Setting the optimal type of equipment to be adopted and the optimal time to replace it

Mihaela Albici

University Constantin Brancoveanu Ramnicu Valcea

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Setting the Optimal Type of Equipment to Be Adopted and the Optimal Time to Replace It

Albici Mihaela, University „Constantin Brancoveanu", Rm. Valcea

Abstract. *The mathematical models of equipment's wear and tear, and replacement theory aim at deciding on the purchase selection of a certain equipment type, the optimal exploitation time of the equipment, the time and ways to replace or repair it, or to ensure its spare parts, the equipment's performance in the technical progress context, the opportunities to modernize it etc.*

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In any economic activity, the equipment (machinery) suffers from a natural process of wear and tear during its usage which is called physical wear and tear. While it is used, the equipment needs ongoing maintenance and repairing of potential damages which pursue the prevention, decrease or delay of the wear and tear process. While the equipment is functioning, technical progress may lead to the emergence of new equipment having the same use scope but higher qualitative and economic performance. It leads to another type of wear and tear in equipment which is called obsolescence [1].

An important aspect in the organizing process of any economic activity is decision making that investment implies both at macro- and micro-economic level. It is either investment referring to limited-function equipment or to unlimited- (long-lasting) prospect objectives, that decision making pursues optimization according to an economic criterion. The mathematical models of equipment's wear and tear, and replacement theory aim at deciding on the purchase selection of a certain equipment type, the optimal exploitation time of the equipment, the time and ways to replace or repair it, or to ensure its spare parts, the equipment's performance in the technical progress context, the opportunities to modernize it etc. Decisional criterion can be total minimum expenses when economic issues are considered, or maximization of benefits reached by equipment exploitation when the pursued goal is its efficaciousness.

Equipment replacement is generally caused by [6]:

- the wear and tear due to its functioning for a certain period of time and in certain circumstances;
- its low-quality functioning as a result of another better equipment's emergence.

The issue of the best policy to maintain equipment ranks first the timely replacement of used equipment with new ones which supposes setting the optimal type of equipment to be adopted in order to replace the one used and setting the best time to replace it [2].

Within the current evolution of science and technique, several types of equipment can be used to execute products, operations or stages in technological processes.

Since company management is forced to purchase new equipment to replace the old one, it faces the need to adopt a decision regarding the optimal equipment type it should choose from among various possible ones.

As equipment is different from one another by its annual maintenance and repairing costs, and by its functioning duration until full wear and tear, it is thought the method to decide upon the equipment type to be adopted should be based on the average size of such costs either per one year, or per one maintenance and repairing duration. In such context, it is believed that the best equipment type to be adopted is the one at minimum average cost to purchase or repair during one year or one maintenance period.

The size of the average purchase, maintenance and repairing cost per one year or one maintenance and repairing period is:

$$k = \frac{K}{m \cdot n} = \frac{1}{m \cdot n} \sum_{i=1}^{m} \left(A_i + \sum_{j=1}^{n} C_{ij} \right),$$

where K – total cost to purchase, maintain and repair;

k – average cost to purchase, maintain and repair per one year or one maintenance and repairing period;

m – number of equipment's purchases during the period under consideration;

i – a certain equipment replacement;

n – number of years to use the equipment between two replacements or the number of periods (cycles of maintenance and repairing between two replacements);

j – a certain concrete year or a certain period (cycle) of maintenance and repairing, and j varies from 1 to n;

A_i - purchase cost during ”*i*” replacement of equipment and i varies from 1 to n;

C_{ij} - maintenance and repairing cost of ”*i*” equipment purchase in j year or during j maintenance and repairing period.[1]

Example: In order to replace used equipment, it is possible for three types of equipment to be purchased having the costs and expenses shown below:

	Equipment type I	Equipment type II	Equipment type III
Purchase cost	60 000	63 000	70 000
Maintenance and repairing expenses during years 1, 2 and 3, those by equipment			
1	7 000	10 000	11 500
2	10 000	12 000	15 000
3	-	15 000	10 000

The average purchase, maintenance and repairing cost is calculated for each type of equipment:

$$k_1 = \frac{1}{2}(60000 + 7000 + 10000) = \frac{1}{2} \cdot 77000 = 38500$$

$$k_2 = \frac{1}{3}(63000 + 10000 + 12000 + 15000) = \frac{1}{3} \cdot 100000 = 33333.3$$

$$k_3 = \frac{1}{3}(70000 + 11500 + 15000 + 10000) = \frac{1}{3} \cdot 106500 = 35500$$

The analysis of the three average annual costs above leads to the conclusion that the best equipment type that should be adopted is the second one as it has the lowest average annual cost of purchase, maintenance and repairing.

[1] Bărbulescu C. (1997) *Industrial Production Management*, vol.II, Sylvi Publishing House, Bucharest, pp.58.

Let us mark by $V(0)$ the equipment's value in the beginning of its activity, a value which decreases annually for various reasons. Let $D(k)$ be the depreciation of year *k* and $V(k)$ the equipment's value in year *k*. There is:

$$V(k+1)=V(k)-D(k+1), 0 \le k \le n-1 \qquad (1)$$

supposing the functioning duration is *n* years. In other words, the equipment's value in late current year equals the equipment's value in late previous year minus the equipment's depreciation during the current year.

Additionally, let $F(k)$ be the functioning expenses in year *k*. Consequently, the total expenses involved in equipment operation from the very beginning until year *k* marked by $C(k)$ are:

$$C(k)=\sum_{s=1}^{k}[D(s)+F(s)] \qquad (2)$$

if their updating is not taken into consideration and :

$$C(k)=\sum_{s=1}^{k}[D(s)+F(s)]\cdot(1+i)^{-s} \qquad (3)$$

if their updating is considered at an annual percentage of $100i$.

Expression $C(k)$ is also called the total cumulated cost of equipment usage or, in other words, the total cumulated cost of equipment usage equals the sum of all annual depreciations of the equipment plus the sum of all annual expenses for equipment operation.

Let us mark by $\overline{C}(k)$ the average annual cost of equipment usage in the first *k* years. The issue arising is to find moment k_0 when the average cost of equipment usage is minimum.

Marking the existing updating by $i>0$ and its lack by $i=0$, one can immediately deduce from relationships (2) and (3):

Proposition 1. The average annual cost of equipment usage in its first *k* years of operation $(k \le n)$ is:

$$\overline{C}(k)=\begin{cases} \dfrac{1}{k}\cdot\sum_{s=1}^{k}[D(s)+F(s)], & if \quad i=0 \\ \dfrac{1}{1-(1+i)^{-k}}\cdot\sum_{s=1}^{k}[D(s)+F(s)](1+i)^{-s}, & if \quad i>0 \end{cases} \qquad (4)$$

Definition 2. It is said that the optimal time to replace equipment is number $k_0 \in \mathbf{N}$ having the following trait:

$$\overline{C}(k_0-1) \ge \overline{C}(k_0) \le \overline{C}(k_0+1) \qquad (5)$$

Observation 3. It is likely that:

i) $\overline{C}(k)$ be monotonously ascendent during equipment lifetime when it is recommended it should be replaced in its early functioning;

ii) $\overline{C}(k)$ be monotonously descendent during equipment lifetime, therefore its replacement is done in the end of its life;

iii) one of inequalities (5) be an equality, therefore there are two consecutive optimal moments to replace the equipment.[2]

Example: A company starts operating a certain type of automatic equipment which costs $V_0 = 1000000$ monetary units (m.u.) and works for ten years with the following estimations:

[2] Purcaru I., Berbec F., Sorin D. (1996) *Financial Mathematics & Business Decisions*, Economic Publishing House, pp. 422

a) its annual degradation is 35, 30, 25, 20, 16, 12, 10, 8, 5, 3 thousand m.u.;
b) its annual operation cost is 10, 12, 14, 18, 22, 28, 32, 37, 40, 44 thousand m.u.

It is known the equipment is not technically obsolete and consequently while operating it can be replaced with an identical one if necessary. Which is the optimal time to replace it if its updating is not taken into account whereas its upgrading by an annual coefficient of $i = 0{,}1$ is considered?

By synthesizing all the calculations of formulae (4) and (5), the result is Table 1 for $i > 0$ and Table 2 for $i = 0$:

Table 1

Operation years s	Annual operation expenses $F(s)$	Annual depreciation $D(s)$	Updating factor $(1+i)^{-s}$	Updated annual operation expenses $F(s)\cdot(1+i)^{-}$	Updated annual depreciation $D(s)\cdot(1+i)^{s}$	Updated total annual expenses $[F(s)+D(s)]\cdot(1+i$	Updated and cumulated total annual expenses $C(k)$	Cumulated updating factor $\frac{i}{1-(1+i)^{-k}}$	Average total annual cost of usage $\overline{C}(k)$	Optimal replacement time k_0
1	10 000	35 000	0.909090	9090.90	31818.15	40909.05	40909.05	1,1	44999.95	
2	12 000	30 000	0.826446	9917.35	247933.8	34710.73	75619.78	0.576189	19999.94	
3	14 000	25 000	0.751314	10518.39	18782.85	29301.24	104921.02	0.402113	11782.40	
4	18 000	20 000	0.683013	12294.23	13660.26	25954.49	130875.51	0.315470	8187.86	
5	22 000	16 000	0.620921	13660.26	9934.73	23594.99	154470.50	0.263797	6224.28	
6	28 000	12 000	0.564473	15805.24	6773.67	22578.92	177049.42	0.229606	5184.25	
7	32 000	10 000	0.513158	16421.05	5131.58	21552.63	198602.05	0.205405	4427.01	
8	37 000	8 000	0.466507	17260.75	3732.05	20992.81	219594.86	0.187443	3934.95	
9	40 000	5 000	0.424097	16963.88	2120.48	19084.36	238679.22	0.173640	3313.80	
10	44 000	3 000	0.385543	16963.89	1156.62	18120.52	256799.74	0.162745	2949.02	

It is easy to notice that $\overline{C}(k)$ is monotonously descendent during the equipment's lifetime, therefore its replacement shall be done in the end of its life.

Table 2

Years s	Annual operation expenses $F(s)$	Annual depreciation $D(s)$	Total annual expenses $D(s)+F(s)$	Total cumulated expenses $C(k)$	Average annual cost of usage $\overline{C}(k)$	Optimal time k_0
1	10 000	35 000	45 000	45 000	40909.05	
2	12 000	30 000	42 000	87 000	38181.78	
3	14 000	25 000	39 000	126 000	35454.51	
4	18 000	20 000	38 000	164 000	34545.42	$k_0 = 4$
5	22 000	16 000	38 000	202 000	34545.42	$k_0 = 5$
6	28 000	12 000	40 000	242 000	36363.60	
7	32 000	10 000	42 000	284 000	38181.78	
8	37 000	8 000	45 000	329 000	40909.05	
9	40 000	5 000	45 000	374 000	40909.05	
10	44 000	3 000	47 000	421 000	42727.23	

The optimal replacement time in this case is in year 4 or year 5.

Observation 4. Irrespective of the situation, the issue always has a solution in the respect of relationship (5). Sensitive differences among the optimal solutions reached with or without any updating can be seen in the equipment whose lifetime is medium and long.

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