Information collection in bargaining

Ming Li

Concordia University

September 2002

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Ming Li†

March, 2007

Abstract

I analyze a bilateral bargaining model with one-sided uncertainty about time preferences. The uninformed player has the option of halting the bargaining process to obtain additional information, when it is his turn to offer. For a wide class of preference settings, the uninformed player does not collect information when he is quite sure about his opponent's type. There exist preference settings in which the uninformed player collects information until he is sufficiently sure about his opponent's type, as long as the information source is accurate enough. With additional assumptions, the uninformed player is more likely to draw signals and is better off, if the information is more accurate.

Key words: bargaining, alternate offers, incomplete information, delay.
JEL codes: C78, D82.

*I thank Larry Samuelson for encouragement and guidance. I also wish to thank Hiroyuki Kasahara, Grigory Kosenok, Akos Magyar, and Lucía Quesada for helpful comments and discussions. All possible errors in this paper are my own.

†Address: 1455 Boul. de Maisonneuve O., Department of Economics, Concordia University, Montreal, QC, H3G 1M8. Tel: +1 (514) 848-2424 Ext. 3922. Fax: +1 (514) 848-4536. Email: mingli@alcor.concordia.ca.

1 Introduction

Incomplete information is widely viewed as an important reason for delays in bargaining. This paper aims to investigate a previously unstudied source of delay in bargaining when there is incomplete information. That is, the uninformed player may not want to make offers at all, unless he gets sufficient information about his opponent's type.[1] He may instead choose to search for additional information. This consideration is especially important when the acquirement of a small amount of additional information may significantly improve the outcome for the uninformed player.

Consider a bilateral bargaining situation in which an informed player bargains with an uninformed player. The informed player could be either "strong" or "weak." The uninformed player prefers to have a "weak" opponent. The uninformed player may only want to start bargaining when he is sufficiently sure that his opponent is weak or strong.

To motivate the exercise in this paper, consider a district attorney prosecuting a case against a crime suspect. The defense has private information about the merits of the case. Both sides prefer to arrive at a deal earlier and avoid going to trial. However, if the district attorney chooses to wait, he may come upon clues that reveal information about the defendant. Therefore, he may not want to offer or accept a deal until he has sufficient information about the case.

Another example is negotiations between two geopolitical entities. Due to historical isolation, one entity is likely to lack information about the other's strengths and weaknesses. Both sides prefer to settle disputes earlier to avoid potential conflicts. In this case, the uninformed side may prefer not to make or accept an offer until it has enough information.

Ausubel, Cramton, and Deneckere (2002) provide a comprehensive survey of the vast literature on bargaining with incomplete information. In one branch of the literature, the uncertainty lies in the informed player's value of the good being bargained upon. The time length between offers is exogenously given. The uninformed player uses ascending offers to screen the informed player. The informed player of the strong type waits longer to arrive at agreements with the uninformed player. This phenomenon is sometimes referred to as the "Coasian Dynamics." The works by Fudenberg, Levine, and Tirole (1985), Grossman and Perry (1986), Gul and Sonnenschein (1988), and Gul, Sonnenschein, and Wilson (1986) are a few examples.

[1] In this paper, the uninformed player is referred to as "he", and the informed player "she."

Note that delays *can* happen in these games if arbitrary dependence on history of play is allowed. Ausubel and Deneckere (1989) show that, when there is no gap between the valuations of the uninformed and informed players, a “folk theorem” is true in the context of durable goods monopoly (which is technically equivalent to bilateral bargaining with one-sided incomplete information), in that the uninformed party’s equilibrium payoff ranges from that when he has full monopoly power and that when he has no such power. Another branch of literature allows the informed player to endogenously choose the amount of delay. The uncertainty is about time preferences, following Rubinstein (1985). In the selected equilibrium, stronger types incur longer delays (and refuse to return to bargaining), so as to capture a bigger proportion of the surplus. This is usually called “strategic delay.” Admati and Perry (1997) initiate this line of research and Cramton (1992) extends it to the case of two-sided uncertainty.

This paper adopts Rubinstein’s (1985) model, in which the private information is about time preferences. In addition, I introduce an outside information source that the uninformed party can use if he halts the bargaining process. I assume that he gets a signal from this information source for each period he stays away from the bargaining table. I call this information collection.

I focus my analysis on pure strategy sequential equilibrium. Alternate-offer bargaining games of incomplete information have a plethora of sequential equilibria. The same is also true in my model. I adopt a version of refinement used by Rubinstein (1985) and Osborne and Rubinstein (1990), called bargaining sequential equilibrium or rationalizing sequential equilibrium. This refinement ensures that for the two-type model without information collection, for any distribution over the two types, there is a unique equilibrium satisfying the refinements.[2] Thus, the uninformed player is able to make a comparison between waiting and offering, since he knows what his payoff would be if he starts the bargaining process. This proves very important for the analysis in this paper.

The results of this paper are fairly intuitive. The uninformed player does not collect information, if his belief about the informed player being the weak type is very high or very low. I identify preference settings in which the uninformed player collects information until he is sufficiently sure about his opponent’s type. I also identify preferences for which it is true that the uninformed player prefers to have a more accurate information source, and is more willing to collect information when the information source is more accurate.

[2] There could still be multiplicity at one or two threshold values. In those cases, the equilibrium with shorter delay is selected.

I view information collection as a potentially important explanation of bargaining delays in many situations that complements those explanations mentioned above, since it works through a very different mechanism.

2 The Model

The model is the standard alternating offers bargaining model of incomplete information with one additional feature, namely, the option for the uninformed player to search for information at periods when he is supposed to make offers. It is an adaptation from the incomplete information alternating-offers bargaining model by Rubinstein (1985).

Two players bargain over the division of a pie of size one. The set of feasible agreements is

$$X = \{(x, 1-x) : 0 \leq x \leq 1\},$$

where x is the proportion of the pie Player 1 gets from the agreement. Thus, each agreement can be identified by x. The players alternate in making offers. Let $T = \{0, 1, 2, ...\}$ be the set of times at which players move. Without information collection, Player 1 makes offers in even periods ($t = 0, 2, 4, ...$), and responds to Player 2's offers in odd periods ($t = 1, 3, ...$), and vice versa for Player 2. If agreement x is reached in period t, then the outcome is denoted as (x, t). The outcome of perpetual disagreement is denoted as PD. I denote the preference relations of players 1 and 2 over $X \times T \cup \{PD\}$ by $\succcurlyeq_1$ and $\succcurlyeq_2$. Following Rubinstein (1985), I assume that the player's preferences can be represented by a utility function of the form

$$u(x)\delta^t,$$

where u is an increasing and concave function and δ can be different for the two players.[3] Furthermore, players are expected-utility maximizers.

All aspects of the game are common knowledge, except Player 2's time preference. In particular, Player 2's time preference can be either $\succcurlyeq_w$("weak") or $\succcurlyeq_s$("strong"), which is her private information. Bargaining with a strong opponent is less favorable for Player 1 than bargaining with a weak opponent. If Player 2 has preference $\succcurlyeq_w$ ($\succcurlyeq_s$), she is called Player 2_w (2_s). The prior distribution of Player 2's preference is

$$Pr(\succcurlyeq_2 = \succcurlyeq_w) = \pi, \; 0 < \pi < 1.$$

[3]In addition, I will discuss the fixed bargaining costs preference, i.e., that represented by the utility function $x - ct$.

The following assumptions are about the comparisons between 2_w and 2_s. The first assumption is that 2_s is more patient than 2_w.

Assumption (C-1): If $x \neq 1$ and $(y, 1) \sim_w (x, 0)$, then $(y, 1) \succ_s (x, 0)$.

In the case of fixed discounting factors δ_w and δ_s, this means that $\delta_w < \delta_s$.

Rubinstein (1982) provides a full characterization of the subgame perfect equilibrium of the game under complete information, given the above assumptions about preferences. Let $(V_i, \hat{V}_i)$ be respectively the complete-information equilibrium divisions when player 1 starts the bargaining and when player 2_i starts the bargaining, where $i = w, s$. Since 2_w is more impatient than 2_s, $V_w > V_s$ and $\hat{V}_w > \hat{V}_s$. When the players have fixed discounting factors δ, δ_w, and δ_s and the same linear utility function,

$$(V_i, \hat{V}_i) = (\frac{1-\delta_i}{1-\delta\delta_i}, \delta\frac{1-\delta_i}{1-\delta\delta_i}), \qquad i = w, s.$$

The last assumption about the time preferences is the following:

Assumption (C-2): $(V_s, 1) \succ_w (\hat{V}_w, 0)$.

This assumption states that Player 2_w prefers the complete information partition between players 1 and 2_s, even if 1 starts the bargaining and there is one period of delay. This rules out the possibility that in equilibrium 2_w sorts herself by making an offer z satisfying $(z, 0) \succcurlyeq_1 (V_w, 1)$ (and thus $(z, 0) \preccurlyeq_w (\hat{V}_w, 0)$) and $(z, 0) \succcurlyeq_w (V_s, 1)$.

Now, I introduce a new component to the bargaining game above. At the beginning of the bargaining game, Player 1 has the option of drawing an exogenously given signal about his opponent's type (the phrases "information collection" and "signal drawing" are interchangeable in this paper). He can get only one signal each period. He can draw as many signals as he wants by not making serious offers in the subsequent periods. After Player 1 starts the bargaining process, he also has the option of halting the bargaining process and drawing additional signals. If he makes offers, however, he does not get to draw the signal. There is no way for Player 2 to start or restart the bargaining process unless Player 1 chooses to.[4] Furthermore, once Player 1 draws a signal, it becomes common knowledge between the two players. This avoids the complication that arises when one has to consider Player 2's beliefs about what signal(s) Player 1 has received. Player 1 updates his beliefs about Player 2's type according to the Bayes' Rule, using the signals he has drawn and possibly the history of

[4]This assumption is made to simplify discussion and in fact not needed. See Section 5.

Table 1: Description of the signal

Signal	2_w	2_s
W	$1-\alpha$	α
S	α	$1-\alpha$

past offers and acceptance behavior of Player 2. Table 1 describes the signal available to Player 1.

The numbers in the table stand for the probability of getting a certain signal given Player 2's true type. I assume $0 < \alpha < 1/2$, thus a smaller α means a more accurate signal. Player 1 does not incur other costs besides the loss of utility from waiting.

With this component added, the representation of histories, strategies and beliefs have to be adjusted accordingly from the Rubinstein (1985) model. A *history* of the game is defined as all the past proposals and responses, plus all the signals Player 1 has drawn. Note that there is no definite relation between the period and the identity of the proposer, except at Period 0. This makes it necessary to specify the identities of the proposer and the responder. Let h^t denote the history at time t *before* a proposal is made or a signal is drawn, and $\tilde{h}^t$ denote the history at time t *after* a proposal is made or a signal is drawn. Now if I use ϕ to denote the null action, then $h^t = (x^0, s^0, A^0, x^1, s^1, A^1, ..., x^{t-1}, s^{t-1}, A^{t-1})$, and $\tilde{h}^t = (x^0, s^0, A^0, x^1, s^1, A^1, ..., x^{t-1}, s^{t-1}, A^{t-1}, x^t, s^t)$, where $x^t = (x_1^t, x_2^t) \in X \times \phi \cup \phi \times X$ is a two-dimensional vector of proposed divisions by both players at period t, $s^t \in \{W, S, \phi\}$ is the signal drawn by Player 1 at period t, and $A^t = (A_1^t, A_2^t) \in Y, N$ is a two-dimensional vector of actions consisting of rejection by one player and the null action by the other. Note that among x_1^t, s^t, and A_1^t, one and only one of them is not equal to ϕ. One and only one of x_2^t and A_2^t is not equal to ϕ, except when $s^t \neq \phi$, in which case both of them are equal to ϕ. Let H^t and $\tilde{H}^t$ be respectively the set of all possible histories at period t before and after a proposal is made or a signal is drawn, H be the union of all H^t and $\tilde{H}^t$, and h be a generic element of H. A *strategy* of a player specifies an action for all possible histories after which the player has to move. Denote the action of signal drawing as D. Player 1's strategy is represented as a sequence of functions $\sigma_1^t : H^t \to X \cup \{D\}$ if $x_2^t = \phi$ (or equivalently, $t = 0$, $s^{t-1} \neq \phi$, or $A_1^{t-1} = N$), and $\sigma_1^t : \tilde{H}^t \to \{Y, N\}$ if $x_2^t \neq \phi$. Similarly, the strategy of player $2_i, (i = w, s)$ is represented as a sequence of functions $\sigma_i^t : H^t \to X$ if $A_2^{t-1} = N$, and

$\sigma_i^t : \tilde{H}^t \to \{Y, N\}$ if $x_1^t \neq \phi$. In the other cases, the null action is taken. I denote the entire strategies of Players 1, 2_w, and 2_s respectively by σ_1, σ_w, and σ_s. A *system of beliefs* of Player 1 in $\Gamma(\pi)$ assigns a probability to the event that Player 2 is 2_w, after every possible history. It is represented by a function $\mu : H \to [0,1]$, and $\mu(h^0) = \pi$.

A sequential equilibrium of bargaining games requires that μ be Bayesian, and Players 1, 2_w, and 2_s act optimally given their beliefs and the other player's strategies. Furthermore, it requires that the *NDOC (Never Dissuaded Once Convinced)* property be satisfied, or, once $\mu(h) = 1$ or 0 for some history h, it remains so for all subsequent histories.

The equilibrium concept I use is the so-called *bargaining (rationalizing) sequential equilibrium* (referred to as "equilibrium" hereafter), which consists a triple of strategies of σ_1, σ_w, and σ_s, and a system of beliefs of Player 1, μ, such that the conditions for sequential equilibrium and the assumptions (B-1) through (B-4) below are satisfied.

The assumptions (B-1) through (B-4) below are adopted to significantly reduce the multiplicity of sequential equilibrium in incomplete-information bargaining games. They are the same as those made by Rubinstein (1985) (as by Osborne and Rubinstein (1990)), with one minor exception in (B-2).

The first assumption says that if Player 2 rejects an offer by Player 1 and counteroffers in such a way that if Player 1 accepts the offer 2_w is worse off than 1's original offer, yet 2_s is better off, then Player 1 puts probability one on the event that Player 2 is 2_s.

Assumption (B-1): If $\mu(h^{t-1}) \neq 1$, $(x^t, 1) \succ_s (x^{t-1}, 0)$, and $(x^{t-1}, 0) \succ_w (x^t, 1)$, then $\mu(\tilde{h}^t) = 0$.

The second assumption says that if Player 2 rejects an offer by Player 1 and counteroffers in such a way that both 2_w and 2_s would be better off if Player 1 accepts the offer, then Player 1 cannot *change* his subjective probability that he is playing against 2_w.[5]

Assumption (B-2): If $(x^t, 1) \succ_s (x^{t-1}, 0)$ and $(x^t, 1) \succ_w (x^{t-1}, 0)$, then $\mu(\tilde{h}^t) = \mu(h^{t-1})$.

[5]This is a strengthened version of the assumption in the original paper. The original assumption is that the subjective probability cannot *increase*.

The third assumption is a "tie-breaking" assumption for Player 1.

Assumption (B-3): Player 1 always accepts an offer x if after rejecting he expects to reach an agreement whereby he is indifferent to x.

The fourth assumption limits Player 2's range of offers, that is, 2_w and 2_s cannot use offers that are rejected by Player 1 with certainty to sort themselves.

Assumption (B-4): When it is Player 2's turn to make an offer, $\sigma_w(h^t) \geq \hat{V}_s$ and $\sigma_s(h^t) \geq \hat{V}_s$.

Now, I discuss the equilibrium of the bargaining game without information collection. Define the pair (x^π, y^π) as

$$\begin{aligned}(x^\pi, 0) &\sim_w (y^\pi, 1), \\ (y^\pi, 0) &\sim_1 \pi(x^\pi, 1) \oplus (1-\pi)(z(x^\pi), 2),\end{aligned}$$

where $z(x)$ is defined as the z that satisfies $(x, 0) \sim_w (z, 1)$. Note that from the above, $y^\pi = z(x^\pi)$.

Rubinstein (1985) proves the following result.

Theorem 0. *(Rubinstein (1985)) For a game starting with Player 1's offer: (i) If π is high enough such that $y^\pi > \hat{V}_s$, then the only bargaining sequential equilibrium outcome is $\langle (x^\pi, 0), (y^\pi, 1) \rangle$. (ii) If π is low enough such that $y^\pi < \hat{V}_s$, then the only bargaining sequential equilibrium outcome is $\langle (V_s, 0), (V_s, 0) \rangle$.*

The theorem states that for a general class of preferences, imposing certain plausible assumptions on Player 1's beliefs, the game never lasts beyond the second period in the equilibrium.[6] Rubinstein also notices "it is not clear whether Player 1 would benefit from having the information about Player 2's type." This paper intends to explore what happens if Player 1 has the option to collect information about Player 2's type. In particular, I would like to study whether and how information collection causes delays.

One important property of the equilibrium outcome is that the pair(x^π, y^π) are both strictly increasing in π. This is intuitive since the more likely Player 2 is 2_w, the

[6]Osborne and Rubinstein (1990) discuss the case of fixed bargaining costs in details in their Chapter 5. The theorem is stated as Proposition 5.7 in the discussion. The equilibrium outcome of the fixed bargaining costs preference is slightly different from that of the preferences above. This paper also explores this special case in subsequent sections.

more favorable the situation is for Player 1. By (A-7), Player 1 maximizes expected value of $u(x)\delta^t$. The payoff of Player 1 can be represented by the following function

$$v_0(\pi) = \begin{cases} u(V_s), & \pi < \pi^*; \\ u(y^\pi)/\delta, & \pi > \pi^*. \end{cases}$$

where π^* is defined as the boundary point at which $y^\pi = \hat{V}_s$, and hence $u(V_s) = u(y^\pi)/\delta$. Thus, v_0 is increasing, and strictly increasing if $\pi > \pi^*$. Note also that $v_0(1) = u(V_w)$. Since the utility functions of the players are concave, which are necessarily continuous, v_0 is also continuous.

In the analysis that follows, I will not distinguish between the prior probability of Player 2 being 2_w and Player 1's belief about Player 2 being 2_w, and will simply call both of them π. Since Player 1's beliefs will be common knowledge throughout the game, it is valid to treat them equally. Every information set where Player 1's belief about Player 2 being 2_w is the same can be treated essentially the same. I will denote them by $\Gamma(\pi)$.

3 Characterization of Equilibrium

First, I establish a lemma that characterizes Player 1's information collection behavior.

Lemma 1. *In the equilibrium, once Player 1 starts the bargaining process, he never halts it to draw additional signals. Formally, if $x_1^T \neq \phi$ for some T, then $s^t = \phi$ for all $t \geq T$. Furthermore, the equilibrium bargaining outcome must be as described in Theorem 0, with π equal to Player 1's updated belief about Player 2 being 2_w when he starts the bargaining process.*

Proof. Suppose Player 1 starts the bargaining process by offering x. There could be three types of possible responses from 2_w and 2_s: i) both reject it; ii) both accept it; iii) 2_s rejects it, and 2_w accepts it (the case in which 2_s accepts while 2_w rejects the offer is impossible, since 2_s is more patient than 2_w, and if 2_w prefers to wait, then 2_s must also prefer to wait). In case ii), the bargaining game is over. In case iii), Player 1 should be able to infer Player 2's type, then no additional signals are to be drawn. In case i), Rubinstein (1985) shows in his Proposition 4 that if 2_w and 2_s both reject the offer, then they must make the same counteroffer.[7] Player 1 should either reject

[7] If the two types make different offers, then no more information collection is needed, the subgame is the same as that in which there is no information collection. So, I may apply Rubinstein's argument.

or accept this counteroffer. If he accepts it, then again the bargaining game is over. If he rejects it, then the game is back to the same situation as it was when he started the bargaining process. In particular, Player 1's subjective probability that Player 2 is 2_w should not change. If Player 1 finds it profitable to draw additional signals now, he should not have started making offers two periods before. This completes the proof of the first part.

Now that Player 1 will not draw more signals once he starts the bargaining, the game is equivalent to the original bargaining game without information collection. So its rationalizing sequential equilibrium must be as specified in Theorem 0. □

This lemma states that if Player 1 ever collects information about Player 2's type, he always does so before he starts to make offers. It lays the foundation for the rest of the paper. Player 1 obtains as many signals as is beneficial to him, and then starts to make offers. Furthermore, once he starts to make offers, the bargaining outcome is described by Theorem 0. In particular, Player 1's payoff must be consistent with what Theorem 0 requires. Thus, in characterizing the equilibrium of this model, I can focus on the decision problem of Player 1 at the beginning of the bargaining game.

Note that there are only two types of player 2 in this model. Since I focus on pure strategy equilibria, whenever the two types make different offers or acceptance/rejection decisions, their types are perfectly revealed. If they both accept the offer, then the game is over. If they both reject it, then they will make the same counteroffer. If the uninformed player accepts the offer, the game is over. If he rejects the offer, then the game reverses to the state when he initially starts to make offers. He would not have started to make offers if he found it optimal to collect information now. As a result, no additional information collection is needed once the uninformed player starts to make offers.[8]

Now I define two functions p^W and p^S, both mapping from $[0,1] \times (0,1/2)$ to $[0,1]$, as follows.

$$p^W(\pi,\alpha) = \pi(1-\alpha)/(\pi(1-\alpha)+(1-\pi)\alpha)$$
$$p^S(\pi,\alpha) = \pi\alpha/(\pi\alpha+(1-\pi)(1-\alpha))$$

[8] Suppose instead there are three types of informed players: weak, strong, and extra strong, where the weak type is extremely impatient while the other two types and the uninformed player are quite patient. In addition, let the weak type happen with a very high probability. Thus, it may be the case that the uninformed player uses an extremely low offer to screen out the weak type and then start collecting information about whether the informed player is strong or extra strong. To provide a complete analysis of the game with more than two types, one needs to solve a multi-type extension of the Rubinstein (1985) model, which is beyond the scope of the paper. However, I believe the intuition provided by the analysis below would carry through to that model.

The function p^W gives Player 1's updated belief about the event that Player 2 is 2_w after seeing the signal "W," given his initial belief π, and p^S is the counterpart when Player 1 sees the signal "S." Some properties of these two functions are in order.

Lemma 2. *The functions p^W and p^S satisfy the following properties.*

1. *$p^W(\pi,\alpha) \geq \alpha$, and $p^S(\pi,\alpha) \leq \alpha$, with strict inequalities when $\pi \neq 0$ or 1. Furthermore, π is a convex combination of $p^W(\pi,\alpha)$ and $p^S(\pi,\alpha)$.*[9]

$$\pi = [\pi(1-\alpha) + (1-\pi)\alpha]p^W(\pi,\alpha) + [\pi\alpha + (1-\pi)(1-\alpha)]p^S(\pi,\alpha).$$

2. *Both p^W and p^S are continuous and strictly increasing in π. The function p^W is decreasing in α, and strictly decreasing in α except when $\pi = 1$ or 0; and p^S is increasing in α, and strictly increasing in α except when $\pi = 1$ or 0.*

3. *Given α, p^W and p^S are mutually inverse functions. That is, $p^W(p^S(\pi)) = p^S(p^W(\pi)) = \pi$.*[10]

4. *Given α, define functions $(p^W)^k(\pi)$ and $(p^S)^k(\pi)$ where $(p^W)^k(\pi) = \underbrace{p^W \circ p^W \circ ... \circ p^W}_{k}(\pi)$ and similarly for $(p^S)^k(\pi)$. Then*

   (a) $(p^W)^k(\pi) = \pi(1-\alpha)^k/(\pi(1-\alpha)^k + (1-\pi)\alpha^k)$,
   $(p^S)^k(\pi) = \pi\alpha^k/(\pi\alpha^k + (1-\pi)(1-\alpha)^k)$;

   (b) $$(p^W)^{k_1}(p^S)^{k_2}(\pi) = \begin{cases} (p^W)^{k_1-k_2}(\pi), & k_1 > k_2 \\ \pi, & k_1 = k_2 \\ (p^S)^{k_2-k_1}(\pi), & k_1 < k_2 \end{cases}.$$

Proof. Most of the statements are straightforward from the definition. I only provide the proof of Part 3.

$$p^W(p^S(\pi)) = \frac{\frac{\pi\alpha}{\pi\alpha+(1-\pi)(1-\alpha)}(1-\alpha)}{\frac{\pi\alpha}{\pi\alpha+(1-\pi)(1-\alpha)}(1-\alpha) + \frac{(1-\pi)(1-\alpha)}{\pi\alpha+(1-\pi)(1-\alpha)}\alpha} = \pi.$$

Similarly for $p^S(p^W(\pi))$. □

[9] By convex combination, I mean the linear (nonnegative) coefficients on the terms sum up to 1. The whole expression does not have to be linear. This is used in a number of occasions in the rest of the paper.

[10] Here, I suppress the dependence of p^W and p^S on α. I will do so in the rest of the paper, where there is no confusion.

These properties are very useful in the analysis. For example, Part 3 implies that if it takes one "W" signal to change belief π to π', then it takes one "S" signal to change it back. By this property and Property 4, I can construct intervals with bounds $\{\pi^k\}_{k=0}^N$, where $\pi^k = (p^S)^k(\pi^0) = (p^W)^{N-k}(\pi^N)$.

Player 1's decision can be described by the following Bellman's equation:

$$v(\pi) = \max\{v_0(\pi),\ \delta(\pi(1-\alpha) + (1-\pi)\alpha)v(p^W(\pi)) + \delta(\pi\alpha + (1-\pi)(1-\alpha))v(p^S(\pi))\} \tag{1}$$

The first argument in the maximum function is Player 1's payoff from starting to offer, and the second argument is his payoff from waiting one period and obtaining a signal. Let us define the second argument to be $\tilde{w}$, a function of π, δ and α. For any given value of π, if the first argument is bigger than or equal to the second one, then Player 1 starts to offer, and plays according to the rationalizing sequential equilibrium specified in Theorem 0. If, on the contrary, the second term is bigger, then Player 1 draws another signal. Then, according to the signal he gets, he updates his beliefs, and makes the decision again according to the Bellman's equation, and so on.

Before proceeding to the analysis of the equilibrium, I normalize Player 1's utility function such that

$$u(0) = 0, \quad u(1) = 1.$$

I can do this normalization because players maximize von Neumann-Morgenstern expected utility, which is unique up to monotonic affine transformation. This normalization is used in all subsequent analysis. First, I establish a lemma that describes Player 1's signal drawing behavior in the equilibrium.

Lemma 3. *In equilibrium, the following is true about Player 1's signal drawing decisions: there exist π_L and π_H in $(0,1)$, such that if $\pi < \pi_L$ or $\pi > \pi_H$, then Player 1 does not draw any signals.*

Proof. I prove the statement by contradiction.

Suppose there is no π_L satisfying the required property. Then there must exist a strictly decreasing nonzero sequence $\{\pi_k\}$ that converges to 0, and that satisfies

$$v(\pi) = \delta(\pi_k(1-\alpha) + (1-\pi_k)\alpha)v(p^W(\pi_k)) + \delta(\pi_k\alpha + (1-\pi_k)(1-\alpha))v(p^S(\pi_k)).$$

Define $M = \left[\frac{\log u(V_s)}{\log \delta}\right] + 1$. Observe that $(p^w)^M$ is continuous, strictly increasing and $(p^w)^M(0) = 0$ by Parts 2 and 4 of Lemma 2. Thus, I can find K such that for all $k > K$, $(p^w)^M(\pi_k) < \pi^*$. Note π^* is the threshold value of π in the definition of the function v_0. Choose one such k. Note that the right hand side of the equation above is a convex combination of $\delta v(p^W(\pi_k))$ and $\delta v(p^S(\pi_k))$. so $\delta v(p^W(\pi_k)) > u(V_s)$ or $\delta v(p^S(\pi_k)) > u(V_s)$. Since $(p^W)^M(\pi_k) < \pi^*$, $p^S(\pi_k) < p^W(\pi_k) < \pi^*$ by Part 1 of Lemma 2. Then Player 1 must get this payoff by signal drawing since by Theorem 0 and Lemma 1, Player 1 can get only $u(V_s)$ without signal drawing. Without loss, suppose $\delta v(P^W(\pi_k)) > u(V_s)$. By the same argument, I have either $\delta^2 v((p^W)^2(\pi_k)) > u(V_s)$ or $\delta^2 v(p^S(p^W(\pi_k))) > u(V_s)$. Proceed by induction for M steps, and I will eventually be able to find m, such that $|m| \leq M$, and $\delta^M v((p^W)^m(\pi_k)) > u(V_s)$, hence, $v((p^W)^m(\pi_k)) > 1$ by the definition of M, which is impossible. So there exists π_L satisfying the desired property.

The argument for the existence of π_H is slightly different. The observation is that Player 1 can never get a payoff higher than $u(V_w)$ by Lemma 1 and Theorem 0. By the continuity of v_0, if π is sufficiently close to 1, then $v_0(\pi) > \delta u(V_w)$. Hence it is impossible that Player 1 draws signals at this π. □

The intuition for Lemma 3 is that if Player 1 is quite sure about Player 2's type, then he does not want to collect information. However, note that it does not show that Player 1's signal drawing region is an interval in the middle. It does not rule out the possibility that two signal drawing regions surround one region without signal drawing. It is desirable to have that when Player 1 is *not* sure about Player 2's type, he does collect information. In the next section, I identify special cases in which this is true.

Now, I show the existence and uniqueness of the equilibrium.

Theorem 1. *There exists a unique function v solving Player 1's Bellman's equation (1). Therefore, the bargaining sequential equilibrium outcome of the bargaining game exists, and is unique.*

Proof. The proof is an application of the contraction mapping theorem.[11] The proof of Theorem 1 in the case where players have fixed bargaining costs can be found in the appendix.

Consider the space of functions on $[0, 1]$ bounded between 0 and 1. Denote it by V. Define the sup norm $\| \|$ on the space. Denote by $T : V \to V$ the function-valued mapping defined by (1), i.e., for $w \in V$,

$$Tw(\pi) = \max\{v_0(\pi),\ \delta(\pi(1-\alpha) + (1-\pi)\alpha)w(p^W(\pi))+ \\ \delta(\pi\alpha + (1-\pi)(1-\alpha))w(p^S(\pi))\}$$

[11]Please refer to Aubin (1993) for a statement of the contraction mapping theorem.

If the mapping T has a unique fixed point in the space V, then the solution to (1) exists and is unique.

Take any two functions $w_1, w_2 \in V$. Let $d = \|w_1 - w_2\|$. Consider $\|Tw_1 - Tw_2\|$. For any $\pi \in [0,1]$, there are three cases: (i) $Tw_1(\pi) = Tw_2(\pi) = v_0(\pi)$; (ii) $Tw_1(\pi) \neq v_0(\pi), Tw_2(\pi) \neq v_0(\pi)$; (iii) $Tw_i(\pi) = v_0(\pi), Tw_j(\pi) \neq v_0(\pi), i \neq j$.

In case (i), $|Tw_1(\pi) - Tw_2(\pi)| = 0 < \delta d$.

In case (ii),

$$|Tw_1(\pi) - Tw_1(\pi)| = \left| \begin{array}{l} \delta(\pi(1-\alpha) + (1-\pi)\alpha)[w_1(p^W(\pi)) - w_2(p^W(\pi))] \\ +\delta(\pi\alpha + (1-\pi)(1-\alpha))[w_1(p^S(\pi)) - w_2(p^S(\pi))] \end{array} \right|$$

By the triangle inequality, I obtain

$$\begin{aligned} |Tw_1(\pi) - Tw_2(\pi)| \leq\; & \delta(\pi(1-\alpha) + (1-\pi)\alpha) \left| w_1(p^W(\pi)) - w_2(p^W(\pi)) \right| \\ & +\delta(\pi\alpha + (1-\pi)(1-\alpha)) \left| w_1(p^S(\pi)) - w_2(p^S(\pi)) \right| \\ \leq\; & \delta d \end{aligned}$$

The last inequality sign comes from the definition of the sup norm.

In case (iii), without loss of generality assume $Tw_1(\pi) = v_0(\pi),\ Tw_2(\pi) \neq v_0(\pi)$, then

$$\delta(\pi(1-\alpha) + (1-\pi)\alpha) w_1(p^W(\pi)) + \delta(\pi\alpha + (1-\pi)(1-\alpha)) w_1(p^S(\pi)) \leq v_0(\pi)$$

and

$$\delta(\pi(1-\alpha) + (1-\pi)\alpha) w_2(p^W(\pi)) + \delta(\pi\alpha + (1-\pi)(1-\alpha)) w_2(p^S(\pi)) > v_0(\pi)$$

Therefore

$$\begin{aligned} |Tw_1(\pi) - Tw_2(\pi)| &= |v_0(\pi) - Tw_2(\pi)| \\ &\leq \delta(\pi(1-\alpha) + (1-\pi)\alpha) \left| w_1(p^W(\pi)) - w_2(p^W(\pi)) \right| \\ &\quad +\delta(\pi\alpha + (1-\pi)(1-\alpha)) \left| w_1(p^S(\pi)) - w_2(p^S(\pi)) \right| \\ &\leq \delta d \end{aligned}$$

I have shown in all three cases $|Tw_1(\pi) - Tw_2(\pi)| \leq \delta d$, so $\|Tw_1 - Tw_2\| \leq \delta d$. Since $0 < \delta < 1$, T is a contraction. Applying the contraction mapping theorem, I conclude that T must have a unique fixed point in V.

Now that the function v exists and is unique, Player 1's equilibrium decision rule must be unique. Whether Player 1 draws signals or not at a certain value of π is fully revealed by the comparison between v and v_0 at this value. If the former is larger, then Player 1 draws signals; while if they are equal, then Player 1 does not. In the equilibrium, starting from the initial prior probability π_0, Player 1 keeps drawing signals and doing Bayesian updating, until he obtains a subjective probability π_e, at which no signal is drawn. Then he starts the bargaining process, arriving at an equilibrium outcome according to Theorem 0, with $\pi = \pi_e$. □

Theorem 1 shows that for the general preferences discussed above, there exists a unique rationalizing sequential equilibrium of pure strategies. The function v fully describes the equilibrium, as shown in the proof. Thus, I will frequently use v in place of the equilibrium. An important implication of Theorem 1 is that if the mapping T is applied to any function belonging to the space V repeatedly, then in the limit the unique solution v is reached. I can then use the limit argument to prove properties of v. This kind of reasoning will be used repeatedly in the subsequent analysis. Corollary 1 summarizes properties of the function v.

Corollary 1.1 The function v is continuous and increasing.

Proof. In the proof, had I defined the space V to be all *continuous* functions bounded between 0 and 1, the contraction mapping theorem would still apply. Since limits of uniformly convergent sequences of continuous functions are also continuous functions, $(V, \| \ \|)$ is a complete metric space. Hence the solution obtained is the same under these two definitions of V. So v is continuous.

I prove monotonicity from the fact that the solution is unique. For any function $w \in V$, the sequence $\{T^n w\}$ converges to the solution v. In particular, we can choose w to be increasing. The first argument of the maximum function, v_0, is increasing in π. For the second argument of the maximum function, $\tilde{w}$, differentiating with respect to π,[12] I get

$$\begin{aligned}\tilde{w}'(\pi) = {} & (1-2\alpha)[w(p^W(\pi)) - w(p^S(\pi))] \\ & +[\pi(1-\alpha) + (1-\pi)\alpha]w'(p^W(\pi))(p^W)'(\pi) \\ & +[\pi\alpha + (1-\pi)(1-\alpha)]w'(p^S(\pi))(p^S)'(\pi)\end{aligned} .$$

By Lemma 2, $p^W(\pi) \geq p^S(\pi)$, and both p^W and p^S are increasing in π. Since w is increasing and $0 < \alpha < 1/2$, all three terms in the expression of $\tilde{w}'$ are nonnegative, which shows $\tilde{w}$ is increasing in π. So Tw is increasing. Therefore for all n, $T^n w$ is increasing in π. Since v is the limit of the sequence $\{T^n w\}$, it is also increasing. □

The function v is Player 1's expected equilibrium payoff. The fact that v is continuous comes from the continuity of preferences and the continuity of the equi-

[12]The argument here assumes differentiability. This is to simplify the analysis. The statement would still be true without the differentiability assumption.

librium outcome without information collection. Monotonicity of v is again intuitive, since Player 1 is in a more favorable position if he faces a weak opponent with higher probability.

4 Special Cases

In this section, I look at some special forms of preferences, and prove properties of the equilibrium under these preferences.

4.1 Fixed Discounting Rate

In this case, players maximize the expectation of discounted value of the share they get of the pie, as opposed to some strictly concave function of the share. Player 1's preference can be described by maximization of the expectation of the function $\delta^t x$. Similarly 2_i $(i = w, s)$ maximizes $\delta_i^t(1-x)$. In this case,

$$V_s = \frac{1-\delta_s}{1-\delta\delta_s}, \quad V_w = \frac{1-\delta_w}{1-\delta\delta_w}, \quad \hat{V}_s = \delta V_s, \quad \hat{V}_w = \delta V_w;$$

$$x^\pi = \frac{(1-\delta_w)(1-\delta^2(1-\pi))}{1-\delta^2+\delta(\delta-\delta_w)\pi}, \quad y^\pi = \frac{\delta(1-\delta_w)\pi}{1-\delta^2+\delta(\delta-\delta_w)\pi};$$

$$\pi^* = \frac{(1-\delta_s)(1+\delta)}{(1-\delta_s)(1+\delta)+(\delta_s-\delta_w)};$$

$$v_0(\pi) = \begin{cases} V_s = \frac{1-\delta_s}{1-\delta\delta_s}, & \pi \le \pi^*, \\ \frac{y^\pi}{\delta} = \frac{(1-\delta_w)\pi}{1-\delta^2+\delta(\delta-\delta_w)\pi} & \pi > \pi^* \end{cases}.$$

In addition, I assume $\delta = \delta_w$. In this case I obtain a simple form of Player 1's payoff function, that is

$$v_0(\pi) = \begin{cases} V_s = \frac{1-\delta_s}{1-\delta\delta_s}, & \pi \le \pi^*, \\ \frac{y^\pi}{\delta} = \frac{\pi}{1+\delta} & \pi > \pi^*. \end{cases} \tag{2}$$

where $\pi^* = \frac{(1-\delta_s)(1+\delta)}{(1-\delta_s)(1+\delta)+(\delta_s-\delta)}$. Note that π^* is increasing in δ. The function consists of two linear sections that intersect each other at π^*. The following lemma proves some general facts about applying the mapping T defined in the proof of Theorem 1 onto functions consisting of two linear sections.

Lemma 4. *Define two linear functions w_1 and w_2 on $[0,1]$,*

$$w_1(\pi) = a_1\pi + b_1, \quad w_2(\pi) = a_2\pi + b_2,$$

where $0 < a_1 < a_2$, $b_2 < b_1$, *and* $a_1 + b_1 < a_2 + b_2 < 1$. *They intersect each other at* $\pi^* = \frac{b_1 - b_2}{a_2 - a_1}$ *(less than 1 since* $a_1 + b_1 < a_2 + b_2$*). Now define function* w *as*

$$w(\pi) = \begin{cases} w_1(\pi) & \pi \leq \pi^*, \\ w_2(\pi) & \pi > \pi^*. \end{cases} \tag{3}$$

Let v_0 *in equation 1 be* w. *The following statements are true:*

1. *There exists* $\pi \in [0, 1]$ *such that* $Tw(\pi) > w(\pi)$, *if and only if the following conditions hold:*

$$\alpha < \frac{1}{2} - \frac{(1 - \delta)(b_1 a_2 - a_1 b_2)}{2\delta[(a_2 + b_2) - (a_1 + b_1)](b_1 - b_2)}, \tag{4}$$

$$\delta[(a_2 + b_2) - (a_1 + b_1)](b_1 - b_2) > (1 - \delta)(b_1 a_2 - a_1 b_2). \tag{5}$$

2. *The difference between* Tw *and* w, $Tw - w$, *is increasing on* $[p^S(\pi^*), \pi^*]$, *and decreasing on* $[\pi^*, p^W(\pi^*)]$, *and attains its maximum at* π^*.

3. *The region in which* $Tw - w > 0$ *is an open interval* (π_1^1, π_2^1) *lying in* $[p^S(\pi^*), p^W(\pi^*)]$.

Proof. In this case, function $\tilde{w}$ can be written as

$$\begin{aligned} \tilde{w}(\pi) &= \delta[\pi(1 - \alpha) + (1 - \pi)\alpha]w(p^W(\pi)) + \delta[\pi\alpha + (1 - \pi)(1 - \alpha)]w(p^S(\pi)) \\ &= \begin{cases} \delta(a_1\pi + b_1), & \pi \leq p^S(\pi^*), \\ \delta\{[\alpha a_1 + (1 - \alpha)a_2 - (1 - 2\alpha)(b_1 - b_2)]\pi + [(1 - \alpha)b_1 + \alpha b_2]\}, & \\ & p^S(\pi^*) \leq \pi \leq p^W(\pi^*), \\ \delta(a_2\pi + b_2), & \pi \geq p^W(\pi^*). \end{cases} \end{aligned}$$

It has three linear sections intersecting their neighbors at $\pi = p^S(\pi^*)$ and $\pi = p^W(\pi^*)$, and $d = \max\{0, \tilde{w} - w\}$.

Let us first prove Parts 2 and 3. First suppose that there exists $\pi = \pi_0$, such that $Tw(\pi) > w(\pi)$. Observe that $\tilde{w} - w$ is negative for $\pi \leq p^S(\pi^*)$ or $\pi \geq p^W(\pi^*)$, since $\delta < 1$ and w is piecewise linear. So $d = 0$ for these cases. Therefore π_0 must lie in $[p^S(\pi^*), \pi^*]$ or $[\pi^*, p^W(\pi^*)]$. Now I argue that $\tilde{w} - w$ must be increasing on $[p^S(\pi^*), \pi^*]$ and decreasing on $[\pi^*, p^W(\pi^*)]$. Since $\tilde{w} - w$ is linear on both intervals, it must be monotonic (including the case that it is constant) on these two intervals. I prove the claim by showing that the other cases are impossible. If $\tilde{w} - w$ is decreasing on $[p^S(\pi^*), \pi^*]$, then it is always negative on this interval. In particular, it is negative at π^*. But since $\tilde{w} - w$ is monotonic on $[\pi^*, p^W(\pi^*)]$, and negative at $\pi = p^W(\pi^*)$, it is always negative on $[\pi^*, p^W(\pi^*)]$. This contradicts the existence of π_0. So $\tilde{w} - w$ is increasing on $[p^S(\pi^*), \pi^*]$. Similarly, it is decreasing on $[\pi^*, p^W(\pi^*)]$. Thus it attains its maximum at π^*. Furthermore, there must exist $\pi_1^1 \in (p^S(\pi^*), \pi^*)$ and $\pi_2^1 \in (\pi^*, p^W(\pi^*))$ at which $\tilde{w} - w$ is zero. The expressions for the two values are

$$\begin{aligned} \pi_1^1 &= \frac{(1-\delta)b_1 + \delta\alpha(b_1 - b_2)}{\delta(1-\alpha)(a_2 - a_1) - \delta(1-2\alpha)(b_1 - b_2) - (1-\delta)a_1}, \\ \pi_2^1 &= \frac{\delta(1-\alpha)(b_1 - b_2) - (1-\delta)b_2}{(1-\delta)a_2 + \delta\alpha(a_2 - a_1) + \delta(1-2\alpha)(b_1 - b_2)}. \end{aligned}$$

In the open interval between these two values $\tilde{w} - w$ is positive. Outside this interval, $\tilde{w} - w$ is negative. Since $d = \max\{0, \tilde{w} - w\}$, the function d must also be increasing on $[p^S(\pi^*), \pi^*]$, decreasing on $[\pi^*, p^W(\pi^*)]$, and attains its maximum at π^*. Furthermore, it is positive only on the open interval (π_1^1, π_2^1). This completes the proof of (ii) and (iii).

Now I prove Part 1. First I show the conditions are necessary. From the above proof, I know that if there exists $\pi \in [0, 1]$, such that $Tw(\pi) > w(\pi)$, then $d = Tw - w$ is maximized at π^*. Then $Tw(\pi^*) - w(\pi^*) > 0$ must hold, which implies

$$\delta(1-2\alpha)[(a_2 + b_2) - (a_1 + b_1)](b_1 - b_2) - (1-\delta)(b_1 a_2 - a_1 b_2) > 0.$$

Thus

$$\alpha < \frac{1}{2} - \frac{(1-\delta)(b_1 a_2 - a_1 b_2)}{2\delta[(a_2 + b_2) - (a_1 + b_1)](b_1 - b_2)},$$

but the right hand side of the above inequality to be positive, which implies that the following inequality must hold

$$\delta[(a_2 + b_2) - (a_1 + b_1)](b_1 - b_2) > (1-\delta)(b_1 a_2 - a_1 b_2).$$

The above two inequalities are also sufficient for the existence of $\pi_0 \in [0, 1]$ such that $Tw(\pi_0) > w(\pi_0)$. Just choose $\pi_0 = \pi^*$. □

Lemma 4 says that if the mapping T is applied on a function w with two linear sections with ascending slopes, with the function itself serving as v_0, then the region in which $Tw > w$ must lie in the middle. Furthermore, the "gain" $Tw - w$ is maximized at the intersection of the two linear sections. This fact is important in the characterization of the equilibrium for the class of preferences I consider in this part. Note that although in most cases, the coefficients a_1, b_1, a_2, b_2 should depend on δ, δ_w, and δ_s, the proof is not affected by such dependence. This is useful in the proof of Theorem 2.

Theorem 2. *If players' preferences are such that without information collection, Player 1's payoff* v_0 *takes the form of* w *in (3),*[13] *then the following are true about Player 1's signal drawing decisions:*

1. *Player 1's signal drawing region is nonempty if and only if (4) and (5) hold;*

2. *Player 1's signal drawing region is in the form of* (π_L, π_H)*, where* $0 < \pi_L < \pi_H < 1$*, if his signal drawing region is nonempty.*

Proof. Since $v_0 = w$, by Theorem 1, if $Tw = w$ then the solution to equation (1) is $v = w$, and Player 1 does not draw signals. The converse is also true. By Lemma 4, this implies that Player 1 draws signals if and only if (4) and (5) hold.

If Player 1 does draw signals, then as shown in the proof of Lemma 4, Tw consists of three linear sections. The middle one is equal to $\tilde{w}$, with slope and intercept being

$$\langle a_2^1, b_2^1 \rangle = \langle \delta\alpha a_1 + \delta(1-2\alpha)(b_1 - b_2), \delta(1-\alpha)b_1 + \delta\alpha b_2 \rangle.$$

The inequality $Tw > w$ holds only on the open interval (π_1^1, π_2^1). From the proof of Lemma 4, if I let $a_1^1 = a_1, b_1^1 = b_1, a_3^1 = a_2$, and $b_3^1 = b_2$, then I claim that $\{a_j^1\}_{j=1}^3$ and $\{a_j^1 + b_j^1\}_{j=1}^3$ are increasing sequences, and $\{b_j^1\}_{j=1}^3$ is a decreasing sequence. The sequence $\{a_j^1\}_{j=1}^3$ is increasing because of the monotonicity property of $\tilde{w} - w$. Since for $j = 1, 2$, $a_j^1\pi + b_j^1$ intersects with $a_{j+1}^1\pi + b_{j+1}^1$ on $(0, 1)$ by Lemma 4, it must be that $b_j^1 > b_{j+1}^1$, and $a_j^1\pi + b_j^1 < a_{j+1}^1\pi + b_{j+1}^1$ for all π to the right of the intersection point. In particular, $a_j^1 + b_j^1 < a_{j+1}^1 + b_{j+1}^1$ holds.

Now I prove by induction the claim "for all k, $T^k w > w$ is only true on an open interval$(\pi_1^k, \pi_{m_k-1}^k)$, where $0 < \pi_1^k < \pi_{m_k-1}^k < 1$."

Suppose $T^k w$ consists of m_k linear sections intersecting with neighboring sections, and let these sections be represented by slope and intercept pairs (a_j^k, b_j^k), $j = 1, 2, ..., m_k$. Suppose that $\{a_j^k\}_{j=1}^{m_k}$, $\{a_j^k + b_j^k\}_{j=1}^{m_k}$ are increasing sequences and $\{b_j^k\}_{j=1}^{m_k}$ is a decreasing sequence. Let the boundary points between these sections be π_j^k, $j = 1, 2, ..., m_k - 1$. The sequences and boundary points are ordered from left to right. Finally, $T^k w = w$ for $\pi \leq \pi_1^k$ and $\pi \geq \pi_{m_k-1}^k$. On the open interval $(\pi_1^k, \pi_{m_k-1}^k)$, $Tw > w$ is true.

Now look at $T^{k+1}w$. Applying to the linear sections of $T^k w$ the same reasoning used in the proof of Lemma 4, the result in Lemma 4 is true for any two linear sections. Thus, after the $k+1$-th iteration, $T^{k+1}w$ also consists of linear sections intersecting with neighboring sections, and the sequences $\{a_j^{k+1}\}_{j=1}^{m_{k+1}}$, $\{b_j^{k+1}\}_{j=1}^{m_{k+1}}$,

[13] To be precise, considering the result of Theorem 0, $a_1 = 0$ and $b_1 = u(V_s)$ for all such preferences.

and $\{a_j^{k+1}+b_j^{k+1}\}_{j=1}^{m_{k+1}}$ preserve the properties of the sequences of $T^k w$. Furthermore, if I denote by $\pi_{i,j}^k$ the intersection between $a_i^k\pi+b_i^k$ and $a_j^k\pi+b_j^k$, then $\pi_{m_{k+1}-1}^{k+1} < p^W(\max_j \pi_{j,m_k}^k) = p^W(\pi_{m_k-1,m_k}^k) < 1$, and $\pi_1^{k+1} > p^S(\min_j \pi_{j1}^k) = p^W(\pi_{21}^k) > 0$. $T^{k+1}w > w$ is true only on $(\pi_1^{k+1}, \pi_{m_{k+1}-1}^{k+1})$. The claim is proved.

The final step is to state that it takes finite steps to get to v for any initial w. This is true due to the reasoning used in the proof of Lemma 3. There exists M great enough, such that $\delta^M < w(\pi)$ for all π.

Now I have shown that $v > w$ only on an open interval (π_L, π_H). Hence, Player 1 only draws signals on this interval. □

The above theorem applies to not only the fixed discounting rate preferences with the assumption $\delta = \delta_w$, but also any preference settings in which Player 1's payoff according to Theorem 0 as a function of π consists of two linear sections intersecting each other. In all these cases, Player 1's signal drawing behavior can be fully characterized by two threshold values of π. In the interval between these two values, Player 1 draws signals; outside this interval, he does not. This result means that starting from any belief, Player 1 trades off between earlier agreement and more favorable agreement with more information. Player 1, the uninformed player, collects information until he is sufficiently sure about Player 2's type. His main objective is to take advantage of the weak type as much as possible.

Condition (4) suggests that Player 1 draws signals if and only if the information is accurate enough, other parameters being equal. Condition (5) is hard to interpret as a_1, b_1, a_2, and b_2 usually depend on δ, δ_w, and δ_s (but not α).

For the fixed discounting rate case with the assumption $\delta = \delta_w$, conditions (4) and (5) take the following form:

$$\alpha < \frac{(2\delta-1)(\delta_s-\delta)-(1-\delta_s)(1-\delta^2)}{2\delta(\delta_s-\delta)}, \tag{6}$$

$$0 < (2\delta-1)(\delta_s-\delta)-(1-\delta_s)(1-\delta^2). \tag{7}$$

But these in turn require the following two conditions:

$$\delta_s > \sqrt{3}/2, \tag{8}$$

$$\delta \in \left(\frac{2\delta_s+1-\sqrt{4\delta_s^2-3}}{2(1+\delta_s)}, \frac{2\delta_s+1+\sqrt{4\delta_s^2-3}}{2(1+\delta_s)}\right). \tag{9}$$

This means that δ_s must be great enough and δ must be in a suitable range. That is, it can be neither too close nor too far from δ_s. If δ is too close to δ_s, then there is not too much to gain from knowing whether Player 2 is strong or weak. However, if δ is too far from δ_s, Player 1 loses too much from waiting, and probably loses too much from knowing that Player 2 is strong. Conditions (6), (8) and (9) are necessary and sufficient conditions for there to be information collection in the equilibrium.

Corollary 2.1. *Given the same condition as that in Theorem 2, the function v is decreasing in α at every $\pi \in [0,1]$, and the lower (upper) threshold of the signal drawing region is increasing (decreasing) in α.*

Proof. **Step 1** First I prove Tw is decreasing in α at every $\pi \in [0,1]$. Observe that the middle linear section generated by T has the following expression:

$$\tilde{w}(\pi, \alpha) = [\delta\alpha a_1 + \delta(1-\alpha)a_2 - \delta(1-2\alpha)(b_1 - b_2)]\pi + \delta(1-\alpha)b_1 + \delta\alpha b_2$$

Differentiating with respect to α gives

$$\frac{\partial \tilde{w}}{\partial \alpha} = -\delta[(a_2 + b_2) - (a_1 + b_1)]\pi - \delta(b_1 - b_2)(1-\pi) < 0,$$

because of the conditions $a_1 + b_1 < a_2 + b_2$ and $b_2 < b_1$. Since $Tw = \max\{v_0, \tilde{w}\}$, for every $\pi \in [0,1]$, Tw is decreasing in α no matter $v_0 \geq w$ or $v_0 < w$ at π. Note that this result does *not* depend on the fact $v_0 = w$.

Step 2 Let T_{α_1} and T_{α_2} denote respectively the mapping T when $\alpha = \alpha_1$ and $\alpha = \alpha_2$, and $\alpha_1 \leq \alpha_2$. Let v_{α_1} and v_{α_2} denote respectively the corresponding solution. By Step 1, I have $T_{\alpha_1} w \geq T_{\alpha_2} w$, hence

$$(T_{\alpha_1})^2 w \geq T_{\alpha_1} T_{\alpha_2} w.$$

Following similar argument to that in Step 1, I can prove

$$T_{\alpha_1} T_{\alpha_2} w \geq (T_{\alpha_2})^2 w.$$

Since they are generated respectively by applying T_{α_1} and T_{α_2} on $T_{\alpha_2} w$, I can use the result of Step 1. For any two linear sections of $T_{\alpha_2} w$, T_{α_1} gives a higher middle section than T_{α_2} does. It is possible that at some point π, $p^W(\pi, \alpha_1)$ is so large or $p^S(\pi, \alpha_2)$ so small (by part 2 of Lemma 2, $p^W(\pi, \alpha_1) > p^W(\pi, \alpha_2)$ and $p^S(\pi, \alpha_1) < p^S(\pi, \alpha_2)$ for all $\pi \in (0,1)$) that $p^W(\pi, \alpha_1)$ is on a different linear section from $p^W(\pi, \alpha_2)$ or $p^S(\pi, \alpha_2)$ is on a different linear section from $p^S(\pi, \alpha_1)$. But whenever this happens, $p^W(\pi, \alpha_1)$ will be on a higher section than if the section that $p^W(\pi, \alpha_2)$ is on is extended to $p^S(\pi, \alpha_1)$. Similarly for $p^S(\pi, \alpha_1)$. For the same linear sections, T_{α_1} gives a higher middle section than T_{α_2} does. Now that $p^W(\pi, \alpha_1)$ or $p^S(\pi, \alpha_1)$ is on a higher linear section, then T_{α_1} is even higher than T_{α_2}, hence $T_{\alpha_1} T_{\alpha_2} w \geq (T_{\alpha_2})^2 w$. Now I have shown that

$$(T_{\alpha_1})^2 w \geq (T_{\alpha_2})^2 w.$$

Proceeding by induction, I can show that $(T_{\alpha_1})^k w \geq (T_{\alpha_2})^k w$ for all k if $\alpha_1 \leq \alpha_2$. Since the limits of the iteration (in fact, it is over in finite steps) are the solutions v_{α_1} and v_{α_2}, $v_{\alpha_1} \geq v_{\alpha_2}$ if $\alpha_1 \leq \alpha_2$. That is, v is decreasing in α at every $\pi \in [0, 1]$.

Recall in Part 2 of Theorem 2, the signal drawing region is an open interval in the middle. Since v is decreasing in α at every $\pi \in [0, 1]$, the lower (upper) threshold of the signal drawing region is increasing (decreasing) in α. □

What the above corollary says is that for the class of preferences that result in v_0 of the form of w in equation (3), the equilibrium has nice properties in terms of comparative statics. That v is decreasing in α at every π shows that the uninformed player is always better off having a more accurate signal about the informed player. Also, since the signal drawing region is also bigger under a smaller α, the uninformed player is more willing to collect information at any initial belief, if the information source is more accurate. The second property is in fact an implication of the first one. Since the uninformed player draws signals if and only if $v > v_0$, and v is always greater for a more accurate signal, Player 1 is more likely to draw signals with a more accurate signal.

4.2 Fixed Bargaining Costs

In this subsection, I look at preferences that can be represented by the utility function $x - ct$ for the division x at period t. In this case, Player 2_w has higher cost (c_H) than Player 1's (c), and 2_s has lower cost (c_L). So for this discussion, I will call them 2_H and 2_L, the prior probability for Player 2 to be 2_H is π. The costs also satisfy $c_1 + c_H + c_L < 1$. The proofs for Lemmas 1 and 3 are slightly different from the ones presented above, while the proof of Theorem 1 is quite different, in that the application of the contraction mapping theorem is not as handy. I put the proof of Theorem 1 for this case in the appendix. For this case, in the bargaining sequential equilibrium outcome without information collection, Player 1's payoff function becomes

$$v_0(\pi) = \begin{cases} (c_H + c_1)\pi + (1 - c_H - c_1), & \pi \geq \pi_0; \\ (c_H + c_1)\pi - c_1\,, & \pi^* \leq \pi < \pi_0; \\ c_L, & \pi < \pi^*. \end{cases}$$

where

$$\pi_0 = 2c_1/\left(c_H + c_1\right),$$

and

$$\pi^* = (c_L + c_1)/(c_H + c_1).$$

The special feature of v_0 in this case is that it has two threshold values, and a discontinuity at one of them, π_0. This jump is a major motivation for information collection. In the appendix, I include the proof of Theorem 1 for this case. Corollaries 1.2 and 1.3 further describe Player 1's equilibrium information collection behavior.

Corollary 1.2 *In the case of fixed bargaining costs described above, the function v is increasing, strictly bigger than v_0 in an interval (ω, π_0) if there are values of π at which $v > v_0$, and equal to v_0 in $(0, \omega) \cup [\pi_0, 1]$. In other words, Player 1's signal drawing decision can be characterized by two thresholds ω and π_0, if he draws signals at all.*

Proof. The proof is constructive. By Theorem 1 and in particular, condition (C), if I start with any $w \in V$, by iteration of T, I am able to get the function v. Let us start with the function w_1 defined by

$$w_1(\pi) = (c_H + c_1)\pi + (1 - c_H - c_1)$$

for all $\pi \in [0, 1]$. This is an upper bound of the payoff Player 1 can get. The corollary can be proved by using the following facts.

1. For all k, $T^k w_1$ is increasing in π.
2. $\{T^k w_1\}$ is a decreasing sequence. So if there exists ω, such that $T^k w_1(\pi) = v_0(\pi)$ for all $\pi \leq \omega$, then this remains true for $T^{k+1} w_1$.
3. For any $\omega < \pi_0$, if $T^k w_1(\omega) = v_0(\omega)$, then $T^k w_1(\pi) = v_0(\pi)$ for all $\pi \leq \omega$. □

This corollary shows that for the case of fixed bargaining costs, Player 1's signal drawing region also lies in the middle. That is, Player 1 collects information until he is quite sure about Player 2's type.

Corollary 1.3 *If $\max\{3c_H/2 + c_1/2, 2c_1 + c_H\} < 1$, then there always exists a $\omega_0 \in [\pi_1, \pi_0)$, such that $v(\pi) > v_0(\pi)$ for all $\pi \in (\omega_0, \pi_0)$, regardless of the value of α. In other words, if this condition is satisfied, there is always a nontrivial region in which Player 1 draws signals.*

Proof. Similar to the Proof of Corollary 1.2. This time let us choose $w_2 = v_0$ as the initial value. By iteration of T, we can again get the function v. The Corollary is proved by using the following facts:

1. $\{T^k w_2\}$ is an increasing sequence.

2. Given the required condition, in the first round of iteration, $Tw_2(\pi_0-) > v_0(\pi_0-)$, regardless of the value of α. □

Corollary 1.3 shows that there is a particular property about the fixed bargaining costs preference. If the cost of waiting is not too large, then there always exist initial beliefs at which there is gain from information collection, regardless of the accuracy of the information source. This result comes from the discontinuity of Player 1's payoff in the probability of Player 2 being 2_H without information collection. As we have seen, at one threshold value of π, Player 1's payoff jumps from almost nothing to almost everything.

It would be interesting to explore how Player 1's information collection behavior changes as α changes, and how Player 1's payoff depends on α. But these problems remain unresolved, due to the lack of an analytical solution.

5 Discussions and Further Research

This paper looks at one particular reason why agreement may be delayed in bargaining: the possibility to collect more information. I identify cases in which delays do happen, and perform comparative statistic analysis on two special cases, namely the fixed-bargaining-cost and fixed-discounting-rate cases.

Note that the setup of this paper is different from that of the models in which the uncertainty is about the valuations of the two parties. In particular, without information collection, there is at most one period of delay in the selected "bargaining sequential equilibrium." With information collection, there is a possibility (albeit small) that the bargaining process drags on for a long period of time.

Although it is not necessary for the result that signals come in each period, they must come frequently enough in order for the result of this paper to have significance. If the information source produces signals sparsely, then it is easy to see that the uninformed party will not delay the bargaining process at all. This points to one direction in which this result could be generalized. The information source can be modelled to generate signals at a fading speed. The question for the uninformed player is then to find the right combination of time and belief to stop. So my result would be subject to the "Coase conjecture" critique should it be applied to the model in which the uncertainty is about valuations. It may be reasonable to argue that offer-making can be done frequently. However, it is less appropriate to argue the same about information collection. Therefore, if the "Coase conjecture" critique is valid, the delay caused by information collection will vanish as the players are allowed to make frequent offers.

It may seem that the result depends on the uninformed player's ability to commit to not returning to the bargaining table. This is not the case. In fact, the informed player can only lure the uninformed player back to bargaining by sweetening the offer she would have otherwise made without the informed player having the information collection option. The weak type and strong type must make the same counteroffer by Rubinstein (1985). However, the strong type would not go along with this sweetened offer, since by Assumption (B-1), by separating herself from the weak type she would be able to achieve a higher payoff.

Appendix: Proof of Theorem 1 for the case of Fixed Bargaining Costs

Theorem 1 *There exists a unique function v solving Player 1's Bellman's equation (1).*

Proof. The proof is an application of the contraction mapping theorem.

First, observe that by definition,

i) $v(\pi) \geq v_0(\pi)$ everywhere in the interval [0,1].

By Theorem 0 and Lemma 1 (also see the proof of Lemma 3),

ii) $v(\pi) \leq (c_H + c_1)\pi + (1 - c_H - c_1)$ everywhere in the interval [0,1].

By Lemma 3,

iii) $v(\pi) = v_0(\pi)$ if $\pi \in [\pi_0, 1]$.

Let the space V be the set of functions satisfying Properties i), ii) and iii). I denote by $T : V \to V$ the function-valued mapping defined by (1), i.e.,

$$\begin{aligned} Tw(\pi) = \max\{v_0(\pi),\ &(\pi(1-\alpha) + (1-\pi)\alpha)w(p^H(\pi)) + \\ &(\pi\alpha + (1-\pi)(1-\alpha))w(p^L(\pi)) - c_1\} \end{aligned}$$

A solution to (1) is equivalent to a fixed point of Tin the space V. Define the sup norm $\|\|$ on the space V, then V is a complete metric space. Also, define

$$\pi_k = (p^L)^k(\pi_0), \quad k = 1, 2, ..., n.$$

Part 1

With the following procedure, I will find $\tilde{\pi} > 0$ and finite $K \in$ N, such that $T^k w(\pi) = u(\pi)$ for all $k \geq K$and $\pi < \tilde{\pi}$. I use an inductive argument.

Step 1.1 Given Property ii) and Part 1 of Lemma 2, I can immediately show that any function $w \in V$ must satisfy the following

$$\begin{aligned} (\pi(1-\alpha) + (1-\pi)\alpha)w(p^H(\pi))\ &+ (\pi\alpha + (1-\pi)(1-\alpha)w(p^L(\pi)) - c_1 \\ &\leq (c_H + c_1)\pi + (1 - c_H - c_1) - c_1 \end{aligned}$$

for every $\pi \in [0, \pi_0)$. So

$$Tw(\pi) \leq \max\{v_0(\pi),\ (c_H + c_1)\pi + (1 - c_H - c_1) - c_1\}$$

for every $w \in V$ and $\pi \in [0, \pi_0)$. Note that the form of v_0 is

$$v_0(\pi) = \begin{cases} (c_H + c_1)\pi + (1 - c_H - c_1), & \pi \geq 2c_1/(c_H + c_1); \\ (c_H + c_1)\pi - c_1, & (c_L + c_1)/(c_H + c_1) \leq \pi < 2c_1/(c_H + c_1); \\ c_L, & \pi < (c_L + c_1)/(c_H + c_1). \end{cases}$$

Now for $\pi \in [0, \pi_0)$, if

$$(c_H + c_1)\pi + (1 - c_H - c_1) - c_1 \geq v_0(\pi),$$

then

$$Tw(\pi) \leq (c_H + c_1)\pi + (1 - c_H - c_1) - c_1. \tag{T-1}$$

But that only requires

$$1 - c_H - c_1 \geq 0$$

for $\pi \in [(c_L + c_1)/(c_H + c_1), \pi_0)$ and

$$(c_H + c_1) \cdot 0 + (1 - c_H - c_1) - c_1 \geq c_L$$

for $\pi \in [0, (c_L + c_1)/(c_H + c_1))$. The first condition holds by assumption. If the second condition fails to hold, there must exist π', such that for all $\pi < \pi'$, $Tw(\pi) = u(\pi) = c_L$. The same is true for all $k > 1$, $T^k w(\pi)$ as well (I omit the calculation). I stop here, and let $\tilde{\pi} = \pi'$ and $K = 1$. If the second condition does hold, then we proceed to Step 1.2.

Step 1.2 Now applying Part 1 of Lemma 2 again, and using (T-1), I obtain

$$\begin{aligned}(\pi(1-\alpha) + (1-\pi)\alpha)Tw(p^H(\pi)) + (\pi\alpha + (1-\pi)(1-\alpha)Tw(p^L(\pi)) - c_1 \\ \leq (c_H + c_1)\pi + (1 - c_H - c_1) - c_1 - c_1\end{aligned}$$

for every $\pi \in [0, \pi_1)$, i.e., $p^H(\pi) \in [0, \pi_0)$. So

$$T^2 w(\pi) \leq \max\{u(\pi), (c_H + c_1)\pi + (1 - c_H - c_1) - 2c_1\}$$

Following the same logic in Step 1, we have if

$$(c_H + c_1)\pi + (1 - c_H - c_1) - 2c_1 \geq v_0(\pi),$$

then for all $\pi \in [0, \pi_1)$,

$$T^2 w(\pi) \leq (c_H + c_1)\pi + (1 - c_H - c_1) - 2c_1. \tag{T-2}$$

Similarly to Step 1.1, this requires that

$$1 - c_H - 2c_1 \geq 0,$$

for $\pi \in [(c_L + c_1)/(c_H + c_1), \pi_1)$ if $(c_L + c_1)/(c_H + c_1) < \pi_1$, and

$$(c_H + c_1) \cdot 0 + (1 - c_H - c_1) - 2c_1 \geq c_L$$

for $\pi \in [0, (c_L + c_1)/(c_H + c_1))$. Note that the first condition is redundant if the second condition holds. The same is true in all subsequent steps if necessary. Now if the second condition fails, I am able to find π' such that $T^2 w(\pi) = u(\pi) = c_L$ for all $\pi < \pi'$. Set $\tilde{\pi} = \pi'$ and $K = 2$. If it does hold, then I proceed to Step 1.3, and so on.

Step 1.K From the above procedure I know that $K = [(1 - c_H - c_L)/c_1]$, and $\tilde{\pi}$ can be chosen accordingly.

Now I have shown for all $w \in V$ and $k > K$, $T^k w(\pi) = c_L$ holds for all $\pi < \tilde{\pi}$. This implies without loss, I can focus on functions that satisfy

$$w(\pi) = c_L \text{ for all } \pi < \tilde{\pi} \qquad (*)$$

in the subsequent analysis.

Part 2

Define $M = \min_{m \in \mathbf{N}}\{(p^H)^m(\tilde{\pi}) \geq \pi_0\}$. I know $M < \infty$ because $(p^H)^m(\pi)$ converges to 1 as $m \to \infty$ for any $\pi > 0$ by Part 4 of Lemma 2. Now consider the mapping T^M. I shall show that it is a contraction. Take any two functions $w_1, w_2 \in V$ satisfying (*). Let $d = \|w_1 - w_2\|$. Observe that $w_1 = w_2$ everywhere on $[0, \tilde{\pi}) \cup (\pi_0, 1]$ Let us take any $\pi \in [\tilde{\pi}, \pi_0)$. There are three cases:

Case 1. If $T^M w_1(\pi) = T^M w_2(\pi) = v_0(\pi)$, then any $0 < \rho < 1$ will make $\left|T^M w_1(\pi) - T^M w_2(\pi)\right| < \rho \cdot d$. Also note it satisfies the (C-M) condition as defined in Case 2.

Case 2. If $T^M w_1(\pi) = v_0(\pi)$, but $T^M w_2(\pi) \neq v_0(\pi)$, then

$$u(\pi) \geq (\pi(1-\alpha) + (1-\pi)\alpha) T^{M-1} w_1(p^H(\pi)) + (\pi\alpha + (1-\pi)(1-\alpha)) T^{M-1} w_1(p^L(\pi)) - c_1$$

and

$$u(\pi) < (\pi(1-\alpha) + (1-\pi)\alpha) T^{M-1} w_2(p^H(\pi)) + (\pi\alpha + (1-\pi)(1-\alpha)) T^{M-1} w_2(p^L(\pi)) - c_1$$

Then

$$\begin{aligned} &\left|T^M w_1(\pi) - T^M w_2(\pi)\right| \\ &< \left| \begin{array}{l} (\pi(1-\alpha) + (1-\pi)\alpha)[T^{M-1} w_1(p^H(\pi)) - T^{M-1} w_2(p^H(\pi))] + \\ \qquad (\pi\alpha + (1-\pi)(1-\alpha))[T^{M-1} w_1(p^L(\pi)) - T^{M-1} w_2(p^L(\pi))] \end{array} \right|. \end{aligned}$$

By the triangle inequality, I obtain

$$\begin{aligned} &\left|T^M w_1(\pi) - T^M w_2(\pi)\right| \\ &\leq (\pi(1-\alpha) + (1-\pi)\alpha) \left|T^{M-1} w_1(p^H(\pi)) - T^{M-1} w_2(p^H(\pi))\right| + \\ &\qquad (\pi\alpha + (1-\pi)(1-\alpha)) \left|T^{M-1} w_1(p^L(\pi)) - T^{M-1} w_2(p^L(\pi))\right| \end{aligned} \tag{C-M}$$

Similarly, the case that $T^M w_1(\pi) \neq v_0(\pi)$, but $T^M w_2(\pi) = v_0(\pi)$ also satisfies (C-M).

Case 3. If $T^M w_1(\pi_H) \neq v_0(\pi_H)$ and $T^M w_2(\pi_H) \neq v_0(\pi_H)$, then by direct substitution (C-M) is satisfied.

Before proceeding, note that the RHS of (C-M) is a convex combination of

$$\left|T^{M-1}w_1((p^H)^1(\pi)) - T^{M-1}w_2((p^H)^1(\pi))\right|$$

and

$$\left|T^{M-1}w_1((p^L)^1(\pi)) - T^{M-1}w_2((p^L)^1(\pi))\right|.$$

Repeat the above analysis for

$$\left|T^{M-1}w_1(p^H(\pi)) - T^{M-1}w_2(p^H(\pi))\right|$$

and

$$\left|T^{M-1}w_1(p^L(\pi)) - T^{M-1}w_2(p^L(\pi)\right|,$$

I obtain two (C-(M-1)) conditions for all three possible cases listed above. Substitute them back into (C-M), I obtain that $\left|T^M w_1(\pi) - T^M w_2(\pi)\right|$ is less than or equal to a convex combination of $\left|T^{M-2}w_1((p^H)^m(\pi)) - T^{M-2}w_2((p^H)^m(\pi))\right|$, $|m| \leq 2$, with the interpretation $(p^H)^m = (p^L)^{-m}$ if $m < 0$ and the identity function if $m = 0$. In particular, the coefficient on $\left|T^{M-2}w_1((p^H)^2(\pi)) - T^{M-2}w_2((p^H)^2(\pi))\right|$ is $\pi(1-\alpha)^2 + (1-\pi)\alpha^2$ and that on $\left|T^{M-2}w_1((p^H)^{-2}(\pi)) - T^{M-2}w_2((p^H)^{-2}(\pi))\right|$ is $\pi\alpha^2 + (1-\pi)(1-\alpha)^2$.

I proceed by induction for M steps, until I obtain $2M$(C-1) conditions, substitute them into the condition gotten from the previous step and obtain that $|T^M w_1(\pi) - T^M w_2(\pi)|$ is less than or equal to a convex combination of expressions in the form of $\left|w_1((p^H)^m(\pi)) - w_2((p^H)^m(\pi))\right|$, where $|m| \leq M$. The coefficient on $\left|w_1((p^H)^M(\pi)) - w_2((p^H)^M(\pi))\right|$ is $\pi(1-\alpha)^M + (1-\pi)\alpha^M$ and the coefficient on $\left|w_1((p^H)^{-M}(\pi)) - w_2((p^H)^{-M}(\pi))\right|$ is $\pi\alpha^M + (1-\pi)(1-\alpha)^M$. By the definition of M, $(p^H)^M(\pi) \geq \pi_0$ and $(p^H)^{-M}(\pi) \leq \tilde{\pi}$ must be true for all $\pi \in [\tilde{\pi}, \pi_0)$. So $\left|w_1((p^H)^M(\pi)) - w_2((p^H)^M(\pi))\right| = \left|w_1((p^H)^{-M}(\pi)) - w_2((p^H)^{-M}(\pi))\right| = 0$ by (*) and Property iii). Further, by the fact

$\left|w_1((p^H)^m(\pi)) - w_2((p^H)^m(\pi))\right| \leq \|w_1 - w_2\| = d$ for all $|m| \leq M$,

I conclude for all $\pi \in [\tilde{\pi}, \pi_0)$,

$$\begin{aligned}
&\left|T^M w_1(\pi) - T^M w_2(\pi)\right| \\
&\leq [1 - (\pi(1-\alpha)^M + (1-\pi)\alpha^M) - (\pi\alpha^M + (1-\pi)(1-\alpha)^M)] \cdot d \;. \\
&= [1 - \alpha^M - (1-\alpha)^M] \cdot d
\end{aligned}$$

Hence,

$\left\|T^M w_1(\pi) - T^M w_2(\pi)\right\| \leq [1 - \alpha^M - (1-\alpha)^M] \cdot d.$ (C)

Since $\alpha \in (0, 1/2)$ and $M < \infty$, I have shown that T^M is a contraction mapping.

Now I apply the contraction mapping theorem and conclude that T^M must have a unique fixed point on V. So does T. □

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