The capitalization of taxes in bond prices: Evidence from the market for Government of Canada bonds

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Abstract

This paper provides estimates of the extent to which corporate and personal income taxes are capitalized in bond prices. The methodology yields estimates of the degree of tax capitalization, rather than an implied tax rate. This makes it straightforward to identify the marginal investor and test for changes in tax capitalization. The empirical approach also makes it unnecessary to jointly estimate the degree of tax capitalization and the entire yield curve. Corporate taxes are found to have been fully capitalized in pre-tax Government of Canada bond yields during the period 1986-1993. Since 1994, taxes have not been capitalized in yields. These results are consistent with the existence of a marginal investor, but the identity of the marginal investor changed from a financial sector firm to a non-taxed entity in the early 1990s.

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1. Introduction

This paper presents estimates of the extent to which personal and corporate taxes are capitalized in bond prices. Tax capitalization can alter market bond yields and after-tax investment returns, so an understanding of the relationship between taxes and bond prices is of fundamental importance to bond holders, such as private investors and portfolio managers, as well as to bond issuers, such as governments and corporations. Further, as noted by Pye (1969) and Robichek and Niebuhr (1970), the capitalization of taxes in bond prices can affect estimates of the term structure, an important input in many asset-pricing models. As well, the extent of tax capitalization can influence the slope of the yield curve and, thus, estimates of inflation expectations and predictions of business cycle fluctuations that depend on the term premium. Identification of the degree to which taxes are capitalized in bond prices is also central to an understanding of tax incidence in financial markets.

In addition to quantifying the capitalization of taxes in bond prices, I use the impact of different types of taxes on bond prices to determine whether a marginal investor exists in the bond market and, if so, to identify the marginal investor type. The existence of a marginal investor is closely related to the issue of tax clienteles. Tax clienteles form when different sub-sets of investors, such as investors belonging to different tax brackets, hold different sub-sets of bonds, implying that investors are not indifferent across all bonds (Schaefer (1982)). Dybvig and Ross (1986) distinguish two types of clientele effects – those in quantities only and those in both prices and quantities. Clientele effects in quantities arise when there is at least one marginal investor who holds (or is indifferent between) all bonds, but other types of investors hold only subsets of bonds. Clientele effects in prices and quantities occur when no investor type is indifferent between all bonds, so there is no price vector for which any investor is willing to hold all bonds. In this latter case, a marginal investor does not exist and, as a result, bond prices do not reflect the demand or tax bracket of any single investor type. This is important because, in the absence of a marginal

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investor, estimates of the term structure using all bond prices will depend on the bond demands and tax rates of multiple investor types and, therefore, on the relative proportion of investors of each type. On the other hand, if bond prices reflect the tax position of one type of investor (the marginal investor), since tax rates generally differ across investor types, profitable trading opportunities could exist for other investors (Green and Ødegaard (1997)). The presence of tax clienteles, whether in quantities or in prices and quantities, signals the existence of constraints on arbitrage that inhibit investors from taking full advantage of these trading opportunities (MacKay, Prisman and Tian (2000)).

The methodology I employ augments the existing literature in several ways. First, in contrast to most previous analysis, this study incorporates data on corporate and personal income tax rates. Rather than use actual tax rate data, it is common in the literature to estimate an *implied* tax rate, a rate that combines the degree of tax capitalization with the tax rate of the marginal investor. The estimation of an implied tax rate makes it difficult to separate the tax capitalization effect from the tax rate and, therefore, to identify the marginal investor type.[1] In contrast, the use of actual tax rate data makes it feasible to directly estimate the degree of tax capitalization. Given this estimate, it is possible to determine whether a marginal investor exists and, if so, to identify which of the three principal types of investors in bonds – individuals, corporations or non-taxed entities – is the marginal investor type.

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[1] For example, Litzenberger and Rolfo (1984a) estimate the implied interest income tax rates associated with different pre-specified capital gains tax rates, where the capital gains tax rates are the rates that apply to different types of taxpayers. Litzenberger and Rolfo then compare their estimates of the implied interest income tax rates to the tax rates faced by taxpayers paying each pre-specified capital gains tax rate. If the estimated rate is close to that of an actual taxpayer, Litzenberger and Rolfo conclude that this type of taxpayer is the marginal investor. An alternative that does not use actual tax rate data or estimate implied tax rates is employed by MacKay, Prisman and Tian (2000). They specify four hypothetical tax brackets, one of which corresponds to tax-exempt investors, while another is close to the tax bracket faced by corporations in Canada. Using Canadian bond price data, MacKay, Prisman and Tian estimate after-tax yield curves for each of these four hypothetical tax brackets. If the estimates associated with a particular tax bracket can explain actual bond prices, they conclude that this is the tax bracket of the representative investor. The results of MacKay, Prisman and Tian show that there exists a representative investor, but the tax bracket of this investor does not appear to correspond to the tax bracket faced by any obvious type of actual taxpayer.

A further advantage of the empirical methodology employed here is that it utilizes data on pairs of bonds, where both bonds in a pair have the same maturity date. The use of these matched bond pairs makes it unnecessary to estimate the entire yield curve. In contrast, most previous analyses use an empirical approach that involves joint estimation of an implied tax rate and an approximation to the yield curve (see, for example, McCulloch (1975), Jordan (1984), Litzenberger and Rolfo (1984a), Ehrhardt, Jordan and Prisman (1995), Green and Ødegaard (1997), Elton and Green (1998), Elton, Gruber, Agrawal and Mann (2001), Liu, Shi, Wang and Wu (2007)). The problem with this joint estimation approach is that mis-specification of the yield curve could lead to poor estimates of the implied tax rate and the degree of tax capitalization.[2]

It is common in the literature to use multiple cross-sections of bond price data to generate period-by-period estimates of the implied tax rate (generally each day or each month).[3] As it does not link the implied tax rate estimates from different cross sections, period-by-period estimation makes it more difficult to test for shifts in behaviour and to identify the impact of changes in tax policy on bond prices. In this study, I employ a panel of bond data. The use of bond price data with a time series component, in conjunction with explicit data on tax rates, allows the degree of tax capitalization to be estimated for the whole sample, as well as for sub-periods, and facilitates tests of structural change in bond market behaviour.

The analysis utilizes a relatively large panel of monthly Government of Canada bond data. An advantage of Canadian data is that the tax code, as it pertains to fixed-income assets, is less complicated than that of the U.S. (MacKay, Prisman and Tian, 2000). This greatly simplifies the derivation of an empirical specification that more accurately approximates tax policy. Further, with

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[2] A related approach, used by Litzenberger and Rolfo (1984b), Ronn and Shin (1997), Elton and Green (1998) and MacKay, Prisman and Tian (2000), exploits the information in bond triplets (three bonds from the same issuer with the same maturity date, but different coupons). The applicability of this methodology is limited by the small quantity of bond triplet data typically available. For example, there are only two triplets in the data set used in the present study.
[3] Green and Ødegaard (1997) and Chittenden and Hein (1999) are exceptions that employ time series data. The latter use cointegration analysis to examine the relationship between the yields on taxable and tax-exempt bonds, while Green and Ødegaard (1997) employ a structural model of the term structure to estimate implied U.S. tax rates.

a simpler tax system, it is easier for investors to determine the tax implications of different

investments, so pricing errors based on tax confusion are less likely. In addition, Government of

Canada bonds are traded in a well-developed secondary market with significant liquidity (Gravelle

(1999)).[4] Another advantage of the data used here is that the 1986 to 2006 sample period includes

relatively large movements in tax rates. The sample is also longer than the samples used in most

previous studies, which may be important as the results in these studies often appear to be sample

period specific. Finally, to my knowledge, the impact of taxes on Canadian bond prices has not

been examined with post-1986 data.

The current study adds to a literature that has yielded mixed results with respect to the

impact of taxes on bond prices and yields.[5] A number of studies find that taxes affect U.S. Treasury

bond yields, although this effect is often found to be small, implying that the degree of tax

capitalization is less than complete (see, for example, McCulloch (1975), Van Horne (1982),

Litzenberger and Rolfo (1984a), Heuson and Lasser (1990), Ehrhardt, Jordan and Prisman (1995),

and Elton and Green (1998) as well as Green and Ødegaard (1997) for the pre-1986 period and

Guenther (1994) for the case of US Treasury bills).[6] Other authors identify a significant tax effect

on bond yields, but note that estimates of the magnitude of this effect can be quite variable (Jordan

(1984), Gay and Kim (1991)). In contrast, Green and Ødegaard (1997) and Liu, Shi, Wang and Wu

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[4] Outstanding Government of Canada bonds and debentures totaled C$87.4 billion in 1986 (17 percent of GDP), were
C$272.7 billion in 2006 (19 percent of GDP), and peaked at C$320.1 billion in 2001 (29 percent of GDP).
[5] There also exists a large empirical literature that examines the capitalization of taxes in *equity* prices. See, for
example, Lang and Shackelford (2000), Sinai and Gyourko (2004) and Dai, Maydew, Shackelford and Zhang (2008).
This literature focuses on different issues than the bond literature, but also yields conclusions that are mixed.
[6] Many of the studies that examine the relationship between taxes and bond prices (or yields) analyze the US municipal
bond "puzzle". This "puzzle" is the small spread between the yields on bonds that earn taxable interest and the yields
on US municipal bonds that are tax exempt for some groups of taxpayers. See, for example, Green (1993), Chalmers
(1998), Chittenden and Hein (1999) and Erickson, Goolsbee and Maydew (2003). The "muni bond puzzle" is a
phenomenon that is peculiar to the US tax system. The US municipal bond market is also quite different from other
bond markets since the majority of US municipal bonds are held by individuals (70 percent according to Ang, Bhansali
and Xing (2007)), a far greater proportion than for most other types of bonds.

(2007) present evidence that taxes did not have a significant impact on Treasury bond prices in the post-1986 period.[7]

Several studies examine tax capitalization in bond yields using data for non-US markets. Litzenberger and Rolfo (1984a) show that taxes have a significant impact on the prices of bonds in Germany, Japan and the UK, and that estimates of implied tax rates reflect the tax status of the major holders of government bonds (corporations in Germany and Japan, individuals in the UK). Eijffinger, Huizinga and Lemmen (1998) find that withholding taxes have a significant impact on government bond yields in industrialized countries. McCallum (1973) shows that the announcement of the introduction of a capital gains tax in Canada raised bond yields, as would be expected if taxes are capitalized in bond prices. In contrast, using Canadian data for 1964-1976, Brennan and Schwartz (1979) find that the effect of personal income taxes on bond yields is small. MacKay, Prisman and Tian (2000) confirm the results of Brennan and Schwartz, but find that taxes induced a segmented equilibrium with clientele effects after 1976, particularly between 1981 and 1986 (the end of their sample).

While much of the literature finds evidence of partial tax capitalization, I find either full tax capitalization or no tax capitalization, depending on the sample period. From 1986 to 1993, financial sector corporate income taxes were almost fully capitalized in the prices of Government of Canada bonds, implying that the marginal investor in the bond market during this period was a financial sector corporation and that pre-tax bond yields moved in response to changes in corporate tax rates. For the period 1994 through 2006, corporate taxes and personal income taxes do not appear to have been capitalized in bond prices. This result is consistent with a tax exempt marginal investor and implies that tax changes had no impact on pre-tax bond yields. The decline in the degree of tax capitalization in the early 1990s occurred following reforms to the Government of

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[7] Similar contradictory results have been found for the U.S. corporate bond market. Evidence that U.S. state corporate taxes are capitalized in corporate bond yields is presented in Elton, Gruber, Agrawal and Mann (2001), while Liu, Shi, Wang and Wu (2007) find that taxes have only a small impact on US corporate bond returns.

Canada bond market which reduced transactions costs and increased transparency and liquidity.

The next section outlines the empirical methodology, while Section 3 describes the data. Section 4 presents the results and indications of the robustness of the estimates. The final section provides a brief conclusion.

2. Empirical Methodology

Consider a bond, bond A, with a par-value of 100, coupon C_A and M periods to maturity. The relationship between the price (P_{At}) of this bond in period t, the discounted cash flow of the payments associated with the bond, and the tax rates faced by different types of investors can be written as:

$$P_{At} = C_A(1-\beta_k\tau_{ikt})\sum_{m=1}^{M}d_t(t_m,\beta_k\tau_{ikt})+100d_t(t_M,\beta_k\tau_{ikt})-(100-P_{At})\beta_k\tau_{gkt}d_t(t_M,\beta_k\tau_{ikt}), \quad (1)$$

where τ_{ik} and τ_{gk} are the tax rates on interest income and capital gains faced by an investor of type k, the β_k parameter represents the extent to which the taxes faced by a type k investor are capitalized in the bond price (so β_k equals one for the marginal investor), and $d_t(t_m, \beta_k\tau_{ikt})$ is the after-tax discount factor in period t for the mth period in the future. That is, $d_t(t_m,\beta_k\tau_{ikt})$ is the price at time t a type k investor will pay for an after-tax claim of one dollar to be delivered m periods in the future. Thus, when r_m is the pre-tax market yield on a zero-coupon bond held for m periods, $d_t(t_m,\beta_k\tau_{ikt}) =$

$$\frac{1}{(1+r_{mt}(1-\beta_k\tau_{ikt}))^m}.$$

As is common in the literature, this specification implies that the tax system treats capital gains and losses symmetrically, that taxes are imposed when income is received, and that investors act as if tax rates are known and expected to be constant through time.[8] The only difference

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[8] This last characteristic is consistent with the assumption in Shiller and Modigliani (1979, 300) "that tax rates and tax laws relating to capital gains are, and are expected to be, unchanging." As in other studies that examine the impact of taxes on bond yields, the specification given in equation (1) assumes that investors plan to hold bonds to maturity and

between equation (1) and the standard bond pricing formula used in other studies, such as in Robichek and Niebuhr (1970) or Green and Ødegaard (1997), is the addition of the parameter β_k.

Re-writing equation (1), so that the bond price appears only on the left-hand side, yields:

$$P_{At} = \frac{C_A(1-\beta_k\tau_{ikt})\sum_{m=1}^{M}d_t(t_m,\beta_k\tau_{ikt})+100(1-\beta_k\tau_{gkt})d_t(t_M,\beta_k\tau_{ikt})}{1-\beta_k\tau_{gkt}d_t(t_M,\beta_k\tau_{ikt})}. \qquad (2)$$

Using this specification, along with data on bond prices, coupons, tax rates, and the discount factors, it would be possible to estimate β_k and determine the extent to which taxes are capitalized in bond prices. One problem with such an approach is that it requires data on the discount factors associated with the entire yield curve, the M different values of $d_t(t_m,\beta_k\tau_{ikt})$ at each time t. An alternative, used in much of the literature, is to simultaneously estimate the implied tax rate $(\beta_k\tau_{ik})$ and an approximation to the yield curve at each point in time. A shortcoming with this alternative procedure is that any errors in the estimation or specification of the yield curve may bias estimates of the implied tax rate and the extent of tax capitalization.

Information on the entire yield curve at every point in time is not required if there exists a second bond (B) with a different coupon than bond A, but the same maturity date and risk characteristics. From equation (2), the price of this second bond can be expressed as:

$$P_{Bt} = \frac{C_B(1-\beta_k\tau_{ikt})\sum_{m=1}^{M}d_t(t_m,\beta_k\tau_{ikt})+100(1-\beta_k\tau_{gkt})d_t(t_M,\beta_k\tau_{ikt})}{1-\beta_k\tau_{gkt}d_t(t_M,\beta_k\tau_{ikt})}. \qquad (3)$$

Since the income streams of both bonds are discounted by the same discount factors, it is possible to use equation (3) to substitute for $\sum_{m=1}^{M}d_t(t_m,\beta_k\tau_{ikt})$ in equation (2). This yields:[9]

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so ignores the issue of tax-timing options discussed in, for example, Constantinides and Ingersoll (1984). The use of equation (1) also implicitly assumes that investors do not have an *a priori* preference for high or low coupon bonds.
[9] It is easy to modify the pricing equations to incorporate accrued interest. However, this modification adds little to the analysis as the accrued interest terms cancel out of equation (4).

$$C_B P_{At} - C_A P_{Bt} = \frac{(C_B - C_A)100(1 - \beta_k \tau_{gkt}) d_t(t_M, \beta_k \tau_{ikt})}{1 - \beta_k \tau_{gkt} d_t(t_M, \beta_k \tau_{ikt})}, \tag{4}$$

where $d_t(t_M, \beta_k \tau_{ikt}) = \dfrac{1}{(1 + r_{Mt}(1 - \beta_k \tau_{ikt}))^M}$. Equation (4) can be used to estimate the tax

capitalization parameter (β_k). It incorporates only one point on the yield curve (r_M) for each

observation, rather than the entire yield curve, which greatly simplifies estimation and should

reduce the likelihood of misspecification bias.[10]

Application of the Model to Alternative Investor Types

Equation (4) is estimated using data on the tax rates of the two principal types of domestic

taxable bondholders in Canada – individuals and financial corporations. Individuals (I) pay tax rate

τ_{iI} on coupon income and tax rate τ_{gI} on capital gains, with τ_{gI} a proportion φ_{gI} of τ_{iI} ($0 < \varphi_{gI} < 1$), so

$\tau_{gI} = \varphi_{gI}\tau_{iI}$. Financial corporations (C) pay the same rate of tax (τ_C) on coupon income and capital

gains, implying $\tau_{iC} = \tau_{gC} = \tau_C$. The corporate tax rate is generally different from the tax rate paid by

individuals on both interest income and capital gains.[11]

Pension funds and government agencies also hold large quantities of Government of Canada

bonds, but pay no tax on both capital gains and coupon income. Non-residents held between one-

third and one-quarter of marketable Government of Canada bonds during the sample period and,

thus, are another important group of bondholders. However, estimation of a tax capitalization

parameter for non-resident bondholders using equation (4) is not feasible because this would require

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[10] Van Horne (1982) also uses pairs of bonds with "similar" maturities. He chooses pairs so that one of the bonds in each pair sells near par, while the other sells at a discount, and uses the yield-to-maturity on the near-par bond as the discount rate in the pricing equation for the discount bond. Heuson and Lasser (1990) use five years of data on pairs of bonds with the same maturity date to calculate the marginal tax rate that equates the after-tax yield-to-maturity on the bonds in each pair. Chittenden and Hein (1999) examine the yields on bonds with the same maturities, but use different types of bonds, so the risk characteristics of the bonds in each pair are unlikely to be the same.

[11] Non-financial sector corporations (corporations that do not trade in bonds as part of their business) pay tax rate τ_C, but pay only a fraction φ_{gC} of this rate on capital gains (where $\varphi_{gC} = \varphi_{gI}$). See Canada Revenue Agency (1984) for a detailed description of the differences between financial and non-financial corporations. Non-financial corporations hold only very small quantities of Government of Canada bonds and, therefore, are considered further only in the robustness section below.

information on non-resident tax rates. As these tax rates vary by country of residence and investor type, and information on these characteristics is unavailable, it is not possible to identify appropriate non-resident tax rates. Since Canada does not impose a withholding tax on the income earned by non-resident holders of Government of Canada bonds issued after 1966, which includes all the bonds used in this study, the tax rate faced by non-resident bond holders could be zero, which would make the tax liability of these bond holders the same as that of non-taxed domestic entities.

Estimates of the parameter β_k derived using data on the tax rates of individuals and financial corporations can be employed to determine the extent to which each type of tax is capitalized in the prices of Government of Canada bonds and to identify the marginal investor type. If a marginal investor exists, bond prices will reflect the tax rates faced by that investor. Thus, if the marginal investor is an individual, so the personal income tax rates τ_{gI} and τ_{iI} are capitalized in bond prices, the tax capitalization parameter for individuals, β_I, would equal one. Alternatively, if a financial-sector corporation is the marginal investor, bond prices will capitalize the corporate tax rate. In this case, the corporate tax capitalization parameter, β_C, would equal one when τ_{gk} and τ_{ik} are replaced by the corporate tax rate, τ_C, in equation (4). On the other hand, if the marginal investor is a tax exempt entity (such as a pension fund), the tax capitalization parameters, β_I and β_C, will both be zero as individual and corporate tax rates will have no impact on bond prices. Finally, if the estimates of β_I and β_C fall between zero and one, there could exist tax clienteles in both prices and quantities, implying the absence of a marginal investor, or the marginal investor could be a non-resident facing a non-zero tax rate that differs from the rates faced by domestic investors.

3. Data

To estimate equation (4), price and coupon data are required for bond pairs, where the bonds in each pair have different coupon rates, but are identical in terms of risk and date of maturity. To ensure that all data are characterized by similar risk characteristics, I employ bond price data for a

single issuer – the Government of Canada. These data are from the annual publications *Canadian Bond Prices* and *FP Bonds—Canadian Prices*, both published by the Financial Post Corporation.[12]

The sample consists of 2190 last-business-day-of-the-month observations on the bid prices of 94 bonds that can be combined to form 49 bond pairs.[13] The span of the data is from 1986 to 2006, although some of the bonds matured during this period, while others were newly issued, so the bond data form an unbalanced panel. All bonds in the sample are not callable, are denominated in Canadian dollars, are not real return bonds, are not extendable or exchangeable, pay interest on a semi-annual basis and have more than 12 months to maturity.[14] The bonds in each pair may differ in terms of amount issued and issue year. Strip bonds and zero-coupon bonds are not included in the data set as they are taxed in a different fashion than coupon bearing bonds. The sample includes, on average, 104 observations per year and almost four years of monthly observations per bond pair. The average coupons of the high and low coupon bonds in each pair are just under 12 and 8 percent, respectively. Appendix A provides additional descriptive statistics.

Equation (4) is estimated using tax rate data for two types of bondholders – individuals and corporations. Data on the highest personal interest income and capital gains tax rates for a resident of Ontario are employed to represent the individual income tax rates, τ_{iI} and τ_{gI}, while data on the combined federal and Ontario general corporate tax rate are used for τ_C.[15] The tax rates applicable

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[12] The characteristics of the bonds (for example, amount issued, issue year, callability) are given in the annual Financial Post publications: *Government Bond Record* and *FP Bonds – Government*.

[13] Ninety-four bonds can be used to construct 49, rather than 47, bond pairs because there are cases in which three bonds share the same maturity date, and two separate pairs can be formed from the observations on these bonds.

[14] This is consistent with the usual definition of fixed rate debt as debt with at least one year to maturity (Harvey, 1999, 29n). Heuson and Lasser (1990), Elton and Green (1998) and Elton, Gruber, Agrawal and Mann (2001) also do not use data for bonds with less than one year to maturity. The issues of risk differentials and differences in callability, which have been important in the US muni-bond-puzzle literature (Chalmers (1998)), are not relevant here as the bond price data correspond to bonds that are issued by one issuer and are not callable. As in Elton, Gruber, Agrawal and Mann (2001), I exclude a small number of observations with obvious pricing (data entry) errors. For example, five observations are not used because the reported price and yield data give inconsistent predictions (by a small number of basis points) with respect to whether the bonds were trading at a discount or a premium.

[15] The personal income tax rate data were taken from Canadian federal and Ontario income tax forms for each year. These forms can be accessed at http://www.cra-arc.gc.ca/formspubs/t1general/allyears-e.html on the website of the Canada Revenue Agency. The corporate income tax rate data were provided by Bev Dahlby and Ergete Ferede. They

in Ontario were employed because Ontario is the largest province in terms of both GDP and population. The tax rate data are presented in Table 1.

The estimating equation, equation (4), includes one point on the zero-coupon yield curve – r_M. The values for r_M were proxied using estimates generated by the Bank of Canada for the business day prior to the business day on which the bond price data are observed.[16]

As in Green and Ødegaard (1997), an error term is added to each of the bond pricing equations, equations (2) and (3). The substitution of equation (3) into (2) to generate equation (4) yields an error in equation (4) that is heteroscedastic. It is also possible that the errors are not independent across time for each bond pair or across the bonds in each time period. As a result, the standard errors reported below are adjusted for heteroscedasticity and two-way clustering (by bond pair and time period).[17] This is done using the methodology proposed in Cameron, Gelbach and Miller (2006) and Thompson (2006).

4. Results

Table 2 reports estimates of the individual and corporate tax capitalization parameters, β_I and β_C, using data for the whole sample, 1986 through 2006, as well as for two sub-periods – 1986-1993 and 1994-2006. As noted by Johnson (2004-05, 19), the behaviour of prices in the Government of Canada bond market appears to have changed in the early 1990s. This change may have been caused by institutional reforms to the operation of the market for Government of Canada

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obtained these data from various issues of *The National Finances* and *Finances of the Nation*, both published by the Canadian Tax Foundation.

[16] Downloaded 1 June 2007 from http://www.bankofcanada.ca/en/rates/yield_curve.html. For a description of the methodology used to generate these data, see Bolder, Johnson and Metzler (2004) and Bank of International Settlements (2005). For the sample period employed, the Bank of Canada provides zero-coupon yields for every maturity from 3 months to 300 months at three month intervals. The maturity chosen to represent r_M for each monthly observation is the maturity that is closest to the M for that observation.

[17] Rather than cluster by bond pair and observation (month), since tax rates generally remain constant for an entire year, it is possible to estimate the standard errors while clustering by bond pair and year, which allows the errors of all the observations in a particular year to be correlated. This procedure increases the magnitude of the estimates of the standard errors, but not by enough to alter any of the conclusions.

bonds at this time.[18] These reforms improved bond market liquidity and transparency, and led to a

reduction in the use of "rules of thumb" and "idiosyncratic" bond pricing formulas by traders

(Whittingham (1996-97), Johnson (2004-5)). Since the period following the early 1990s may have

been characterized by different behaviour than the earlier period, estimates are provided for the two

sub-samples as well as for the full sample. Johnson (2004-5) suggests that the exact point at which

to split the sample is unclear. If the break point between the two sub-samples is moved one year in

either direction, the values given in Table 2 are generally less than one standard error, and never

more than two standard errors, from the new estimates.

 As shown in Table 2, over the whole sample, the estimate of the personal income tax

capitalization parameter, β_I, is .1809, while the estimate of the corporate tax capitalization

parameter , β_C, is .5350.[19] As these two parameter estimates are significantly different from both

zero and one, these results imply that no single type of domestic bondholder – an individual, a

corporation or a non-taxed entity – was the marginal investor in the Government of Canada bond

market for the entire sample period.

 As can be seen from Table 2, the estimates for the two sub-samples differ strongly from

each other as well as from the estimates for the whole sample. Tests that the tax capitalization

parameters do not vary across the two sub-samples reject the hypothesis of parameter constancy for

both β_I and β_C .[20] For the earlier sub-sample, 1986 through 1993, the estimate of the personal

income tax capitalization parameter, β_I, is .3190 and remains significantly different from both zero

and one. In contrast, the corporate tax capitalization parameter (β_C) is estimated to be .9193. This

estimate is significantly different from zero (as well as significantly different from the estimate

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[18] See Branion (1995), Whittingham (1996-97), Halpern and Rumsey (1997, 2000), Gravelle (1999), Harvey (1999) and
Chouinard and Lalani (2001-02).
[19] Although the estimating equation is non-linear, the parameter estimates are robust to different starting values. For
example, the estimates converge to the same values when the parameter starting value is set to either zero or one.
[20] The t-statistics for the structural change tests of the individual and corporate tax capitalization parameters, β_I and β_C,
are 12.63 and 8.32, respectively.

for β_I), but is not significantly different from one. Thus, for the earlier period, the results suggest that the marginal investor in the bond market was a financial-sector corporation and that changes in the corporate tax rate lead to compensating movements in bond yields.

For the sub-period 1994 through 2006, estimates of the two tax capitalization parameters (β_I and β_C) are near zero (and negative) and both parameters are significantly different from one. These estimates imply that, during this later period, neither corporate nor personal income taxes were capitalized in bond prices, so the marginal investor in the bond market was an investor with a zero tax rate, such as a pension fund or government agency. While the negative estimates for the tax capitalization parameters seem somewhat unusual, Green and Ødegaard (1997) find negative implied tax parameter estimates for the US in the post-1986 period and also interpret these estimates as indicating the absence of tax capitalization in bond prices.

4.1 Year-by-Year Estimates of the Tax Capitalization Parameters

As an indication of the robustness of the tax capitalization parameter estimates presented in Table 2, I re-estimated equation (4) separately for each year of the sample.[21] The year-by-year corporate and individual tax capitalization parameter estimates are illustrated in Figures 1a and 1b. For the period 1986 through 1993, the results presented in Figure 1a show that five of the eight estimates of the corporate tax capitalization parameter are not significantly different from one (six using a 99 percent confidence interval). From 1994 onwards, all the estimates of the corporate tax capitalization parameter are close to zero, and all are significantly different from one. Thus, the results in Figure 1a are generally consistent with the results of Table 2 which imply that corporate taxes were capitalized in bond prices in the pre-1994 period, but had little impact on bond prices after 1993.

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[21] The data set does not include enough observations to estimate the model for each month. However, neither τ_{il} nor τ_C change within any year of the sample.

The year-by-year estimates of the personal income tax capitalization parameter (β_1) given in Figure 1b are all significantly less than one, are almost always less than .5 in magnitude, and are very close to zero after 1993. As with the results of Table 2, these annual estimates imply that individual taxpayers are not the marginal investor in the Government of Canada bond market. The post-1993 tax capitalization parameter estimates of approximately zero for both financial corporations and individuals illustrated in Figures 1a and 1b are consistent with the marginal investor during this period having a tax rate of zero.

4.2 Robustness of the Estimates[22]

To assess the robustness of the results, I re-estimated the model using several alternative specifications. These included limiting the data to bonds of the most actively traded maturities (36 to 120 months), using data for only one month (June) of each year, excluding provincial and federal surtaxes from individual tax rates, modifying the model to incorporate the pre-1994 individual lifetime capital gains deduction, and utilizing data for off-the-run bonds only. These changes yield estimates of the tax capitalization parameters and standard errors that differ very little from those presented in Table 2 (generally the parameter estimates differ by less than one standard error). I also re-estimated the model using the tax rates faced by non-financial corporations and found the results to be very similar to the results for individuals, which is not surprising as both pay tax on the same proportion of capital gains and, like individuals, non-financial corporations hold only a very small proportion of Government of Canada bonds (Table 3).

To allow for possible errors in bond pricing and liquidity effects, I modified the specification of the estimating equation to allow the price of each bond to differ from the present discounted value of the bond's income stream in three ways – by a constant parameter; by a constant parameter and a linear function of the log of the issue amount of the bond; and by a

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[22] See Appendix B for a more complete discussion and description of the robustness tests and results.

constant parameter and a linear function of the number of years since the bond was issued. For the whole sample, and in both sub-samples, the estimates of β_I and β_C for these three variants of the model are similar to those presented in Table 2. Thus, the conclusions that follow from the estimates reported in Table 2 do not appear to be the result of a failure to account for errors in bond pricing or liquidity effects.

4.3 Explanations for the Decline in the Degree of Tax Capitalization in the Early 1990s

There are a number of possible explanations for the decline in the degree of tax capitalization after 1993. One explanation is that there was an increase in the proportion of bonds held by tax-exempt bondholders. The percentage of Government of Canada marketable bonds held by tax-exempt domestic entities rose, on average, by 1.9 percentage points, from 35.2 percent in 1986-1993 to 37.1 percent in 1994-2006 (Table 3).[23] On the other hand, the average holdings of non-residents, a group that also pays no Canadian taxes, fell by nine percentage points between the same two sub-periods. Further, the average holdings of banks and other taxable financial institutions rose by a large amount (8.0 percentage points). Thus, there is little evidence to suggest that an increase in the share of bonds held by non-taxed investors caused the change in tax capitalization.

A second possible explanation for the decrease in the extent of tax capitalization is that an increase in the quantity of outstanding Canadian government debt made the market more liquid, which facilitated arbitrage. Green and Ødegaard (1997, 628) suggest that one of the reasons for the disappearance of tax capitalization in the U.S. during the mid-1980s was that "the number of bonds and the range of maturities issued by the U.S. Treasury increased dramatically." However, the rapid

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[23] These include trusteed pension funds and social security funds, the Bank of Canada, and government and financial government business enterprises. It may be reasonable to also include mutual funds in this category as mutual fund earnings flow through to unit holders and, thus, fund managers may not take taxes into account when making investment decisions. The inclusion of mutual funds in the non-taxed group would increase the change in the average holdings of these entities by a further 3.7 percentage points, not enough to outweigh the increase in the holdings of taxable financial institutions.

increase in Canadian federal government debt began in the mid-1980s, long before the decline in

the estimates of the tax capitalization parameters illustrated in Figures 1a and 1b.[24]

A further possible explanation is that a major change in tax policy caused the reduction in

tax capitalization. Green and Ødegaard (1997) found that implied tax rates were positive in the U.S.

bond market during the pre-1986 period, but fell to zero following the 1986 U.S. tax reform. In

contrast, the decline in the estimated Canadian tax capitalization parameters did not occur at the

same time as a large corporate or personal income tax policy change.

It has also been suggested that the failure to find a tax capitalization effect could be because

large interest rate movements make it difficult to detect tax effects in asset markets (MacKay,

Prisman and Tian (2000)). However, interest rates were more variable during the 1986 to 1993

period, for which a significant tax capitalization effect is identified, than in the years following

1993.

A final possible explanation is that the decline in tax capitalization followed from reforms to

the operation of the bond market. During the late 1980s and early 1990s, limited liquidity and

transparency in the bond market, as well as high transaction costs, may have discouraged secondary

market trading by some types of investors. A set of institutional changes implemented in the early

1990s reduced transactions costs and increased both transparency and liquidity. The introduction

of benchmark-bond issues in the early 1990s (Harvey, 1999), the rise in the proportion of

marketable debt in the form of bonds (Gravelle (1999)), and the removal of tax disincentives for

foreign repo traders (Morrow (1994-1995)) all increased market liquidity. Price discovery was

made easier and more transparent when the Bank of Canada moved to an all auction format for

bond issues and adopted a regular bond auction calendar (Branion, 1995). A regulatory change that

facilitated the reconstitution of strip bonds made arbitrage easier, increased liquidity in the bond

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[24] From a base of C$87 billion in 1986, the quantity of Government of Canada bonds outstanding rose to C$185 billion
in 1993, a rise of C$14 billion per year on average, with the increase in each year exceeding C$10 billion.

market and narrowed spreads between benchmark and other issues (Branion (1995), Halpern and Rumsey (1997, 2000)). Finally, the clearing and settlement of Government of Canada bond purchases became electronic, a change that likely reduced transaction costs (Branion (1995)). By improving transparency and liquidity, and lowering transaction costs, these reforms may have caused a change in the pattern of trading by bond market participants that altered the extent of tax capitalization.

5. Concluding Comments

The estimates presented above quantify the extent to which corporate and personal income taxes are capitalized in bond prices. The methodology augments the existing literature in several ways. First, the empirical analysis incorporates actual tax rate data, which makes it possible to directly estimate the degree of tax capitalization and identify the marginal investor type. Second, as the empirical methodology employs data on matched bond pairs, it is not necessary to estimate the entire yield curve jointly with the tax capitalization parameter. Finally, the use of time series data facilitates tests for structural change.

The empirical findings indicate near complete capitalization of the financial sector corporation income tax in bond prices during the period 1986-1993, implying that changes in corporate tax rates caused changes in pre-tax bond yields during this period. This evidence suggests that there existed a marginal investor during this period and that this investor paid corporate taxes, rather than personal income taxes, which is, perhaps, not surprising given the relatively small share of Government of Canada bonds held by individuals (see Table 3).

For the period 1994-2006, the tax capitalization parameter estimates fall to near zero. This result implies that, during this period, the marginal investor faced a zero tax rate, the tax rate of pension funds and government agencies. As there is no withholding tax on interest payments to non-resident holders of Government of Canada bonds, the marginal investor could also have been a

non-resident investor facing a zero tax rate in their home country (such as a foreign pension fund or government agency).

For both the 1986-1993 and 1994-2006 periods, there is evidence of a marginal investor type – a financial sector corporation in the first period and a non-taxpaying entity in the second. The differing tax treatment of different investor-types, in conjunction with the finding of a single type of marginal investor, implies that not all investors would want to hold all bonds. In the terminology of Dybvig and Ross (1986), there are clientele effects in quantities, but not in both prices and quantities. This suggests that frictions exist in the Canadian bond market that inhibit arbitrage.[25]

The estimates indicate a significant shift in the degree of tax capitalization in the early 1990s. Estimates that do not account for this shift would lead to a conclusion of partial tax capitalization and the absence of a marginal investor. Once account is taken of the shift in the tax capitalization parameter, the evidence does not support this conclusion.

The change in the identity of the marginal investor and the shift in the extent of tax capitalization after 1993 do not appear to be correlated with movements in bond holdings, nor are they consistent with several other explanations. It is possible, however, that this change was induced by the introduction in the early 1990s of several institutional reforms to the operation of the market for Government of Canada bonds that improved market liquidity, efficiency and transparency. These reforms are expected to have reduced the risk and transaction costs of trading and, as a consequence, may have induced investors not subject to tax to trade more actively. This is consistent with the suggestion of Shleifer and Vishny (1997) that arbitrage in financial markets may be less constrained when there is less risk. Nevertheless, the evidence in support of this explanation is circumstantial, so further research is required.

As the estimates for the more recent period indicate that bond prices do not capitalize either

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[25] Prisman and Tian (1994, 303) note that, in Canada, short sales "are difficult and costly to implement and may constitute the frictions needed to support an equilibrium with clientele effects."

personal or corporate taxes, after-tax bond returns for individuals and corporations change as tax rates change, causing the burden of tax changes to be borne by these non-tax exempt holders of bonds, rather than bond issuers. As a consequence, tax changes should not affect government financing costs through the tax capitalization channel.

As pre-tax bond yields did not incorporate tax effects in the period 1994-2006, estimates of expected inflation and predictions of business cycle fluctuations based on the term premium are not likely to have been directly affected by tax rates during this period. On the other hand, for the pre-1994 sub-sample, the significant effect of corporate taxes on bond yields implies that, in this earlier period, tax rates may have had an impact on business cycle and inflation forecasts.

The results also have implications for estimates of the yield curve. Tax-related coupon effects are considered to be of sufficient importance that Bank of Canada estimates of the Government of Canada yield curve exclude observations for bonds that trade at large deviations from par (Bank of International Settlements, 2005, 21). The estimates presented here indicate that, at least since the mid-1990s, there has been little capitalization of taxes in bond prices, implying that pre-tax yields are largely insensitive to movements in corporate or personal income tax rates. As a consequence, yield curve estimates are unlikely to be biased by taxes or the coupons of the bonds included in a particular sample. Elton and Green (1998) suggest that this type of evidence justifies the use of all available bond data when estimating the yield curve.

Table 1: Tax Rates (percent)

| | Corporate Tax Rate | Personal Income Tax Rates | |
| | | | |

Year	τ_C	Capital Gains (τ_{gI})	Interest Income (τ_{iI})
1986	53.30	27.71	55.42
1987	52.07	26.265	52.53
1988	47.95	30.7593	46.139
1989	44.34	31.4553	47.183
1990	44.34	36.1703	48.227
1991	44.34	36.8358	49.1144
1992	44.34	37.3295	49.7727
1993	44.34	39.2588	52.345
1994	44.34	39.8895	53.186
1995	44.62	39.8895	53.186
1996	44.62	39.6894	52.9192
1997	44.62	38.7324	51.6432
1998	44.62	37.7161	50.28818
1999	44.62	36.5661	48.7548
2000	44.62	35. 8947 (Jan) 31.9064 (Feb-Sep) 23.9298 (Oct-Dec)	47.8596
2001	42.12	23.2048	46.4096
2002	38.62	23.2048	46.4096
2003	36.62	23.2048	46.4096
2004	36.12	23.2048	46.4096
2005	36.12	23.2048	46.4096
2006	36.12	23.2048	46.4096

Correlation Matrix

	τ_C	τ_{gI}	τ_{iI}
τ_C	1		
τ_{gI}	.43192	1	
τ_{iI}	.682455	.598691	1

Table 2: Estimates of the Tax Capitalization Parameters

	Sample Period		
	1986-2006	**1986-1993**	**1994-2006**
Part A: Individuals			
Individual tax capitalization parameter (β_I)	.1809 (.0515)	.3190 (.0275)	-.0384 (.0094)
Hypothesis Tests[†]			
Test that $\beta_I = 1$	15.90	24.76	110.47
Test that $\beta_I = 0$	3.51	11.62	4.09
Part B: Corporations			
Corporate tax capitalization parameter (β_C)	.5350 (.1718)	.9193 (.1176)	-.0858 (.0231)
Hypothesis Tests[†]			
Test that $\beta_C = 1$	2.71	.69*	47.00
Test that $\beta_C = 0$	3.11	7.81	3.72
Number of Observations:	2190	1257	933

Notes:

Standard errors are shown in brackets below each coefficient estimate. The standard errors are calculated using a variance estimator that provides heteroscedasticity consistent two-way cluster-robust inference, where clustering is by time period (month) and bond pair. See Thompson (2006) and Cameron, Gelbach and Miller (2006) for details.

[†] The test statistics are t-statistics calculated using the two-way cluster-robust standard errors.

* The hypothesis is not rejected using a 95 percent confidence interval.

Table 3:   **Share of Holdings of Outstanding Government of Canada Marketable Bonds (percent of total, period averages)**

	1986-1993	1994-2006	Change
Persons and unincorporated business[a]	5.7	1.0	-4.7
Mutual funds[b]	2.8	6.5	3.7
Non-financial corporations including government business enterprises[c]	1.0	.9	-0.1
Bank of Canada[d]	7.0	6.9	-0.1
Government and financial government business enterprises[e]	6.5	9.6	3.1
Trusteed pension plans and social security funds[f]	21.7	20.6	-1.1
Chartered banks and near-banks[g]	7.8	14.9	7.1
Life insurance and other financial institutions[h]	13.5	14.4	0.9
Non-residents[i]	34.0	25.0	-9.0

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Source: Statistics Canada, *National Balance Sheet Accounts, by Sectors*, CANSIM II Table 3780004.
[a] Cansim series: V33483-V33484.
[b] Cansim series V34179.
[c] Cansim series V33236.
[d] Cansim series V33689.
[e] Excludes social security funds. Cansim series: V32597+V34323-V34589.
[f] Cansim series: V34085+V34589.
[g] Cansim series: V33736.
[h] Includes life insurance business; segregated funds of life insurance companies; mortgages; and total other financial institutions. Excludes mutual funds. Cansim series V34023+V34056+V34114-V34179.
[i] Cansim series: V34623.

Shares are calculated by dividing by total holdings (Cansim series: V34786-V33484).

Figure 1a: Year-By-Year Corporate Tax Capitalization Parameter Estimates
(with a 95 percent confidence interval)

◆ Estimate of Corporate Tax Capitalization Parameter
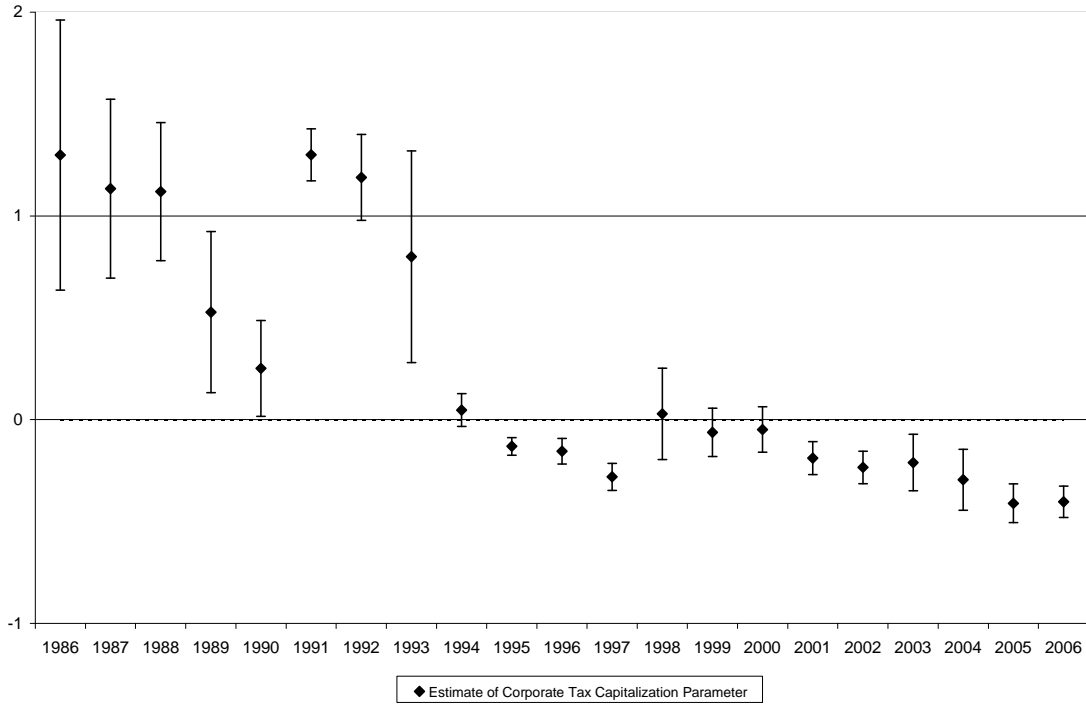
Figure 1b: Year-By-Year Personal Income Tax Capitalization Parameter Estimates
(with a 95 percent confidence interval)

◆ Estimate of Personal Income Tax Capitalization Parameter
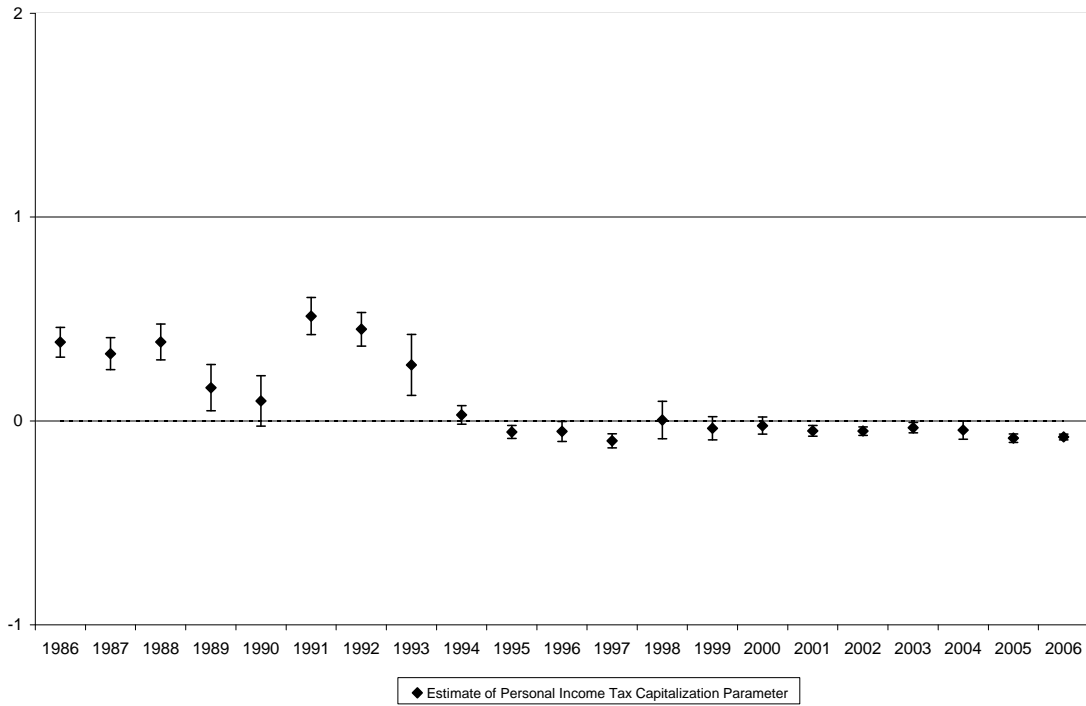
Appendix A: Descriptive Statistics

Number of bonds: 94
Number of bond pairs: 49
Number of bond pair observations: 2190
Sample period: 1986-2006

	Average	Range
Coupon of high coupon bond in pair:	11.824	4.25 – 15.00
Coupon of low coupon bond in pair:	7.875	3.0 – 13.0
Amount issued of each bond (millions $CDN):	2752.8	22.5 – 15000
Yield to maturity of high coupon bond in pair:	7.670	2.279 – 13.549
Yield to maturity of low coupon bond in pair:	7.514	2.268 – 13.223
Months to maturity:	51.1	13 – 132
Personal income tax rate on other income excluding surtaxes (percent):	44.2	40.16 – 51.0
Personal income tax rate on other income including surtaxes (percent):	49.3	46.1 – 55.4
Personal income tax rate on capital gains excluding surtaxes (percent):	28.3	20.1 – 34.4
Personal income tax on capital gains including surtaxes (percent):	31.7	23.2 – 39.9
Personal income tax on capital gains as a share of tax on other income (percent):	64.2	50 – 75

Appendix B: Robustness of the Results[26]

To assess the robustness of the results, I re-estimated the model using several alternative specifications. Branion (1995, 14) and Jones and Fabozzi (1992, 180) note that the most actively traded Government of Canada bonds are of intermediate maturities, in the range of 3 to 10 years. Estimation using only data for bonds with maturities of 36 to 120 months yields estimates of the tax capitalization parameters and standard errors that differ very little from those presented in Table 2 (generally the parameter estimates differ by less than one standard error) even though the number of observations over the whole sample and in each of the two sub-periods is reduced by almost half.

Since both personal and corporate tax rates generally do not vary during a calendar year, there may be little additional information generated by use of observations for more than one month of each year. Further, tax rates are likely to be known with more certainty after the release of the budget, which generally occurs in the first third of the year. As a result, I re-estimated the tax capitalization parameters, β_I and β_C, using data for June only, where the choice of June was made *ex ante*. Despite a large reduction in the size of the sample, the new estimates of the parameters and standard errors are very similar to those reported in Table 2 and cause none of the conclusions to change.[27]

The marginal personal income tax rates used to generate the results in Table 2 and Figure 1b include both provincial and federal surtaxes. Since these surtaxes are paid only by taxpayers with the highest incomes, the model was re-estimated using the highest personal tax rate *exclusive* of federal and provincial government surtaxes as the income tax rate.[28] This change yielded parameter estimates that were extremely close to those reported in Table 2 (all within one standard error).

The Canadian government introduced a lifetime capital gains deduction for individuals in 1985, a deduction which was capped at $100,000 in 1988 and eliminated in 1994 (Zeng (2002)). As this capital gains deduction may have had an impact on individual bond demand, it could have affected the estimates of the individual tax capitalization parameter (β_I) for the period 1986-1993. To examine this possibility, I estimated parameter β_I for the period 1986 to 1993 with the capital gains tax rate set to zero, as would be the case if individual taxpayers could take advantage of the lifetime capital gains deduction. While this change causes the estimate of β_I to fall, it remains significantly different from both zero and one, which is consistent with the results reported in Table 2.

Non-financial corporations pay tax at the corporate rate, but, unlike financial corporations, pay tax on only a fraction φ_{gC} of capital gains ($0 < \varphi_{gC} < 1$ and $\varphi_{gC} = \varphi_{gI}$). Estimation of equation (4) using the appropriate tax rates for non-financial corporations yields tax capitalization parameter estimates that are very close to those presented in Table 2 for individuals.[29] These results imply that, as with individuals, non-financial corporations are not the marginal investor in the market for Government of Canada bonds. This is not surprising as these corporations hold only one percent of outstanding marketable Government of Canada bonds (Table 3).

The bonds employed include both on-the-run and off-the-run bonds. There may be differences in the market liquidity and transactions costs associated with trading these two types of bonds, differences that could be reflected in bond prices. To test if the mixture of on-the-run and off-the-run bonds is important to the results, I re-estimated the tax capitalization parameters using data

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[26] This appendix presents a summary of the robustness results. The estimates underlying this summary are available from the author.

[27] There are 183 June observations in the full sample, 1986-2006, and 105 and 78 observations in the 1986-1993 and 1994-2006 sub-samples, respectively.

[28] The combined federal and provincial personal income surtax averaged a little over 5 percentage points during the sample period and varied from approximately 2 to 9 percentage points.

[29] For the non-financial corporations case, equation (4) is modified by replacing τ_{ik} and τ_{gk} with τ_C and $\varphi_{gC}\tau_C$, respectively.

only for bonds that had been issued for at least two years (and so are likely to be off-the-run). The new parameter estimates are almost identical to the estimates given in Table 2, which is not surprising as only a small percentage of the observations in the sample correspond to bonds issued in the previous two years.

Finally, to allow for possible errors in bond pricing and liquidity effects, I generalized equation (1) in the text to allow the price of Bond A (P_A) to differ by a function $Z(X_A)$ from the present discounted value of the stream of income payments associated with the bond:

$$P_{At} = C_A(1-\beta_k\tau_{ikt})\sum_{m=1}^{M}d_t(t_m,\beta_k\tau_{ikt})+100d_t(t_M,\beta_k\tau_{ikt})-(100-P_{At})\beta_k\tau_{gkt}d_t(t_M,\beta_k\tau_{ikt})+Z(X_{At}),\text{ (A1)}$$

where X_{At} is a vector of the characteristics of Bond A that may proxy liquidity, for example, and all other variables are defined as for equation (1). The reason for including the $Z(X_{At})$ term is to avoid forcing the effects of systematic pricing errors and liquidity effects into the estimates of the tax capitalization parameter. If the Z function is excluded from the estimating equation, pricing errors and liquidity effects could, potentially, be reflected in the β_k parameter estimate. Re-writing equation (A1) so that the price appears only on the left-hand side yields:

$$P_{At} = \frac{C_A(1-\beta_k\tau_{ikt})\sum_{m=1}^{M}d_t(t_m,\beta_k\tau_{ikt})+100(1-\beta_k\tau_{gkt})d_t(t_M,\beta_k\tau_{ikt})+Z(X_{At})}{1-\beta_k\tau_{gkt}d_t(t_M,\beta_k\tau_{ikt})}. \tag{A2}$$

If there exists a second bond (Bond B) with the same maturity date, but a different coupon, the price of this bond is given by:

$$P_{Bt} = \frac{C_B(1-\beta_k\tau_{ikt})\sum_{m=1}^{M}d_t(t_m,\beta_k\tau_{ikt})+100(1-\beta_k\tau_{gkt})d_t(t_M,\beta_k\tau_{ikt})+Z(X_{Bt})}{1-\beta_k\tau_{gkt}d_t(t_M,\beta_k\tau_{ikt})}. \tag{A3}$$

Using equation (A3) to substitute for $\sum_{m=1}^{M}d_t(t_m,\beta_k\tau_{ikt})$ in (A2) yields:

$$C_BP_{At}-C_AP_{Bt} = \frac{(C_B-C_A)100(1-\beta_k\tau_{gkt})d_t(t_M,\beta_k\tau_{ikt})+C_BZ(X_{At})-C_AZ(X_{Bt})}{1-\beta_k\tau_{gkt}d_t(t_M,\beta_k\tau_{ikt})}. \tag{A4}$$

This model can be estimated in the same fashion as equation (4) once an explicit form for $Z(X_t)$ is specified. Equation (A4) is more flexible than equation (4) in that it allows for the systematic over- or under-pricing of bonds.

I chose specifications for $Z(X_t)$ that allow the price of each bond to differ from the present discounted value of the bond's income stream in three ways – by a constant parameter; by a constant parameter and a linear function of the log of the issue amount of the bond; and by a constant parameter and a linear function of the number of years since the bond was issued.[30] For the whole sample, and in both sub-samples, the estimates of β_I and β_C for these three variants of the model are similar to those presented in Table 2 and, in not a single case, is there a change in the conclusions with respect to the extent of tax capitalization or the identity of the marginal investor type. Thus, the conclusions that follow from the estimates reported in Table 2 do not appear to be the result of a failure to account for errors in bond pricing or liquidity effects.

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[30] Green and Ødegaard (1997) suggest the number of years since issue as a proxy for liquidity.

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