, transfers can be used to avoid envy-motivated destruction in equilibrium, which is in line with the evidence on the role of redistributive mechanisms in developing societies.

A dynamic extension of the model explores the evolution of the economy through different envy regimes. Starting at any initial conditions, the economy converges to the long-run KUJ steady state, driven by the decline of inequality along the transition path. This transition can be delayed or accelerated by factors that affect the strength of social comparisons and the relative attractiveness of productive and destructive effort.

This paper lays a foundation for understanding the changing role of envy in the process of development. This line of research is pursued in Gershman (2011b) by incorporating the basic model in a simple endogenous growth framework, in which resources are limited and productivity is driven by learning-by-doing and knowledge spillovers. As rising productivity expands the production possibilities frontier for everyone, the society experiences an endogenous transition from the fear equilibrium to the KUJ equilibrium causing a qualitative change in the relationship between envy and economic performance.

Appendix

Proof of Lemma [1.](#page-10-2) As follows from [\(7\)](#page-10-3), $d_2^* = 0$ iff $Y_2/Y_1 \geq \tau \theta (1 - d_1)(1 + \tau d_1)$. This clearly holds, if $d_1 = 0$, since $\tau \theta < 1$ and $Y_2 > Y_1$. If $d_1 = (\sqrt{\tau \theta Y_2/Y_1} - 1)/\tau > 0$, the above inequality is equivalent to

$$
\sqrt{\frac{Y_2}{Y_1}} \geqslant \frac{\theta(1+\tau)\sqrt{\tau\theta}}{1+\tau\theta^2}.
$$

Since $Y_2 > Y_1$, this holds, if the right-hand side is less or equal than unity, that is, θ $\sqrt{\tau\theta} \leqslant (1+\tau\theta^2)/(1+\tau).$ Since $\tau < 1/\theta$, sup $\{\theta$ $\langle \sqrt{\tau \theta} \rangle$ = inf{ $(1 + \tau \theta^2)/(1 + \tau)$ } = θ . Hence, θ $\sqrt{\tau \theta} \leq (1 + \tau \theta^2)/(1 + \tau)$, given that $\tau\theta$ < [1](#page-10-2) and $Y_2 > Y_1$, and Lemma 1 gives the unique second-stage equilibrium.

Proof of Lemma [2.](#page-11-1) Agent 2 is solving [\(10\)](#page-11-2) subject to [\(8\)](#page-10-4) and [\(9\)](#page-10-5). Consider first the case $\tau \theta Y_2 \leq Y_1$, in which $d_1^* = 0$ and

$$
U^{2} = \frac{(Y_{2} - \theta Y_{1})^{1-\sigma}}{1-\sigma} - \frac{Y_{2}}{K_{2}}
$$

It is strictly concave in Y_2 , and the first-order conditions yield the following optimum:

$$
Y_2 = \begin{cases} K_2^{1/\sigma} + \theta Y_1, & \text{if } Y_1 \ge \tilde{C}_1; \\ Y_1 \cdot \frac{1}{\tau \theta}, & \text{if } Y_1 < \tilde{C}_1, \end{cases} \tag{A1}
$$

.

.

where \tilde{C}_1 is defined in Lemma [2.](#page-11-1) That is, the left derivative of U^2 at point $Y_2 = Y_1/\tau \theta$ is positive (negative) iff $Y_1 < \tilde{C}_1$ ($Y_1 > \tilde{C}_1$). Next, consider the destructive case, $\tau \theta Y_2 > Y_1$, in which, as follows from [\(8\)](#page-10-4)-[\(10\)](#page-11-2),

$$
U^{2} = \frac{1}{1 - \sigma} \cdot \left(\sqrt{\frac{Y_{1} Y_{2}}{\tau \theta}} \cdot (1 + \theta^{2}) - \frac{\theta(\tau + 1)}{\tau} Y_{1} \right)^{1 - \sigma} - \frac{Y_{2}}{K_{2}}
$$

Again, U^2 is strictly concave in Y_2 , and the interior optimum is uniquely defined by the first-order condition

$$
\left(\sqrt{\frac{Y_1 Y_2}{\tau \theta}} \cdot (1 + \theta^2) - \frac{\theta(\tau + 1)}{\tau} Y_1\right)^{-\sigma} \cdot \left(\frac{1 + \theta^2}{2\sqrt{\tau \theta}} \cdot \sqrt{\frac{Y_1}{Y_2}}\right) - \frac{1}{K_2} = 0.
$$
\n(A2)

It follows that the right derivative of U^2 at point $Y_2 = Y_1/\tau \theta$ is positive (negative) iff $Y_1 < \widehat{C}_1$ $(Y_1 > \widehat{C}_1)$, where \widehat{C}_1 is defined in Lemma [2](#page-11-1) and $\widehat{C}_1 < \widetilde{C}_1$, since $0 < \theta < 1$. Hence, allowing destruction is the best response iff $Y_1 < \widehat{C}_1$.

To rewrite [\(A2\)](#page-32-0) in terms of consumption note that if destruction takes place

$$
\begin{cases}\nC_1 = (1 - d_1^*)Y_1 = \frac{1 + \tau}{\tau} Y_1 - \sqrt{\frac{\theta}{\tau} Y_1 Y_2}; \\
C_2 = p_2^* Y_2 = \sqrt{\frac{Y_1 Y_2}{\tau \theta}}.\n\end{cases}\n\implies\n\begin{cases}\nY_1 = \frac{1 + \tau}{\tau} (C_1 + \theta C_2); \\
Y_2 = \theta (1 + \tau) \cdot \frac{C_2^2}{C_1 + \theta C_2}.\n\end{cases}\n\tag{A3}
$$

Substituting this into $(A2)$ yields (12) . Next, applying the implicit function theorem to (12) gives

$$
\frac{dC_2}{dC_1} = \frac{\theta + \frac{1}{\sigma} \cdot \frac{C_2 - \theta C_1}{C_1 + \theta C_2}}{1 + \frac{1}{\sigma} \cdot \frac{C_2 - \theta C_1}{C_1 + \theta C_2} \cdot \frac{C_1}{C_2}} > 0,
$$
\n(A4)

since the optimal response requires $C_2 > \theta C_1$. The latter also implies that $dC_2/dC_1 < C_2/C_1$. Finally,

$$
\frac{d^2C_2}{dC_1^2} = \left(\sigma + \frac{C_2 - \theta C_1}{C_1 + \theta C_2} \cdot \frac{C_1}{C_2}\right)^{-1} \cdot \frac{C_2 - C_1 \cdot \frac{dC_2}{dC_1}}{C_2(C_1 + \theta C_2)} \cdot \left[\frac{(1 + \theta^2) \left(C_1 \frac{dC_2}{dC_1} - C_2\right)}{C_1 + \theta C_2} - \frac{\frac{dC_2}{dC_1}(C_2 - \theta C_1)}{C_2}\right] < 0,
$$

since $C_2 > \theta C_1$ and $0 < dC_2/dC_1 < C_2/C_1$. Hence, $C_2^d(C_1)$ is strictly increasing and concave.

Proof of Lemma [3.](#page-12-2) Agent 1 is solving [\(13\)](#page-12-3) subject to [\(8\)](#page-10-4) and [\(9\)](#page-10-5). In the KUJ case, $\tau \theta Y_2 \leq Y_1$,

$$
U^{1} = \frac{(Y_{1} - \theta Y_{2})^{1-\sigma}}{1-\sigma} - \frac{Y_{1}}{K_{1}}
$$

It is strictly concave in Y_1 , and the first-order conditions yield the following optimum:

$$
Y_1 = \begin{cases} K_1^{1/\sigma} + \theta Y_2, & \text{if } Y_2 \leq \widehat{C}_2; \\ \tau \theta Y_2, & \text{if } Y_2 > \widehat{C}_2, \end{cases}
$$
 (A5)

.

where \hat{C}_2 is defined in Lemma [3.](#page-12-2) In the destructive case, $\tau \theta Y_2 > Y_1$, expressions [\(8\)](#page-10-4), [\(9\)](#page-10-5), and [\(13\)](#page-12-3) yield

$$
U^{1} = \frac{1}{1 - \sigma} \cdot \left(\frac{1 + \tau}{\tau} Y_{1} - 2\sqrt{\frac{\theta}{\tau} Y_{1} Y_{2}}\right)^{1 - \sigma} - \frac{Y_{1}}{K_{1}}.
$$

Assumption $\sigma > 1$ is sufficient for U^1 to be strictly concave in Y_1 . In particular, it can be shown by simple differentiation that the sign of d^2U^1/dY_1^2 coincides with the sign of $h(x) \equiv -(1+\sigma)x^2 + \xi(2\sigma+1/2)x - \sigma\xi^2$, where $x \equiv \sqrt{\theta Y_2/\tau Y_1}$ and $\xi \equiv (\tau + 1)/\tau$. Then, it is easy to show that $h_{\text{max}} \propto 1 - 8\sigma < 0$ and so, U^1 is concave under $\sigma > 1$.

Note that U^1 is differentiable at point $Y_1 = \tau \theta Y_2$ with $dU^1(\tau \theta Y_2)/dY_1 = ((\tau - 1)\theta Y_2)^{-\sigma} - 1/K_1$. Hence, there are only two cases. If $Y_2 \leq \widehat{C}_2$, the global optimum is given by the interior optimum in the KUJ region. If $Y_2 > \hat{C}_2$, there global optimum is in the destructive region and it is given by the first-order condition

$$
\left(\frac{1+\tau}{\tau}Y_1 - 2\sqrt{\frac{\theta}{\tau}Y_1Y_2}\right)^{-\sigma} \cdot \left(\frac{1+\tau}{\tau} - \sqrt{\frac{\theta}{\tau} \cdot \frac{Y_2}{Y_1}}\right) - \frac{1}{K_1} = 0.
$$
\n(A6)

Finally, using [\(A3\)](#page-32-1) rewrite equation [\(A6\)](#page-33-0) in terms of consumption levels to get [\(15\)](#page-12-4). Applying the implicit function theorem to [\(15\)](#page-12-4) gives

$$
\frac{dC_1}{dC_2} = \frac{\theta - \frac{\theta}{\sigma} \cdot \frac{C_1 - \theta C_2}{C_1 + \theta C_2}}{1 - \frac{\theta}{\sigma} \cdot \frac{C_1 - \theta C_2}{C_1 + \theta C_2} \cdot \frac{C_2}{C_1}} > 0,
$$
\n(A7)

since $\sigma > 1$ and the optimal response requires $C_1 > \theta C_2$. The latter also implies that $dC_1/dC_2 < C_1/C_2$. Finally,

$$
\frac{d^2C_1}{dC_2^2} = \frac{\theta}{\sigma} \left(1 - \frac{\theta}{\sigma} \cdot \frac{C_1 - \theta C_2}{C_1 + \theta C_2} \cdot \frac{C_2}{C_1} \right)^{-1} \left[\frac{\theta \left(\frac{dC_1}{dC_2} \cdot \frac{C_2}{C_1} - 1 \right)^2}{C_1 + \theta C_2} + \frac{(C_1 - \theta C_2) \left(\frac{dC_1}{dC_2} + \theta \right) \left(1 - \frac{dC_1}{dC_2} \cdot \frac{C_2}{C_1} \right)}{(C_1 + \theta C_2)^2} \right] > 0,
$$

since $\sigma > 1$, $C_1 > \theta C_2$, and $dC_1/dC_2 < C_1/C_2$. Hence, $C_1^d(C_2)$ is strictly increasing and convex.

Proof of Proposition [1.](#page-14-1) Private outputs in the KUJ equilibrium are given by [\(16\)](#page-13-2). For this to be an equilibrium two conditions must hold: $Y_1^{\text{KUJ}} \geq \tilde{C}_1$ and $Y_2^{\text{KUJ}} \leq \hat{C}_2$. This yields $k \geq \tilde{k}$, where \tilde{k} is defined in Proposition [1.](#page-14-1) In the fear equilibrium private outputs are given by (17) . For Y_2^{F} to be the best response of Agent 2 it must be the case that $\widehat{C}_1 \leqslant Y_1^{\rm F} < \widetilde{C}_1$, which yields $\hat{k} \leqslant k < \tilde{k}$ with \hat{k} defined in Proposition [1.](#page-14-1) The borderline fear equilibrium happens when $\tau \theta \hat{C}_2 = \hat{C}_1$, that is, when $k = \hat{k}$. If $\tau \theta \hat{C}_2 < \hat{C}_1$, that is, $k < \hat{k}$, the only possibility is the destructive equilibrium. To prove its existence note that: 1) from Lemma [2,](#page-11-1) $C_2^d(C_1)$ is strictly increasing and concave which implies that $C_2^d(C_1^d(\hat{C}_2)) > \hat{C}_2$; 2) from Lemma [3,](#page-12-2) $C_1^d(C_2)$ is strictly increasing and convex which implies that the inverse of C_1^d at point \hat{C}_1 is greater than $C_2^d(\widehat{C}_1) = \widehat{C}_1/\tau\theta$. Hence, the intermediate value theorem guarantees existence of a destructive equilibrium (C_1^D, C_2^D) such that $C_1^D < \hat{C}_1$ and $C_2^D > \hat{C}_2$. Finally, direct comparison of [\(A4\)](#page-32-2) and [\(A7\)](#page-33-1) shows that the slope of the inverse of C_1^d is always steeper than that of C_2^d , which ensures single crossing.

Proof of Proposition [2.](#page-16-0) Results for the KUJE and the FE follow directly from (16) and (17) . For the DE, consider the system defining the equilibrium:

$$
\begin{cases}\nf_1 \equiv C_1 - \theta C_2 - \psi [C_1/(C_1 + \theta C_2)]^{1/\sigma} = 0; \\
f_2 \equiv C_2 - \theta C_1 - \phi [(C_1 + \theta C_2)/C_2]^{1/\sigma} = 0.\n\end{cases}
$$

Using the equilibrium conditions, it can be shown that

$$
[D_{C}f]^{-1} = \frac{1}{\Delta} \begin{bmatrix} 1 + \frac{1}{\sigma} \cdot \frac{C_{2} - \theta C_{1}}{C_{1} + \theta C_{2}} \cdot \frac{C_{1}}{C_{2}} & \theta - \frac{\theta}{\sigma} \cdot \frac{C_{1} - \theta C_{2}}{C_{1} + \theta C_{2}} \\ \theta + \frac{1}{\sigma} \cdot \frac{C_{2} - \theta C_{1}}{C_{1} + \theta C_{2}} & 1 - \frac{\theta}{\sigma} \cdot \frac{C_{1} - \theta C_{2}}{C_{1} + \theta C_{2}} \cdot \frac{C_{2}}{C_{1}} \end{bmatrix},
$$

where C is the vector of consumption levels, f is the vector of f_1 and f_2 , and $\Delta \equiv \det(D_C f) = 1 \theta^2 + [(\mathcal{C}_1 - \theta \mathcal{C}_2)^2(\mathcal{C}_2 - \theta \mathcal{C}_1)]/[\sigma(\mathcal{C}_1 + \theta \mathcal{C}_2)\mathcal{C}_1\mathcal{C}_2] > 0$. It follows from this expression that all elements of $[D_C f]^{-1}$ are positive since $\sigma > 1$ and $\theta C_1 < C_2 < C_1/\theta$ in equilibrium.

For part (a), note that, by the implicit function theorem, $D_{\lambda}C = -[D_{C}f]^{-1} \cdot D_{\lambda}f$ and

$$
D_{\lambda}f = -\left[\begin{array}{c} \frac{\psi_{\lambda}}{\psi}(C_1 - \theta C_2) \\ \frac{\phi_{\lambda}}{\phi}(C_2 - \theta C_1) \end{array}\right] = \frac{1}{\sigma} \left[\begin{array}{c} -\frac{1}{\lambda}(C_1 - \theta C_2) \\ \frac{1}{1-\lambda}(C_2 - \theta C_1) \end{array}\right]
$$

.

It follows that the sign of $\partial C_1/\partial \lambda$ coincides with the sign of

$$
\left[1+\frac{1}{\sigma}\cdot\frac{C_2-\theta C_1}{C_1+\theta C_2}\cdot\frac{C_1}{C_2}\right](C_1-\theta C_2)-\frac{\theta\lambda}{1-\lambda}\cdot\left[1-\frac{1}{\sigma}\cdot\frac{C_1-\theta C_2}{C_1+\theta C_2}\right](C_2-\theta C_1).
$$

Dividing [\(12\)](#page-11-3) by [\(15\)](#page-12-4) and denoting $x \equiv C_1/C_2 \in (\theta, 1)$, in equilibrium we have

$$
\frac{\lambda}{1-\lambda} = \left(\frac{x-\theta}{1-\theta x}\right)^{\sigma} \cdot \frac{\tau(1+\theta^2)}{2\theta(1+\tau)^2} \cdot \frac{(x+\theta)^2}{x}.
$$

Plugging this expression in the previous equation and making transformations, we get

$$
\frac{\sigma(\theta+x)+x(1-\theta x)}{\sigma(\theta+x)-(x-\theta)}-\frac{\tau(1+\theta^2)}{2(1+\tau)^2}\cdot\left(\frac{x-\theta}{1-\theta x}\right)^{\sigma-1}\cdot\frac{(x+\theta)^2}{x}.
$$

The first term is always greater than 1. The second term is increasing in x and at $x = 1$ simplifies to $\tau (1+\theta^2)(1+\theta)^2/[2(1+\tau)^2]$. The latter expression is maximized at $\tau = 1$ in which case it is equal to $(1 + \theta^2)(1 + \theta)^2/4 \leq 1 \ \forall \ \theta \in (0, 1)$. Hence, the second term is always less than 1 and $\partial C_1/\partial \lambda > 0$.

Although the sign of $\partial C_2/\partial \lambda$ is ambiguous, the sign of $\partial C/\partial \lambda$ can be determined, as it coincides with the sign of

$$
\frac{\sigma(1+\theta)(x+\theta)+(1-\theta x)(1+x)}{\sigma(1+\theta)(x+\theta)-\theta(x-\theta)(1+1/x)}-\frac{\tau(1+\theta^2)}{2(1+\tau)^2}\cdot\left(\frac{x-\theta}{1-\theta x}\right)^{\sigma-1}\cdot\frac{(x+\theta)^2}{x}.
$$

As above, the first term of this expression is always greater than 1, while the second is always less than 1. Hence, $\partial C/\partial \lambda > 0$.

For part (b), note that $D_{\theta}C = -[D_C f]^{-1} \cdot D_{\theta}f$ and

$$
D_{\theta}f = -\left[\begin{array}{c} C_2 \cdot \left(\frac{1}{\sigma} \cdot \frac{C_1 - \theta C_2}{C_1 + \theta C_2} - 1\right) \\ -C_1 + \frac{1}{\sigma} \cdot \frac{C_2 - \theta C_1}{C_1 + \theta C_2} \cdot \frac{(1 - \theta^2)C_1 - 2\theta^3 C_2}{\theta (1 + \theta^2)} \end{array}\right].
$$

It follows that the sign of $\partial C_1/\partial \theta$ is determined by the sign of

$$
C_2 + \theta C_1 - \frac{1}{\sigma} \frac{C_2 - \theta C_1}{C_1 + \theta C_2} \left(\frac{(1 - \theta^2)C_1 - 2\theta^3 C_2}{1 + \theta^2} - C_1 \right).
$$

Since $(1 - \theta^2)C_1 - 2\theta^3C_2 - (1 + \theta^2)C_1 = -2\theta^2C_1 - 2\theta^3C_2 < 0$, we get $\partial C_1/\partial \theta > 0$. As discussed in the main text, the sign of $\partial C_2/\partial \theta$ is generally ambiguous.

For part (c), note that $D_{\tau}C = -[D_{C}f]^{-1} \cdot D_{\tau}f$ and

$$
D_{\tau}f = -\left[\begin{array}{c} \frac{\psi_{\tau}}{\psi}(C_1 - \theta C_2) \\ \frac{\phi_{\tau}}{\phi}(C_2 - \theta C_1) \end{array}\right]
$$

with both elements positive, since $\psi_{\tau} < 0$ and $\phi_{\tau} < 0$. It follows that the elements of $D_{\tau}C$ are negative meaning that C_1 and C_2 are decreasing in τ .

Proof of Proposition [3.](#page-18-2) As follows from [\(17\)](#page-13-3), the utilities in the FE are given by

$$
U_{\mathcal{F}}^1 = \left(\frac{1}{1-\sigma} - \frac{\tau}{\tau - 1}\right) K_1^{\frac{1-\sigma}{\sigma}}, \qquad U_{\mathcal{F}}^2 = \left(\frac{1}{1-\sigma} \cdot \left[\frac{1-\tau\theta^2}{\theta(\tau-1)}\right]^{1-\sigma} - \frac{1}{\theta(\tau-1)} \cdot \frac{K_1}{K_2}\right) K_1^{\frac{1-\sigma}{\sigma}}.\tag{A8}
$$

In the KUJE, as follows from [\(16\)](#page-13-2), they are

$$
U_{\text{KUJ}}^{i} = \left(\frac{1}{1-\sigma} - \frac{1}{1-\theta^{2}}\right) \cdot K_{i}^{\frac{1-\sigma}{\sigma}} - \frac{\theta}{1-\theta^{2}} \cdot \frac{K_{j}^{\frac{1}{\sigma}}}{K_{i}}, \quad i, j = 1, 2, \ i \neq j. \tag{A9}
$$

For Agent [1,](#page-14-1) $U_{\text{KUJ}}^1 < U_{\text{F}}^1$ iff $k < [\theta(\tau - 1)/(1 - \tau \theta^2)]^{\sigma}$ which, as shown in Proposition 1, is always true in the FE. For Agent 2, $U_{\text{KUJ}}^2 < U_{\text{F}}^2$ iff

$$
\left[\frac{1-\tau\theta^2}{\theta(\tau-1)}\cdot\frac{1}{1-\theta^2}\right]k < -\frac{1}{\sigma-1}\cdot\left[\frac{1-\tau\theta^2}{\theta(\tau-1)}\right]^{1-\sigma} + \left(\frac{1}{\sigma-1} + \frac{1}{1-\theta^2}\right)k^{\frac{\sigma-1}{\sigma}}.\tag{A10}
$$

Let $L(k)$ and $R(k)$ denote the left-hand side and the right-hand side of [\(A10\)](#page-35-0), respectively. Note that: 1) $R(k)$ is strictly increasing and concave, 2) $L(\tilde{k}) = R(\tilde{k})$, and 3) $L'(\tilde{k}) > R'(\tilde{k})$. Hence, either $L(k) < R(k)$ $\forall k \in (\hat{k}, \tilde{k})$ or $\exists! \bar{k} \in (\hat{k}, \tilde{k})$, such that $L(k) < R(k)$ iff $k > \bar{k}$. The former is true iff $L(\hat{k}) < R(\hat{k})$ which boils down to the condition

$$
\left(\frac{1}{1-\theta^2} + \frac{1}{\sigma - 1}\right) \left(\frac{1+\theta^2}{2}\right)^{\frac{\sigma - 1}{\sigma}} - \frac{1}{\sigma - 1} - \frac{1+\theta^2}{2(1-\theta^2)} > 0.
$$

Let $z \equiv (1 + \theta^2)/2 \in (1/2, 1)$ and $a \equiv (\sigma - 1)/\sigma \in (0, 1)$. Then, after re-arranging this condition becomes

$$
g(z) \equiv [a+2(1-a)(1-z)]z^{a} + (2-3a)z - 2(1-a) > 0.
$$

Note that: 1) $g(1/2) = (1/2)^a + a/2 - 1 < 0$ since it is strictly convex in a on the $(0, 1)$ interval and is equal to zero at its endpoints, 2) $g(1) = 0, 3$ $g'(1) = a(a-1) < 0$, and 4) $g''(z) < 0$. Hence $\exists ! \ \bar{z} \in (1/2, 1)$, such that $g(z) > 0$ iff $z > \overline{z}$. This implies that $\exists! \bar{\theta} \in (0,1)$, such that $L(k) < R(k)$ (and thus, $U_{\text{KUJ}}^2 < U_{\text{F}}^2$) $\forall k \in (\hat{k}, \tilde{k}) \text{ iff } \theta > \bar{\theta}$. If $\theta < \bar{\theta}$, $\exists ! \bar{k} \in (\hat{k}, \tilde{k})$, such that $U_{\text{KUJ}}^2 < U_{\text{F}}^2$ iff $k > \bar{k}$, which completes the proof.

Proof of Proposition [4.](#page-20-0) It follows immediately from [\(A8\)](#page-35-1) that $dU_F^1/dk > 0$ since $K_1 = \lambda K = kK/(1+k)$. As for Agent 2,

$$
U_{\rm F}^2 = \left(\frac{1}{1-\sigma} \cdot \left[\frac{1-\tau\theta^2}{\theta(\tau-1)}\right]^{1-\sigma} - \frac{1}{\theta(\tau-1)} \cdot k\right) \left(\frac{k}{1+k}\right)^{\frac{1-\sigma}{\sigma}} K^{\frac{1-\sigma}{\sigma}}.
$$

Differentiating with respect to k and re-arranging terms we obtain that $dU_F^2/dk > 0$ iff $h(k) \equiv \sigma k^2 + k - \delta <$ 0, where δ is defined in Proposition [4.](#page-20-0) Next, $h(\hat{k}) > 0$ iff

$$
\sigma \left(\frac{1+\theta^2}{2}\right)^{2\sigma} \left[\frac{\theta(\tau-1)}{1-\tau\theta^2}\right]^\sigma + \frac{1+\theta^2}{2} > 1-\tau\theta^2.
$$

If the above condition holds, $h(k) > 0$ and hence, $dU_F^2/dk < 0 \ \forall k \in (\hat{k}, \tilde{k})$. Otherwise $\exists ! k^*$, such that h(k^{*}) = 0, namely k^{*} = ($\sqrt{1+4\sigma\delta}-1$)/2σ. Moreover, k^{*} < k̃ since h(k̄) ∝ σ[θ(τ-1)/(1-τθ²)]^σ +τθ² > 0. Finally $(A9)$ can be re-written in terms of k as

$$
U_{\text{KUJ}}^1 = -\left(\frac{k}{1+k}\right)^{\frac{1-\sigma}{\sigma}} \cdot \left(\frac{1}{\sigma-1} + \frac{1}{1-\theta^2} + \frac{\theta}{1-\theta^2} \cdot \frac{1}{k}\right);
$$

$$
U_{\text{KUJ}}^2 = -\left(\frac{K}{1+k}\right)^{\frac{1-\sigma}{\sigma}} \cdot \left(\frac{1}{\sigma-1} + \frac{1}{1-\theta^2} + \frac{\theta}{1-\theta^2} \cdot k^{\frac{1}{\sigma}}\right),
$$

which implies that $dU_{\text{KUJ}}^1/dk > 0$ and $dU_{\text{KUJ}}^2/dk < 0$.

Proof of Lemma [4.](#page-22-1) Using (8) and (9) , rewrite (22) as

$$
v\left((1+\theta^2)\sqrt{\frac{(Y_1+T)(Y_2-T)}{\tau\theta}}-\frac{\theta(\tau+1)}{\tau}(Y_1+T)\right)\longrightarrow \max_T \qquad s.t. \quad 0 \leq T \leq \overline{T}.
$$

The objective function is strictly concave in T. The first-order condition for interior solution, T^* , is

$$
\frac{1+\theta^2}{2\sqrt{\tau\theta}}\cdot\frac{Y_2-Y_1-2T^*}{\sqrt{(Y_1+T^*)(Y_2-T^*)}}=\frac{\theta(\tau+1)}{\tau},\quad 0
$$

It is straightforward to check that $v'(\overline{T}) \geq 0$ iff $\tau \theta \leq (1-\theta)/(1+\theta)$ which is the condition for full transfer. On the other hand, $v'(0) < 0$ iff $[(1+\theta^2)/2]$ √ $\overline{\tau\theta}$] $(1-x^2) < x[\theta(\tau+1)/\tau]$, where $x \equiv \sqrt{Y_1/Y_2}$. Solving this inequality yields the condition $Y_1/Y_2 \ge \mu$, where $\sqrt{\mu} \equiv \left[\sqrt{\tau^2(1+\theta^2)^2 + \theta^2(\tau+1)^2\tau\theta} - \theta(\tau+1)\sqrt{\tau\theta}\right] / [\tau(1+\theta^2)]$ θ^2), and $0 < \mu \leq \tau \theta$ if $\tau \theta \leq (1 - \theta)/(1 + \theta)$.

Proof of Lemma [5.](#page-22-2) In the KUJ case, $\tau \theta Y_2 \leq Y_1$, as established in the proof of Lemma [2,](#page-11-1) the optimal action of Agent 2 is [\(A1\)](#page-32-3). In the case with full transfer, $p_2^* = 1$, $d_1^* = 0$, $Y_2 - \overline{T} = (Y_1 + Y_2)/(1 + \tau \theta)$, $Y_1 + \overline{T} = \tau \theta (Y_1 + Y_2)/(1 + \tau \theta)$, and so, the utility function of Agent 2 is given by

$$
U^{2} = \frac{\gamma_{2}(Y_{1} + Y_{2})^{1-\sigma}}{1-\sigma} - \frac{Y_{2}}{K_{2}},
$$

where γ_2 is defined in Lemma [5.](#page-22-2) It is strictly concave in Y₂ and the unique optimum is given by Y₂ $(\gamma_2 K_2)^{1/\sigma} - Y_1$, which is interior iff $Y_1 < \check{Y}_1$, where \check{Y}_1 is defined in Lemma [5,](#page-22-2) and $\check{Y}_1 < \widetilde{Y}_1$, as direct comparison shows. This yields [\(23\)](#page-22-3).

Proof of Lemma [6.](#page-23-3) In the KUJ case, $\tau \theta Y_2 \leq Y_1$, as established in the proof of Lemma [3,](#page-12-2) the optimal action of Agent 1 is [\(A5\)](#page-33-2). Now consider the case in which Agent 2 makes a full transfer under a credible threat of destruction, $\tau \theta Y_2 > Y_1$. The utility of Agent 1 is

$$
U^{1} = \frac{\gamma_{1}(Y_{1} + Y_{2})^{1-\sigma}}{1-\sigma} - \frac{Y_{1}}{K_{1}},
$$

where γ_1 is defined in Lemma [5.](#page-22-2) This yields the following solution:

$$
Y_1 = \begin{cases} b_1 \equiv \tau \theta Y_2, & \text{if } Y_2 < \check{Y}_2 / (1 + \tau \theta); \\ b_2 \equiv (\gamma_1 K_1)^{1/\sigma} - Y_2, & \text{if } \check{Y}_2 / (1 + \tau \theta) \le Y_2 < \check{Y}_2; \\ b_3 \equiv 0, & \text{if } Y_2 \ge \check{Y}_2, \end{cases} \tag{A11}
$$

where \check{Y}_2 is defined in Lemma [5.](#page-22-2) First, note that if the optimum is not interior in the KUJ region, that is, if $Y_2 > \hat{C}_2$, the global optimum is determined by [\(A11\)](#page-37-2) since the left derivative of U^1 at point $Y_1 = \tau \theta Y_2$ is less than the right derivative. In particular, $dU^1(\tau \theta Y_2^-)/dY_1 = \gamma_1(1+\tau \theta)^{-\sigma}Y_2^{-\sigma} - 1/K_1$, while $dU^1(\tau \theta Y_2^+) / dY_1 = (\tau \theta Y_2 - \theta Y_2)^{-\sigma} - 1/K_1$, and, after re-arrangement, the former is smaller than the latter iff $\gamma_1 < [(1 + \tau \theta)/\theta(\tau - 1)]^{\sigma}$ which always holds since $\theta(\tau - 1) < 1 + \tau \theta$.

Consider for concreteness only the case $\check{Y}_2 < \hat{C}_2$ (the other case can be analyzed in exactly the same way) which holds if

$$
\gamma_1^{1/\sigma} < \frac{1}{\theta(\tau - 1)}.\tag{A12}
$$

Then, as follows from [\(A11\)](#page-37-2), $Y_2 \geq \widehat{C}_2$ implies that the best response will be b₃. If, however, $Y_2 < \widehat{C}_2$, the optimum in the KUJ region is interior and needs to be compared to the best response in the transfer region. In the former case

$$
U_{\text{KUJ}}^1 = \frac{\sigma}{1 - \sigma} K_1^{\frac{1 - \sigma}{\sigma}} - \frac{\theta Y_2}{K_1}.
$$

If $\check{Y}_2 \leq Y_2 < \hat{C}_2$, b_3 is the best response in the transfer region and the corresponding utility is

$$
U_{\rm FT}^1 = \frac{\gamma_2 Y_2^{1-\sigma}}{1-\sigma}.
$$

Then, it is easy to show that $U_{\text{FT}}^1 > U_{\text{KUJ}}^1$ in the region $\check{Y}_2 \leq Y_2 < \widehat{C}_2$ if the following condition holds:

$$
[1 - \theta(\sigma - 1)]\gamma_1^{1/\sigma} < \sigma. \tag{A13}
$$

In particular, $U_{\text{FT}}^1 > U_{\text{KUJ}}^1$ iff

$$
\frac{\gamma_1 Y_2^{1-\sigma}}{\sigma-1} - \frac{\theta Y_2}{K_1} < \frac{\sigma}{\sigma-1} K_1^{\frac{1-\sigma}{\sigma}}.
$$

The left-hand side of this expression, $L(Y_2)$ is strictly decreasing in Y_2 , which implies that the above inequality always holds in the region $\check{Y}_2 \leq \check{Y}_2 < \hat{C}_2$ if it holds at point $Y_2 = \check{Y}_2$. The latter yields condition [\(A13\)](#page-37-1). Assume that it holds along with [\(A12\)](#page-37-0), in which case $Y_1 = 0$ is the best response for $Y_2 \geq Y_2$.

Next, consider the region $\check{Y}_2/(1+\tau\theta) \leq Y_2 < \check{Y}_2$, in which b_2 is the best response in the case with transfer and the corresponding utility is

$$
U^1_{\rm FT}=\frac{\sigma}{1-\sigma}\gamma_1^{1/\sigma}K_1^{\frac{1-\sigma}{\sigma}}+\frac{Y_2}{K_1}.
$$

Since $(A13)$ holds and KUJ optimum is interior (that is, b_1 cannot be a best response) it must be the case that in this region there exists a value of Y_2 such that $U_{\text{FT}}^1 = U_{\text{KUJ}}^1$. Direct computation shows that this value is \widetilde{Y}_2 , as defined in Lemma [5,](#page-22-2) where $\widetilde{Y}_2/(1+\tau\theta) < \widetilde{Y}_2 < \widetilde{Y}_2$. So, the global optimum is the KUJ best response, if $Y_2 \leqslant Y_2$, and b_2 , otherwise. Putting everything together yields the statement of Lemma [5.](#page-22-2)

Proof of Proposition [5.](#page-23-2) For [\(16\)](#page-13-2) to be an equilibrium, as follows from Lemma [5](#page-22-2) and Lemma [6,](#page-23-3) the following conditions must hold: $Y_1^{\text{KUJ}} \geq \widetilde{Y}_1$ and $Y_2^{\text{KUJ}} \leq \widetilde{Y}_2$. The former yields the condition $k \geq$ $(\gamma_2/\gamma_1)^{\sigma/(\sigma-1)}$, while the latter leads to $k \geq \omega$. As follows from the lemmas below, ω is well-defined and exceeds $(\gamma_2/\gamma_1)^{\sigma/(\sigma-1)}$, that is, a KUJ equilibrium exists iff $k \geq \omega$.

Lemma A. Under the full transfer condition, $\tau \theta \leq (1 - \theta)/(1 + \theta)$, the parameter ω is well-defined, that is, $\sigma(1-\gamma_1^{1/\sigma})(1-\theta) > \theta(\sigma-1)$. The latter can be re-arranged as $(\sigma-\theta)/(1-\theta) < \sigma\gamma_1^{1/\sigma}$. Define $a \equiv (\sigma - 1)/\sigma \in (0, 1)$. Then, the preceding inequality turns into

$$
\frac{1-\theta(1-a)}{1-\theta}<\left[\frac{1+\tau\theta}{\theta(\tau-1)}\right]^a.
$$

The left-hand side of this expression, $L(a)$, is linear in a, with $L(0) = 1$, $L(1) = 1 - \theta$, and $L'(0) =$ $\theta/(1-\theta)$. The right-hand side, $R(a)$, is strictly increasing and convex in a, with $R(0) = 1 = L(0)$, $R(1) = (1+\tau\theta)/(\theta(\tau-1)) > L(1)$, and $R'(0) = \ln[(1+\tau\theta)/\theta(\tau-1)]$. Note that $R'(0)$ is strictly decreasing in τ and, hence, under the full-transfer assumption $\tau < [1 - \theta]/[\theta(1 + \theta)]$, inf $R'(0) = \ln[2/(1 - 2\theta - \theta^2)]$. The latter is well-defined since $\theta < \sqrt{2}-1$, as follows from the full-transfer assumption together with $\tau > 1$. Finally, $\ln[2/(1-2\theta-\theta^2)]$ monotonically increases in θ on the segment [0, √ $(2-1)$, with a minimum value ln 2 at $\theta = 0$, while $L'(0)$ is monotonically increasing with sup $L'(0) = 1/$ $^{\prime}$ $^{\prime}$ $\overline{2} < \ln 2$. Hence, $L'(0) < R'(0)$ for all $a \in (0,1)$ and, given the other properties outlined in the lemma, $L(a) < R(a)$ for all $a \in (0,1)$. Thus, ω is well-defined.

Lemma B. It is always true that $\omega > (\gamma_2/\gamma_1)^{\sigma/(\sigma-1)}$. Given the definitions of ω , γ_1 , and γ_2 , as well as a from Lemma A, this inequality can be re-written as

$$
\left[\frac{1+\tau\theta}{\theta(\tau-1)}\right]^a<1+\frac{a(1+\theta)}{\theta(\tau-1)}.
$$

The left-hand side is a strictly increasing convex function of a , while the right-hand side is linear in a . Moreover, the two parts coincide at the endpoint of the $[0, 1]$ segment. It follows that the above inequality holds for all $a \in (0,1)$.

For [\(25\)](#page-24-1) to be an equilibrium the following conditions must hold: $Y_1^{\text{FT}} \leqslant Y_1$ and $Y_2^{\text{FT}} \geqslant Y_2$. The former is always true, while the latter yields the condition $k \leq \gamma_2/\gamma_1$. It follows that FT and KUJ equilibria coexist iff $\omega \leq k \leq \gamma_2/\gamma_1$, where $\omega < \gamma_2/\gamma_1$ is the existence assumption. If $k = \gamma_2/\gamma_1$, the best-response functions partially overlap yielding a continuum of equilibria.

Finally, it can be shown that the fear equilibrium without transfers (as in the basic model) does not exist. For [\(17\)](#page-13-3) to be an equilibrium, condition $Y_2^{\text{F}} = \hat{C}_2 \leqslant \tilde{Y}_2$ must be satisfied, but restriction [\(A12\)](#page-37-0) rules this out since $\tilde{Y}_2 < \tilde{Y}_2 < \tilde{C}_2$. Putting everything together yields the statement of Proposition [5.](#page-23-2)

Proof of Proposition [6.](#page-25-0) It follows from [\(16\)](#page-13-2) and [\(25\)](#page-24-1) that

$$
\frac{C_1^{\text{KUJ}}}{C_2^{\text{KUJ}}} = \frac{k^{1/\sigma} + \theta}{1 + \theta k^{1/\sigma}}, \qquad \frac{C_1^{\text{FT}}}{C_2^{\text{FT}}} = \tau \theta.
$$

Then, inequality is higher in FT equilibrium iff $k > [(1 - \tau \theta^2)/\theta(\tau - 1)]^{-\sigma}$, and it is sufficient to show that $\omega > [(1 - \tau \theta^2)/\theta(\tau - 1)]^{-\sigma}$ to prove part 1. From definition of ω and after re-arrangement, the latter holds iff $(1+\theta)/[\theta(\tau-1)] > \sigma(\gamma_1^{1/\sigma}-1)/(\sigma-1)$. Denote the right-hand side of this inequality as $L(a) \equiv ([(1+\tau\theta)/\theta(\tau-1)]^a - 1/a$, where $a \equiv (\sigma-1)/\sigma \in (0,1)$. This function is strictly increasing if $L'(a) > 0$, which holds iff

$$
q(a) \equiv \left[\frac{1+\tau\theta}{\theta(\tau-1)}\right]^a \cdot \left(a\ln\left[\frac{1+\tau\theta}{\theta(\tau-1)}\right] - 1\right) + 1 > 0.
$$

The function $q(a)$ is strictly increasing since

$$
q'(a) = a \left[\frac{1 + \tau \theta}{\theta(\tau - 1)} \right]^a \cdot \ln^2 \left[\frac{1 + \tau \theta}{\theta(\tau - 1)} \right] > 0.
$$

Hence, $\inf q = \lim_{a\to 0} q(a) = 0$. This implies that $L'(a) > 0$. Furthermore, $\lim_{a\to 1} L(a) = (1+\theta)/[\theta(\tau-1)],$ which completes the proof of part 1.

It follows from [\(16\)](#page-13-2) and [\(25\)](#page-24-1) that $Y^{KUI} > Y_2^{FT}$ iff $(k^{1/\sigma} + 1)/(1 - \theta) > \gamma_2^{1/\sigma}$. Since $k > \omega$,

$$
\frac{k^{1/\sigma}+1}{1-\theta} > \frac{\omega^{1/\sigma}+1}{1-\theta} = \frac{\sigma\gamma_1^{1/\sigma}-1}{\sigma(\gamma_1^{1/\sigma}-1)-\theta(\sigma\gamma_1^{1/\sigma}-1)}.
$$

As can be easily shown, the right-hand side of this expression exceeds $\gamma_2^{1/\sigma}$ iff $1 - \theta < h(a)$, where

$$
h(a) \equiv \frac{1}{1 + L(a)} + \left[\frac{1 - \tau \theta^2}{1 + \tau \theta}\right]^a
$$

and $f(a)$ is defined as above. Since $L'(a) > 0$, $h'(a) < 0$. Furthermore, $\lim_{a\to 1} h(a) = 1 - \theta$, completing the proof of part 2.

As follows from [\(2\)](#page-7-2), [\(5\)](#page-8-3), and Proposition [5,](#page-23-2) in the FT equilibrium the utilities of agents are given by:

$$
U_{\text{FT}}^1 = \frac{\gamma_1(\gamma_2 K_2)^{\frac{1-\sigma}{\sigma}}}{1-\sigma}, \qquad U_{\text{FT}}^2 = \frac{\sigma \gamma_2^{1/\sigma} K_2^{\frac{1-\sigma}{\sigma}}}{1-\sigma}.
$$

In the KUJ equilibrium they are given by [\(A9\)](#page-35-2). Direct comparison shows that $U_{\text{KUJ}}^1 > U_{\text{FT}}^1$ iff (1 – $\theta^2 \gamma_1 \gamma_2^{(1-\sigma)/\sigma} k > (\sigma - \theta^2) k^{1/\sigma} + \theta(\sigma - 1)$, which is the condition in Proposition [6.](#page-25-0) Similarly, $U_{\text{KUJ}}^2 > U_{\text{FT}}^2$ iff

$$
k^{1/\sigma} < \frac{\sigma \gamma_2^{1/\sigma} - 1}{\sigma - 1} \cdot \frac{1 - \theta^2}{\theta} - \frac{1}{\theta}.
$$

Since the left-hand side is strictly increasing in k , it is sufficient to prove that the above inequality holds at $k = \gamma_2/\gamma_1$ to establish that it holds $\forall k \in (\omega, \gamma_2/\gamma_1)$. After re-arranging the terms, at $k = \gamma_2/\gamma_1$ the inequality takes the form

$$
a\theta^2 + (1 - \theta^2) < \left[\frac{1 + \tau\theta}{1 - \tau\theta^2}\right]^a - a\theta \left[\frac{\theta(\tau - 1)}{1 - \tau\theta^2}\right]^a.
$$

Denote the right-hand side of this inequality as $R(a)$. Then, $R'(a) > 0$ iff

$$
q(a) \equiv \left[\frac{1+\tau\theta}{\theta(\tau-1)}\right]^a \ln\left[\frac{1+\tau\theta}{1-\tau\theta^2}\right] - \theta a \ln\left[\frac{\theta(\tau-1)}{1-\tau\theta^2}\right] - \theta > 0.
$$

The function $q(a)$ is strictly increasing since

$$
q'(a) = \left[\frac{1+\tau\theta}{\theta(\tau-1)}\right]^a \ln^2\left[\frac{1+\tau\theta}{1-\tau\theta^2}\right] - \theta\ln\left[\frac{\theta(\tau-1)}{1-\tau\theta^2}\right] > 0,
$$

as $\theta(\tau-1)/(1-\tau\theta^2) < 1$. Hence, inf $R = \lim_{a\to 0} R(a) = \ln[(1+\tau\theta)/(1-\tau\theta^2)] - \theta > 0$. The latter inequality holds because: 1) $\ln[(1+\tau\theta)/(1-\tau\theta^2)]-\theta > \ln[1/(1-\theta)]-\theta$, since $\tau > 1$, and 2) $\ln[1/(1-\theta)]$ is a strictly convex function in θ with the slope of 1 at $\theta = 0$, that is, it always lies above θ . Hence, $R'(a) > 0$. Furthermore, inf $R = \lim_{a\to 0} R(a) = 1$, while $\sup\{a\theta^2 + 1 - \theta^2\} = 1$, which completes the proof of part 3.

Proof of Lemma [8.](#page-27-2) The functional forms of $g_i(k_t)$, $i = 1, 2$, follow directly from [\(16\)](#page-13-2), [\(17\)](#page-13-3), and [\(29\)](#page-26-4). The form of $g_3(k_t)$ follows from [\(12\)](#page-11-3), [\(15\)](#page-12-4) and (29). Next, it is straightforward to establish by differentiation that $g_1(k)$ and $g_2(k)$ are strictly increasing and concave, given that $\beta < 1$ and $\sigma(1-\beta) > 1$. Furthermore, 1) $g_1(1) = 1$, 2) $g_1(\tilde{k}) = g_2(\tilde{k}) > \tilde{k}$, and 3) $g_2(\hat{k}) = g_3(\hat{k}) > \hat{k}$. The second property holds iff $\tau\theta > (\tilde{k})^{1-\beta} = [\theta(\tau-1)/(1-\tau\theta^2)]^{\sigma(1-\beta)}$ which is always trues since $\tau\theta > \theta(\tau-1)/(1-\tau\theta^2)$ and $\sigma(1-\beta) > 1$. The third property holds iff $\tau \theta > (\hat{k})^{1-\beta}$ which is always true, as follows from property 2 and the fact that $\hat{k} < \tilde{k}$. Together these properties imply that $k_{t+1} > k_t$ for $k_t \geq \hat{k}$.

Finally, if there exists a steady state \bar{k} in the destructive segment, it is given by $g_3(\bar{k}) = \bar{k}$. Making a substitution $\kappa = \bar{k}^{1-\beta}$ and rearranging terms yields the equation defining the steady state:

$$
\frac{\kappa^{2-\beta}}{(\kappa+\theta)^{2(1-\beta)}} = \chi \left(\frac{\kappa-\theta}{1-\theta\kappa}\right)^{\sigma(1-\beta)},\tag{A14}
$$

where $\chi \equiv [\tau (1 + \theta^2)/2\theta (1 + \tau)^2]^{1-\beta}$ and in equilibrium $\kappa > \theta$. It is easy to show that $R(\kappa)$, the righthand side of $(A14)$, is strictly increasing and convex. Also, $L(\kappa)$, the left-hand side of $(A14)$, is strictly increasing. To show that it is concave for $\kappa > \theta$, note that the sign of $L''(\kappa)$ is determined by the sign of $\hat{L}(\kappa) \equiv (\kappa + \theta)(2-\beta)(\beta\kappa + \theta(1-\beta)) - \kappa(3-2\beta)(\beta\kappa + \theta(2-\beta))$. At $\kappa = \theta$ this expression is negative, since β < 1. Moreover, it is strictly decreasing for the same reason. Hence, $L''(\kappa)$ < 0 for $\kappa > \theta$. This implies that there exists at most one solution to [\(A14\)](#page-40-0). If there is no solution, the proof is finished. Assume now that \bar{k} exists. In this case $\bar{k} > \hat{k}$. To see this, note first that $\hat{k} < (\tau \theta)^{1/(1-\beta)}$. Next, $\kappa > \tau \theta$ since $L(\tau\theta) > R(\tau\theta)$. Hence, $\bar{k} \equiv \kappa^{1/(1-\beta)} > (\tau\theta)^{1/(1-\beta)} > \hat{k}$, that is, $k_{t+1} > k_t$ for $0 < k_t < \hat{k}$.

Proof of Lemma [9.](#page-27-3) It follows from the properties of $g_1(k_t)$ that k_t monotonically converges to 1 in the KUJ region. The expression for $\Delta K_i = 0$ comes from [\(30\)](#page-27-1) and may be rewritten as $K_{jt} =$ $K_{it}[(\rho K_{it}^{1-\beta-1/\sigma}-1)/\theta]^{\sigma}$, where $\rho \equiv (1-\theta^2)/\alpha$. Then, by differentiation we get that $dK_{jt}/dK_{it} > 0$ and $d^2K_{jt}/dK_{it}^2 > 0$, since $\sigma(1-\beta) > 1$. This implies the stated properties of the $\Delta K_i = 0$ loci. The expression for \bar{K} follows from solving $\Delta K_1 = \Delta K_2 = 0$ and [\(16\)](#page-13-2) then gives \bar{Y} .

Proof of Lemma [10.](#page-28-1) The equations for $\Delta K_i = 0$ come from [\(30\)](#page-27-1). Since in the fear region $K_{1t+1} =$ $\alpha \tau K_1^{1/\sigma+\beta}/(\tau-1)$ and $\sigma(1-\beta) > 1$, K_{1t} converges to \bar{K}_1 . Plugging this in $\Delta K_2 = 0$ yields \bar{K}_2 . The output levels follow from [\(17\)](#page-13-3). To see that the long-run fear equilibrium is in the KUJ region, note that the implied steady-state level of inequality is $[\tau \theta]^{1/(1-\beta)}$ which exceeds $\tilde{k} = [\theta(\tau-1)/(1-\tau \theta^2)]^{\sigma}$, since $\sigma(1-\beta) > 1$ and $\theta(\tau-1)/(1-\tau\theta^2) < \tau\theta < 1$.

Equilibria in the model with symmetric destruction opportunities. As mentioned in section [3,](#page-7-0) the assumption about "predator and prey" relationship between Agent 1 and Agent 2 is without loss of generality. Combining the proofs of lemmas [2](#page-11-1) and [3](#page-12-2) we get that in the general case the best responses of both agents are symmetric and have the following form:

$$
C_i^*(C_j) = \begin{cases} \bar{C}_i^d(C_j), & \text{if } C_j > \check{C}_j; \\ K_i^{1/\sigma} + \theta C_j, & \text{if } \tilde{C}_j < C_j \leq \check{C}_j; \\ C_j \cdot \frac{1}{\tau \theta}, & \text{if } \hat{C}_j \leq C_j < \tilde{C}_j; \\ C_i^d(C_j), & \text{if } C_j < \hat{C}_j, \end{cases}
$$

where $i, j = 1, 2, i \neq j$, the threshold values are given by

$$
\widehat{C}_j \equiv \frac{\tau \theta}{1 - \tau \theta^2} \left(\frac{1 + \theta^2}{2} K_i \right)^{1/\sigma}, \quad \widetilde{C}_j \equiv \frac{\tau \theta}{1 - \tau \theta^2} K_i^{1/\sigma}, \quad \widetilde{C}_j \equiv \frac{1}{\theta(\tau - 1)} K_i^{1/\sigma},
$$

and the functions $C_i^d(C_j)$, $\overline{C}_i^d(C_j)$ are implicitly defined by

$$
C_i^d(C_j): C_i - \theta C_j = \phi \left(\frac{C_j + \theta C_i}{C_i}\right)^{1/\sigma}, \quad \phi_i \equiv \left(\frac{1+\theta^2}{2\theta(1+\tau)} K_i\right)^{1/\sigma},
$$

$$
\bar{C}_i^d(C_j): C_i - \theta C_j = \psi \left(\frac{C_i}{C_i + \theta C_j}\right)^{1/\sigma}, \quad \psi_i \equiv \left(\frac{1+\tau}{\tau} K_i\right)^{1/\sigma}.
$$

It follows from the proof of Proposition [1](#page-14-1) and the fact that $K_1 < K_2$ that the equilibria of this game in general formulation are fully characterized in Proposition [1.](#page-14-1) Figure [14](#page-41-0) depicts the alternative equilibria under symmetric best responses (cf. figures [3](#page-13-0) and [4\)](#page-14-0).

Figure 14: Equilibria in the symmetric envy game.

References

- Banerjee, Abhijit, "Envy," in Bhaskar Dutta, Shubhashis Gangopadhyay, Debraj Ray, and Dilip Mookherjee, eds., Economic Theory and Policy: Essays in Honour of Dipak Banerjee, New York: Oxford University Press, 1990, pp. 91–111.
- Barnett, Richard C., Joydeep Bhattacharya, and Helle Bunzel, "Choosing to Keep up with the Joneses and Income Inequality," Economic Theory, December 2010, $45(3)$, 469–496.
- Belk, Russell W., "Marketing and Envy," in Richard H. Smith, ed., *Envy: Theory and* Research, Oxford University Press, 2008, chapter 12, pp. 211–226.
- Benhabib, Jess, Alberto Bisin, and Matthew Jackson, eds, Handbook of Social Economics, Vol. 1, Elsevier, 2011.
- Boskin, Michael J. and Eytan Sheshinski, "Optimal Redistributive Taxation when Individual Welfare Depends upon Relative Income," Quarterly Journal of Economics, November 1978, 92 (4), 589–601.
- Bowles, Samuel and Yongjin Park, "Emulation, Inequality, and Work Hours: Was Thorsten Veblen Right?," Economic Journal, November 2005, 115 (507), F397–F412.
- Cancian, Frank, Economics and Prestige in a Maya Community, Stanford, CA: Stanford University Press, 1965.
- Carroll, Christopher D., Jody R. Overland, and David N. Weil, "Comparison Utility in a Growth Model," Journal of Economic Growth, December 1997, 2 (4), 339–367.
- Clanton, Gordon, "Jealousy and Envy," in Jan E. Stets and Jonathan H. Turner, eds., Handbook of the Sociology of Emotions, Berlin: Springer, 2006, chapter 18, pp. 410– 442.
- Clark, Andrew E. and Claudia Senik, "Who Compares to Whom? The Anatomy of Income Comparisons in Europe," Economic Journal, May 2010, 120, 573–594.
- , Paul Frijters, and Michael Shields, "Relative Income, Happiness and Utility: An Explanation for the Easterlin Paradox and Other Puzzles," Journal of Economic Literature, June 2008, 46 (1), 95–144.
- Clark, Anrew E. and Andrew J. Oswald, "Comparison-Concave Utility and Following Behaviour in Social and Economic Settings," Journal of Public Economics, October 1998, 70 (1), 133–155.
- D'Arms, Justin and Alison Duncan Kerr, "Envy in the Philosophical Tradition," in Richard H. Smith, ed., Envy: Theory and Research, Oxford University Press, 2008, chapter 3, pp. 39–59.
- Demsetz, Harold, "Toward a Theory of Property Rights," American Economic Review, May 1967, 57 (2), 347–359.
- Dow, James, "The Image of Limited Production: Envy and the Domestic Mode of Production in Peasant Society," Human Organization, Winter 1981, 40 (4), 360–363.
- Elster, Jon, "Envy in Social Life," in Richard J. Zeckhauser, ed., Strategy and Choice, Cambridge, MA: MIT Press, 1991, chapter 3, pp. 49–82.
- Falk, Armin and Markus Knell, "Choosing the Joneses: Endogenous Goals and Reference Standards," Scandinavian Journal of Economics, October 2004, 106 (3), 417–435.
- Fernández de la Mora, Gonzalo, Egalitarian Envy. The Political Foundations of Social Justice, New York: Paragon House Publishers, 1987.
- Fernández, Raquel, "Does Culture Matter?," in Jess Benhabib, Alberto Bisin, and Matthew Jackson, eds., *Handbook of Social Economics*, Vol. 1, Elsevier, 2011, chapter 11, pp. 481–510.
- Fliessbach, K., B. Weber, P. Trautner, T. Dohmen, U. Sunde, C.E. Elger, and A. Falk, "Social Comparison Affects Reward-Related Brain Activity in the Human Ventral Striatum," Science, 2007, 318 (5854), 1305–1308.
- Foster, George, "The Anatomy of Envy: A Study in Symbolic Behavior," Current Anthropology, April 1972, 13 (2), 165–202.
- , Tzintzuntzan: Mexican Peasants in a Changing World, New York: Elsevier, 1979.
- Frank, Robert H., "The Demand for Unobservable and Other Nonpositional Goods," American Economic Review, March 1985, 75 (1), 101–116.
- , Falling Behind: How Rising Inequality Harms the Middle Class, Berkeley: University of California Press, 2007.
- and Ori Heffetz, "Preferences for Status: Evidence and Economic Implications," in Jess Benhabib, Alberto Bisin, and Matthew Jackson, eds., Handbook of Social Economics, New York: Elsevier, 2011, chapter 3.
- Gershman, Boris, "The Economic Origins of the Evil Eye Belief," 2011. Working Paper, Brown University.
- , "From Fear to Competition: Envy in the Process of Development," 2011. Working Paper, Brown University.
- Graham, Carol, Happiness Around the World. The Paradox of Happy Peasants and Miserable Millionaires, Oxford University Press, 2010.
- Grossman, Herschel I. and Minseong Kim, "Swords or Plowshares? A Theory of the Security of Claims to Property," Journal of Political Economy, December 1995, 103 (6), 1275–88.
- and \equiv , "Predation and Production," in Michelle R. Garfinkel and Stergios Skaperdas, eds., The Political Economy of Conflict and Appropriation, Cambridge, MA: Cambridge University Press, 1996, chapter 4, pp. 57–72.
- Hopkins, Ed, "Inequality, Happiness and Relative Concerns: What Actually Is Their Relationship?," Journal of Economic Inequality, December 2008, 6 (4), 351–372.
- and Tatiana Kornienko, "Running to Keep in the Same Place: Consumer Choice as a Game of Status," American Economic Review, September 2004, 94 (4), 1085– 1107.
- Knell, Markus, "Social Comparisons, Inequality, and Growth," Journal of Institutional and Theoretical Economics, December 1999, 155 (4), 664–695.
- Ljungqvist, Lars and Harald Uhlig, "Tax Policy and Aggregate Demand Management under Catching Up with the Joneses," American Economic Review, June 2000, 90 (3), 356–366.
- Luttmer, Erzo F., "Neighbors as Negatives: Relative Earnings and Well-Being," Quarterly Journal of Economics, August 2005, 120 (3), 963–1002.
- Matt, Susan J., Keeping Up with the Joneses: Envy in American Consumer Society, 1890–1930, Philadelphia: University of Pennsylvania Press, 2003.
- Mitsopoulos, Michael, "Envy, Institutions and Growth," Bulletin of Economic Research, July 2009, 61 (3), 201–222.
- **Mui, Vai-Lam**, "The Economics of Envy," Journal of Economic Behavior \mathcal{C} Organization, May 1995, 26 (3), 311–336.
- Neumark, David and Andrew Postlewaite, "Relative Income Concerns and the Rise in Married Women's Employment," Journal of Public Economics, October 1998, 70 $(1), 157-183.$
- Oswald, Andrew J., "Altruism, Jealousy and the Theory of Optimal Non-linear Taxation," Journal of Public Economics, February 1983, 20 (1), 77–87.
- Platteau, Jean-Philippe, Institutions, Social Norms, and Economic Development, Harwood Academic Publishers, 2000.
- Robson, Arthur J. and Larry Samuelson, "The Evolutionary Foundations of Preferences," in Jess Benhabib, Alberto Bisin, and Matthew Jackson, eds., Handbook of Social Economics, New York: Elsevier, 2011, chapter 4.
- Rustichini, Aldo, "Dominance and Competition," Journal of the European Economic Association, April–May 2008, 6 (2–3), 647–656.
- Schoeck, Helmut, *Envy: A Theory of Social Behavior*, New York: Harcourt, Brace, and World, 1969.
- Schor, Juliet B., The Overworked American: The Unexpected Decline of Leisure, New York: Basic Books, 1991.
- Scott, James C., The Moral Economy of the Peasant: Rebellion and Subsistence in Southeast Asia, New Haven, CT: Yale University Press, 1976.
- Smith, Richard H. and Sung H. Kim, "Comprehending Envy," Psychological Bulletin, January 2007, 133 (1), 46–64.
- Solnick, Sara J. and David Hemenway, "Are Positional Concerns Stronger in Some Domains than in Others?," American Economic Review Papers and Proceedings, May 2005, 95 (2), 147–151.
- van de Ven, Niels, Marcel Zeelenberg, and Rik Pieters, "Leveling Up and Down: The Experiences of Benign and Malicious Envy," Emotion, June 2009, 9 (3), 419–429.
- Veblen, Thorstein B., "Some Neglected Points in the Theory of Socialism," Annals of the American Academy of Political and Social Science, November 1891, 2 (3), 57–74.
- Wolf, Eric R., "Types of Latin American Peasantry: A Preliminary Discussion," American Anthropologist, June 1955, 57 (3, Part 1), 452–471.
- Zizzo, Daniel J., "Money Burning and Rank Egalitarianism with Random Dictators," Economics Letters, November 2003, 81 (2), 263–266.
- , "The Cognitive and Behavioral Economics of Envy," in Richard H. Smith, ed., Envy: Theory and Research, Oxford University Press, 2008, chapter 11, pp. 190–210.