Direct Mechanisms, Menus and Latent Contracts

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JEL Classification: D82

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July 30, 2007

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Abstract

In common agency games, one cannot characterize all equilibria by considering only direct mechanisms. In an attempt to overcome this difficulty, Peters [Econometrica, 2001] and Martimort and Stole [Econometrica, 2002] identified a class of indirect mechanisms (namely, menus) which are able to characterize every equilibrium. Unfortunately, menus are difficult to handle, and several methodologies have been proposed in the literature. Here, it is shown that, even if authors consider menus rather than simpler mechanisms, many equilibria described in the literature could have been characterized by direct incentive compatible mechanisms. Use of more sophisticated mechanisms was not necessary in these cases.

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1 Introduction

The restriction to direct incentive compatible mechanisms is a cornerstone of contract theory. It provides a simple and elegant method for characterizing arbitrary equilibria in any principal-agent model, even with very complex communication between the players. Because of its tractability, the principal-agent model has been very successful, and it has revitalized many economic fields: Regulation, redistribution, insurance and others.[1] Multiagent games have provided the basis for auction theory and the theory of the provision of public goods.

Unfortunately, the restriction to direct incentive compatible mechanisms causes some loss of generality in multi-principal games. Intuitively, simple contracts fail to be general because the structure of the game involves endogenous information. For a principal, relevant information includes not only the type of the agent (for example his/her willingness to pay in a case of a duopoly) but also the message that the agent sends to other principals; the message sent sets a particular agreement between a principal and the agent, which could modify the agent's willingness to pay for the products of other principals.

A strategy for overcoming this limitation is to give up the concept of "direct mechanism" or any of its generalizations, and consider the Taxation Principle. This principle was introduced by Hammond [1979], Guesnerie [1981] and Rochet [1986], and states that there is no loss of generality in considering menus, or nonlinear prices. Peters [2001] and Martimort and Stole [2002] show that an equivalent of the Taxation Principle (they call it Delegation Principle) makes it possible to characterize any equilibrium of any common agency game. The problem with this approach is that the concept of menu is large for common agency games, and, even if it simplifies the game, equilibria remain hard to characterize. To reach tractable problems, other *ad hoc* assumptions are added to restrict the menu set.

The present paper does not question the validity of the differing further assumptions made in the literature. We welcome assumptions (differentiability or continuity) if they allow ready characterization of equilibria in this class of games. The cost of these assumptions is probably a loss of generality.[2] Nevertheless, the author does not believe that restrictions invalidate the results obtained with menus. The methodologies used to find a fixed-point in common agency games in which menus are allowed are criticized. The present paper shows that, in almost all models of the common agency literature, equilibria characterized by menus could have been characterized by direct mechanisms.

[1] See Laffont and Martimort [2002] for a complete survey.

[2] In common agency games, some equilibria may be sustained by discontinuous menus; see Laffont and Tirole [1993] ch 17.

The basic intuition is that menus can characterize a large set of equilibria because a principal, by using a menu, can create sophisticated rewards.

Given menus, it might seem that analysis of common agency games is simply a matter of computation. Unfortunately, though the use of menus may be helpful in this class of game, it does not permit ready characterization of equilibria. Below, it is argued that common methodologies used in the literature characterize only a restricted set of pure strategy equilibria. Let us now consider the "latent contract" concept, which gives insight into the main result.

The next section presents a basic common agency model. Section 3 defines direct mechanisms and menus. Section 4 introduces the concept of latent contracts. In section 5 some examples are presented from the literature. Section 6 sets out conclusions.

2 The Model

Consider a scenario in which there are a number of principals (indexed by $i \in N = \{1,...,n\}$) contracting with one agent (denoted by index, 0). The agent's type is drawn from a compact set Θ having a probability distribution $F(.)$ that is common knowledge. The principal i also makes an action: He has to decide which allocation $y_i \in Y_i$ to implement. The implemented allocation is observable and contractible. This means that a principal can write a contract which specifies his chosen allocation. Let us make the stronger assumption that a principal is not able to contract on a probability distribution over Y_i.

The payoff to principal $i \in \{1,\ldots,N\}$ is represented by the vNM utility function

$$V_i : \prod_{k \in N} Y_k \times \Theta \to \mathbb{R}_+,$$

and for the agent the payoff is represented by the function

$$U : \prod_{k \in N} Y_k \times \Theta \to \mathbb{R}_+.$$

Preferences could be more general; the restriction to von Neumann-Morgenstern utility functions is not critical for any of the following results, but merely makes the model simpler. Moreover, applications in the literature invariably consider this class of preferences.

The principals compete through mechanisms. Each principal's mechanism is a map from M_i to Y_i, where M_i is the message space. Each set M_i (for any principal i) is

compact, and each map σ_i is measurable. For clarity, we require any mechanisms σ_i to be such that the image set $\sigma_i(M_i)$ is a compact set. We denote by Σ_i the set of all available map σ_i to principal i when his message space is M_i. We denote by $M = \times_{i\in N} M_i$ the collection of the chosen message space, and Σ is the collection of Σ_i.

Denote by $\sigma_i(m_i)$ the decision y_i in Y_i that the agent gets by sending the message m_i to the principal i. We explicitly assume that the rule $\sigma_i(.)$ is enforceable. Once a principal has announced a mechanism, he commits himself to respect his own rule; if he receives a message m_i, he cannot choose a decision different from $\sigma_i(m_i)$. Finally, since the sets $\sigma_i(M_i)$ are compact, it follows that

$$\underset{(m_i,m_{-i})\in M_i\times M_{-i}}{\arg\max} \quad U(\sigma_i(m_i),\sigma_{-i}(m_{-i}),\theta) \neq \emptyset. \tag{1}$$

The agent's pure strategy is to choose a message $m_i \in M_i$ for each principal. Hence, a pure strategy for the agent is a map $\sigma_0 : \Theta \times \Sigma \to M$. We denote by Σ_0 the collection of all these possible pure strategies.

3 Direct Mechanisms and Menus

In the game $\Gamma_{\mathcal{M}}$ we have made only made standard assumptions concerning the sets M_i. The message spaces may be quite complex. We can simplify the game by considering *direct mechanisms*. For each principal the message space M_i is given, and coincides with the agent's type space Θ. We thereby restrict the strategy spaces of the principals.

The strategy of principal i is the map $\tilde{\sigma}_i : \Theta \times E \to Y_i$; we let $\tilde{\Sigma}_i$ be the strategy space for principal i, and $\tilde{\Sigma}$ be the collection of all such strategy profiles.

The strategy of the agent is then a map $\tilde{\sigma}_0 : \Theta \times \tilde{\Sigma} \to \Theta^N$, and $\tilde{\Sigma}_0$ denotes the collection of all such maps.

Given Θ, the common agency game induced by direct mechanism is the array:

$$\Gamma_\Theta = \left\{\Theta, \left(\tilde{\Sigma}_i\right)_{i\in N}, \tilde{\Sigma}_0, U(.,\theta), (V_i(.,\theta))_{i\in N}, F(.)\right\}.$$

Direct mechanisms have an obvious appeal – the message spaces are simple and given. But, to be useful, we need more than a simplification of the message space. To apply the traditional principal-agent methodology, we need also incentive compatibility.

Definition 1 *A collection of strategies* $\left((\tilde{\sigma}_i^*)_{i\in N}, \tilde{\sigma}_0^*\right)$ *is an incentive compatible equilibrium of the game* Γ_Θ *if it satisfies the following two conditions:*

$$\forall \theta \in \Theta, \text{ if } \quad \tilde{\sigma}_0^*\left((\tilde{\sigma}_i^*)_{i\in N}, \theta\right) = (\theta,\ldots,\theta,\tilde{e}^*(\theta)), \text{ then } \quad \forall i \in N, t_i = \theta. \tag{2}$$

$$\forall i \in N, \quad \hat{\sigma}_i \in \tilde{\Sigma}_i \text{ is such that } \tilde{\sigma}_0^* \left(\hat{\sigma}_i, \tilde{\sigma}_{-i}^*, \theta\right) = \left(t_1(\theta), \ldots, t_N(\theta), \tilde{e}(\theta)\right), \quad \text{with } t_i(\theta) = \theta$$

then $\hat{\sigma}_i$ *is not a profitable deviation:*

$$\int_\Theta v_i\left(\hat{\sigma}_i(\theta), \tilde{\sigma}_{-i}(t_{-i}(\theta)), \theta, \tilde{e}(\theta)\right) dF(\theta) \leq \int_\Theta v_i\left(\tilde{\sigma}_i(\theta), \tilde{\sigma}_{-i}(\theta), \theta, \tilde{e}^*\right) dF(\theta). \tag{3}$$

When is it possible to restrict attention to direct incentive compatible mechanisms? In other words, is it always the case that, for any equilibrium σ^* of the game $\Gamma_{\mathcal{M}}$, there exists a incentive compatible equilibrium $\tilde{\sigma}^*$ of the game Γ_Θ such that the two equilibria are outcome equivalent? For common agency games this is not so (see, for instance, Peck [1997], Martimort and Stole [2002]). In games with multiple principals, equilibria may exist whose outcomes cannot be supported in equilibrium in the corresponding direct mechanism game.

Peters [2001] and Martimort and Stole [2002] show that even if one cannot restrict attention to direct incentive compatible mechanisms, a modified version of the Taxation Principle applies. This principle states that, without loss of generality:

- One can restrict the set of message space and consider the sets of all compact subsets of Y_i rather than M_i.
- The map σ_i is the identity over the chosen subset of Y_i.

Given $Y = \times_{i \in N} Y_i$, the common agency game induced by menu is the array

$$\Gamma_Y = \left\{\Theta, (Z_i)_{i \in N}, \overline{\Sigma}_0, U(.,\theta), (V_i(.,\theta))_{i \in N}, F(.)\right\},$$

where $\overline{\Sigma}_0$ is defined as Σ_0. Here Z_i denotes the set of all compact subsets of Y_i, and T_i is a generic element of Z_i. We will use the obvious notation $T = \times_{i \in N} T_i$.

Theorem 1 *The three following statements can be established:*

- *For every equilibrium* $\left(\sigma^*, \sigma_0^*\right)$ *of the game* $\Gamma_{\mathcal{M}}$ *there exists a an outcome equivalent equilibrium* $\left(T^*, \overline{\sigma}_0^*\right)$ *of the game* Γ_Y.
- *For any equilibrium* $\left(T^*, \overline{\sigma}_0^*\right)$ *of the game* Γ_Y *there exists an outcome equivalent equilibrium* $\left(\pi^*, \sigma_0^*\right)$ *of the game* $\Gamma_{\mathcal{M}}$.
- *For any incentive compatible pure strategy equilibrium* $\left(\tilde{\sigma}^*, \tilde{\sigma}_0^*\right)$ *of the game* Γ_{Θ^N}, *there exists a pure strategy equilibrium of the game* $\Gamma_{\mathcal{M}}$ *such that the two equilibria are outcome equivalent.*

The two first statements have been shown by Peters [2001] and Martimort and Stole [2002]. The third is a result from Peters [2003]. The last statement of the theorem is very general, and it does not rely on any assumption about the action space available to the agent. It applies to all common agency models in the literature to date.

4 Latent contracts

This section demonstrates how "latent contracts" can help to characterize a larger set of equilibria.[3] By latent contract or or latent decision is meant any decision reachable by the agent but never implemented at equilibrium, whatever the type of the agent. For example, if a principal uses a incentive compatible direct mechanism, this mechanism does not involve any "latent contract". This former mechanism is a map from the type set Θ to the decision set Y_i, denoted $\tilde{\sigma}_i$. By definition, for any decision $\tilde{y}_i$ in the image set $\tilde{\sigma}_i(\Theta)$, there is a type $\theta \in \Theta$ such that $\tilde{\sigma}_i(\theta) = \tilde{y}_i$.

Definition 2 *We say that a menu T_i contains latent decisions if, given the strategies of the other players T_{-i} and $\overline{\sigma}_o^*$,*

$$\exists y_i \in T_i, \quad \forall \theta \in \Theta, \quad \overline{\sigma}_{0i}^*(\theta) \neq y_i, \tag{4}$$

where $\overline{\sigma}_{0i}^*(\theta)$ is the projection of $\overline{\sigma}_0^*$ over T_i.

From this definition, we can reach the following theorem:

Theorem 2 *Consider the game Γ_Y and a pure strategy equilibrium. If principals offer menus without latent decisions, then there exists an output equivalent incentive compatible equilibrium in the direct mechanisms game Γ_Θ.*

Proof Consider an equilibrium $\left(T_i^*, T_{-i}^*, \overline{\sigma}_0^*\right)$ of the game Γ_Y. We wish to construct an output equivalent equilibrium $(\tilde{\sigma}_i, \tilde{\sigma}_{-i}, \tilde{\sigma}_0)$ in the game Γ_Θ.

By assumption, menus $\left(T_i^*, T_{-i}^*\right)$ do not involve latent decisions. One can consider the agent's equilibrium best reply $\overline{\sigma}_0^*$, which is, for every collection of menu $T \in Z$, a function from set $T \times \Theta$ to the set T. We can construct unambiguously the following direct mechanisms denoted $\tilde{\sigma}_i$.

$$\forall i \in N, \forall \theta \in \Theta, \quad \tilde{\sigma}_i^*(\theta) = \overline{\sigma}_{0i}^*\left(T_i^*, T_{-i}^*, \theta\right), \tag{5}$$

where $\overline{\sigma}_{0i}^*$ is defined as above. We have constructed the strategies $\left(\tilde{\sigma}_i^*, \tilde{\sigma}_{-i}^*\right) \in \tilde{\Sigma}$ of the principals in the game Γ_Θ. Let us construct the agent's best reply, denoted $\tilde{\sigma}_0$. For all $\tilde{\sigma} \in \tilde{\Sigma}$ and for all $\theta \in \Theta$, we denote by $\tilde{\sigma}_i(\Theta)$ the image set of the mapping $\tilde{\sigma}_i$. Define the best reply mapping of the agent as follows:

$$\forall \tilde{\sigma} \in \tilde{\Sigma}, \forall \theta \in \Theta, \quad \tilde{\sigma}_0^*(\tilde{\sigma}, \theta) = \overline{\sigma}_0^*\left(\left(\tilde{\sigma}_i(\Theta)\right)_{i \in N}, \theta\right). \tag{6}$$

[3] Latent contracts were introduced by Hellwig [1983], and the concept is widely used in the literature on foundations of competitive equilibrium; see for example Bisin et al. [1999].

For all collections of direct mechanism $\tilde{\sigma} \in \tilde{\Sigma}$, we have

$$\tilde{\sigma}_0^*(\tilde{\sigma},\theta) \in \underset{(h_1,\ldots,h_N)\in\Theta^N}{\arg\max}\ U\left((\tilde{\sigma}_i(h_i))_{i\in N},\theta\right). \tag{7}$$

Suppose not:

$$\exists\,(h_1,\ldots,h_N)\in\Theta^N,\ \exists\theta\in\Theta,\quad U\left((\tilde{\sigma}_i(h_i))_{i\in N},\theta\right) > U\left((\tilde{\sigma}_i(\tilde{\sigma}_{i0}^*(\tilde{\sigma},\theta)))_{i\in N},\theta\right). \tag{8}$$

By construction, $U\left((\overline{\sigma}_{i0}(\tilde{\sigma}_i(\Theta)))_{i\in N},\theta\right) = U\left(\left(\tilde{\sigma}_i\left(\tilde{\sigma}_{0i}^*(\tilde{\sigma},\theta)\right)\right)_{i\in N},\theta\right)$. Consequently,

$$U\left((\tilde{\sigma}_i(h_i))_{i\in N},\theta\right) > U\left((\overline{\sigma}_{i0}^*(\tilde{\sigma}_i(\Theta)))_{i\in N},\theta\right). \tag{9}$$

Since by construction we have $\forall i\in N,\ \tilde{\sigma}_i(h_i)\in\tilde{\sigma}_i(\Theta)$, we generate a contradiction. Moreover, we have:

$$\forall\theta\in\Theta,\quad \tilde{\sigma}_0^*(\tilde{\sigma}^*,\theta) = (\theta,\ldots,\theta), \tag{10}$$

because, by definition for all $\theta\in\Theta$ and for every principal i, $\tilde{\sigma}_i(\theta) = \overline{\sigma}_{0i}^*(T^*,\theta)$. Hence the candidate equilibrium is incentive compatible.

Supppose that principals play $\tilde{\sigma}^*$, and principal i deviates toward $\tilde{\sigma}_i\in\tilde{\Sigma}_i$ (all other players keep their strategies). The agent's best reply is then $\tilde{\sigma}_0^*\left(\tilde{\sigma}_i,\tilde{\sigma}_{-i}^*,\theta\right)$. The "no latent decision assumption" implies that

$$\forall i\in N,\quad \forall Y_i\in T_i^*,\quad \exists\theta\in\Theta \text{ such that } \overline{\sigma}_{0i}^*(T_i^*,\theta) = y_i. \tag{11}$$

Hence,

$$\forall i\in N,\quad \tilde{\sigma}_i^*(\Theta) = T_i^*. \tag{12}$$

Using the definition of $\tilde{\sigma}_0^*$, we can state that

$$\forall\, j\in N,\ \forall i\in N,\ \forall\sigma_i\in\Sigma_i,\quad \tilde{\sigma}_{0j}^*\left(\tilde{\sigma}_i,\tilde{\sigma}_{-i}^*,\theta\right) = \overline{\sigma}_{0j}^*\left(\tilde{\sigma}_i(\Theta),T_{-i}^*,\theta\right). \tag{13}$$

Under the "no latent decision" condition:

$$\forall i\in N,\ \forall y_i\in T_i^*,\ \exists h\in\Theta,\ \text{such that } \tilde{\sigma}_i^*(h) = y_i. \tag{14}$$

Hence, for all $\theta\in\Theta$, there exists $(h_i(\theta),h_{-i}(\theta))\in\Theta^N$ such that

$$\left(\tilde{\sigma}_i(h_i(\theta)),\left(\tilde{\sigma}_j^*\left(h_j(\theta)\right)\right)_{j\in N\setminus\{i\}}\right) = \left(\overline{\sigma}_{0i}^*\left(\tilde{\sigma}_i(\Theta),T_{-i}^*,\theta\right),\left(\overline{\sigma}_{0j}^*\left(\tilde{\sigma}_i(\Theta),T_{-i}^*,\theta\right)\right)_{j\in N\setminus\{i\}}\right). \tag{15}$$

Moreover, by definition $(h_i(\theta),h_{-i}(\theta)) = \tilde{\sigma}_0^*(\tilde{\sigma}_i,\theta)$.

Suppose that for principal i the deviation is strictly profitable:

$$\int_{\Theta} V_i\left(\left(\tilde{\sigma}_{0j}^{*}\left(\tilde{\sigma}_i,\tilde{\sigma}_{-i}^{*},\theta\right)\right)_{j\in N},\theta\right)dF\left(\theta\right)>\int_{\Theta} V_i\left(\left(\tilde{\sigma}_{0j}^{*}\left(\tilde{\sigma}_i^{*},\tilde{\sigma}_{-i}^{*},\theta\right)\right)_{j\in N},\theta\right)dF\left(\theta\right). \tag{16}$$

Equation (16) can now be rewritten as

$$\int_{\Theta} V_i\left(\left(\overline{\sigma}_{0j}^{*}\left(\tilde{\sigma}_i\left(\Theta\right),T_{-i}^{*},\theta\right)\right)_{j\in N},\theta\right)dF\left(\theta\right)>\int_{\Theta} V_i\left(\left(\overline{\sigma}_{0j}^{*}\left(T_i^{*},T_{-i}^{*},\theta\right)\right)_{j\in N},\theta\right)dF\left(\theta\right), \tag{17}$$

which is a contradiction.

We conclude that $\left(\tilde{\sigma}_i^{*},\tilde{\sigma}_{-i}^{*},\tilde{\sigma}_0^{*}\right)$ is an equilibrium of the game Γ_{Θ}. By construction, this equilibrium is output equivalent to the equilibrium $\left(T_i^{*},T_{-i}^{*},\overline{\sigma}_0^{*}\right)$. □

Consider now the following example.

Example 1 Each principal ($i=1,2$) must make a decision (y_i^1 or y_i^2 with $i=1,2$). The corresponding payoffs are given by the following matrix:

	y_2^1	y_2^2
y_1^1	$(2,2,3)$	$(0,3,1)$
y_1^2	$(1,0,1)$	$(1,1,2)$

Table 1: Common Agency with complete information

where the first element in each cell refers to the payoff of Principal 1, the second element to the payoff of Principal 2, and the last element to the agent's payoff.

If we consider that principals are using direct mechanisms (which are take-it or leave-it offers since information is complete), there is only one pure strategy equilibrium: Principal 1 plays y_1^2 and principal 2 plays y_2^2. Agent's payoffs are not relevant, since the the agent plays no role.

If principals are allowed to use menus, so that they offer subsets of $\left\{y_i^1,y_i^2\right\}$, then there are two equilibria. In the first equilibrium, principal 1 offers the degenerated menu $\left\{y_1^2\right\}$ and the second principal offers the menu $\left\{y_2^2\right\}$. Although the agent has no relevant choice, it can nevertheless be confirmed that we have a regular equilibrium. In the second equilibrium, principal 1 offers the menu $\left\{y_1^1,y_1^2\right\}$ and principal 2 offers the menu $\left\{y_2^1,y_2^2\right\}$. The agent chooses y_1^1 from principal 1 and y_2^1 from principal 2. The outcome $\left(y_1^1,y_2^1\right)$ is finally implemented.

The outcome $\left(y_1^1, y_2^1\right)$ cannot be supported by an equilibrium if principals use only direct mechanisms. If principal 1 offers $\left\{y_1^1\right\}$, the direct mechanism $\left\{y_2^1\right\}$ is not the best reply for principal 2. He gets more by offering $\left\{y_2^2\right\}$.

The outcome $\left(y_1^1, y_2^1\right)$ can be implemented because menus $\left\{y_2^1, y_2^2\right\}$ and $\left\{y_1^1, y_1^2\right\}$ embed latent decisions: y_1^2 and y_2^2 are not chosen by the agent, but they are crucial because they prevent deviations. □

Let us consider a second example.

Example 2 The type of the agent is θ_1 with probability $p_1 = 1/2$, and θ_2 with probability $1 - p_1$. Payoffs are given by the following matrices:

θ_1

	y_2^1	y_2^2
y_1^1	$(2,2,3)$	$(0,3,1)$
y_1^2	$(1,0,1)$	$(1,1,2)$

θ_2

	y_2^1	y_2^2
y_1^1	$(0,1,2)$	$(1,0,1)$
y_1^2	$(0,3,1)$	$(2,2,3)$

Table 2: Common Agency with incomplete information

This game has an equilibrium in the menu game. Each principal proposes the menu $\left\{y_i^1, y_i^2\right\}$ $(i = 1,2)$; the agent chooses $\left(y_1^1, y_2^1\right)$ if his type is θ_1, and chooses $\left(y_1^2, y_2^2\right)$ if his type is θ_2. Since the set of possible menus is very small, we can check that for principal 1 no deviations (which are the singletons $\left\{y_1^1\right\}$ and $\left\{y_1^2\right\}$) are profitable. The same holds for principal 2.

The outcome can also be supported as an equilibrium in the direct mechanism game. For principal 1 the former strategy can be reproduced in the following way: he plays y_1^1 if the agent sends the message θ_1, and plays y_1^2 if the agent announces θ_2. Principal 2 plays the same strategy (y_2^1 if θ_1 and y_2^2 if θ_2). The agent can reach any cell by misreporting his type. By analogy with the menu game, it is best for the agent to announce his real type. Using a similar argument, one can check that the strategies described are also best replies for the principals. □

Consider a last example taken from the literature.

Example 3 [Biais and Mariotti [2005]] There are two principals (indexed by $i = 1,2$), and their decision spaces are $Y_1 = Y_2 = Y = \mathbb{R} \times [0,1]$. A generic decision is denoted by (t,q). The two principals have the same utility functions $\theta q - t$, where θ is the information of the agent, $\theta \in [0,1]$. The distribution function of θ over $[0,1]$ is denoted by F.

Agent preferences are represented by the utility function $t - \lambda\theta q$, where the variable $\lambda \in (0,1)$; this is common knowledge. The agent is constrained to accept contracts (t_1, q_1) and (t_2, q_2) such that $q1 + q2 \leq 1$.

Assume that $E(\theta) < \lambda$, where $E(\theta) = \int_0^1 \theta F(\theta) d\theta$. Then an equilibrium exists in which each principal offers the menu $M = \{(t,q) \in Y | t = qE(\theta)\}$. If the two principals offer this menu, the agent (whatever his type) will choose the allocation $q = 1$ and $t = E(\theta)$. If one principal deviates, and offers a unique contract (t', q'), then

- If (t', q') is below the line $t = qE(\theta)$, the agent will accept this contract and the deviating principal makes zero profit, as he did at equilibrium. The deviation is not profitable.
- If (t', q') is above the line $t = qE(\theta)$, the agent will always accept the contract offered. Whatever his type, the agent will buy a quantity $q = 1 - q'$ from ***OK? - EDITOR*** the other principal. Since $t'/q' < E(\theta)$, the deviating principal makes losses.

This argument can easily extended to any kind of menu.

If we now look at the best direct mechanism against the menu M, it is obvious that the degenerated mechanism gives the contract $(E(\theta), 1)$ to every type. It is also clear that, if one principal plays that mechanism, it is not the best reply for the other principal to play that same mechanism. He should offer the contract (t', q'), where $t' = \lambda q'$. This contract is accepted by the agent only when his type is $\theta = 1$, in which case it provides positive profit to the principal.

The equilibrium characterized by Biais and Mariotti [4] is efficient, so that if we ignore it, we may reach wrong conclusions. □

In the preceding example, the menus $\{y_1^1, y_1^2\}$ and $\{y_2^1, y_2^2\}$ do not embed latent decisions. For principal 1, y_1^1 (resp. y_1^2) is chosen when the agent's type is θ_1 (resp. θ_2). Similarly, for principal 2, item y_2^1 is chosen if the agent is of type θ_1, and y_2^2 is chosen when the type is θ_2. Menus do not embed latent decisions, so that the equilibrium can be sustained by direct mechanisms. Moreover, at equilibrium, the agent reveals his true type.

5 Applied Common Agency Models

Let us now focus on examples taken from the literature.

To characterize equilibria in the set of menus is not a trivial exercise. Martimort [1996] and Martimort and Stole [2002] have introduced a sophisticated methodology.

- They consider that principal i uses direct mechanisms $\tilde{\sigma}_i^*$ to reply to the menus T_{-i}^* and to the agent's strategy.
- From the best direct mechanism, one can deduce a menu.
- If this is done for every principal, and if each principal is playing the menu derived from the best direct mechanism, we finally obtain an equilibrium.

At equilibrium, menus do not involve latent decisions; each item is chosen by some agent. This method provides no gain over the traditional method; any equilibrium characterized using this methodology can be characterized by the simple use of direct mechanisms.

Here is an example that shows how this methodology fails to characterize any equilibrium of a common agency game.

Example 4 The type of the agent is θ_1 with probability $p_1 = 1/2$, and θ_2 with probability $1 - p_1$. Payoffs are given by the following matrices:

θ_1

	y_2^1	y_2^2
y_1^1	$(2,2,3)$	$(0,3,1)$
y_1^2	$(1,0,1)$	$(1,1,4)$

θ_2

	y_2^1	y_2^2
y_1^1	$(4,2,2)$	$(2,3,0)$
y_1^2	$(1,0,1)$	$(1,1,5)$

Table 3: Common Agency with incomplete information

This common agency game has one pure strategy equilibrium. The first principal (P1) plays the menu $\{y_1^1, y_1^2\}$ and the second principal plays the menu $\{y_2^1\}$, and gets an expected utility of 2. Clearly, the first principal has no profitable deviation. The second principal has two possible deviations in the menu game: $\{y_2^2\}$ and $\{y_2^1, y_2^2\}$. If the second principal plays $\{y_2^2\}$ or the menu $\{y_2^1, y_2^2\}$ then his expected utility is 1.

Using the Martimort-Stole algorithm we cannot characterize the equilibrium described above. If player 2 is playing the menu $\{y_2^1\}$, the unique best reply of principal 1 is to play the direct mechanism (with incentive compatible revelation of type):

$$\sigma_1(\theta_1) = \sigma_1(\theta_2) = y_1^1.$$

This mechanism is equivalent to the menu $\left\{y_1^1\right\}$. The best reply to the menu $\left\{y_1^1\right\}$ for principal 2 (in the set of direct mechanisms) is to play the mechanism

$$\sigma_2(\theta_1) = \sigma_2(\theta_2) = y_2^2,$$

or equivalently the menu $\left\{y_2^2\right\}$, and not the menu $\left\{y_2^1\right\}$. By construction, menus characterized by the Martimort-Stole algorithm do not embed latent decisions; they support equilibria which can also be supported by direct mechanisms. □

This methodology has been used, explicitly or implicitly, in several papers: Biais et al. [2000] Calzolari [2004], Laffont and Pouyet [2004], Martimort and Stole [2003] and Khalil et al. [2005], and in several other unpublished papers: Olsen and Osmundsen [2003], Diaw and Pouyet [2004] or Calzolari and Scarpa [2004]. These authors are able to characterize regular and realistic equilibria. Nevertheless, these equilibria could have been characterized using the standard methods of mechanisms design. Moreover, the authors may not succeed in characterizing all of the equilibria of the communication game.

When Martimort and Stole [2003] consider a complete information version of their game (i.e., when $|\Theta| = 1$; roughly speaking, when their model is qualitatively similar to our first example) by using menus, they are able to characterize equilibria that could not have been characterized by direct mechanisms. (Direct mechanisms are take-it or leave-it offers in that case.) They do not then use the former ***CLARIFY - EDITOR*** methodology. For complete information games, their analysis of their model is invaluable, as it does not rely on the former methodology.[4]

Martimort [1992] proposes an original methodology. He proposes focusing on direct mechanisms, and extends the type set: the agent can report a type belonging to the set $\tilde{\Theta}$, with $\Theta \subset \tilde{\Theta}$. At equilibrium, whatever his type is, the agent is reporting his true type, but the fact that he can report $\tilde{\theta} \notin \Theta$ (a type which does not exist) and get $y_i\left(\tilde{\theta}\right)$ (an outcome that cannot be reached if he reports any $\theta \in \Theta$), extends his possible strategies. The outcome $y_i\left(\tilde{\theta}\right)$ is never reach at equilibrium. The possibility of reporting an "absurd type" makes some deviations of the other principals unprofitable, as in examples 1 and 3. Clearly, this methodology is able to characterize equilibria that cannot be characterized if we restrict our attention to direct mechanisms. The decision $y_i\left(\tilde{\theta}\right)$ is clearly a "latent decision". Let us reconsider our last example.

Example 5 The payoffs are given by the following matrices:

[4] They use the term "singleton contracts" instead of take-it or leave-it offers.

θ_1

	y_2^1	y_2^2
y_1^1	$(2,2,3)$	$(0,3,1)$
y_1^2	$(1,0,1)$	$(1,1,4)$

θ_2

	y_2^1	y_2^2
y_1^1	$(4,2,2)$	$(2,3,0)$
y_1^2	$(1,0,1)$	$(1,1,5)$

Table 4: Common Agency with incomplete information

The equilibrium can be also characterized in the following way. The first principal plays the direct mechanism

$$\tilde{\sigma}_1 : \left\{\theta_1, \theta_2, \tilde{\theta}\right\} \to \left\{y_1^1, y_1^2\right\}$$

$$\begin{cases} \tilde{\sigma}_1(\theta_1) = y_1^1, \\ \tilde{\sigma}_1(\theta_2) = y_1^1, \\ \tilde{\sigma}_1\left(\tilde{\theta}\right) = y_1^2, \end{cases}$$

where $\tilde{\theta}$ is an absurd type.

The second principal plays the direct mechanism

$$\sigma_2 : \left\{\theta_1, \theta_2, \tilde{\theta}\right\} \to \left\{y_1^1, y_1^2\right\}$$

$$\begin{cases} \sigma_2(\theta_1) = y_2^1, \\ \sigma_2(\theta_2) = y_2^1. \end{cases}$$

The direct mechanisms $\tilde{\sigma}_1$ and σ_2 constitute an equilibrium. The best strategy for the agent is to reveal his type. (We do not describe the agent's strategy, as it is very long and is not necessary.) The first principal has no profitable strategy; he gets his maximum payoff in each state of nature. For the second principal there are many possible direct mechanisms. But the second principal cannot get a payoff greater than 2: in every state of nature, if y_2^2 is implemented, the agent will report the type $\tilde{\theta}$ to the first principal (whatever his real type) and the second principal will get a payoff of value 1. Thus, principal 2 has no profitable deviations.

If principal 1 plays the direct mechanism

$$\sigma_1 : \left\{\theta_1, \theta_2, \tilde{\theta}\right\} \to \left\{y_1^1, y_1^2\right\}$$

$$\begin{cases} \sigma_1(\theta_1) = y_1^1, \\ \sigma_1(\theta_2) = y_1^1, \end{cases}$$

then the second principal has a profitable deviation; playing the mechanisms $\overline{\sigma}_2(\theta) = y_2^2$ for every θ gives him a payoff of value 3. □

The weakness of this approach is that there is no theory of how to determine the set $\tilde{\Theta}$ and of how to construct the mechanisms σ_i for values of θ which are not in Θ. Indeed, Martimort [1992] characterizes the equilibrium using the Martimort-Stole algorithm.

6 Conclusion

Almost all of the literature on common agency with incomplete information focuses on equilibria that can be characterized by direct mechanisms.[5] Some papers explicitly apply the Revelation Principle even if it is not applicable. By doing this they may characterize only a subset of all equilibria, and miss some realistic equilibria. Other articles use different mathematical tools and more complex mechanisms, but without characterizing a larger set of equilibria.

We still lack a simple, general, systematic approach for characterizing all of the equilibria of a large class of common agency games. The complexity of the existing methodology (menu or extended types) indicates that this will be a demanding task.

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