# Quantitative and credit easing policies at the zero lower bound on the nominal interest rate

Meixing Dai

BETA, University of Strasbourg, France

2011

# Quantitative and credit easing policies at the zero lower bound on the nominal interest rate

**Abstract**: Using a New-Keynesian model extended to include credit, money and reserve markets, we examine the dynamics of inflation and output gap under some monetary policy options adopted when the economy is hit by large negative real, financial and monetary shocks. Relaxing the assumption that market interest rates are perfectly controlled by the central bank using the funds rate operating procedure, we have shown that the equilibrium at the zero lower bound on the nominal discount rate is stable (or cyclically stable, depending on monetary and financial parameters) and constitutes a liquidity trap, making the central bank's communication skills useless in the crisis management. While the quantitative easing policy allows attenuating the effects of crisis, it is not always sufficient to restore the normal equilibrium. Nevertheless, quantitative and credit easing policies coupled with the zero discount rate policy could stabilize the economy and make central bank's communication potentially credible during the crisis.

Key words: Zero lower bound (ZLB) on the nominal interest rate, zero interest rate policy, liquidity trap, quantitative easing policy, credit easing policy, dynamic stability.

JEL Classification: E43, E44, E51, E52, E58.

---

* BETA, University of Strasbourg, 61, avenue de la Forêt Noire – 67085 Strasbourg Cedex – France. Phone: (33) 03 68 85 21 31; Fax: (+33) 03 68 85 20 71; E-mail: dai@unistra.fr.

## 1. Introduction

During the recent global financial and economic crisis, we have witnessed a certain number of most prominent central banks in the world, e.g. the Federal Reserve, the European central bank (ECB), the Bank of England and the Bank of Japan, have brought down the discount and funds rates to a level near to zero, and massively inject the central-bank liquidity into the money and credit markets under what is called quantitative and credit easing policies.

The monetary policy experience of Japan during the 1990s and 2000s has stimulated a vivid interest among economists on the “liquidity trap” in the sense of Keynes, i.e. whatever is the quantity of central-bank liquidity injected into the money market, it is absorbed without a decrease of the nominal money-market interest rate notably because the latter has encountered the zero lower bound (ZLB). The interest has grown larger after the burst of the Internet bubble in 2000 because many economists doubt that the USA could enter a deflation crisis.

Various proposals have been advanced to make monetary policy effective in the event of the ZLB on nominal interest rates. One widely shared belief among economists is that pre-emptive monetary easing is important to minimize the likelihood that interest rates will fall to zero. Studies on the issue of pre-emptive monetary easing include, among others, Orphanides and Wieland (2000), Reifschneider and Williams (2000), Kato and Nishiyama (2005), Adam and Billi (2006, 2007), Nakov (2008), and Oda and Nagahata (2008).[1] For Benhabib *et al*. (2002), a route to avoiding self-fulfilling liquidity traps is to modify monetary policy, by switching from Taylor rule to a money growth rate target letting interest rates be market determined, when the economy seems to be headed toward a low-inflation spiral. This change of policy regime may be effective when fiscal policy is not Ricardian. Buiter and Panigirtzoglou (2003) and Buiter (2009) have proposed the use of Gesell taxes on monetary balances, which can be interpreted as a negative

[1] These authors consider the optimal commitment or discretionary policy in terms of interest rate rules using three types of theoretical frameworks, i.e. purely forward-looking, purely backward-looking, or “hybrid” forward- and backward-looking structural model.

interest rate on money, as a way to avoid liquidity traps.[2] This point of view is contested by Benhabib *et al*. (2002) who argue that if a liquidity trap is understood as a situation where the opportunity cost of holding money (instead of bonds) becomes zero, a Gesell tax clearly does not eliminate it but simply pushes the nominal interest rate on bonds at which it occurs below zero.

Another important consensus among economists is that, when nominal interest rates have fallen to zero, "expectations management" which acts on the formation of private-sector expectations about future monetary policy is important. A relatively large literature about "expectations management" is developed since the work of Krugman (1998), who argues that, even when the nominal interest rate hits the ZLB, the central bank could still stimulate the current level of output by raising expectations of future inflation.[3] Most economists working on the issue of interest rate ZLB share this point of view and suggest that the Bank of Japan commits to policies that would raise future inflation. However, raising inflation expectations and committing to reducing the policy interest rates in the future are not separate issues since it is by committing to lower future policy rates that the central bank affects future inflation at the ZLB (Eggertsson and Woodford, 2003, 2004; Jung *et al*., 2005; Adam and Billi, 2006, 2007; Nakajima, 2008; Walsh, 2009). Three alternative policy proposals involving a yen depreciation are advanced. The first calls for an aggressive base money expansion when the nominal rate reaches zero (Orphanides and Wieland, 2000). The second suggests that the central bank switches to an exchange rate-based Taylor rule when the ZLB is encountered, with the exchange rate adjusted in response to inflation and output gap (McCallum, 2000, 2001). The last proposal, due to Svensson (2001, 2003), calls for a depreciation followed by an exchange rate peg and an announced price-level target. However, these proposals have limited utility if several large economies simultaneously enter into a liquidity trap. Altering the composition of assets on the central bank's

---

[2] Buiter (2009) has made two other proposals, i.e. (1) abolishing currency; (2) decoupling the *numéraire* from the currency/medium of exchange/means of payment and introducing an exchange rate between the *numéraire* and the currency. The exchange rate is set over time to achieve a forward discount or expected depreciation of the currency vis-à-vis the *numéraire* when the nominal interest rate in terms of the *numéraire* is set at a negative level for monetary policy purposes.

[3] Another strand of research on the ZLB has considered Keynesian fiscal stimulus (Posen, 1998; Bernanke, 2000; Kuttner and Posen, 2001; Seidman, 2001; Benhabib *et al*., 2002; Lewis and Seidman, 2008).

balance sheet offers another potential lever for monetary policy while the effectiveness of such policies is a contentious issue (Bernanke and Reinhart, 2004). Auerbach and Obstfeld (2005) have shown that an open-market purchase of government debt can counteract deflationary price tendencies when the ZLB is encountered. Furthermore, the central bank can also alter monetary policy by changing the size of its balance sheet through buying or selling securities to affect the overall supply of reserves and the money stock. Therefore, even if the overnight interest rate becomes pinned at zero, the central bank can still expand the quantity of reserves beyond what is required to hold the overnight rate at zero or a very low level. Such policy, commonly referred to as “quantitative easing” is experimented firstly in Japan and currently in the United-States, the euro zone and the United-Kingdom.

There is some evidence that quantitative easing can stimulate the economy even when interest rates are near zero. The quantitative easing policy that leads to an expansion of the money supply at the ZLB will affect the economy as long as the rise in the money supply is expected to persist (Sellon, 2003). According to Spiegel (2006), in the case of Japan, the real effects of quantitative easing appear to be principally associated with some measurable declines in longer-term interest rates.[4] These have been associated both with changes in agents’ expectations of future interest rate levels and with purchases of “nonstandard” assets, such as longer-term government bonds. Since quantitative easing and other unconventional monetary policies often occurred simultaneously, it is difficult to discriminate between them.

The Fed has gone much further down the path of quantitative easing. In particular, it focuses on expanding the asset side of its balance sheet in order to lower interest rates on the credit markets. In such a policy, compared to what the Fed has traditionally done through the open-market operations or discount, the range of assets accepted is much broader, they have much

[4] The quantitative easing policy aided weaker Japanese banks and generally encouraged greater risk-tolerance in the Japanese financial system. This could have positive effects on the real economy in the short-run even though the magnitudes of these effects are very uncertain. However, in strengthening the performance of the weakest Japanese banks, quantitative easing may have had the undesired impact of delaying structural reform and could negatively impact the long term growth. For a review of Japanese experience of zero interest rate policy coupled with quantitative easing policy, see Spiegel (2006).

longer maturities and the number of financial institutions that have access to the central-bank liquidity has been significantly increased following a relaxation of criteria and a change in the status of some institutions. The Bank of England, the ECB to a lesser extent, has followed the practice of the Fed. Even though market observers initially use the term “quantitative easing”, the Fed Chairman Ben Bernanke (2009) has preferred to use the term “credit easing”. The difference between quantitative easing and credit easing does not reflect any doctrinal disagreement with the Japanese approach, but rather the differences in financial and economic conditions between the two episodes. The new term allows the Fed Chairman to better communicate with the public on unconventional policy measures and to make a difference with a monetary policy involving only the injection of central-bank liquidity through the increase of banking reserves.[5] Policies which go beyond the quantitative easing policy such as buying private-sector credit instruments or lending by the central bank have been previously discussed in some studies (Clouse *et al.*, 2003; Sellon, 2003). However, such policies are not yet discussed in a theoretical framework which clearly distinguishes quantitative and credit easing policies.

We remark that using similar framework as the literature on inflation targeting and interest rate rules, theoretical studies about the ZLB on the nominal interest rate do not make explicit the links between monetary policy and extremely negative financial and monetary shocks and hence are not wholly satisfactory for studying the underlying transmission mechanism of the effects of zero interest rate, and quantitative and credit easing policies. Discussions about the quantitative easing policy are made generally without using models except Auerbach and Obstfeld (2005), while the latter use a framework which cannot be used to discuss how quantitative and credit easing policies could interact. Furthermore, most theoretical frameworks do not explicitly consider the operating procedure of the central bank by not distinguishing the overnight, longer term interbank and credit market interest rates. When discussing about the ZLB on the nominal interest rate, most economists talk in effect about the funds rate targeted by the central bank. The

[5] When the ZLB is encountered, communication is one of most important instruments available to the central bank. Before the current crisis, Bernanke *et al.* (2004) have shown that there is in the USA some evidence that central bank communications can help to shape public expectations of future policy actions.

ZLB on the funds rate becomes a problem because we are concerned with the market interest rates which can be higher if market operators perceive that the monetary policy of lowering funds rate is not sufficient to restore the economic growth and the confidence on the financial markets. Hence, it is very important to make the distinction between the funds rate and other market interest rates.

This paper provides a framework where several policies options, such as zero interest rate policy, and quantitative and credit easing policies used in the current financial and economic crisis, could be simultaneously examined. We extend a New Keynesian framework (Clarida *et al.*, 1999) to a model of policy analysis with credit, money and reserve markets where the funds rate operating procedure is explicitly integrated. It offers a more realistic view about how the interest rate policy is first put in place through the targeting of very short-run interest rate, contrary to the existing monetary policy literature which assumes that the central bank directly controls the interest rate affecting the aggregate demand. Thus, it allows understanding why these policies become suddenly necessary under extreme financial stress. It clarifies the links between the inflation-targeting regime and these policies when important financial and monetary shocks hit the economy. Our objective is to examine the dynamics of inflation and output gap when some or all of these policy options are adopted and how these variables will behave when a particular exit strategy is adopted.

The remainder of the paper is structured as follows. The next section presents the New Keynesian model extended to include credit, money and reserve markets. The third section presents the dynamics of inflation rate and output gap under the standard inflation-targeting regime. The fourth section examines the dynamic stability of the economy when it is hit by large persistent real, financial and monetary shocks such that the ZLB on the nominal discount rate is attained. In the fifth section, we examine the inflation and output-gap dynamics under quantitative and credit easing policies. The last section summarizes our findings.

## 2. The model

The supply and demand sides of the economy are described by a stylized new-Keynesian model:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \lambda x_t + \varepsilon_{\pi t}\,, \qquad \text{with } 0 < \beta < 1\,,\ \lambda > 0\,, \tag{1}$$

$$x_t = \mathrm{E}_t x_{t+1} - \varphi(i_{ct} - \mathrm{E}_t \pi_{t+1}) + \varepsilon_{xt}\,, \qquad \text{with } \varphi > 0\,, \tag{2}$$

where $\pi_t$ $(\equiv p_t - p_{t-1})$ denotes the rate of inflation, $p_t$ the (log) general price level, $x_t$ the output gap (i.e., the log deviation of output from its flexible-price level), $i_t^c$ the nominal credit market interest rate at which non-financial private sector can borrow from banks.

Equation (1) represents the New-Keynesian Phillips curve, where the inflation rate is related to the expected future inflation rate ($\mathrm{E}_t \pi_{t+1}$) and current marginal cost, which is affected by the output gap. The inflation shock, $\varepsilon_{\pi t}$, is due to productivity disturbances.

Equation (2) is the expectational IS curve which relates the current output gap to the expected future output gap ($\mathrm{E}_t x_{t+1}$) and the real credit market interest rate. The latter is defined as the difference between the nominal credit-market interest rate $i_{ct}$ and the expected future rate of inflation, i.e. $(i_{ct} - \mathrm{E}_t \pi_{t+1})$.

We assume that the individual saver can save at $i_{ct}$ if she directly buys bonds emitted by firms, which offer a rate of return equal to $i_{ct}$. Furthermore, for simplicity, savers are assumed to save in a deposit account bearing no interests at banks and hence the intertemporal arbitrage between present consumption and saving depends only on $i_{ct}$. The aggregate demand shock, $\varepsilon_{xt}$, reflects either productivity disturbances which affect the flexible-price level of output or, equivalently, changes in the natural real interest rate. Without explicitly introducing asset prices, we admit that $\varepsilon_{xt}$ could also include wealth shocks affecting the aggregate demand.

The money market equilibrium condition is given by

$$m_t - p_t = l_1 x_t - l_2 i_{mt} + \varepsilon_{lt}\,, \qquad \text{with } l_1, l_2 > 0\,, \tag{3}$$

where $i_{mt}$ is the nominal interest rate determined on the money market at which the banks can refinance, $m_t$ represents the log nominal money supply, $\varepsilon_{lt}$ is a random money demand shock.

The money supply is endogenous but it is imperfectly elastic as the banking system will increase or decrease the internal money taking account of nominal interest rate and will not always satisfy the demand of this money (or credit) if it is expected to be unwarranted by collaterals. Similarly, the central bank provides a limited quantity of central-bank liquidity on the reserve market to a limited number of banks by accepting certain categories of assets as collaterals. Instead, if the central bank desires, control can be exercised over a narrow monetary aggregate such as base money (including reserves and currency), and its variations are then associated with these in broader measures of money supply.

The link between the total money supply and the base money is modeled as follows:

$$m_t = b_t + h_1 i_{mt} + \omega_t , \qquad h_1 > 0 \ . \tag{4}$$

where $b_t$ is the base money in log terms, and money multiplier ($m_t - b_t$ in log terms) is assumed to be an increasing function of the nominal money-market interest rate, and $\omega_t$ is a money-multiplier disturbance. The money supply function is similar to that adopted by Modigliani *et al*. (1970), and McCallum and Hoehn (1983).

We assume that the central bank indirectly targets money and credit market interest rates through the funds rate targeting procedure. Under this operating procedure, the central bank indirectly targets $i_{ct}$ or $i_{mt}$, longer term interest rates, by targeting in the first place the funds rate ($i_{ft}$), a very short-run or overnight interest rate. More precisely, the central bank controls the discount rate, $i_{dt}$, and conducts open market operations in order to affect the supply of reserves in the banking system to target the funds rate. We assume that the access to the central-bank liquidity at the discount window is submitted to nonprice rationing, so that $i_{ft} \neq i_{dt}$.[6] Similarly, we assume that the access to inside liquidity created within the banking and financial system here is subject to non-price rationing so that we generally have $i_{mt} \neq i_{ft} \neq i_{dt}$.

[6] In the absence of nonprice rationing at the discount window, the funds rate would never rise above the discount rate since a bank would never pay more for reserves than it would have to pay at the discount window (Goodfriend, 1983).

Adopting a simplified description of the reserve market to establish the link between the base money $b_t$ and the discount rate $i_{dt}$, the money supply under the funds rate targeting procedure can be rewritten as (Appendix A)

$$m_t = \tilde{b}_t + h_1 i_{mt} - h_2 i_{dt} + \varepsilon_{mt}, \qquad h_1, h_2 > 0, \tag{5}$$

where $\tilde{b}_t$ represents the currency in log terms but could also include the component of the reserves that the central bank can discretionarily control by adjusting the ratio of obligatory reserves, and $\varepsilon_{mt}$ represents shocks affecting the base money under the funds rate targeting procedure as well as these affecting the monetary multiplier. According to (5), the central bank, by controlling the discount rate, has not a strict control over the money supply since the latter is affected by the money-market interest rate and a random shock. However, in order to modify the behaviors of private agents and their inflation expectations, control can be exercised by the central bank over $\tilde{b}_t$, a component of base money which do not depend on the discount rate. The equilibrium condition on the money market (3) is rewritten as

$$\tilde{b}_t + h_1 i_{mt} - h_2 i_{dt} + \varepsilon_{mt} - p_t = l_1 x_t - l_2 i_{mt} + \varepsilon_{lt}. \tag{6}$$

In the following, we assume that $h_1 - h_2 + l_2 > 0$. This is justified on the ground that the supply of liquidity by the banking sector is most likely determined by the difference ($i_{mt} - i_{dt}$), i.e. the net gain obtained from providing more liquidity while refinancing it at the discount rate. Thus, an increase of equal amount in $i_{dt}$ and $i_{mt}$ will not (or modestly) affect the money supply but will significantly reduce the money demand, *ceteris paribus*.

The model is completed by a credit market equilibrium condition in the spirit of Bernanke and Blinder (1988):

$$-f_1 i_{mt} + f_2 i_{ct} = j_1 x_t - j_2 i_{ct} + \varepsilon_{ct}, \qquad \text{with } f_1, f_2, j_1, j_2 > 0, \tag{7}$$

where $\varepsilon_{ct}$ denotes a random shock that includes both credit supply and credit demand shocks. Equation (7) gives the credit-market clearing condition and it allows determining $i_{ct}$ for given $i_{mt}$. The supply of loans decreases with $i_{mt}$ and increases with $i_{ct}$. The demand of loans decreases with $i_{ct}$ and it is an increasing function of output gap $x_t$ due to transactions demand for credit, which might arise, for example, from working capital or liquidity considerations. We admit that $f_1 - f_2 - j_2 < 0$, i.e. an increase of identical amount in $i_{mt}$ and $i_{ct}$ will leave the credit supply stable or decreasing less (because the lending margin in absolute terms is unchanged and only the margin in relative terms is reduced) than the credit demand.

Some modifications relative to the model of Bernanke and Blinder have been introduced. Public bonds are not included in the present model since its rate of return could stay relatively stable in the event of important negative financial shocks affecting private sectors. The private bonds are assumed to be a perfect substitute to bank lending. Another modification consists to assume that the longer term money-market interest rate $i_{mt}$, instead of long term public bonds, affects both the demand and supply of liquidity on the money market. For simplicity, we have assumed that $i_{mt}$ does not affect consumption and investment decisions. Despite these simplifications, by giving a special attention to reserve, money and credit markets, we can quite realistically expose how the central bank's interest rate policy makes its way into the economy. Such a framework is more adapted for examining how the inflation expectations behave when the ZLB on the nominal interest rate is encountered.

When the central bank sets $i_{dt}$, it must recursively determine the target of $i_{dt}$ using equations (6)-(7) once the target of credit market interest rate is known. Thus, given that the money-market equilibrium condition (6) determines the value for $i_{dt}$ in order to attain the target of other interest rates, it follows that the money supply cannot be endogenously determined using (6) as it is usually assumed in the inflation targeting literature (Woodford, 1998; King, 2000).

In the inflation-targeting literature, it is assumed that the money supply automatically adjusts to the money demand so that the money market can be ignored without serious consequences. In this model, by assuming that market interest rates and discount rate are distinct, the central bank will not always be able to control the market interest rates without manipulating the money supply. In other words, the money supply is partially endogenous and does not automatically adjust to satisfy the money demand except when the central bank maintains the risk premium on the money-market interest rate over the discount rate, $\rho_{mt} = i_{mt} - i_{dt}$, at a fixed level. This opens the door to quantitative or/and credit easing policies, considered as useful tools to target market interest rates in critical situations, i.e. when the discount rate cannot be decreased anymore due to the ZLB on the nominal interest rate.

The model is closed with the specification of central bank's objective function, which translates the behavior of the target variables into a welfare measure to guide the policy choice. We assume that this objective function is over the target variables $x_t$ and $\pi_t$, and takes the form:

$$L^{CB} = \frac{1}{2}\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i [\alpha x_{t+i}^2 + (\pi_{t+i} - \pi^T)^2], \tag{10}$$

where the parameter $\alpha$ is the relative weight on output deviations. The central bank's loss depends on output gap variability around of zero and inflation variability around of its constant target $\pi^T$ which can be zero or positive. Since $x_t$ is the output gap, the loss function takes potential output as the target. The strategy of flexible inflation targeting is implemented through an optimal nominal interest rate rule, which is deduced from the optimal inflation-targeting rule of the central bank which acts to minimize fluctuations of output gap and inflation rate around their respective target under discretion.

The time sequence of events is as follows: 1) Workers form inflation expectations and negotiate wages taking account of all available information about the economy. 2) Shocks realize. 3) The central bank sets the discount rate following an optimal interest rate rule. 4) Firms decide their production and prices.

The minimization of loss function (10) subject to the Phillips curve (1) leads to the following targeting rules in the sense of Svensson (2002):

$$x_t = -\frac{\lambda}{\alpha}(\pi_t - \pi^T), \tag{11}$$

$$\mathrm{E}_t x_{t+1} = -\frac{\lambda}{\alpha}(\mathrm{E}_t \pi_{t+1} - \pi^T). \tag{12}$$

Equations (1)-(2) and targeting rules (11)-(12) allow defining the following instrument rule in the sense of Svensson (2002):

$$i_{ct}^{T}=\left[1+\frac{\lambda(\alpha\beta-\alpha-\lambda^{2})}{\alpha\varphi(\alpha+\lambda^{2})}\right]\mathrm{E}_{t}\pi_{t+1}+\frac{\lambda^{3}}{\alpha\varphi(\alpha+\lambda^{2})}\pi^{T}+\frac{\lambda}{\varphi(\alpha+\lambda^{2})}\varepsilon_{\pi t}+\frac{1}{\varphi}\varepsilon_{xt}\,, \tag{13}$$

The optimal target of credit-market interest rate, $i_{ct}^{T}$, implied by the minimization of central bank's loss function, must react positively to the expected future rate of inflation if $\alpha\varphi(\alpha+\lambda^{2})+\lambda(\alpha\beta-\alpha-\lambda^{2})>0$. It reacts positively to a variation in $\pi^{T}$, and shocks $\varepsilon_{t}^{x}$ and $\varepsilon_{t}^{\pi}$. The credit-market interest rate is indirectly controlled by the central bank through the funds rate targeting procedure. The latter affects then the longer term money-market interest rate before affecting the credit-market interest rate.

Using (7) and (13), we deduce that the optimal target of money-market interest rate $i_{mt}^{T}$:

$$i_{mt}^{T}=\frac{f_{2}+j_{2}}{f_{1}}\overbrace{\left\{\left[1+\frac{\lambda(\alpha\beta-\alpha-\lambda^{2})}{\alpha\varphi(\alpha+\lambda^{2})}\right]\mathrm{E}_{t}\pi_{t+1}+\frac{\lambda^{3}}{\alpha\varphi(\alpha+\lambda^{2})}\pi^{T}+\frac{\lambda}{\varphi(\alpha+\lambda^{2})}\varepsilon_{\pi t}+\frac{1}{\varphi}\varepsilon_{xt}\right\}}^{i_{ct}^{T}}-\frac{1}{f_{1}}(j_{1}x_{t}+\varepsilon_{ct}), \tag{14}$$

which shows that $i_{mt}^{T}$ is positively related to $i_{ct}^{T}$ and it depends on the structural parameters of the credit market and shocks affecting the latter.

Using (6) and (14), we obtain the optimal target of discount rate

$$\begin{aligned}i_{dt}^{T}&=\frac{h_{1}+l_{2}}{h_{2}}i_{mt}^{T}-\frac{l_{1}}{h_{2}}x_{t}+\frac{1}{h_{2}}(\tilde{b}_{t}-p_{t}+\varepsilon_{mt}-\varepsilon_{lt})\\&=\frac{h_{1}+l_{2}}{h_{2}}\left\{\frac{f_{2}+j_{2}}{f_{1}}\left\{\left[1+\frac{\lambda(\alpha\beta-\alpha-\lambda^{2})}{\alpha\varphi(\alpha+\lambda^{2})}\right]\mathrm{E}_{t}\pi_{t+1}+\frac{\lambda^{3}}{\alpha\varphi(\alpha+\lambda^{2})}\pi^{T}+\frac{\lambda}{\varphi(\alpha+\lambda^{2})}\varepsilon_{\pi t}+\frac{1}{\varphi}\varepsilon_{xt}\right\}-\frac{1}{f_{1}}(j_{1}x_{t}+\varepsilon_{ct})\right\}\\&\quad-\frac{l_{1}}{h_{2}}x_{t}+\frac{1}{h_{2}}(\tilde{b}_{t}-p_{t}+\varepsilon_{mt}-\varepsilon_{lt}).\end{aligned} \tag{15}$$

Equation (15) allows then determining the target for average funds rate (see Appendix A).

In normal situation, when the financial markets function smoothly, the ZLB on the nominal discount rate will not be hit. Assume that the central bank sets the discount rate to attain the other interest rate targets under the funds rate operating procedure. Using (13)-(15), the equilibrium risk premiums are defined as:

$$\rho_{ct} = i_{ct}^T - i_{mt}^T$$
$$= \frac{f_1 - f_2 - j_2}{f_1}\left\{\left[1 + \frac{\lambda(\alpha\beta - \alpha - \lambda^2)}{\alpha\varphi(\alpha + \lambda^2)}\right]\mathrm{E}_t\pi_{t+1} + \frac{\lambda^3}{\alpha\varphi(\alpha + \lambda^2)}\pi^T + \frac{\lambda}{\varphi(\alpha + \lambda^2)}\varepsilon_t^\pi + \frac{1}{\varphi}\varepsilon_t^x\right\} - \frac{1}{f_1}(j_1 x_t + \varepsilon_{ct}),$$
$$\rho_{mt} = i_{mt}^T - i_{dt}^T$$
$$= \frac{h_2 - h_1 - l_2}{h_2}\left\{\frac{f_2 + j_2}{f_1}\left\{\left[1 + \frac{\lambda(\alpha\beta - \alpha - \lambda^2)}{\alpha\varphi(\alpha + \lambda^2)}\right]\mathrm{E}_t\pi_{t+1} + \frac{\lambda^3}{\alpha\varphi(\alpha + \lambda^2)}\pi^T + \frac{\lambda}{\varphi(\alpha + \lambda^2)}\varepsilon_t^\pi + \frac{1}{\varphi}\varepsilon_t^x\right\} - \frac{1}{f_1}(j_1 x_t + \varepsilon_{ct})\right\}$$
$$- \frac{l_1}{h_2}x_t + \frac{1}{h_2}(\tilde{b}_t - p_t + \varepsilon_{mt} - \varepsilon_{lt}).$$

The equilibrium risk premiums depend on the parameters characterizing the structure of goods, credit, money and reserve markets, inflation expectations, inflation target, output gap, as well as shocks affecting these markets. They could be kept at relatively low level when the economy is in expansion but could be enlarged to a high level, incompatible with the optimal interest rate policy defined under the inflation-targeting regime.

Equations (13)-(15) capture well the complexity of indirect market interest rate targeting through the funds rate operating procedure. The transmission mechanism is imperfectly observable by the central bank since it could be hit by numerous unanticipated shocks affecting the goods, credit, money and reserve markets. Furthermore, shocks affecting the interest rate policy could also interfere with the transmission mechanism. In some circumstances, negative disturbances in financial and corporate sectors can create dislocation on financial markets and enlarge the difference between the discount rate, and the money- and credit-market interest rates such that the discount and funds rates hit the ZLB while the market interest rates are still too high for the economy to recover from a severe depression.

## 3. Inflation and output dynamics in the benchmark model

In the inflation-targeting literature, it is a usual practice to assume that all interest rates are equal. Therefore, a funds rate targeting procedure is equivalent to the one directly targeting the interest rate which directly affects the aggregate demand. This assumption could be justified if all financial assets are perfect substitutes, all financial and monetary markets function perfectly and there are no frictions or major disturbances which are out of the control of the central bank.

Under this kind of assumptions, the model is reduced to (1)-(2). Subject to these two equations, the central bank minimizes the loss function (10). This leads to the targeting rules (11)-(12) and the optimal credit-market interest rate rule (13). If the central bank sets the credit market interest rate following (13), the targeting rules (11)-(12) will be verified. Thus, the difference equation for the inflation rate is deduced using (1) and (11):

$$\mathrm{E}_t \pi_{t+1} = \frac{1}{\beta}(1+\frac{\lambda^2}{\alpha})\pi_t - \frac{\lambda^2}{\alpha\beta}\pi^T - \frac{1}{\beta}\varepsilon_\pi \,. \tag{16}$$

The eigenvalue of the difference equation (16) is greater than unity. Consequently, as there is one forward-looking variable, i.e. $\mathrm{E}_t \pi_{t+1}$, the equilibrium is stable. The dynamics of output gap is determined by that of inflation rate according to the targeting rule (12). We remark that the inflation dynamics is governed by a very simple mechanism and depends uniquely on parameters characterizing the Phillips curve and central bank preferences.

## 4. Inflation dynamics when the ZLB on the discount rate is hit

We observe that the ZLB on the nominal discount and funds interest rates are hit in Japan during the 2000 and now in USA during the current financial crisis. One common point between these two circumstances is that both countries are exposed to colossal speculative bubbles on several asset markets. Therefore, the ZLB on the nominal interest rate cannot be appropriately examined if such shocks can not be taken into account in the model. This is the case in the standard New-Keynesian model where the ZLB on the nominal discount and funds interest rates is often confused with the ZLB on the credit-market interest rate. The later has neither been effectively observed in Japan nor in the USA or any other country. The present model introduces the money, credit and reserve markets and hence allows to consider the effects due to the burst of the bubbles in the prices of real and financial assets through the shocks affecting different markets. It is to notice that real, financial and monetary shocks that lead the economy into a liquidity trap and the central bank to practice unconventional monetary policies appear generally after an extended period of sustained rapid economic expansion and the burst of great speculative asset price bubbles formed during the period.

During the last two decades, using an interest rate policy, central banks in many countries have achieved the "great moderation" characterized by moderate and stable inflation rate and less fluctuation of output growth around its potential. A benign macroeconomic environment of great moderation could encourage financial agents to abandon their prudent approach of investing, lending and other financing decisions which balance macroeconomic and idiosyncratic risks, and to progressively espouse exuberant approaches (Carney, 2009). Thus, they take the maximum risk exposure compatible with the existing regulations, thinking that they all have the chance of quitting the sinking Titanic ship before the others or that the monetary and fiscal authorities will save them.

The asset bubbles created under this state of spirit could favour the success of monetary and fiscal policies. Therefore, policymakers might seek to create bubbles and might not want to take account of the potential damages induced by their burst because such damages will only materialize in the future and create difficulties for future policymakers. When the bubbles attain the extreme limit, menacing hence the central bank's principal objectives of price stability and output stabilization, they will be indirectly pricked by policymakers through creating monetary or fiscal conditions unfavourable for them to continue to grow rapidly. Their burst generates great financial and monetary as well as real shocks because a large number of agents have been excessively involved, often with large leverage based on overpriced collateral. The failure of some great financial institutions becomes a dilemma for the monetary and fiscal authorities. Avoiding any potential bankruptcy among these institutions envoys a bad message according to which the speculative feast could continue. On the opposite, not bailing out them could induce financial panics, amplifying hence the shocks generated by the burst of large asset bubbles.

When such bubbles are pricked, to avoid dramatic consequences on the real economy (i.e. deep economic recession and deflation), setting the nominal discount rate at zero is inevitable. In the case of Japan, the zero nominal discount rate is not sufficient to avoid the deflation and it does not provide a sufficient stimulus for the economy. Therefore, the question is under what conditions the deflation (or, on the contrary, the hyperinflation) can be avoided and how will behave the expected inflation rate and output gap once such a policy is implemented.

The ZLB on the nominal interest rate is hit when the target of discount rate determined by (15) is less than or equal to zero, i.e. $i_{dt}^{T} \leq 0$. The zero interest rate policy corresponds to a suboptimal equilibrium if setting the discount rate at zero cannot bring the credit-market interest rate to a level which is optimal for the central bank because the optimal target of discount rate is smaller than zero. To make the monetary policy effective, large negative shocks on goods, credit, money and reserve markets imply a need for simultaneously implementing zero discount rate, and quantitative and credit easing policies.

When shocks affecting money and credit markets generate major dislocations on goods and labor markets, the dynamics of inflation expectations will be greatly modified. The expectations of future inflation and output gap will be determined quite differently compared to these under a normally functioning inflation-targeting regime where they, always independent of financial and monetary shocks, will be determined by the central bank's targets except when inflation shocks are persistent. When the ZLB on the nominal discount rate is hit, the optimal targeting rules in the sense of Svensson (2002) will not be verified. The current and expected inflation rates and output gaps will depend on the functioning of credit and money markets because the latter determine the credit-market interest rate which directly affects the aggregate demand. Therefore, shocks affecting credit, money and reserve markets will be transmitted through this channel to the inflation rate and output gap. This mechanism is unconceivable in standard New-Keynesian models where credit, money and reserve markets are absent and where the ZLB on the nominal discount rate is confounded with that on the credit-market interest rate.

Knowing that, when the ZLB on the nominal discount rate is hit, the equilibrium conditions on the credit and money markets have important role to play in determining the equilibrium level of current and future inflation rates and output gaps, the central bank and private agents who want to form good inflation expectations cannot neglect the developments on these markets, particularly, when the functioning of credit and money markets is such that any inadequate monetary policy response to these shocks can put the economy either on diverging inflationary or deflationary paths for a long time.

The model is solved by determining recursively the money-market interest rate $i_{mt}$ using (6) for $i_{dt} = 0$ and then the nominal credit-market interest rate $i_{ct}$ using (7) and finally substituting the solution of $i_{ct}$ into the IS equation (2). The resulting equation and the Phillips curve (1) allow solving for equilibrium values of expected and current inflation rates and output gaps and determining if a crisis equilibrium is stable or not.

Using (6), the money-market interest rate is expressed by:

$$i_{mt} = \frac{1}{h_1 + l_2}(p_t - \tilde{b}_t + l_1 x_t + \varepsilon_{lt} - \varepsilon_{mt})\,. \tag{17}$$

In a liquidity trap, $\varepsilon_{lt}$ is likely to be important and $\varepsilon_{mt}$ likely to be small. Even though the output gap could be negative, its impact could be insignificant for bringing down the nominal money-market interest rate, a median term rate, to zero.

Using (7) and (17), the credit-market interest rate is given as

$$i_{ct} = \frac{1}{f_2 + j_2}\left[\frac{f_1}{h_1 + l_2}(p_t - \tilde{b}_t + l_1 x_t + \varepsilon_{lt} - \varepsilon_{mt}) + j_1 x_t + \varepsilon_{ct}\right]. \tag{18}$$

During a period where the economy is affected by large negative financial shocks, the credit demand is likely to decrease but the credit supply could decrease further as banks become more prudent or impose more restrictive conditions on credit distribution, so that $\varepsilon_{ct}$ is likely to be positive and could be more than compensating the effects of negative output gap on the equilibrium credit-market interest rate.

Substituting $i_{ct}$ determined by (18) into (2), and taking account of (1), we obtain the inflation rate and output gap as function of expected inflation rate, output gap, current price level, current stock of currency and different shocks:

$$\mathrm{E}_t\pi_{t+1} = \frac{1}{\beta}(\pi_t - \lambda x_t - \varepsilon_{\pi t}), \tag{19}$$

$$\mathrm{E}_t x_{t+1} = -\frac{\varphi}{\beta}\pi_t + \left[1 + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2} + \frac{f_1 l_1 \varphi}{(f_2 + j_2)(h_1 + l_2)}\right]x_t + \frac{f_1\varphi(p_t - \tilde{b}_t)}{(f_2 + j_2)(h_1 + l_2)} + \Sigma_t^{\varepsilon}, \tag{20}$$

where $\Sigma_t^{\varepsilon} = \frac{f_1\varphi}{(f_2+j_2)(h_1+l_2)}(\varepsilon_{lt} - \varepsilon_{mt}) + \frac{\varphi}{\beta}\varepsilon_{\pi t} + \frac{\varphi}{f_2+j_2}\varepsilon_{ct} - \varepsilon_{xt}$. Since $p_t = p_{t-1}(1+\pi_t)$, the system constituted of (19)-(20) has two forward-looking variables, $E_t x_{t+1}$ and $E_t\pi_{t+1}$, and one predetermined variable, $p_{t-1}$. Equation (20) could be rewritten as:

$$\begin{aligned}\mathrm{E}_t x_{t+1} - x_t = &-\frac{\varphi}{\beta}(\pi_t - \pi_{t-1}) + \frac{f_1\varphi\pi_t}{(f_2 + j_2)(h_1 + l_2)} \\ &+ \left[1 + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2} + \frac{f_1 l_1\varphi}{(f_2 + j_2)(h_1 + l_2)}\right](x_t - x_{t-1}) - \frac{f_1\varphi\Delta\tilde{b}_t}{(f_2 + j_2)(h_1 + l_2)} + \Delta\Sigma_t^{\varepsilon}.\end{aligned} \tag{21}$$

The private sector is assumed to form rational inflation expectations conditional on information available at $t$. The equilibrium value of $\pi_{t-1}$, $x_{t-1}$, $\pi_t$, $x_t$, $E_t\pi_{t+1}$, and $E_t x_{t+1}$ can be solved in accordance with the method of undetermined coefficients (McCallum, 1983). Since these solutions are quite cumbersome and they are not central to the dynamic analysis of inflation and output gap, they are not given in this paper. We pay instead our attention to the complementary solutions.

Denote the eigenvalues of the difference equations (19) and (21) by $e$. We can show that they must satisfy the following relation (Appendix B):

$$F(e) = \overbrace{\left\{\begin{array}{l} e^3 - \left\{1 + \frac{1}{\beta} + \left[1 + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2} + \frac{f_1 l_1\varphi}{(f_2 + j_2)(h_1 + l_2)}\right]\right\}e^2 \\ + \left\{(1 + \frac{1}{\beta})\left[1 + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2} + \frac{f_1 l_1\varphi}{(f_2 + j_2)(h_1 + l_2)}\right] + \frac{1}{\beta}\right\}e \\ - \frac{1}{\beta}\left[1 + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2} + \frac{f_1 l_1\varphi}{(f_2 + j_2)(h_1 + l_2)}\right] \end{array}\right\}}^{\equiv\Theta(e)} - \overbrace{\left[\frac{\lambda\varphi}{\beta}\left(\frac{1}{\beta} - \frac{f_1}{(f_2 + j_2)(h_1 + l_2)}\right)e - \frac{\lambda}{\beta}\frac{\varphi}{\beta}\right]}^{\equiv\Phi(e)} = 0. \tag{22}$$

The nature of the dynamic system will depend on the roots of the polynomial $F(e) = \Theta(e) - \Phi(e)$. Since the exact solutions of these roots will not be interpretable due to their complexity, to study the magnitude of these roots, it turns out convenient to use a mixture of graphical and analytical techniques. Therefore, to find $e$ solving $F(e) = 0$ is equivalent to find $e$ solving the equation $\Theta(e) = \Phi(e)$. The left hand of this equation, $\Theta(e)$, is a cubic function and the right hand of this equation $\Phi(e)$ linear function. Assume that the financial system is sufficiently developed, algebraic analysis combined with graphical method shows that two kinds of dynamics can be distinguished (Appendix C). [7]

In the event where $f_1 < \frac{(f_2 + j_2)(h_1 + l_2)}{\beta}$, the credit supply is insufficiently elastic with regard to money-market interest rate. In this case, there is one positive eigenvalue which is smaller than unity but two eigenvalues greater than unity. Given that the dynamic system has two forward-looking variables and one predetermined variable, the crisis equilibrium is stable.

If $f_1 > \frac{(f_2 + j_2)(h_1 + l_2)}{\beta}$, i.e. the credit supply is sufficiently elastic with regard to the money-market interest rate, there is one positive eigenvalue which is smaller than unity. There are two complex eigenvalues with a positive real part greater than unity. Hence, the crisis equilibrium is cyclically stable.

In both cases considered above, the crisis equilibrium is stable. Consequently, it could form a liquidity trap so that a temporary injection of liquidity will not be able to modify the expectations about future inflation and output gap and hence the crisis equilibrium. As the crisis equilibrium is stable, it is impossible to pull the economy out of financial and economic crisis by talking optimistically to financial market operators and the general public because there does not exist a diverging trajectory leading the economy out of the crisis equilibrium.

Since the crisis equilibrium is a liquidity trap, the central bank must react quickly and vigorously to avoid this equilibrium to be anchored in private expectations by using policy

---

[7] A well-developed financial system is considered as a condition necessary for the adoption of inflation targeting. The conditions corresponding to this assumption are given in Appendix C.

measures which could counterbalance the effects of violent financial and monetary shocks. Several possibilities offered to the central bank. One is to act on the monetary market through massively increasing the reserves or/and currency. Another is to increase the credit supply by buying private bonds to fully compensate the effects of real, financial and monetary shocks on the credit-market interest rate.

The actions on the money market may not be enough if the decrease in money-market interest rate is not (or only very partially) transmitted to the credit market, i.e. if $f_1$ tends to be very small. In this case, there might need a very great increase in the reserves and/or currency, which could be incompatible with the mandate of the central bank and could disturb the market expectations, leading hence to criticism of the central banker.

## 5. Quantitative and credit easing policies and dynamic stability

In many industrial countries, the burst of large bubbles in real and financial asset prices formed during the decade of “great moderation” could be a long process and could affect the economic equilibrium during a long period. To avoid that their burst leads to developments similar to these observed in Japan during the decades 1990 and 2000, where the prices of real state and stocks are still largely lower than the highest levels attained at the end of 1989, appropriate policy responses are necessary to answer to the waves of real, financial and monetary shocks linked to such a burst.

Absorbing large negative disturbances on the goods market may require a low credit-market interest rate which may not be within the reach of the central bank when the financial and monetary markets are also affected by large negative disturbances and the ZLB on the nominal discount and funds interest rates is encountered. By manipulating only the discount rate and targeting funds rate through open market operations to indirectly affect the lending interest rates, the central bank has no credible instrument of anchoring the inflation expectations besides the cheap talk about its firm intention to attain its inflation target and to stabilize the economy. The real challenge appears whenever the economy is deviating from the normal equilibrium, where the inflation rate and output gap are stabilized, and is converging to the crisis equilibrium. If this is the case, non-orthodox monetary policy, such as quantitative easing policy, must be used to ease the tension on the money market. Furthermore, the credit easing policy could be needed in order to ease the conditions on the credit market, through for example strengthening banks' balance sheet and/or buying private debts on the credit market by the central bank or Treasury.

These two monetary policies could be clearly distinguished in our framework. Due to the ZLB on the nominal discount rate as well as the malfunctioning of money and credit markets, the zero interest rate policy does not allow the realization of optimal money- and credit-market interest rates. In this case, the effective money- and credit-market interest rates determined by the equilibrium conditions on money and credit markets will be higher than their respective optimal target given by (14) and (13). If no measure is taken to directly affect the equilibrium conditions on the money and credit markets, the targeting rules (11)-(12) will not be effective.

In the following, we first analyze the inflation and output gap dynamics under the quantitative easing policy as practiced by the Bank of Japan. Then, we turn to analyze the dynamics of these variables under a combination of quantitative and credit easing policies as in the case of the Fed, the Bank of England or the ECB.

### *5.1. Quantitative easing policy*

The quantitative easing policy defined in the sense of Ben Bernanke is only directed to the money market. It consists to inject an important quantity of reserve which is greater than what is necessary to keep the funds interest rate at zero. By targeting the liquidity in the banking and financial system, the quantitative easing policy is used to induce an increase in the supply on the credit markets through the reduction of the money-market interest rate. The abundance of liquidity in the banking system is such that banks will not try to retain it by fear of its penury. This allows stimulating interbank lending and could bring down the money- and credit- market interest rates. The objective of the quantitative easing policy is to bring down the credit interest rate to a target level which is compatible with the normal equilibrium in the absence of financial and monetary shocks. Denote by $q^e$ the quantity of base monetary in the form of reserves injected by the central bank into the banking system, the equilibrium condition on the money market (6) is rewritten as:

$$q^e + \tilde{b}_t + h_1 i_{mt} - h_2 i_{dt} + \varepsilon_{mt} - p_t = l_1 x_t - l_2 i_{mt} + \varepsilon_{lt} \,. \tag{23}$$

By targeting the liquidity on the money market, the central banks can reduce the effects of excessively adverse money supply and demand shocks which drive the money-market interest rate to a too high level incompatible with the realization of inflation and output-gap targets. For $i_{dt} = 0$ and $q^e > 0$, (23) gives the money-market interest rate as:

$$i_{mt} = \frac{1}{h_1 + l_2}(-q^e - \tilde{b}_t + p_t + l_1 x_t + \varepsilon_{lt} - \varepsilon_{mt}) \,. \tag{24}$$

Consider in the following three scenarios.

*Positive optimal target of money-market interest rate*

If the target of money-market interest rate given by (14) is positive, i.e. $i_{mt}^T \geq 0$, the central bank can use the quantitative easing policy to achieve the objectives of macroeconomic stabilization and to ensure the dynamic stability of inflation expectations as under the standard inflation-targeting regime by setting $q^e$ to ensure that $i_{mt} = i_{mt}^T$. If this is the case, the quantitative easing policy is fully efficient in the sense that the targeting rules (11)-(12) are verified and the effects of shocks affecting aggregate demand, and credit, money and reserve markets are fully counterbalanced. The inflation and output dynamics is similar to that in the benchmark case corresponding to the standard inflation-targeting regime described in section 3. In this scenario, the economy is confronted a financial and economic crisis which does not constitute a liquidity trap.

*Insufficient responses of the central bank to large adverse shocks*

If shocks affecting negatively the aggregate demand, and money and credit supplies, and positively the demand for liquidity are extremely large and persistent, the central bank may not be able to counterbalance them. In effect, responding to such large shocks by injecting very large quantity of central-bank liquidity into the banking and financial sectors could lead to the criticism of the central bank for sowing seeds for future asset price bubbles and moral hazards in these sectors. This could induce the central bank, instead of restoring the optimal equilibrium, to conduct an incomplete quantitative easing policy in the sense that the realized money market interest rate given by (24) is higher than its target level, i.e. $i_{mt} > i_{mt}^{T}$. Given $i_{mt}$ determined by (24), the nominal credit-market interest rate is

$$i_{ct} = \frac{1_1}{(f_2 + j_2)}\left[\frac{f_1}{(h_1 + l_2)}(-q^e - \tilde{b}_t + p_t + l_1 x_t + \varepsilon_{lt} - \varepsilon_{mt}) + j_1 x_t + \varepsilon_{ct}\right], \tag{25}$$

which is higher than the optimal target of credit-market interest rate determined by (13) but lower than that determined by (18).

The difference equation for inflation rate is always given by (19). Substituting $i_{ct}$ given by (25) into (2), and taking account of (1), we obtain the difference equation for output gap:

$$\begin{aligned} \mathrm{E}_t x_{t+1} = -\frac{\varphi}{\beta}\pi_t + \frac{f_1\varphi}{(f_2 + j_2)(h_1 + l_2)} p_t + \left[1 + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2} + \frac{f_1 l_1 \varphi}{(f_2 + j_2)(h_1 + l_2)}\right] x_t \\ - \frac{f_1\varphi}{(f_2 + j_2)(h_1 + l_2)}\tilde{b}_t - \frac{f_1\varphi}{(f_2 + j_2)(h_1 + l_2)} q^e + \Sigma_t^{\varepsilon}. \end{aligned} \tag{26}$$

The only difference between (20) and (26) is found in the constant terms, with the presence in (26) of the supplementary terms including $q^e$ representing the quantitative easing policy. If $q^e$ is not specified as function of endogenous variables, the property of the dynamic adjustment of inflation rate and output gap described by (19) and (26) is identical to that described by the system of difference equations (19) and (20) examined in the case where the central bank practices only the zero discount rate policy. Using the previous results concerning the stability of the crisis equilibrium, we conclude that a partial quantitative easing policy cannot draw the economy out of the liquidity trap even it is accompanied by an excellent central bank communication.

*The existence of a ZLB for nominal money-market interest rate*

The nominal money-market interest rate hits itself the ZLB when the optimal target of money-market interest rate is negative under the effects of shocks, i.e. $i_{mt}^{T} < 0$. The central bank sets it at zero since $i_{mt}$ cannot fall below zero. It follows from (7) that the nominal credit-market interest rate is given by

$$i_{ct} = \frac{j_1}{f_2 + j_2} x_t + \frac{1}{f_2 + j_2} \varepsilon_{ct}, \tag{27}$$

which is also higher than $i_{ct}^{T}$ given by (13) but below that given by (18). Under these conditions, the money market becomes a liquidity trap in the sense of Keynes.

Rearranging the terms in (1) leads to

$$\mathrm{E}_t \pi_{t+1} = \frac{1}{\beta}(\pi_t - \lambda x_t - \varepsilon_{\pi t}) \tag{28}$$

Substituting $i_{ct}$ given by (27) into the IS equation (2) and using (28) to eliminate $\mathrm{E}_t \pi_{t+1}$ in the resulting equation yield:

$$\mathrm{E}_t x_{t+1} = -\frac{\varphi}{\beta}\pi_t + (1 + \frac{\varphi j_1}{f_2 + j_2} + \frac{\lambda\varphi}{\beta}) x_t + \frac{\varphi}{\beta}\varepsilon_{\pi t} - \varepsilon_{xt} + \frac{\varphi}{f_2 + j_2}\varepsilon_{ct}. \tag{29}$$

We remark that, in the present case, the dynamic system (28)-(29) and hence the equilibrium depends on shocks affecting the credit market through the presence of the term $\frac{\varphi}{f_2 + j_2}\varepsilon_{ct}$ in (29), contrary to what happens under the standard inflation-targeting regime where the central bank is able to directly target the interest rate affecting the aggregate demand (see (16)). Furthermore, the dynamic adjustment depends on parameters characterizing financial conditions, i.e. $\varphi$ , $f_2$ and $j_2$. The eigenvalues of the system of difference equations (28)-(29) must satisfy

$$\begin{vmatrix} \frac{1}{\beta} - e & -\frac{\lambda}{\beta} \\ -\frac{\varphi}{\beta} & (1 + \frac{\varphi j_1}{f_2 + j_2} + \frac{\lambda\varphi}{\beta}) - e \end{vmatrix} = 0,$$

which leads to the polynomial

$$e^2 - \left(1 + \frac{1}{\beta} + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2}\right)e + \frac{1}{\beta}\left(1 + \frac{\varphi j_1}{f_2 + j_2}\right) = 0\,. \qquad (30)$$

Solving (30) gives

$$e_1 = \frac{1}{2}\left(1 + \frac{1}{\beta} + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2} + \sqrt{\Delta}\right) \text{ and } e_2 = \frac{1}{2}\left(1 + \frac{1}{\beta} + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2} - \sqrt{\Delta}\right)\ ;$$

where $\Delta = (1 + \frac{1}{\beta} + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2})^2 - 4\frac{1}{\beta}(1 + \frac{\varphi j_1}{f_2 + j_2}) = (\frac{\lambda\varphi}{\beta})^2 + 2(\frac{\lambda\varphi}{\beta})(1 + \frac{1}{\beta} + \frac{\varphi j_1}{f_2 + j_2}) + (1 + \frac{\varphi j_1}{f_2 + j_2} - \frac{1}{\beta})^2 > 0$. It is easy to show that $e_1 > 1$, and $e_2 < 1$ if $(1-\beta)j_1 < \lambda(f_2 + j_2)$ and $e_2 > 1$ if $(1-\beta)j_1 > \lambda(f_2 + j_2)$.

In practice, we can have either $e_2 > 1$ or $e_2 < 1$. However, prominent studies of monetary policy implications of New Keynesian model, including that of Clarida *et al*. (1999), impose that $\beta = 1$ or arbitrarily near to unity. Admitting that $\beta$ is arbitrarily near to unity as King (2000), we have therefore one stable eigenvalue ($e_1 > 1$) and one unstable ($e_2 < 1$). Given that there are two forward-looking variables, the equilibrium under the quantitative easing policy with the money-market interest rate hitting the ZLB is saddle-point stable. Therefore, when the nominal money-market interest rate hits itself the ZLB, the central bank needs very good communication skills to convince that its quantitative easing policy is effective and can pull the economy out of the crisis equilibrium. Otherwise, the private expectations could take a bad trajectory and drag the economy back into the liquidity trap.

The quantitative easing policy, accompanied by a good communication, could be effective for avoiding the liquidity trap except when it is only too timidly applied. The communication is crucial since the economy could borrow a diverging adjustment leading to the crisis equilibrium as well as a trajectory leading to an equilibrium where the effects of adverse shocks are moderated. However, if these shocks have large negative effects on the aggregate demand and are persistent, the quantitative easing policy must be maintained as long as the effects of these shocks continue to affect the economy. An exit strategy from this policy too prematurely applied could make unsuccessful the anchoring of private expectations at the good equilibrium and hence induce the economy to return to the crisis equilibrium.

### *5.2. Simultaneous use of quantitative and credit easing policies*

As we have discussed above, it may not be sufficient for the central bank to uniquely applying the quantitative easing policy to the banking system by inundating the latter with excessive central-bank liquidity when the ZLB on the nominal discount rate is hit. The gravity of the economic and financial situation could incite the politicians to ask the central bank to apply the credit easing policy in order to avoid the systemic risk induced by important bankruptcies due to severe credit crunch. Furthermore, the money-market interest rate could hit itself the ZLB while the credit-market interest rate still stay at a high level implying a dangerously low inflation rate (or even deflation) and a collapse in output. Hence, it is necessary to extend the quantitative easing policy to principal operators on the supply side of the credit market.

Applying the credit easing policy has some different implications in terms of interest rate policy. Notably, this means that the central bank is using temporarily a credit-market interest rate procedure which is implicitly assumed in the inflation-targeting literature. We consider two scenarios in the following.

*Credit-market interest rate equal to its optimal target*

Assume that the central bank practices simultaneously the quantitative and credit easing policies. The equilibrium condition on the money market is given by (23). Denote by $q^c$ the quantity of credit assets bought by the central bank under the credit easing policy. The equilibrium condition on the credit market (7) is rewritten as

$$q^c - f_1 i_{mt} + f_2 i_{ct} = j_1 x_t - j_2 i_{ct} + \varepsilon_{ct} \,, \tag{31}$$

By coupling these two policy measures with the zero discount rate policy, the central bank is always able to bring down the credit-market interest rate to its optimal target level determined by

(13), i.e. $i_{ct} = i_{ct}^{T}$. In this case, the inflation and output dynamics will be identical to that under the benchmark inflation-targeting regime.

*Credit-market interest rate different from its optimal target*

If the quantitative easing and credit easing policies coupled with the zero discount rate policy are not designed to ensure that $i_{ct} = i_{ct}^{T}$ or to make the values of $q^c$ and $q^e$ dependent on endogenous variables, the dynamics of inflation and output gap could be similar to that described by equations (19) and (20) (or (21)) with an important difference, i.e. the new dynamic system has a quasi-normal equilibrium that the central bank desires to attain while the system (19)-(20) corresponds to a bad equilibrium of liquidity trap.

Assume that it is not necessary for the quantitative easing policy to bring the money-market interest rate down to zero. The money-market interest rate is determined using (23). Substituting $i_{mt}$ given by (24) into (31) and rearranging the terms yield

$$i_{ct} = \frac{1}{f_2 + j_2}\left[ -q^c + \frac{f_1}{h_1 + l_2}(-q^e - \tilde{b}_t + p_t + l_1 x_t + \varepsilon_{lt} - \varepsilon_{mt}) + j_1 x_t + \varepsilon_{ct} \right]. \tag{32}$$

The difference equation for inflation rate is given by (19). Substituting $i_{ct}$ given by (32) into (2), and taking account of (1), we obtain the difference equation for output gap as:

$$\begin{aligned} \mathrm{E}_t x_{t+1} = &-\frac{\varphi}{\beta}\pi_t + \frac{f_1\varphi}{(f_2 + j_2)(h_1 + l_2)} p_t + \left[ 1 + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2} + \frac{f_1 l_1 \varphi}{(f_2 + j_2)(h_1 + l_2)} \right] x_t \\ &- \frac{f_1\varphi}{(f_2 + j_2)(h_1 + l_2)}\tilde{b}_t - \frac{\varphi}{f_2 + j_2} q^c - \frac{f_1\varphi}{(f_2 + j_2)(h_1 + l_2)} q^e + \Sigma_t^{\varepsilon}. \end{aligned} \tag{33}$$

Since the only difference between (20) and (33) is found in the constant terms, with the presence in (33) of the supplementary terms including $q^e$ and $q^c$ representing respectively quantitative and credit easing policies, the property of the dynamic adjustment of inflation rate and output gap described by (19) and (33) is identical to that described by (19) and (20). It means that as long as the central bank keeps using simultaneously the quantitative and credit easing policies, the equilibrium that the central bank battles for is stable and credible. It is not difficult in this case to anchor private expectations by making public announce of a prolonged period of zero discount rate policy coupled with quantitative and credit easing policies. However, as we have emphasized before, if the public believes that the shocks are more or less permanent and these policies are only temporary even though they will be maintained for an extended period, private expectations will not be well anchored by the announces about these policies and/or by any reaffirmation of the central bank's inflation and output gap targets.

## 6. Conclusions

This paper examines inflation and output gap dynamics when unconventional monetary policies, such as zero discount rate policy and quantitative and credit easing policies, are adopted by a central bank to avoid the collapse of financial and economic system after having observed that manipulating the discount rate to target the funds rate is not anymore sufficient to stabilize the economy due to the ZLB on the nominal discount rate.

By extending the standard New-Keynesian model to include credit, money and reserve markets, we have enriched the transmission mechanism of monetary policy in the way that the central bank using the funds rate operating procedure, i.e. targeting the very short-run interbank interest rate, controls only indirectly the market interest rates which effectively affects the investment and consumption. This new framework has the advantage of allowing the reintegration of shocks affecting money, credit and reserve markets into the monetary policy analysis. It can be easily used to analyze the dynamic behavior of inflation and output-gap when a combination of unconventional policies is implemented.

We have shown that, when large adverse real, financial and monetary shocks lead the central bank to set the nominal discount rate at zero, the crisis equilibrium which could be a liquidity trap is stable. This implies that the communication by the central bank cannot bring back the economy on a trajectory leading to the optimal equilibrium where the inflation and output gap are stabilized, and the effects of financial and monetary shocks are fully neutralized.

To avoid the liquidity trap, the quantitative easing policy defined as the injection of liquidity in the banking sector could be an effective measure as long as the central bank can inject all the amount of liquidity to drive to a sufficiently low level the money-market interest rate in order to reduce the credit-market interest rate, and the money-market interest rate is not hitting itself the ZLB. In the opposite case, the quantitative easing policy does not allow the economy to come back to the optimal equilibrium. Furthermore, the temporary equilibrium at the ZLB on the money-market interest rate is saddle-point stable. This implies a good communication of the central bank is extremely important for keeping the economy at the temporary equilibrium or bringing it on a trajectory leading to higher inflation rate and output gap. Otherwise, the economy can return to the crisis equilibrium.

By combining the zero discount rate and quantitative easing policies with the credit easing policy, which is destined to bring down the credit-market interest rate to a sufficiently low level to neutralize the effects of financial and monetary shocks as well as demand shocks on the economy, the central bank can stabilize the inflation expectations and hence the economy. However, the success of the central bank will depend on the degree of persistence of these shocks, how long the central bank is able to keep the discount rate at zero and how long the quantitative and credit easing policies will be maintained in place. If the quantitative easing and credit easing policies are only temporary measures, which are removed before the effects of initial adverse shocks are counterbalanced by the effects of other favorable shocks, the economy can return to the crisis equilibrium.

## REFERENCES:


Adam, K., Billi, R.M. (2006), "Optimal monetary policy under commitment with a zero bound on nominal interest rates," *Journal of Money Credit and Banking* 38 (7), 1877-905.

Adam, Klaus & Roberto Billi (2007), "Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates," *Journal of Monetary Economics* 54, 728-752.

Auerbach, Alan J. & Maurice Obstfeld (2005), "The Case for Open-Market Purchases in a Liquidity Trap," *American Economic Review*, Volume 95, Issue 1, 110-137.

Benhabib, J., S. Schmitt-Grohe & M. Uribe (2002), "Avoiding Liquidity Traps," *Journal of Political Economy* 110, 535-563.

Bernanke, B.S. (2000), "Japanese monetary policy: A case of self-induced paralysis?" In: R. Mikitani and A.S. Posen, Editors, *Japan's financial crisis and its parallels to U.S. experience*, Institute for International Economics, Washington, DC (2000).

Bernanke, B.S., V.R. Reinhart and B.P. Sack (2004), "Monetary policy alternatives at the zero bound: An empirical assessment," *Brookings Papers on Economic Activity* **2**, 1–78.
Bernanke, Ben S. & Blinder, Alan S. (1988), "Credit, Money, and Aggregate Demand," *American Economic Review*, vol. 78(2), 435-439.
Bernanke, Ben S. (2009), "The Crisis and the Policy Response", Speech at the Stamp Lecture, London School of Economics, London, England, January 13, 2009.
Bernanke, Ben S. and Vincent R. Reinhart (2004), "Conducting Monetary Policy at Very Low Short-Term Interest Rates," American Economic Review, *AEA Papers and Proceedings May*, vol. 94(2), 85-90.
Buiter W. H. & N. Panigirtzoglou (2003), "Overcoming the Zero Bound on Nominal Interest Rates with Negative Interest on Currency: Gesell's Solution," *Economic Journal*, 113, 723-746.
Buiter, Willem H. (2009), "Negative Nominal Interest Rates: three ways to overcome the zero lower bound," *The North American Journal of Economics and Finance* 20 (3), 213-238.
Carney, Mark (2009), "Some considerations on using monetary policy to stabilize economic activity," *Remarks* by Mr Mark Carney, Governor of the Bank of Canada, to a symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, Wyoming, 22 August 2009.
Clarida, Richard, Jordi Gali & Mark Gertler (1999), "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, vol. 37, 1661–1707.
Clouse, James; Henderson, Dale; Orphanides, Athanasios; Small, David H.; and Tinsley, P.A. (2003), "Monetary Policy When the Nominal Short-Term Interest Rate is Zero," *Topics in macroeconomics*: Vol. 3 : Iss. 1, Article 12.
Eggertsson G.B. and M. Woodford (2004), "Policy options in a liquidity trap," *American Economic Review Papers and Proceedings* 94 (2) 76–79.
Eggertsson, G.B. and M. Woodford (2003), "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, No. 1, 139-211.
Goodfriend, Marvin (1983), "Discount window borrowing, monetary policy, and the post-October 6, 1979 Federal Reserve operating procedure," *Journal of Monetary Economics* 12, 343-356.
Jung, T., Y. Teranishi and T. Watanabe (2005), "Optimal monetary policy at the zero-interest-rate bound," *Journla of Money, Credit, and Banking* **37** (5), 813–835.
Kato, R. and S. Nishiyama (2005), "Optimal monetary policy when interest rates are bounded at zero," *Journal of Economic Dynamics and Control* 29 (1–2), 97–133.
King, Robert G. (2000), "The new IS-LM model: language, logic, and limits," *Economic Quarterly- Federal Reserve Bank of Richmond*, issue Sum, 45-103.
Krugman, P. (1998), "It's Baaack! Japan's slump and the return of the liquidity trap," *Brookings Papers on Economic Activity* 2, 137-87.
Kuttner, K.N. and A.S. Posen (2001), "The great recession: Lessons for macroeconomic policy from Japan," *Brookings Papers on Economic Activity* 2, 93–160.
Lewis, Kenneth A. and Laurence S. Seidman (2008), "Overcoming the zero interest-rate bound: A quantitative prescription, " *Journal of Policy Modeling*, Vol. 30, Iss. 5, 751-760.
McCallum, Bennet T. (1983), "On non-uniqueness in rational expectations models: An attempt at perspective," *Journal of Monetary Economics*, 11(2), 139-168.
McCallum, Bennett T. & James G. Hoehn (1983), "Instrument Choice for Money Stock Control with Contemporaneous and Lagged Reserve Requirements: Note," *Journal of Money, Credit and Banking*, Vol. 15, No. 1, 96-101.
McCallum, Bennett T. (2000), "Theoretical analysis regarding a zero lower bound on nominal interest rates," *Journal of Money, Credit and Banking* 32(4), 870-904.
McCallum, Bennett T. (2001), "Inflation targeting and liquidity traps." In: Loayze, N. and R. Soto (eds.), *Inflation Targeting: Design, Performance, Challenges.* Central Bank of Chile.
Modigliani, Franco, Robert Rasche & J. Philip Cooper (1970), "Central Bank Policy, the Money Supply, and the Short-Term Rate of Interest," *Journal of Money, Credit and Banking*, Vol. 2(2), 166-218.
Nakajima, Tomoyuki (2008), "Liquidity trap and optimal monetary policy in open economies," *Journal of the Japanese and International Economies*, Vol. 22, Issue 1, 1-33.
Nakov, Anton (2008), "Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rate," *International Journal of Central Banking*, Vol. 4(2), 73-127.
Oda, Nobuyuki and Takashi Nagahata (2008), "On the function of the zero interest rate commitment: Monetary policy rules in the presence of the zero lower bound on interest rates," *Journal of the Japanese and International Economies*, Vol. 22, Issue 1, 34-67.
Orphnides, A., and V. Wieland (2000), "Efficient monetary policy design near price stability," *Journal of the Japanese and International Economy* 14, 327-365.
Posen, A.S. (1998), *Restoring Japan's economic growth*, Institute for International Economics, Washington, DC (1998).

Reifschneider, David, and John C. Williams (2000), "Three Lessons for Monetary Policy in a Low-Inflation Era," *Journal of Money, Credit, and Banking* vol. 32, n° 4, Part 2, 936-66.
Seidman, L.S. (2001), "Reviving fiscal policy," *Challenge* 44(3), 17-42.
Sellon, Gordon H., Jr. (2003), "Monetary Policy and the Zero Bound: Policy Options When Short-Term rates Reach Zero," *Federal Reserve Bank of Kansas City Economic Review*, Fourth quarter, 5-43.
Spiegel, Mark M. (2006), "Did Quantitative Easing by the Bank of Japan 'Work'?" *RBSF Economic Letter,* Number 2006-28, 1-3.
Svensson, Lars E. O. (2001), "The Zero Bound in an Open Economy: A Foolproof Way of Escaping from a Liquidity Trap," *Bank of Japan Monetary and Economic Studies* 19, 277-312.
Svensson, Lars E.O. (2002). Inflation Targeting: Should It Be Modeled as an Instrument Rule or a Targeting Rule?" *European Economic Review* 46, 771-780.
Svensson, Lars E.O. (2003), "Escaping from a liquidity trap and deflation: The foolproof way and others," *Journal of Economic Perspectives* 17(4), 144–166.
Walsh, Carl E. (2003), *Monetary Theory and Policy*, the MIT Press.
Walsh, Carl E. (2009), "Using Monetary Policy to Stabilize Economic Activity," Paper prepared for the symposium "Financial Stability and Macroeconomic Policy," Jackson Hole, Wyoming. August.
Woodford, Michael (1998), "Doing without Money: Controlling Inflation in a Post-Monetary World," *Review of Economic Dynamics* 1, 173-219.

## Appendix A: The money supply under the funds rate targeting procedure

The monetary base $MB_t$ is constituted of the total reserve $TR_t$ and currency $C_t$ , the latter is exogenously fixed by the central bank. The relationship is expressed in log terms as:

$$b_t \equiv \log MB_t = \frac{TR^*}{MB^*}\log TR_t + \frac{C^*}{MB^*}\log C_t \equiv \psi\chi_t + (1-\psi)c_t, \tag{A.1}$$

where $\chi_t \equiv \log TR_t$ , $c_t \equiv \log C_t$ , $\psi \equiv \frac{TR^*}{MB^*}$ , and the superscript asterisk designs the steady state.

In order to clarify the link between the $b_t$ and the discount rate, a simplified description of the reserve market is adopted. We assume that the central bank uses a funds rate targeting procedure. Under this procedure, it controls the discount rate $i_{dt}$ and conducts open market operations to affect the supply of reserves in the banking system to target the funds rate, $i_{ft}$ . The latter is the interest rate banks in need of reserves pay to borrow from banks with surplus reserves. Thus, total reserves and hence the base money $b_t$ will depend on $i_{dt}$ .

Reserve demand is assumed to be a negative function of $i_{ft}$ and it arises mainly from the requirement that banks hold reserves equal to a specified fraction of their deposit liability. For simplicity, other factors such as aggregate income and prices are treated as part of the error term, $\nu_t^d$ , i.e. a disturbance of reserve demand. The function of total reserve demand is:

$$\chi_t^d \equiv \log TR_t^d = -\phi i_{ft} + \nu_t^d \,. \tag{A.2}$$

The total supply of reserves in the banking system can be expressed as the sum of the reserves that banks have borrowed from the central bank ($BR_t$) and nonborrowed reserves ($NBR_t$), i.e. $TR_t^s = BR_t + NBR_t$. Rewriting this relation in log terms gives

$$\chi_t^s \equiv \log TR_t^s \approx \frac{BR^*}{TR^*}\log BR_t + \frac{NBR^*}{TR^*}\log NBR_t = \gamma\chi_t^b + (1-\gamma)\chi_t^{nb}\,, \tag{A.3}$$

where $\gamma \equiv \frac{BR^*}{TR^*}$, $\chi_t^b \equiv \log BR_t$ and $\chi_t^{nb} \equiv \log NBR_t$.

The reserve market is in equilibrium whenever we have

$$\chi_t^d = \chi_t^s \,. \tag{A.4}$$

Similarly to Walsh (2003), we postulate a simple reserve borrowing function:[8]

$$\chi_t^b = \zeta(i_{ft} - i_{dt}) + \nu_t^b\,, \qquad\qquad \zeta > 0\,, \tag{A.5}$$

The parameter $\zeta$ specifies how a variation in $i_{ft}$ affects reserve borrowings and it depends on how such a variation affects expectations of future funds rate levels not modelled here. Due to non-price rationing of access to the central-bank liquidity, we have $i_{ft} \neq i_{dt}$. The shock $\nu_t^b$ represents other factors affecting reserve borrowings.

Under the funds rate targeting procedure, the central bank entirely compensates the effects on $i_{ft}$ of shocks affecting the reserve demand and the borrowed reserves by varying the non-borrowed reserves. Hence, the funds rate is determined by the discount rate. Nonetheless, the funds rate targeting is imperfect since it is subject to a monetary policy shock $\nu_t^s$. The nonborrowed reserves are given by:[9]

$$\chi_t^{nb} = \frac{1}{1-\gamma}\nu_t^d - \frac{\gamma}{1-\gamma}\nu_t^b + \nu_t^s \,. \tag{A.6}$$

[8] The specification of functions $BR_t$ and $NBR_t$ follows Walsh (2003). They are rewritten in logs terms here. In more elaborated reserve market model, the total supply of reserves could also depend on future interest rates (Walsh, 1982; Goodfriends, 1983).

[9] For the implications of other operating procedures and a brief history of operating procedures used by the Fed and some other central banks, see Walsh (2003, pages 451-71).

Using (A.2)-(A.3) and (A.5)-(A.6), we rewrite the equilibrium condition on the reserve market (A.4) as:

$$-\phi i_{ft} = \gamma\zeta(i_{ft} - i_{dt}) + (1-\gamma)\nu_t^s . \tag{A.7}$$

Solving (A.2) and (A.7) for $i_t^f$ and $\chi_t$ in terms of $i_t^d$ and shocks results to:

$$i_{ft} = \frac{\zeta\gamma}{\phi + \zeta\gamma} i_{dt} - \frac{1-\gamma}{\phi + \zeta\gamma}\nu_t^s , \tag{A.8}$$

$$\chi_t = -\frac{\phi\zeta\gamma}{\phi + \zeta\gamma} i_{dt} + \frac{\phi(1-\gamma)}{\phi + \zeta\gamma}\nu_t^s + \nu_t^d . \tag{A.9}$$

Substituting $\chi_t$ given by (A.9) into (A.1) and assuming that $c_t = \bar{c}$ (i.e. the amount of currency is given at period *t*), the monetary base is expressed therefore as:

$$b_t = -\frac{\phi\zeta\gamma\psi}{\phi + \zeta\gamma} i_{dt} + \frac{\phi\psi(1-\gamma)}{\phi + \zeta\gamma}\nu_t^s + \psi\nu_t^d + (1-\psi)\bar{c} , \tag{A.10}$$

Using (A.10) and (4), the money supply is written as:

$$m_t = -\frac{\phi\zeta\gamma\psi}{\phi + \zeta\gamma} i_{dt} + \frac{\phi\psi(1-\gamma)}{\phi + \zeta\gamma}\nu_t^s + \psi\nu_t^d + (1-\psi)\bar{c} + h_1 i_{mt} + \omega_t , \tag{A.11}$$

Define $\bar{b}_t \equiv (1-\psi)\bar{c}$, $h_2 \equiv \frac{\phi\zeta\gamma\psi}{\phi+\zeta\gamma}$ and $\varepsilon_{mt} \equiv \frac{\phi\psi(1-\gamma)}{\phi+\zeta\gamma}\nu_t^s + \psi\nu_t^d + \omega_t$. This leads to (5). ■

## Appendix B. The derivation of equation (22)

Assume that the solutions of inflation and output gap are of the following form:

$$x_{t+1} = me^{t+1};\ x_t = me^t;\ x_{t-1} = me^{t-1};$$

$$\pi_{t+1} = ne^{t+1};\ \pi_t = ne^t;\ \pi_{t-1} = ne^{t-1};$$

where *m* and *n* are coefficients to be determined. Since $x_t = me^t = e^{-1}x_{t+1} \Rightarrow \mathrm{E}_t x_{t+1} = e^{-s}\mathrm{E}_t x_{t+s+1}$, $\pi_t = ne^t = e^{-1}\pi_{t+1} \Rightarrow \mathrm{E}_t\pi_{t+1} = e^{-s}\mathrm{E}_t\pi_{t+s+1}$, forward-looking expectations imply that the equilibrium is stable if $e > 1$. Upon substituting the trial solutions into the reduced (homogenous) version of (19) and (21), we obtain:

$$ne^{t+1} = \frac{1}{\beta}(ne^t - \lambda me^t) , \tag{B.1}$$

$$me^{t+1} - me^{t} = -\frac{\varphi}{\beta}(ne^{t} - ne^{t-1}) + \frac{f_1 \varphi ne^{t}}{(f_2 + j_2)(h_1 + l_2)} + \left[1 + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2} + \frac{f_1 l_1 \varphi}{(f_2 + j_2)(h_1 + l_2)}\right](me^{t} - me^{t-1}). \tag{B.2}$$

Cancelling the common factors, $e^{t-1} \neq 0$ or $e^{t} \neq 0$, in (B.1)-(B.2) and arranging the terms yield:

$$(e - \frac{1}{\beta})n + \frac{\lambda}{\beta} m = 0\,, \tag{B.3}$$

$$\left[\frac{\varphi}{\beta}(e-1) - \frac{f_1 \varphi e}{(f_2 + j_2)(h_1 + l_2)}\right] n + \left\{e^2 - e - \left[1 + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2} + \frac{f_1 l_1 \varphi}{(f_2 + j_2)(h_1 + l_2)}\right](e-1)\right\} m = 0\,. \tag{B.4}$$

To avoid the trivial solutions $m = n = 0$, which would result in trivial complementary solutions $\pi_{t-1} = \pi_t = \pi_{t+1} = x_{t+1} = x_t = x_{t-1} = 0$ as well, the determinant of the coefficient matrix of (B.3)-(B.4) is required to vanish. It follows that we must have:

$$\begin{vmatrix} (e - \frac{1}{\beta}) & \frac{\lambda}{\beta} \\ \frac{\varphi}{\beta}(e-1) - \frac{f_1 \varphi e}{(f_2 + j_2)(h_1 + l_2)} & e^2 - e - \left[1 + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2} + \frac{f_1 l_1 \varphi}{(f_2 + j_2)(h_1 + l_2)}\right](e-1) \end{vmatrix} = 0\,. \tag{B.5}$$

Developing (B.5) leads to

$$F(e) = (e - \frac{1}{\beta})\left\{e^2 - e - \left[1 + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2} + \frac{f_1 l_1 \varphi}{(f_2 + j_2)(h_1 + l_2)}\right](e-1)\right\} - \frac{\lambda}{\beta}\left[\frac{\varphi}{\beta}(e-1) - \frac{f_1 \varphi e}{(f_2 + j_2)(h_1 + l_2)}\right] = 0\,.$$

Rearranging the terms, the function $F(e)$ can be decomposed as in (22). ■

## Appendix C. Determination of the eigenvalues of the dynamic system ((19) and (21)) through graphical resolution of (22)

*Determining the roots of* $\Theta(e) = 0$ *and* $\Phi(e) = 0$: It is a simple matter to determine the roots of these equations since $\Phi(e)$ is linear and the cubic function $\Theta(e)$ is built in the way that

$$\Theta(e) = (e-1)(e - \frac{1}{\beta})\left[e - \left(1 + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2} + \frac{f_1 l_1 \varphi}{(f_2 + j_2)(h_1 + l_2)}\right)\right].$$

The cubic equation $\Theta(e) = 0$ has three solutions which are respectively

$$e_1 = 1\,,\; e_2 = \frac{1}{\beta}\,,\; e_3 = \left[1 + \frac{\lambda\varphi}{\beta} + \frac{\varphi j_1}{f_2 + j_2} + \frac{f_1 l_1 \varphi}{(f_2 + j_2)(h_1 + l_2)}\right].$$

It is straightforwardly to see that

$$e_2 \begin{cases} < e_3 \text{ if } \varphi > \dfrac{(1-\beta)(f_2 + j_2)(h_1 + l_2)}{\lambda(f_2 + j_2)(h_1 + l_2) + \beta j_1(h_1 + l_2) + \beta f_1 l_1}; \\ > e_3 \text{ if } \varphi < \dfrac{(1-\beta)(f_2 + j_2)(h_1 + l_2)}{\lambda(f_2 + j_2)(h_1 + l_2) + \beta j_1(h_1 + l_2) + \beta f_1 l_1}. \end{cases}$$

The parameter $\varphi$ is a measure of the development of financial intermediation. It shows how the aggregate demand is affected by a change in the real interest rate.

The linear equation $\Phi(e) = 0$ has one solution:

$$e_4 = 1 + \frac{\beta f_1}{(f_2 + j_2)(h_1 + l_2) - \beta f_1}.$$

It follows that

$$e_4 \begin{cases} > 0 \text{ if } f_1 < \dfrac{(f_2 + j_2)(h_1 + l_2)}{\beta}, \\ < 0 \text{ if } f_1 > \dfrac{(f_2 + j_2)(h_1 + l_2)}{\beta}. \end{cases}$$

*Graphing the functions* $\Theta(e)$ *and* $\Phi(e)$ *to determine the stability condition:* A graph of these functions provides the easiest way of determining the nature of the roots of the cubic polynomial $F(e) = \Theta(e) - \Phi(e) = 0$. To limit the cases, we assume that the financial system is sufficiently developed such that:

$$\varphi > \max[\varphi_1; \varphi_2],$$

With $\varphi_1 = \frac{(1-\beta)(f_2 + j_2)(h_1 + l_2)}{\lambda(f_2 + j_2)(h_1 + l_2) + \beta j_1(h_1 + l_2) + \beta f_1 l_1}$ and $\varphi_2 = \frac{f_1 \beta^2 (f_2 + j_2)(h_1 + l_2)}{[\lambda(f_2 + j_2)(h_1 + l_2) + \beta j_1(h_1 + l_2) + \beta f_1 l_1][(f_2 + j_2)(h_1 + l_2) - \beta f_1]}$, implying that we have $e_2 < e_3$ and $e_4 < e_3$. The latter is evidently true if $e_4 < 0$ and it is also true if $e_4 > 0$ since under the above condition, we have

$$e_3 - e_4 = \frac{[\lambda\varphi(f_2 + j_2)(h_1 + l_2) + \beta\varphi j_1(h_1 + l_2) + \beta f_1 l_1 \varphi][(f_2 + j_2)(h_1 + l_2) - \beta f_1] - f_1 \beta^2 (f_2 + j_2)(h_1 + l_2)}{\beta(f_2 + j_2)(h_1 + l_2)[(f_2 + j_2)(h_1 + l_2) - \beta f_1]} > 0.$$

Figure 1 represents the case where $e_4 > 0$, $\forall f_1 < \frac{(f_2 + j_2)(h_1 + l_2)}{\beta}$. It contains three functions. One of the solid lines is the cubic function $\Theta(e)$, which highlights the fact that it has three roots $e_1$, $e_2$ and $e_3$. Another solid line is the linear function $\Phi(e)$ represented under the condition that $f_1 < \frac{(f_2+j_2)(h_1+l_2)(1-\beta)}{\beta} \equiv z$, with a root whose value is between $e_1$ and $e_2$. The dashed line is the representation of the linear function $\Phi(e)$ under the condition $f_1 > z$, with a root whose value is between $e_1$ and $e_3$.

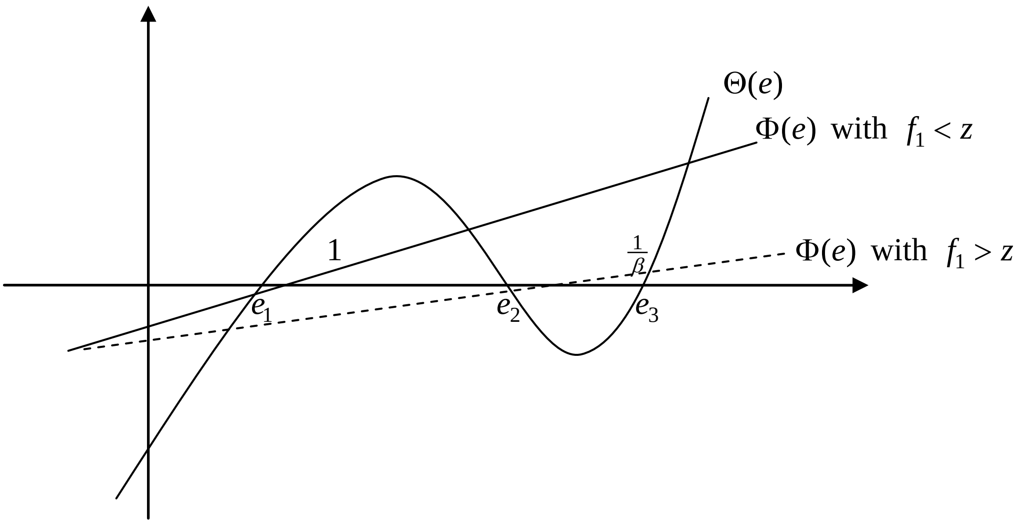


**Fig. 1**. The case where $(f_2 + j_2)(h_1 + l_2) - \beta f_1 > 0$.

The cubic function $\Theta(e)$ cuts the vertical axe at a point lower than the linear function $\Phi(e)$ because:

$$-\frac{1}{\beta}\left[1+\frac{\lambda\varphi}{\beta}+\frac{\varphi j_1}{f_2+j_2}+\frac{f_1 l_1 \varphi}{(f_2+j_2)(h_1+l_2)}\right] < -\frac{\lambda}{\beta}\frac{\varphi}{\beta}\ .$$

It follows that $F(e)$ has only one root between zero and unity and two roots above unity.

Figure 2 represents the case where $e_4 < 0$, $\forall f_1 > \frac{(f_2+j_2)(h_1+l_2)}{\beta}$. It contains the cubic line $\Theta(e)$ and the line $\Phi(e)$ with negative slope. Since these curves have only one intersection point between zero and unity, it follows that there are only one positive real solution smaller than unity but two complex solutions.

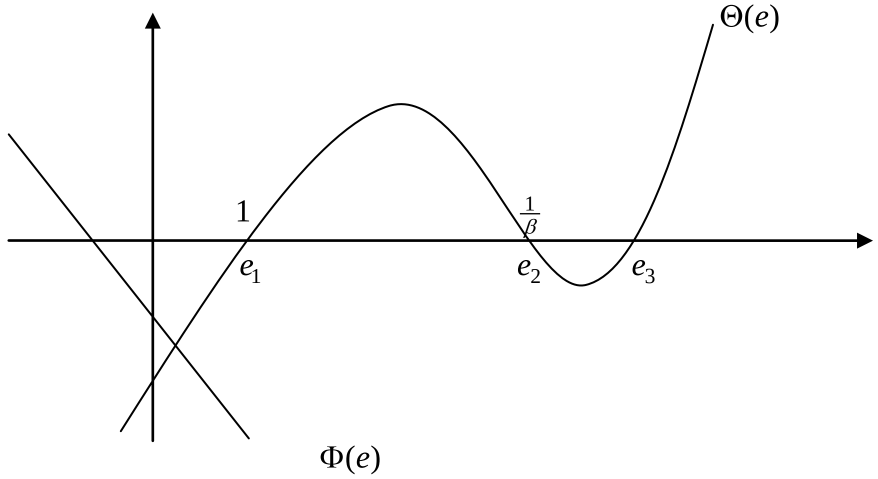


**Fig. 2**. The case where $(f_2 + j_2)(h_1 + l_2) - \beta f_1 < 0$.

To show that the complex solutions have a real part greater than unity, we denote the unique real solution by $e^*$. Then, the function $F(e)$ is decomposed as follows

$$F(e) = (e - e^*)\left\{\begin{array}{l} e^2 - \left\{(1-e^*) + \dfrac{1}{\beta} + \left[1 + \dfrac{\lambda\varphi}{\beta} + \dfrac{\varphi j_1}{f_2 + j_2} + \dfrac{f_1 l_1 \varphi}{(f_2 + j_2)(h_1 + l_2)}\right]\right\} e \\ + \left\{\begin{array}{l} -\left\{(1-e^*) + \dfrac{1}{\beta} + \left[1 + \dfrac{\lambda\varphi}{\beta} + \dfrac{\varphi j_1}{f_2 + j_2} + \dfrac{f_1 l_1 \varphi}{(f_2 + j_2)(h_1 + l_2)}\right]\right\} e^* \\ + \left\{(1 + \dfrac{1}{\beta})\left[1 + \dfrac{\lambda\varphi}{\beta} + \dfrac{\varphi j_1}{f_2 + j_2} + \dfrac{f_1 l_1 \varphi}{(f_2 + j_2)(h_1 + l_2)}\right] + \dfrac{1}{\beta}\right\} + \left\{\dfrac{\lambda}{\beta} \dfrac{f_1 \varphi}{(f_2 + j_2)(h_1 + l_2)} - \dfrac{\lambda}{\beta}\dfrac{\varphi}{\beta}\right\} \end{array}\right\} \end{array}\right\}.$$

Since the coefficient associated with $e^2$ is unity and the composite coefficient associated with $e$ is greater than two, it follows that the real part of complex solutions is greater than unity. ■