The failure of the revenue equivalence principle: multiple objects, information acquisition and favoritism

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1 Introduction

This chapter briefly reviews the motivations behind the models of my thesis and discusses the main findings. A celebrated result of auction theory is the revenue equivalence principle which states that with independent private values and a single unit for sale all selling (or procurement) mechanisms that give the object to the bidder with the highest valuation generate the same revenue. The present thesis explores in each chapter a different deviation from the revenue equivalence principle and compares selling (or procurement) mechanisms that would be equivalent otherwise. In chapter two the equivalence between the first-price auction and the descending auction fails if more than one unit is for sale. In chapter three and four the equivalence between all four of the standard auction formats fails in the case that bidders are not fully informed about their private valuation and may acquire additional information in the course of the auction. The fifth chapter theoretically analyzes the differences between optimal auctions and negotiations that can only arise in the presence of favoritism. Each chapter is written in such a way that it can be read on its own.

The model in the second chapter entitled *"On the strictly descending multi-unit auction"* is joint work with Alexander Rasch and Achim Wambach.[1] It analyzes the bidding behavior in the descending auction if more than one unit is for sale. If one unit is for sale the revenue equivalence principle implies that this format is revenue-equivalent to the first-price auction. The model in chapter two demonstrates

[1] I presented the model at the *"Spring Meeting of Young Economists 2010"* conference, the *"Jornadas de Economía Industrial 2010"* conference, and the Brown-Bag Seminar of the Economics Department at the University of Cologne.

that this equivalence breaks down if more than one unit is for sale. We show that an equilibrium of the descending auction exists by providing a characterization of an equilibrium in terms of initial value problems. Moreover, we demonstrate that any symmetric equilibrium is inefficient. This is different in the first-price auction which unique symmetric equilibrium is efficient. This observation is not only of pure theoretical interest as our results are applicable to many real-world situations like sales in fresh produce markets, sales of event tickets, allocation of overbooked airline seats and the car-scrappage schemes during the economic crisis in the years 2008 and 2009.

The model in the third chapter entitled *"Information acquisition during a descending auction"* is joint work with Achim Wambach.[2] It compares the descending and the first-price auction in the case that bidders are not fully informed about their valuation for the object for sale and may acquire this information during or before the auction. The motivation behind this work is twofold. First, information acquisition is an important feature of real-life auctions.[3] Hence, the effects of information acquisition in standard auctions are of great interest when designing auctions. Second, the information acquisition incentives in a descending auction are different from the information acquisition incentives in the first-price auction. Hence, the comparison of these – otherwise equivalent – formats is of particular interest. The descending auction is an dynamic auction and the bidders can observe dropping price before deciding to acquire information, whereas in the first-price auction the acquisition decision has to be made prior to submitting a bid to the auctioneer. Our model shows that this feature of the descending auction makes information acquisition more desirable. This leads to the fact that if the cost of information acquisition is sufficiently low, the descending auction is more efficient than the first price auction. However, the first-price auction generates more revenue thus should be preferred by

[2] I presented the model at the *"Meeting of the European Economic Association 2012"* and the Micro-Theory-Lunch Seminar at Yale University.

[3] For example, spectrum auctions, real estate auctions, procurement auction and mergers and acquisitions.

sellers.

The fourth chapter entitled *"Regret and excess information acquisition in auctions"* is joint work with Alexander Rajko. This chapter compares – theoretically and experimentally – the second-price auction with the English auction with information acquisition. We build on previous theoretical work of Compte and Jehiel (2007) who showed that if the number of bidders is sufficiently large, the dynamic English auction with information acquisition generates more revenue than the static second-price auction. To test whether the predictions derived by Compte and Jehiel (2007) are behaviorally stable, we conduct an experiment directly attuned to their model. We find that compared to the theoretical predictions in both auction formats bidders overinvest in information and bid significantly to low if they remain uniformed. Both effects contribute to the fact that the difference in revenue between formats vanishes in our experiment. We proceed by arguing that bidders regret serves as a good explanation for the observed behavior. If a bidder is fully informed about his valuation the ex-ante optimal bids in the English auction and the second-price auction are also optimal ex-post, whereas without information the ex-ante optimal bid may turn out to be very negative ex-post. In this case the bidder may experience regret and if this feeling of regret is anticipated by the bidder, his willingness to pay for information increases. We incorporate regret in the initial model and show that the observed data is in line with the regret hypothesis.

The fifth and last chapter entitled *"Auctions vs. negotiations: The case of favoritism"* is joint work with Achim Wambach.[4] The model in this chapter compares auctions and negotiation in the presence of favoritism. Common wisdom suggests that public auctions are the more transparent processes than private negotiations. Hence, if favoritism is an issue auctions should be used instead of negotiations. However, this argument overlooks that even though auctions are conducted publicly the auctioneer may set up the rules of the auction in way that benefits one of the bidders.

[4]I presented the model at the Brown-Bag Seminar at Penn State University, the Micro-Theory Reading Group at Yale University, the workshop of the Research Group *"Design and Behavior"* at the University of Cologne, and the *"Jornadas de Economía Industrial 2012"* conference.

Moreover, even though negotiations are conducted privately the outcome of the negotiation process has to be justified publicly. Thus, auctions are not favoritism proof and the manipulation power in a negotiation is not unlimited. We provide a precise definition of both processes and derive conditions under which either of the processes may generate a higher revenue in the presence of favoritism. More precisely, if the expected punishment in the case of the detection of the manipulation is low and the number of seller sufficiently high the negotiation outperforms the auction with probability one in the parameter space. Moreover, with small expected punishment the negotiation is more efficient than the auction for any number of sellers. This is due to the fact that even though the auction generates more revenue whenever both mechanisms are manipulated or whenever both mechanisms are not manipulated, the negotiation is less likely to be manipulated if the expected punishment is small.

The last chapter has the most potential for future research. Thus, we will briefly comment on the possible extensions of the present work. Despite several efforts in the past, the question of how auctions compare to negotiation is widely unresolved. Even the most basic question of how to draw the line between auctions and negotiations has not yet been answered. What are the precise characteristics of an auction? What are the precise characteristics of an negotiation? Is difference merely semantic? The literature up to today took the approach to define auctions and negotiations in the context of a particular posed question.[5] The results are then crucially dependent on the fine details of the proposed environment and the posed question. Hence, a generalization is often not possible. Future research should be pointed towards identifying general principles and finding solutions that are robust to changes in the environment.

As long as a general characterization is not available for the comparison of auctions and negotiations one can work with the current definitions that are used by practitioners or that are given by the current legislation on public procurement.

[5]In fact in this thesis we draw the line between auctions and negotiations based on the transparency of each of the formats.

Those definitions can be used to pose auctions and negotiations as different optimal mechanism design problems and explore whether the definitions used in practice yield restrictions that lead to different optimal solutions. More generally, one could also pose the question wether the set of feasible mechanisms is restricted. One example of this approach is the clause in European public procurement laws that auctions have to be discrimination free. It is desirable to translate the notion of discrimination freedom into a mechanism design problem and solve for the optimal discrimination free mechanism. In contrast to auctions in this setting, negotiations can be modeled as any (possibly discriminating) mechanism. Thus, the question of whether auctions and negotiations yield different results reduces to: is the optimal discriminatory outcome achievable with a discrimination free mechanisms?

2 On the strictly descending multi-unit auction.

We analyze the bidding behavior in a strictly descending multi-unit auction where the price decreases continuously without going back to the initial start price once an object is sold. We prove that any symmetric equilibrium in the multi-unit descending auction is inefficient. We derive a symmetric equilibrium for general distribution functions as well as an arbitrary number of bidders and objects. Moreover, equilibrium bidding is characterized by a set of initial value problems. Our analysis thus generalizes previous results in the literature.

2.1 Introduction

Bulow and Klemperer (1994) analyze a discriminatory descending multi-unit Dutch auction with single-unit demand. They assume that if demand exceeds supply, the price clock starts again at the initial price without selling any object (i.e., the auction is not strictly descending).[1] This assumption ensures an efficient outcome which simplifies the analysis through the use of the revenue-equivalence theorem: if multiple units are for sale to N risk neutral bidders with independent private values and unit demand, all mechanisms that allocate the objects efficiently yield the same bidder surplus and seller revenue. However, their efficiency result hinges on the crucial assumption that the price goes up again if demand exceeds supply. This is an important aspect as an upward adjustment of the price is not always possible.

[1] See also Goeree, Offerman, and Schram (2006) for an experimental investigation.

Here, we drop this assumption—just like Martínez-Pardina and Romeu (2011) who, independently of our work, developed a version similar to our setup and who speak of a "single-run descending-price auction".[2]

Our contribution to the literature is fourfold: by generalizing previous results in the literature for any distribution function and an arbitrary number of objects, we can (i) show that any symmetric equilibrium is inefficient. We then derive (and prove) (ii) the equilibrium bidding strategies and (iii) characterize the equilibrium structure using the respective initial value problems. Last, we (iv) provide a closed-form solution for the uniform distribution in a situation with two objects for sale and three bidders. When comparing the strictly descending multi-unit auction to its (efficient) first-price sealed-bid counterpart, the inefficiency of the descending auction implies that the revenue in the sealed-bid auction is larger. This is a result worth stressing as it has been argued that bidding is the same in both formats which is based on the incorrect reasoning that bidders do not update their beliefs during the auction.[3]

In their closely related article, Martínez-Pardina and Romeu (2011) characterize the conditions for a monotone, symmetric equilibrium under the strictly descending auction for the case with N bidders and two units for sale. They propose a numerical solution method to obtain the bidding functions for the uniform distribution as well as for a generic distribution function. The authors also show that the strictly descending auction may result in faster sales and a smaller variance in prices which means that risk-averse and/or impatient sellers may have a preference for this type of auction. However, they derive a necessary equilibrium condition but do not show whether the solution to this condition indeed constitutes equilibrium bidding.

Our results explain observations in real-world markets that use this kind of auction (e.g., fish markets off the coast of Valencia, Spain, and fresh produce mar-

[2] Bulow and Klemperer (1994) mention this extension and give a short account of the case with a uniform distribution in Section VI.

[3] See, e.g., Krishna (2009), Sections 12.2.1 and 13.1.

kets[4]). Other examples which have the above-mentioned characteristics are the sale of concert tickets which are often sold on a first-come-first-serve basis or the airline-overbooking problem where airlines look for volunteers among stranded passengers who accept a monetary compensation in exchange for their seat.[5]

Another example was the introduction of the car-scrappage or so-called "cash for clunkers" schemes in an effort to mitigate the adverse consequences of the economic crisis in the years 2008 and 2009.[6] This scheme was introduced in a number of countries in order to help boost demand for new automobiles: owners of older cars could apply for a lump-sum state-paid allowance when they purchased a new vehicle. In Germany, for example, new owners received € 2,500 from January 14, 2009. The program was designed as a first-come-first-serve plan and scheduled to end when the maximum sum of € 5 billion in allowances was reached which occurred in September 2, 2009.[7] While the program was effective, newspapers, magazines, as well well authorities announced the latest numbers of applications on a regular basis such that potential buyers of new cars were well informed about the amount of money that was still available. The authority in charge issued a daily statement that reported the overall amount of applications received so far. Thus, like in a descending auction,

[4]See Martínez-Pardina and Romeu (2011).

[5]The use of auctions as a solution to the airline-overbooking problem was suggested by Simon (1968). This solution is now at the heart of the so-called Volunteer Auction Scheme, a voluntary bumping plan for airlines mandated by the Civil Aeronautics Board (CAB) in 1978. Simon (1994) recalled the developments that had followed his first article and that had led to the airline auction scheme. There, he mentioned that "a cruder version is for the airline to cry a price and to ask for traders". Effectively applying a reverse version of a strictly descending multi-unit auction, the airline personnel starts off with announcing a low price. If there are no or not sufficiently many volunteers who accept the current price in exchange for their seat, then the staff increases the price until a sufficiently high number of volunteers is found. This is actually what is being done by airlines for practical reasons (see, e.g., Rothstein (1985)).

[6]This application of our results is similar to Bulow and Klemperer (1994) who do not explicitly analyze auctions but are interested in investment decisions in order to explain frenzies and crashes.

[7]Note that the initial amount of € 1.5 billion was raised at the end of March due to high demand. After September 2, another 15,000 new applicants had the opportunity to register for a waiting list in case earlier applications were withdrawn. See the official website `http://www.bafa.de/bafa/de/wirtschaftsfoerderung/umweltpraemie/index.html` for details. Similarly, in the United States where the scheme was officially known as Car Allowance Rebate System (CARS), the program was extended from $1 billion to $3 billion and ended early as well.

owners of an old car would prefer to enter the program (auction) later but are afraid that at the point of entry, the budget (units) has (have) been allocated.

The chapter is organized as follows. In the next section, we set up the model and analyze equilibrium bidding behavior in the third section. In Section 2.4, we illustrate the results for the special case of uniformly distributed valuations. The last section concludes. Proofs are relegated to the appendix.

2.2 Model

Consider a setting where K identical objects are for sale to N risk-neutral bidders (with $N > K$) who all wish to purchase a single unit. Let $\mathcal{N}$ denote the set of bidders, i.e., $\mathcal{N} = \{1, .., N\}$. The reserve price for each of the units is denoted by r. Bidder i assigns a value of X_i to any of the K objects. This value represents the maximum price bidder i is willing to pay for any of the units. The valuation is independently and identically distributed on the unit interval $[0, 1]$ according to an absolutely continuous distribution function F. It is assumed that the corresponding density function f is continuous on the real interval $[0, 1]$ (where $F' \equiv f$). Bidder i only knows his own realization x_i of X_i which is not affected by the valuation of the other bidders (independent private values). Each bidder wants to buy only one of the objects for sale (single-unit demand) and maximizes expected profits. Except for the realized values, everything else is common knowledge.

Following Bulow and Klemperer (1994), we use the efficient discriminatory first-price sealed-bid (or pay-as-bid) multi-unit auction as the benchmark of our analysis. In this format, a bidder wins one of the objects paying bid b if his bid is among the K highest bids. We compare its outcome with the discriminatory multi-unit open strictly descending (or Dutch) auction. In this multi-unit Dutch auction, a price clock starts at 1 and decreases continuously. Bidders decide when to stop the clock. A winning bidder has to pay the price b at which he stopped the price clock. The bidder who stopped the auction at price b to get one of the (remaining) objects then leaves the auction. The clock then continues at b and the remaining bidders may stop the clock at any time. If more than one bidder stop the clock at the same time, either all bidders obtain a good (if sufficiently many objects are still available) or there is a lottery among those who stopped the clock where each bidder has the same probability of being chosen. Whenever we speak of the descending auction in the following, we refer to this version of the strictly descending Dutch auction just described (unless otherwise stated).

2.3 Equilibrium bidding

In order to derive the equilibrium bidding strategy, consider the following history h_k at a given price b when there are still k units available:

$$h_k = (b, b_{k+1}, \ldots, b_K)$$

where b_j (with $j \in \{k+1, \ldots, K\}$) denotes the price at which object j was sold. In this situation, a pure measurable strategy σ is a mapping

$$\sigma(x_i, h_k) : [0,1]^{K-k+2} \longrightarrow \{0,1\}.$$

The mapping specifies whether bidder i should stop the clock for a given history h_k ($\sigma(x_i, h_k) = 1$) or not ($\sigma(x_i, h_k) = 0$).

As a first general result, we observe that any equilibrium of the discriminatory descending auction is inefficient.

Proposition 1. *In the discriminatory strictly descending multi-unit auction with single unit-demand and independent private values,*

(i) there is simultaneous bidding on all *objects but the first with positive probability in equilibrium and*

(ii) the allocation of the objects is inefficient with positive probability in any symmetric equilibrium.

Proof. See the appendix. □

Efficiency means that those bidders with the K highest valuations receive the K units on offer. However, equilibrium bidding in the strictly descending multi-unit auction results in a set of bidders with different valuations who all have the same probability of winning the auction as they stop the price clock at the same time. Therefore, this format does not ensure that those bidders whose valuations for the objects are highest win the auction with certainty. The important observation here is that interestingly, pooling occurs for any object but the first. Note that simultaneous bidding means that bidders who submit a bid at the current price may not win any object for sale. Hence, if no simultaneous bidding took place, then it would never be optimal for a bidder to bid first because the bidder could gain by waiting and bidding second at a strictly lower price.

As the structure of the game is symmetric, we consider a symmetric bidding equilibrium. Furthermore, if the bidding strategy is monotonous in the sense that higher valuation types wait no longer than lower valuation types, then one would expect that the bidding behavior depends explicitly only on the current price and on the price at which the last item was sold.[8] Hence, we propose the following bidding structure:

$$\sigma(x_i, h_k) = \begin{cases} 1 & \text{if } b \leq \beta_k(x, b_{k+1}) \\ 0 & \text{if } b > \beta_k(x, b_{k+1}). \end{cases}$$

We propose a bidding function β which is a set of K functions $\beta = \{\beta_1, \beta_2, \ldots, \beta_K\}$

[8]The equilibrium strategies implicitly also depend on the number of active bidders and remaining units.

with

$$\beta_k(x, b_{k+1}) = \begin{cases} \bar{\beta}_k(x) & \text{if } x \leq c_k(b_{k+1}) \\ b_{k+1} & \text{if } x > c_k(b_{k+1}) \end{cases} \tag{2.1}$$

for any $k < K$ and

$$\beta_K(x) = \bar{\beta}_K(x). \tag{2.2}$$

$\bar{\beta}_k(x) \leq b_{k+1}$ denotes a differentiable, increasing function and $c_k(b_{k+1})$ is a cut-off-value, i.e., all types $x > c_k(b_{k+1})$ bid as soon as the auction continues after a bidder purchased object $k+1$ at price b_{k+1}. Note that the bidding function $\bar{\beta}_1(x)$ is the well established equilibrium bidding strategy for the case with $N - K + 1$ bidders competing for a single object.

The following proposition establishes existence:

Proposition 2. *For any reserve price $r > 0$ and any number of units $K < N$ in the discriminatory strictly descending multi-unit auction with single-unit demand and independent private values, there exists an equilibrium bidding function $\beta = \{\beta_1, \beta_2, \ldots, \beta_K\}$ where β_k (with $k \in \{1, 2, \ldots, K\}$) is given by expressions* (2.1) *and* (2.2)*, $\bar{\beta}_k(x)$ is given by the well defined solution to an initial value problem and $c_k(b_{k+1})$ is given by a root of a continuous function.*[9]

Proof. See the appendix. □

We use complete induction over K to prove this result. Starting at $K = 1$, the problem is just a single-unit descending auction and the equilibrium is well known. To perform the inductive step, we first look at a situation in which the first object was sold for a price b_K. The problem then reduces to a descending auction with

[9] The (standard) assumption $r > 0$ is a technical assumption that simplifies the proof to a great extent. The underlying reasoning should hold for the case where $r = 0$ (see example below). Alternatively, one could assume that the distribution F of valuations has an atom at 0.

$K-1$ units—for which the existence of an equilibrium is ensured by the induction hypothesis—and a starting price of b_K. It is shown that an equilibrium in cut-off strategies exists for the subsequent subgame, i.e., all bidders with a valuation above the cut-off value bid at the starting price. Making use of this result, we show that an increasing, differentiable bidding function for the first object exists by considering the difference in expected utility underlying the decision problem of bidding at the current price b or waiting for the auction to continue for one more (infinitesimal) tick of the price clock. We can then derive a differential equation for the bidding function and show that a unique solution to this equation exists. Furthermore, it is shown that every solution of this differential equation constitutes an equilibrium bidding function for the first object.

The structure of the bidding behavior with a cut-off-value function $c_{K-1}(b_K)$ such that all types $x \in (c_{K-1}(b_K), \bar{x}_K]$ accept price b_K simultaneously implies that the discriminatory strictly descending multi-unit auction —unlike its first-price sealed-bid counterpart— is not efficient. This inefficiency of the discriminatory format has implications for seller revenues: under the assumption that the virtual valuation (defined as $x-(1-F(x))/f(x)$) is increasing, it holds that the expected revenue in the discriminatory multi-unit first-price sealed-bid auction exceeds the expected revenue in its descending counterpart.[10]

2.4 Example

Turning to the case where bidders' valuations are uniformly distributed on $[0,1]$, Bulow and Klemperer (1994) show that if the distribution of values is uniform and an equilibrium exists, a bidder with valuation x stops the clock for the first object at the same price he submits in the discriminatory sealed-bid auction with x as the maximum valuation, i.e., a first-price sealed-bid auction where valuations are distributed according to $\tilde{F}(y) = F(y)/F(x)$. The bidding strategy for the first object

[10] See also Martínez-Pardina and Romeu (2011).

is thus $\bar{\beta}_K(x) = E[Y_K^{(N-1)}|Y_1^{(N-1)} < x]$.[11] Figure 2.1 illustrates the relationship of the discriminatory sealed-bid and descending auction for the uniform distribution and $N = 3$ and $K = 2$. The bidding behavior is illustrated for the three cases where there are still all $K = 2$ objects available and where the price clock shows a price of 1/3, 2/3, and 1, respectively.

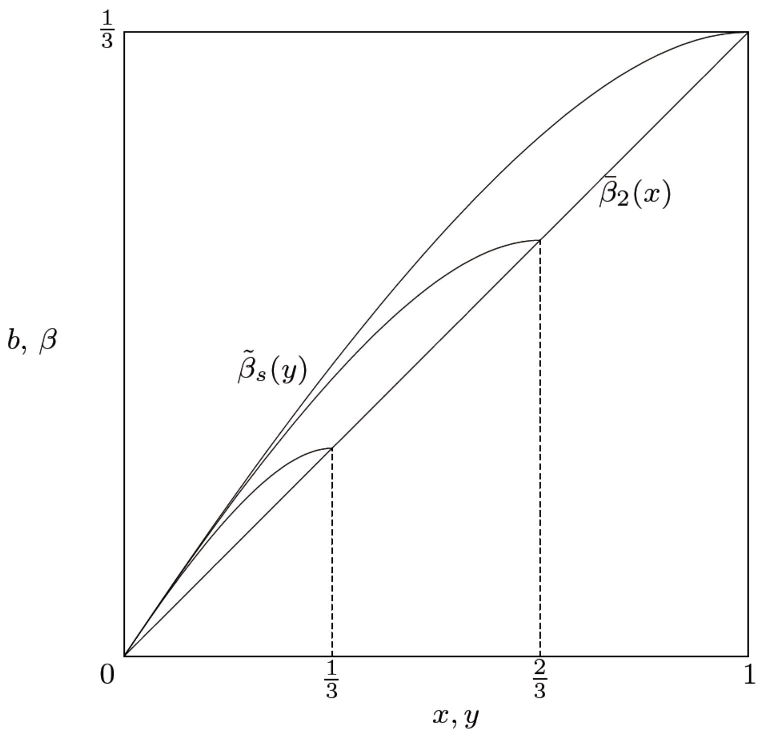

Figure 2.1: Equilibrium bidding in the descending ($\bar{\beta}_2(x) = x/3$) and the sealed-bid auction ($\tilde{\beta}_s(y)$, maximum valuations of 1/3, 2/3, and 1) for $N = 3$ and $K = k = 2$.

Now suppose that the first object is sold at a price $b_K = \bar{\beta}_K(x)$. The remaining bidders may then decide whether to buy the remaining object at price b_K or wait for the auction to continue.[12] In this situation, all bidders with a valuation x below

[11] Note that $Y_K^{(N-1)}$ denotes the K-highest order statistic of $N-1$ independent draws and equilibrium bidding is characterized as $\beta(x) = E[Y_K^{(N-1)}|Y_1^{(N-1)} < x]$ in a first-price sealed-bid auction (see Krishna (2009)). Note further that the above-mentioned relation only holds for the uniform distribution. If the distribution of values is not uniform, this result no longer holds. It can be shown that bidding for the first object in the descending auction is different from $E[Y_K^{(N-1)}|Y_1^{(N-1)} < x]$.

[12] For more on the buy-now option, see Mathews and Katzman (2006).

a cut-off value $c_{K-1}(b_K)$ (see condition (2.4) in the appendix) wait for the auction to continue and bid in the auction according to $\bar{\beta}_{K-1}(x) = E[Y_{K-1}^{(N-2)}|Y_1^{N-2} < x]$. Moreover, all bidders with a valuation $x \geq c_{K-1}(b_K)$ bid instantly.[13] Figure 2.2 illustrates this behavior for the case with $N = 3$ and $K = 2$ and where a bidder accepted price b_2 for the first unit. Note that as $E[Y_K^{(N-1)}|Y_1^{(N-1)} < x] \leq E[Y_K^{(N-1)}|Y_K^{(N-1)} < x]$, bidders wait longer in the descending auction before they stop the clock for the first unit compared to the sealed-bid auction (except for the type with the maximum valuation). However, if someone buys this unit before them, then they rush for the second unit such that simultaneous bidding might occur. So the second unit might be sold at a larger price in the descending auction compared to a sealed bid auction. Still, as argued before, the expected revenue is always smaller in the descending auction.

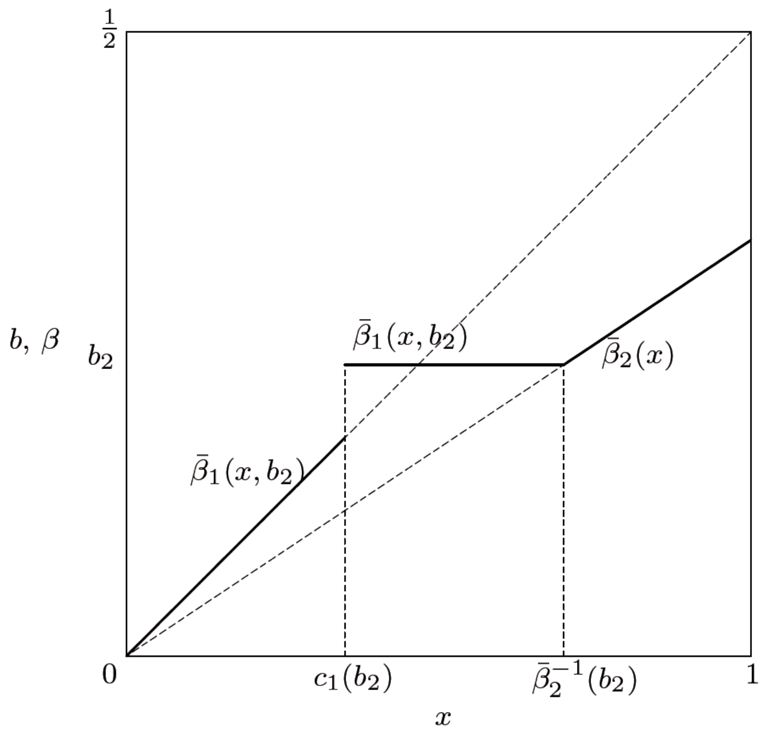

Figure 2.2: Equilibrium bidding strategies and simultaneous bidding for $N = 3$ and $K = 2$ ($\bar{\beta}_1(x, b_2) = x/2\ \forall x < c_1(b_2) = b_2/2$).

[13] This observation is what Bulow and Klemperer (1994) call a frenzy. Analogously, the ensuing decrease of the price clock where none of the remaining bidders is willing to bid corresponds to a crash.

2.5 Summary

In this chapter, we analyze bidders' optimal strategy in the discriminatory strictly descending multi-unit auction with independent private values and single-unit demand. Contrary to the arguments brought forward in the auction literature, the well established equilibrium in the first-price sealed-bid auction is not an equilibrium in the descending format. Analyzing the bidding behavior in the strictly descending multi-unit auction reveals that an equilibrium exists where once an object is sold, there is a set of bidder types who immediately accept the price in order to win the next object, i.e., simultaneous bidding occurs with positive probability. In this case, each of these bidders only receives the object with a certain probability which implies that the descending auction is not efficient. This inefficiency also has implications for the revenue comparison as revenue is always higher in the first-price sealed-bid auction.

2.6 Appendix

Proof of *Proposition 1*

Proof. Suppose the current price is $\bar{b}$ and all K objects are still for sale. The support of the distribution of bidder j's valuation is then known as

$$A_{j,K}(\bar{b}) := \bigcap_{b \geq \bar{b}} \{x \,|\sigma_{j,K}(x, b) = 0\}.$$

As we are only concerned with measurable strategies $\sigma_{j,K}$, the set $A_{j,K}(\bar{b})$ is also measurable. The new distribution of the valuation of bidder j is $P(\cdot \,\big|A_{j,K}(\bar{b}))$. Suppose bidder i with valuation x_i stops the auction as the price clock reaches b' in equilibrium. It then follows that for all $\epsilon > 0$, there exists at least one $j \in \mathcal{N} \setminus \{i\}$ such that $P(A_{j,K}(b') \setminus A_{j,K}(b' - \epsilon)) > 0$. Otherwise bidder i would gain by waiting for the price clock to decrease to price $b' - \epsilon$ to purchase a remaining object with probability 1 at a price lower than b' . Now suppose the probability for simultaneous bidding on all of the remaining objects is 0, i.e., there exists an $\epsilon > 0$ and an $\mathcal{I} \subset \mathcal{N} \setminus \{i\}$ with $|\mathcal{I}^c| \leq K - 2$ such that for all $j \in \mathcal{I}$ and all $b \in [b' - \epsilon, b']$ with $B_{j,K-1}(b) := \{x \mid \sigma_{j,K-1}(x, b, b) = 1\}$, it holds that $P(B_{j,K-1}(b) \,|A_{j,K}(b)\,) = 0$. Then, if bidder i does not bid at b' but waits for the price clock to reach price $b' - \epsilon$, he faces the risk that one of the other bidders will buy the object at a price $\bar{b} < b'$. However, in this case, $P(B_{j,K-1} \,|A_{j,K}(b)\,) = 0 \; \forall j \in \mathcal{I}$ implies that as soon as the auction continues at $\bar{b}$, at least one object will be available with probability 1. Bidder i can then purchase the object with probability 1 at a price strictly lower than b'. Thus, the proposed bidding strategy would not be optimal which is a contradiction.

In a symmetric equilibrium, this implies that $B_{j,K-1}(b) = B_{l,K-1}(b) \; \forall l, j \in \mathcal{N}$ and $P(B_{j,K-1} \,|A_{j,K}(b)\,) > 0 \; \forall j \in \mathcal{N} \setminus \{i\}$, i.e., the equilibrium is inefficient with positive probability. □

Proof of *Proposition 2*

Proof. To prove the proposition, we will use complete induction over K. The induction hypothesis can be stated as follows.

Induction hypothesis: *For each $k \leq K$, there exists an equilibrium in the discriminatory multi-unit descending auction where the bidding functions are given by expressions* (2.1) *and* (2.2).

We start the induction with $K = 1$. In this case, the problem reduces to a single-unit descending auction. From Krishna (2009) it is known that an equilibrium for this game exists and that this equilibrium is characterized by $\beta_1(x) = E[Y_1^{(N-1)} | Y_1^{(N-1)} < x]$.

To complete the induction, we have to show that if the induction hypothesis is true for all $k \leq K - 1$, it is also true for K. We divide the proof of the inductive step into four parts. More precisely, we show the following:

(i) If the first object is sold at a price b, the subsequent subgame has an equilibrium in cut-off strategies.

(ii) The necessary condition for a bidding function for the first object to be part of an equilibrium constitutes an initial value problem (IVP).

(iii) The IVP from part (ii) has a unique solution.

(iv) The solution to the IVP from part (iii) constitutes the equilibrium bidding function for the first object.

Ad (i): suppose K objects are for sale to N bidders and the first object was sold at a price b. In an increasing equilibrium the valuation of the bidder who has won the object becomes common knowledge. Denote this valuation by $\bar{x}$. Each of the remaining $N-1$ bidders can observe that one of the objects was sold and is faced with the following decision problem: he can either bid for one of the $K-1$ remaining objects or wait and observe how many objects will be bought at price b and whether the auction will continue. We will consider an equilibrium in cut-off strategies in which each bidder with a valuation $x \geq c_{K-1}(b)$ bids immediately and bidders with valuation $x < c_{K-1}(b)$ wait for the auction to continue. If a bidder with valuation x decides to wait, either all the other objects are bought at b and the expected utility of waiting is 0 or $n < K-1$ objects are being bought and the expected utility is the same as in the descending auction with $K-1-n$ items conditional on the highest valuation bidder being $c^* := c_{K-1}(b)$. We will refer to this expected utility as $U^w_{K-1-n}(x, c^*)$.[14] If a bidder decides to bid immediately, he may not be the only one to do so. More precisely, each bidder with a valuation higher than c^* will also

[14]Note that $U^w_{K-1-n}(x, c^*)$ depends on the reservation price r and therefore c^* will also depend on r.

bid immediately. Therefore, the expected utility of bidding immediately is given by

$$\begin{aligned}U^b_{K-1}(x,c^*) = &\left(\sum_{n=0}^{K-2}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2}\right)(x-b)+\\ &\left(\sum_{n=K-1}^{N-2}\frac{K-1}{n+1}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2}\right)(x-b).\end{aligned} \quad (2.3)$$

The expected utility of waiting depends on how many of the objects were bought at price b and amounts to

$$U^w_{K-1}(x,c^*) = \left(\sum_{n=0}^{K-2}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2}U^w_{K-1-n}(x,c^*,b)\right).$$

From the induction hypothesis we know that an equilibrium in each subsequent subgame exists. The necessary condition for c^* to be an equilibrium cut-off value is

$$\begin{aligned} & U^b_{K-1}(c^*,c^*) = U^w_{K-1}(c^*,c^*)\\ \Leftrightarrow\ & U^b_{K-1}(c^*,c^*) - U^w_{K-1}(c^*,c^*) = 0\\ \Leftrightarrow\ & \left(\sum_{n=0}^{K-2}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2}\right)(c^*-b-U^w_{K-1-n}(c^*,c^*,b))+ \end{aligned} \quad (2.4)$$

$$\left(\sum_{n=K-1}^{N-2}\frac{K-1}{n+1}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2}\right)(c^*-b) = 0. \quad (2.5)$$

Substituting $c^* = 0$ in the left-hand side of equation (2.4) yields $-b$. Substituting $c^* = \bar{x}$ yields $\bar{x} - b - U^w_{K-1}(\bar{x},\bar{x},b)$. If this expression is negative, the type with the highest valuation would never bid immediately, independent of the cut-off value. The problem reduces to a discriminatory descending auction with $K-1$ objects because every bidder would like to wait for the auction to continue.[15] If $\bar{x}-b-U^w_{K-1}(\bar{x},\bar{x},b)$ is non-negative, we can conclude from the intermediate value theorem that there exists

[15]In this case, the induction hypothesis ensures existence of an equilibrium.

a c^* that solves equation (2.4). To show that c^* indeed constitutes an equilibrium, we prove that $U^b_{K-1}(x, c^*) = U^w_{K-1}(x, c^*)$ is increasing in x. Differentiating with respect to x yields

$$\left(\sum_{n=0}^{K-2} \binom{N-2}{n} \left(1 - \frac{F(c^*)}{F(\bar{x})}\right)^n \left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2}\right) \left(1 - \frac{\partial U^w_{K-1-n}(x, c^*, b)}{\partial x}\right) + \left(\sum_{n=K-1}^{N-2} \frac{K-1}{n+1} \binom{N-2}{n} \left(1 - \frac{F(c^*)}{F(\bar{x})}\right)^n \left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2}\right).$$

Clearly, $\sum_{n=K-1}^{N-2} (K-1)/(n+1) \binom{N-2}{n} (1 - F(c^*)/F(\bar{x}))^n (F(c^*)/F(\bar{x}))^{N-n-2} \geq 0$ holds for all $c^* \in [0, \bar{x}]$. Therefore, in order to show that the derivative is positive, it suffices to show that $1 - \partial U^w_{K-1-n}(x, c^*, b)/\partial x \geq 0$ holds for each $n \leq K-1$ and each $x \in [0, \bar{x}]$. The induction hypothesis ensures the existence of an equilibrium in each of the subsequent subgames. Therefore, the revelation principle may be applied to each subgame. However, in a direct mechanism, the derivative of the expected utility of type x with respect to x is equal to his probability to win one of the objects.[16] Therefore, we can deduct $1 \geq \partial U^w_{K-1-n}(x, c^*, b)/\partial x$. This concludes part (i) of the inductive step.

Ad (ii): we have shown that if the first object is sold for a price b, an equilibrium in cut-off strategies for the subsequent subgame exists. To conclude the proof, we will show that an increasing, differentiable equilibrium bidding function $\bar{\beta}_K(x)$ for the first object exists. Suppose such a function exists, all but bidder 1 follow the strategy $\bar{\beta}_K(x_i)$, and bidder 1 intends to stop the auction at b. As $\bar{\beta}_K(x)$ is increasing, it cannot be optimal for bidder 1 to stop the auction at a price b higher than $\bar{\beta}_K(1)$. Hence, there exists a $z \in [0, 1]$ such that $b = \bar{\beta}_2(z)$. In order to derive the properties of the optimal bidding strategy, we need to consider the difference in expected utility underlying the following decision problem. Suppose the price on the price clock has reached $b = \bar{\beta}_K(z)$ and the bidder has to decide whether to stop the clock or let the auction continue. If he stops the clock, he gains $x_1 - \bar{\beta}_K(z)$ for

[16] See Myerson (1981) for a proof.

sure. Now suppose that he lets the auction continue for one more tick of the price clock. He then faces a trade-off of paying less for the unit and gaining $x_1 - \bar{\beta}_K(z-\epsilon)$ compared to the risk of not receiving the object at all. Two cases are relevant: first, with probability $(F(z-\epsilon)/F(z))^{N-1}$, none of the other bidders stop the clock. In this case, he gains $x_1 - \bar{\beta}_K(z-\epsilon)$ with probability 1. Second, with probability $1-(F(z-\epsilon)/F(z))^{N-1}$, one of the other bidders stops the clock at a price $\bar{\beta}_K(\tilde{z})$ with $\tilde{z} \in [z-\epsilon, z]$. As we have shown above, an equilibrium in cut-off strategies exists in the subsequent game. Therefore, bidder 1 would either bid immediately and expect a utility of $U^b_{K-1}(x_1, c(\bar{\beta}_K(\tilde{z}))$ or he would wait and expect a utility of $U^w_{K-1}(x_1, c(\bar{\beta}_K(\tilde{z}))$. Thus, the change in expected utility from waiting for another tick of the price clock either amounts to

$$\begin{aligned}\Delta U^b = &\left(x_1 - \bar{\beta}_K(z-\epsilon)\right)\left(\frac{F(z-\epsilon)}{F(z)}\right)^{N-1} \\ &+ \left(1-\left(\frac{F(z-\epsilon)}{F(z)}\right)^{N-1}\right) E\left[U^b_{K-1}(x_1, c(\bar{\beta}_K(\tilde{z}))\middle|\, z-\epsilon \le \tilde{z} \le z\right] \\ &- \left(x_1 - \bar{\beta}_K(z)\right)\end{aligned}$$

if it is optimal to submit a bid immediately in the subsequent subgame or to

$$\begin{aligned}\Delta U^w = &\left(x_1 - \bar{\beta}_K(z-\epsilon)\right)\left(\frac{F(z-\epsilon)}{F(z)}\right)^{N-1} \\ &+ \left(1-\left(\frac{F(z-\epsilon)}{F(z)}\right)^{N-1}\right) E\left[U^w_{K-1}(x_1, c(\bar{\beta}_K(\tilde{z}))\middle|\, z-\epsilon \le \tilde{z} \le z\right],\end{aligned}$$

if it is optimal to wait in the subsequent subgame. Note that for any continuous function g and any random variable Y, which is distributed according to a distribution function F with continuous density f,

$$\lim_{\epsilon \to 0} E[g(Y)|x-\epsilon \le Y \le x] = g(x)$$

holds.

As $\partial U^{\{b,w\}}(z)/\partial z = \lim_{\epsilon \to 0^+} \Delta U^{\{b,w\}}/\epsilon$, the marginal change in expected utility can be written as

$$\frac{\partial U^b(z)}{\partial z} = -\bar{\beta}'_K(z) + \left(x_1 - \bar{\beta}_K(z)\right) \frac{(N-1)f(z)F(z)^{N-2}}{F(z)^{N-1}} - \frac{(N-1)f(z)F(z)^{N-2}}{F(z)^{N-1}} U^b_{K-1}(x_1, c(\bar{\beta}_K(z))) \quad (2.6)$$

if it is optimal to submit a bid and as

$$\frac{\partial U^w(z)}{\partial z}(z) = -\bar{\beta}'_K(z) + \left(x_1 - \bar{\beta}_K(z)\right) \frac{(N-1)f(z)F(z)^{N-2}}{F(z)^{N-1}} - \frac{(N-1)f(z)F(z)^{N-2}}{F(z)^{N-1}} U^w_{K-1}(x_1, c(\bar{\beta}_K(z))) \quad (2.7)$$

if it is optimal to wait. In a symmetric equilibrium, the expected profit is maximized at $z = x_1$. Moreover, if the price clock shows $\bar{\beta}_K(x_1)$ in equilibrium, it is always optimal for bidder 1 to bid immediately in the subsequent subgame and expression (2.6) is relevant.[17] Thus, the first-order condition is $U'(x_1) = 0$. Rearranging equation (2.6) then gives

$$\bar{\beta}'_K(x_1) = \left(x_1 - \bar{\beta}_K(x_1)\right) \frac{(N-1)f(x_1)F(x_1)^{N-2}}{F(x_1)^{N-1}} - \frac{(N-1)f(x_1)F(x_1)^{N-2}}{F(x_1)^{N-1}} U^b_{K-1}(x_1, c(\bar{\beta}_K(x_1))). \quad (2.8)$$

[17] Why will bidder 1 bid immediately? Suppose the opposite is true. As $\bar{\beta}_K(x)$ is increasing, x_1 is the highest valuation in the auction. As we have shown above, the subsequent game has an equilibrium in cut-off strategies. Therefore, none of the other bidders would like to bid immediately and bidder 1 would receive the object with probability 1 and pay a price less than $\bar{\beta}_K(x_1)$. This implies that $\bar{\beta}_K(x)$ is not optimal.

Substituting expression (2.3) into the right-hand side of equation (2.8) yields

$$\bar{\beta}_K'(x_1) = \left(x_1 - \bar{\beta}_K(x_1)\right)\left(\frac{(N-1)f(x_1)F(x_1)^{N-2}}{F(x_1)^{N-1}} - \frac{(N-1)f(x_1)F(x_1)^{N-2}}{F(x_1)^{N-1}}\right.$$
$$\left(\sum_{n=0}^{K-2}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2} + \right.$$
$$\left.\left.\sum_{n=K-1}^{N-2}\frac{K-1}{n+1}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2}\right)\right). \quad (2.9)$$

Together with $\bar{\beta}_K(r) = r$, expression (2.9) constitutes an initial value problem.

Ad(iii): if $r > 0$, it follows that $F(r) > 0$ and therefore

$$\left(\frac{(N-1)f(x_1)F(x_1)^{N-2}}{F(x_1)^{N-1}} - \frac{(N-1)f(x_1)F(x_1)^{N-2}}{F(x_1)^{N-1}}\right.$$
$$\left(\sum_{n=0}^{K-2}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2} + \right.$$
$$\left.\left.\sum_{n=K-1}^{N-2}\frac{K-1}{n+1}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2}\right)\right) \quad (2.10)$$

is bounded for all $x_1 \in [r, 1]$. Hence, we can deduct that the right-hand side of equation (2.9) is globally Lipschitz-continuous. From the Picard-Lindelöf theorem[18] it then follows that the initial value problem has a unique, differentiable solution $\bar{\beta}_K(x_1)$. We have found that the first-order condition yields a unique, differentiable function. It remains to be shown that a solution of equation (2.9) is increasing and indeed optimal. Note that

$$\left(\sum_{n=0}^{K-2}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2} + \right.$$
$$\left.\sum_{n=K-1}^{N-2}\frac{K-1}{n+1}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2}\right) \quad (2.11)$$

is a probability and therefore smaller than or equal to 1. Hence, expression (2.10)

[18] A version of this theorem can be found in Coddington and Levinston (1955).

is positive and together with $x_1 - \bar{\beta}_K(x_1) > 0$, it follows that $\bar{\beta}'_K(x_1)$ is positive and thus $\bar{\beta}_K(x_1)$ is increasing.

Ad (iv): suppose that z is such that equation (2.6) is relevant. Plugging expression (2.9) into equation (2.6) yields

$$\begin{aligned}\frac{\partial U^b(z)}{\partial z} &= -\left(z - \bar{\beta}_K(z)\right)\Bigg(\frac{(N-1)f(z)F(z)^{N-2}}{F(z)^{N-1}} - \frac{(N-1)f(z)F(z)^{N-2}}{F(z)^{N-1}} \\ &\Bigg(\sum_{n=0}^{K-2}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2} + \\ &\sum_{n=K-1}^{N-2}\frac{K-1}{n+1}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2}\Bigg)\Bigg) \\ &+\left(x_1 - \bar{\beta}_K(z)\right)\frac{(N-1)f(z)F(z)^{N-2}}{F(z)^{N-1}} - \\ &\qquad \frac{(N-1)f(z)F(z)^{N-2}}{F(z)^{N-1}}U^b_{K-1}(x_1, c(\bar{\beta}_K(z))).\end{aligned} \tag{2.12}$$

Using expression (2.3), equation (2.12) simplifies to

$$\begin{aligned}\frac{\partial U^b(z)}{\partial z} &= (x_1 - z)\Bigg(\frac{(N-1)f(z)F(z)^{N-2}}{F(z)^{N-1}} - \frac{(N-1)f(z)F(z)^{N-2}}{F(z)^{N-1}} \\ &\Bigg(\sum_{n=0}^{K-2}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2} + \\ &\qquad \sum_{n=K-1}^{N-2}\frac{K-1}{n+1}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2}\Bigg)\Bigg).\end{aligned}$$

As noted before, expression (2.10) is positive. Therefore, if $z < x_1$, then $\partial U^b(z)/\partial z > 0$ and if $z > x_1$, then $\partial U^b(z)/\partial z < 0$. It is thus clear that $z = x_1$ maximizes the expected utility. It remains to be checked whether $z = x_1$ is also optimal if equation (2.7) is relevant. It is clear that equation (2.7) can only be relevant if $z > x_1$.[19]

[19] If it is optimal to bid immediately in the subsequent auction when the price is $\bar{\beta}_K(x_1)$, it is also optimal to bid immediately at any other lower price.

Plugging expression (2.9) into equation (2.7) yields

$$\begin{aligned}\frac{\partial U^w(z)}{\partial z} &= -\left(z - \bar{\beta}_K(z)\right)\left(\frac{(N-1)f(z)F(z)^{N-2}}{F(z)^{N-1}} - \frac{(N-1)f(z)F(z)^{N-2}}{F(z)^{N-1}}\right.\\ &\left(\sum_{n=0}^{K-2}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2} + \right.\\ &\left.\left.\sum_{n=K-1}^{N-2}\frac{K-1}{n+1}\binom{N-2}{n}\left(1-\frac{F(c^*)}{F(\bar{x})}\right)^n\left(\frac{F(c^*)}{F(\bar{x})}\right)^{N-n-2}\right)\right)\\ &+\left(x_1 - \bar{\beta}_K(z)\right)\frac{(N-1)f(z)F(z)^{N-2}}{F(z)^{N-1}} - \\ &\qquad \frac{(N-1)f(z)F(z)^{N-2}}{F(z)^{N-1}}U^w_{K-1}(x_1, c(\bar{\beta}_K(z))). \qquad (2.13)\end{aligned}$$

If waiting in the subsequent subgame is optimal, then

$$U^b_{K-1}(x_1, c(\bar{\beta}_K(z))) \leq U^w_{K-1}(x_1, c(\bar{\beta}_K(z)))$$

holds. Therefore, the right-hand side of equation (2.12) is greater than the right-hand side of equation (2.13). It then follows that $\partial U^b(z)/\partial z < 0$ if $z > x_1$.

Summing up, we have shown that a unique solution to the initial value problem exists if $r > 0$ and that this solution indeed constitutes the equilibrium bidding function for the first object. □

3 Information acquisition during a descending auction

We compare the effects of information acquisition during a descending auction with its static counterpart, the first-price sealed-bid auction. In a framework with heterogeneous prior information, we show that an equilibrium with information acquisition exists in both auction formats. We show that everything else equal information acquisition is more desirable in the dynamic auction. Moreover, we characterize a set of parameter values where more information is acquired in the dynamic auction in equilibrium. If the costs of information acquisition are sufficiently low, the descending auction is more efficient than the first-price sealed-bid auction although it generates less revenue.

3.1 Introduction

Information acquisition has generated significant interest in the recent literature on auctions. This is mostly due to two facts. First, the assumption that by spending resources bidders can learn more about their valuation seems to fit many real-life situations better than standard models.[1] Second, from a theoretical point of view, the effect of endogenous information acquisition on dynamic mechanisms is still to a large degree unresolved.

If bidders can acquire information about their valuation, dynamic formats and sealed-bid auction formats allow for different acquisition strategies. In a sealed-bid

[1]For example, in spectrum auctions, corporate takeovers or procurement with unknown project costs information about ones valuation is costly.

auction, bidders have to decide on information acquisition *prior* to bidding whereas in a dynamic auction, information acquisition can take place *during* the auction.

Our objective is to compare a dynamic descending auction to its sealed-bid counterpart – the first-price auction – in a framework where bidders have heterogeneous prior information with respect to their expected valuations.[2] We establish that an equilibrium in both formats exists and provide a range of parameters where bidders acquire more information in the Dutch auction. Our main insight is that if the costs of information acquisition are relatively low, the dynamic auction is ex-ante more efficient even though the sealed-bid auction generates more revenue.[3]

To get some intuition for the efficiency result, observe that in our setting, there are two sources of inefficiency: the inefficiency due to the costs of information acquisition and the inefficiency due to a possible misallocation of the object. In the first-price auction – if the costs of information is sufficiently low– all bidders will simultaneously acquire information before bidding and the subsequent allocation will be efficient. On the other hand, the option to wait for a lower price induces the bidders in the descending auction to acquire information sequentially. The bidder with the most favorable combination of expected valuation and costs of information acquisition acquires information first. If this information turns out to be favorable, she will buy the object right away and no further information acquisition takes place. While this saves on information costs, the resulting allocation may be inefficient. However, as it turns out the costs of information acquisition saved by the other bidders outweighs the efficiency loss in equilibrium. Hence, the descending auction is the more efficient

[2] If information acquisition is not an issue, both formats are strategically equivalent as they can both be modeled as identical normal form games (Klemperer, 2004; Milgrom, 1989; Krishna, 2009). Due to this apparent similarity with the first-price sealed-bid auction, the descending auction seems to have been somewhat neglected in the literature. For example, Ausubel and Crampton (2006) in their work for practitioners on "Dynamic auctions in procurement" do not consider the descending auction as a potential dynamic auction.

[3] After finishing this work, the authors were informed of the work by Miettinen (2010) who also analyzes information acquisition during a descending auction. As he considers bidders who are completely informed or completely uninformed, the effects described here, namely that bidders with different expected values have different interests in acquiring information, do not arise in his setup.

mechanism in terms of allocative efficiency and money spent on information if the costs of information acquisition is sufficiently low.

Even though the option to wait for the price to decrease renders the descending auction more efficient, it also leads to lower buying prices and therefore to a lower revenue. In particular, bidders wait until the costs of information acquisition equals the expected loss from not buying the object at the equilibrium price for his true valuation. If a bidder acquires information and realizes that the optimal stopping price has passed, she stops the clock right away. Waiting for lower prices makes high bids less likely and the subsequent immediate stopping causes a more concentrated distribution of bids and hence more intense competition. The first effect drives the expected prices down and outweighs the second effect which drives prices up.

We are the first to demonstrate that a dynamic auction with information acquisition may generate less revenue. However, due to the enhanced efficiency, the descending auction may still be a desirable format for auction designers.

Related Literature

Some authors have analyzed the incentives to acquire information before the auction (or the mechanism) has started and derived the efficient and the revenue-optimal auctions.[4] However, most of the previous work concerning information acquisition *during* a dynamic auction has focused on the comparison of the English auction and the second-price sealed-bid auction.

Compte and Jehiel (2007) show in a model where bidders are either fully informed or completely uninformed that the ascending auction leads to more acquisition of information and to a higher revenue for the auctioneer. The reason why more information is acquired is that bidders in an ascending auction can observe how many competitors are still active. This allows them to condition their information acquisition on the degree of competitiveness, i.e., they only buy information if there

[4] See Bergemann and Välimäki (2002), Persico (2000), and Shi (2011) among others.

are few remaining competitors. This is different in the descending auction as exit does not occur. The descending auction ends as soon as someone accepts the clock price. Here, information acquisition occurs because bidders can wait until the clock price has reached a relevant region where information acquisition becomes profitable.

Rezende (2005) also focuses on the comparison of the English auction and the second-price sealed-bid auction. Contrary to Compte and Jehiel (2007), bidders can only observe the clock price but not the drop-out points of other bidders. The value of information is then in avoiding to buy the good at a price higher than the bidder's valuation. Thus, information is acquired only if the price is sufficiently high. Roughly speaking, this leads to higher drop-out prices and therefore to higher revenues in the English auction. As we have argued above, this is the opposite of the effect of information acquisition in the descending auction where bidder wait for the price to decrease.

Endogenous information acquisition is not the sole framework in which the strategic considerations in a descending auction differ from the strategic considerations in a first-price auction. Carare and Rothkopf (2005) analyze a model where bidders incur costs if they delay bidding. In Katok and Kwasnica (2008), bidders are impatient in that they prefer to leave the auction earlier. Both papers come to the conclusion that in longer lasting Dutch auctions, bidders might bid more aggressively. In a framework of non-expected utility, differences between the two auction formats have also been obtained by Karni (1988) and Grimm and Schmidt (2000).

In the next section, we provide a characterization of the equilibrium of the descending auction and a proof of its existence. In section 3.3, we compare the first-price sealed-bid auction with the descending auction in terms of information acquisition strategies, efficiency, and revenue. We conclude in Section 3.4.

3.2 The descending auction: equilibrium analysis

In this section, we set up the model and analyze the equilibrium bidding and information acquisition behavior in the descending auction.

3.2.1 The model

There are n bidders competing in a descending auction for one indivisible object. Before the auction starts each bidder privately observes her expected valuation $\hat{v}_i \in [0, \bar{v}]$ where $i \in \{1, 2, ..., n\}$. The expected valuations $\hat{v}_i$ are independently and individually distributed according to a common distribution function $F_{\hat{v}}$. At any point in the auction, bidders can at costs c_i learn their true valuation v_i. The costs of information acquisition, $c_i \in [\underline{c}, \bar{c}]$, is known to player i, but cannot be observed by anybody else. However, it is commonly known that c_i is independently and individually distributed according to a common distribution function F_c.[5] The true valuation $v_i = \hat{v}_i + x$ is a sum of the expected valuation and a white noise $x \in [-\Delta, \Delta]$ with distribution function $L(x)$, i.e., $\int_{-\Delta}^{\Delta} x dL(x) = 0$. The other bidders cannot observe whether the bidder has learned her true valuation or not. All distributions are assumed to be absolutely continuous and to have densities that are bounded above and away from zero everywhere on their support.

A strategy of a bidder in a descending auction takes the following form: as the clock price is decreasing, and given that acceptance by any bidder implies that the auction ends, a bidder has to specify when to acquire information if at all and when to stop the clock given her information acquisition decision.

To simplify the exposition of the equilibrium properties, it turns out to be useful to define two functions. First, define $\beta(v)$ as the clock price at which a bidder who has learned her true valuation for free in the beginning of the auction stops the clock.

[5]Assuming that c_i and $\hat{v}_i$ are private information is a good approximation to most real-life application. We will comment on the technical implications of the assumption that the distributions of c_i and $\hat{v}_i$ are not degenerate in Subsection 3.2.3.

Second, denote by $H(v)$ the probability for a single bidder that in equilibrium the clock price will reach $\beta(v)$, assuming that this bidder does not accept the clock price before. In equilibrium it must hold that:

$$v = \text{argmax}_{v'} H(v')(v - \beta(v')). \tag{3.1}$$

3.2.2 When to acquire information

We are now in a position to state properties of the bidding function and the information acquisition function.

Proposition 3. *Let $x^*(c)$ be implicitly defined by*

$$c = \int_{x^*(c)}^{\Delta} (x - x^*(c))dL(x) = Prob(x \geq x^*(c))E[x - x^*(c)|x \geq x^*(c)]. \tag{3.2}$$

The following holds true:

(i) the bidding function of a bidder who remains uninformed is given by $\beta(\hat{v})$;

(ii) if a bidder with expected valuation $\hat{v}$ acquires information, she will do so when the clock price has reached $\beta(\hat{v} + x^*(c))$;

(iii) if at the clock price of information acquisition p she learns that $\beta(\hat{v} + x) \geq p$, she will stop the clock immediately;

(iv) if at the clock price of information acquisition p she learns that $\beta(\hat{v} + x) < p$, she will stop the clock at $\beta(\hat{v} + x)$;

Proof. The proof is relegated to the appendix. □

Bidders in the descending auction bid optimally given their available information: the expected valuation for bidders who remain uniformed (case i) and the true valuation for bidders who acquire information (case iv). However, after a bidder acquires information she may realize that the price clock is already below the optimal bid. In this case she stops the price clock right away (case iii).

The optimal timing for information acquisition is then determined by the price at which the costs of acquiring information is equal to the benefit from being informed. The costs of information acquisition is c. The benefit is to avoid the expected loss from not buying the good at the current clock price even though it would be optimal given the true valuation is larger than $\hat{v} + x^*(c)$. This consideration is reflected in equation (3.2).

It may appear surprising that independent of the number of bidders, independent of the distribution function, and independent of who acquires information in equilibrium, a bidder with expected valuation $\hat{v}$ will acquire information (if she acquires information) when the clock price in the auction has reached a level which a bidder with valuation $\hat{v} + x^*(c)$ will accept. However, note that all the factors just mentioned influence the bidding function $\beta(v)$.

3.2.3 Equilibrium existence

The next step is to determine which types will acquire information in equilibrium. For this purpose we specify the best reply to a given symmetric profile of information acquisition decisions by the other bidders. Denote by $B \subset [0, \bar{v}] \times [\underline{c}, \bar{c}]$ the set of all types that buy information and by B^c the set of all types that stay uninformed. From *Proposition 3* we know that every bidder $(\hat{v}, c) \in B$ will acquire information if the clock price has reached $\beta(\hat{v} + x^*(c))$. Such a bidder bids like a bidder with valuation $\hat{v} + x$ if she learns that her true valuation $\hat{v} + x$ is below $\hat{v} + x^*(c)$ while in case her true valuation is above $\hat{v} + x^*(c)$, she bids like a bidder with valuation $\hat{v} + x^*(c)$. A bidder that decides to stay uniformed bids like a $\hat{v}$ valuation bidder. Let $\mathcal{F}$ be the set of all absolutely continuous distributions over $[-\Delta, \bar{v} + \Delta]$ and $\mathcal{B}$ the collection of all measurable subsets of $[-\Delta, \bar{v} + \Delta] \times [\underline{c}, \bar{c}]$. Define by $T : \mathcal{B} \to \mathcal{F}$ a map between these spaces. T gives the distribution of valuations that would arise if a bidder acquires information if his type is in B, i.e.,

$$T(B)(v) = \int_B \text{Prob}\left[\hat{v} + \min\{x, x^*(c)\} \leq v \,|\hat{v}, c\right] dF_c dF_{\hat{v}} + \int_{B^c} \text{Prob}\left[\hat{v} \leq v \,|\hat{v}, c\right] dF_c dF_{\hat{v}}. \quad (3.3)$$

For a given profile of information acquisition decisions, we can write $H(v) = (T(B)(v))^{n-1}$. The bidding function in the descending auction can then be calculated to give[6]

$$\beta(v) = v - \frac{1}{H(v)} \int_{-\Delta}^{v} H(s) ds. \quad (3.4)$$

The value of information once the clock price reaches $\beta(\hat{v} + x^*(c))$ is given by the expected profit conditional on immediate information acquisition minus the expected profit in case the bidder does not acquire information, i.e.,

$$(1 - L(x^*(c))) \left[\hat{v} + E[x|x \geq x^*(c)] - \beta(\hat{v} + x^*(c))\right] + \int_{-\Delta}^{x^*(c)} \frac{H(\hat{v} + x)}{H(\hat{v} + x^*(c))} (\hat{v} + x - \beta(\hat{v} + x)) dL(x) - c - \frac{H(\hat{v})}{H(\hat{v} + x^*(c))} (\hat{v} - \beta(\hat{v})). \quad (3.5)$$

By using equation (3.4) and the definition of $x^*(c)$, the value of information can be calculated to give

$$r_H(\hat{v}, c) = \frac{1}{H(\hat{v} + x^*(c))} \left[(1 - L(x^*(c))) \int_{\hat{v}}^{\hat{v}+x^*(c)} H(s) ds - \int_{-\Delta}^{x^*(c)} \left(\int_{\hat{v}+x}^{\hat{v}} H(s) ds \right) dL(x) \right]. \quad (3.6)$$

A bidder with type $(\hat{v}, c)$ will (will not) acquire information if $r_H(\hat{v}, c)$ is positive (negative). Define the best response to a given information acquisition profile R :

[6] See, e.g., Krishna (2009), p. 19.

$\bar{\mathcal{F}} \to \mathcal{B}$ as follows:[7]

$$R(F) = \{(\hat{v}, c) \mid r_{F^{n-1}}(\hat{v}, c) \geq 0\}. \tag{3.7}$$

In a symmetric equilibrium, the information acquisition profiles have to be mutual best responses. Hence, in order to find a symmetric equilibrium, we have to find a fixed point of $T \circ R$, i.e., a $F^* \in \mathcal{F}$ with

$$F^* = T(R(F^*)). \tag{3.8}$$

The following proposition establishes that a fixed point of $T \circ R$ exists which ensures the existence of a pure-strategy equilibrium in the descending auction.[8]

Proposition 4. *There exists a symmetric pure-strategy equilibrium in the descending auction.*

Proof. The proof is relegated to the appendix. □

The proof is an application of Schauder's fixed point theorem which states that if $T \circ R$ is a continuous map from a convex subset $\mathcal{H}$ of a Hausdorff t.v.s. onto itself and $T \circ R(\mathcal{H})$ is contained in a compact subset of $\mathcal{H}$, then $T \circ R$ has a fixed point in $\mathcal{H}$.[9]

The proof of *Proposition 4* relies on the fact that the distribution functions of the expected valuations, the costs, and the true valuations are absolutely continuous.[10] However, an equilibrium of the descending auction may also exist if one or more of these distribution functions are not absolutely continuous but in this case it may fail to be in pure strategies. For example, Miettinen (2010) considers a situation where $F_{\hat{v}}$ collapses to an atom and F_c is a discrete distribution function with two

[7] Let $\bar{\mathcal{F}}$ denote the closure of $\mathcal{F}$.

[8] The underlying idea of the proof is inspired by Rezende (2005).

[9] For a proof of this version of Schauder's Theorem, see Cauty (2001).

[10] If one of the distributions is not absolutely continuous, $T \circ R$ fails to be continuous and Schauder's Theorem is not applicable.

mass points at 0 and c. He finds an equilibrium where the bidders with acquisition costs c mix with respect to their information acquisition strategy. However, his derivation relies strongly on the fact that the uniformed bidders are homogeneous with respect to their prior information, i.e. $F_{\hat{v}}$ is degenerate. As soon as we drop this assumption, the information acquisition strategies become intractable. This is due to the fact that the incentives to acquire information are not monotonic in $\hat{v}$ and there is no straightforward way to find a F^* that satisfies (3.8).[11]

3.3 Comparison with the sealed-bid auction

In the following section we compare the dynamic auction with its sealed-bid counterpart. To this end, we analyze information acquisition strategies, efficiency and revenues.

The main difference between both formats is that in the sealed-bid auction, the bidders have to decide before the auction begins whether they will acquire information. Hence, they cannot condition their information acquisition strategy on the current clock price. Apart from this aspect we can proceed in the same way as in the previous section by defining T_s as the distribution of valuations that would arise if a bidder buys information if her type is in B, i.e.,

$$T_s(B)(v) = \int_B \text{Prob}[\hat{v} + x \leq v \,|\hat{v}, c] dF_c dF_{\hat{v}} + \int_{B^c} \text{Prob}[\hat{v} \leq v \,|\hat{v}, c] dF_c dF_{\hat{v}}. \quad (3.9)$$

Denoting $H_s = T_s(B)^{n-1}$, the bidding function in the sealed-bid auction is

$$\beta_s(v) = v - \frac{1}{H_s(v)} \int_{-\Delta}^{v} H_s(s) ds. \quad (3.10)$$

[11]We obtained some numerical results by directly iterating the operator $T \circ R$ with different initial distributions $F_{\hat{v}}$. The resulting fixed points were not instructing towards finding an analytical solution. Moreover, we derived a mixed strategy equilibrium for $F_{\hat{v}} = U[0, 1]$ and different costs of information c. However, the results were not generalizable and very sensitive to the choice of c.

The value of information before the auction starts is equal to

$$
\begin{aligned}
r^s_{H_s}(\hat{v}, c) = \int_{-\Delta}^{\Delta} H_s(\hat{v} + x)(\hat{v} + x - \beta_s(\hat{v} + x))dL(x) - c - H_s(\hat{v})(\hat{v} - \beta_s(\hat{v})) \\
= \int_{-\Delta}^{\Delta} \int_{\hat{v}}^{\hat{v}+x} H_s(s)dsdL(x) - c. \quad (3.11)
\end{aligned}
$$

The best response to a given information acquisition profile is

$$
R_s(F) = \left\{ (\hat{v}, c) \mid r^s_{F^{n-1}}(\hat{v}, c) \geq 0 \right\}. \quad (3.12)
$$

An equilibrium of the sealed-bid auction is a fixed point of $T_s \circ R_s$ in $\mathcal{F}$.[12]

In general it is difficult to establish a comparison of the sealed-bid and the descending auction as Schauder's fixed point theorem does not provide any uniqueness or structural results. Thus, comparing equilibria where some types acquire information and some types remain uninformed in both auction formats is analytically intractable.[13] Hence, in what follows we circumvent this issue and focus on parameter values for which all of the bidders acquire information in equilibrium for at least one of the formats. Fortunately, this still leaves us with a variety of interesting results concerning information acquisition strategies, efficiency and revenue.

3.3.1 Comparison of information acquisition strategies

Our first major result concerns the incentives to acquire information in the descending auction and the sealed-bid auction. We show that everything else equal, the incentive to acquire information is higher in the descending auction than in the first-price auction. This is due to the fact that in the descending auction, a bidder can wait in order to acquire information only if it becomes relevant. Thus, from an

[12]The existence of a fixed point can be established in exactly the same manner as in *Proposition* 4.

[13]Numerical results may be obtained by directly iterating the operators $T \circ R$ and $T_s \circ R_s$. Revenue and efficiency results may then be established directly for certain distributions and parameters. Such a numerical exercise, however, is beyond the scope of this chapter.

ex-ante point of view, the costs of information are not given by c but by $H(\hat{v}+x^*(c))c$ as they only need to be paid if no one else has ended the auction before the information acquisition becomes relevant. Hence, the possibility to delay the information acquisition decision in the descending auction makes information acquisition more desirable in the following sense: given a distribution of the other bidders bids, a bidder who acquires information in the first-price auction also acquires information in the Dutch auction, given no other bidder has stopped the clock. We summarize this general result in the following proposition:

Proposition 5. *For any $H \in \mathcal{F}$, $R_s(H) \subset R(H)$.*

Proof. The proof is relegated to the appendix. $\square$

We can use *Proposition* 5 to characterize equilibrium information acquisition, i.e. we can find parameter values where more information is acquired in equilibrium in the descending auction than in the first-price sealed-bid auction.

Proposition 6. *The following holds true:*

(i) for each n, $F_{\hat{v}}$ (with $F_{\hat{v}}(\Delta) < 1$), and $L(x)$, there exists a $\underline{c}$ and a F_c such that an equilibrium without information acquisition exists in a first-price sealed-bid auction but not in a descending auction;

(ii) if there exists an equilibrium without information acquisition in the descending auction, there also exists an equilibrium of the first-price sealed-bid auction without information acquisition;

(iii) if there exists an equilibrium in which every bidder acquires information in the first-price auction, there also exists an equilibrium of the descending auction in which every bidder acquires information with positive probability.

Proof. The proof is relegated to the appendix. $\square$

3.3.2 Comparison of efficiency

The comparison of efficiency and revenue will be made for the case that every bidder acquires information in equilibrium. The following lemma establishes that such an equilibrium always exists if the costs of information acquisition are sufficiently low:

Lemma 1. *For each n, $F_{\hat{v}}$, and $L(x)$, there exists a $\bar{c}$ such that for all F_c with a support of $[\underline{c}, \bar{c}]$, such that $B = [0, \bar{v}] \times [\underline{c}, \bar{c}]$ in the sealed-bid and in the descending auction in equilibrium.*[14]

Proof. $r^s_H(\hat{v}, 0) > 0$ and $r_H(\hat{v}, 0) > 0$ for all $\hat{v} \in [0, \bar{v}]$ and all $H \in \mathcal{F}$. Moreover, r^s_H and r_H are continuous in c. Hence, there exists a $\bar{c}$ such that all types from $\bar{B} := [0, \bar{v}] \times [\underline{c}, \bar{c}]$ acquire information in equilibrium, i.e., $T(\bar{B})$ is a fixed point of $T \circ R$ and $T_s(\bar{B})$ is a fixed point of $T_s \circ R_s$. □

We compare the efficiency of both formats for all equilibria where all bidders acquire information before the auction in a first-price auction, and when the price is sufficiently low in the descending auction. If every bidder acquires information in the first-price auction before the auction begins, the allocation will be efficient, i.e., the bidder with the highest true valuation wins the contest. However, an inefficiency arises due to the costs of information acquisition as all bidders acquire information. These costs amount to $nE[c]$ in equilibrium.

In the descending auction, there are two types of potential inefficiencies: the inefficiency due to the costs of information acquisition and the inefficiency due to a possible misallocation of the object. Information will be acquired by at least the bidder with the highest value of $\hat{v}_i + x^*(c_i)$. More information is only acquired if the true valuation v_i for this bidder turns out to be smaller than $\hat{v}_j + x^*(c_j)$ for some other bidder j. In addition there is an efficiency loss due to the possibly inefficient allocation. This loss occurs if v_i is larger than $\hat{v}_j + x^*(c_j)$ but at the same time

[14]In the descending auction bidders only acquire information if the price is sufficiently low and if the object is still available.

$v_j > v_i$ for some other bidder j who has not acquired information yet. However, the expected efficiency loss from the possible misallocation (i.e. $v_j - v_i$) is less than the costs of one more bidder acquiring information. This is due to the fact that each bidder trades off the costs of information and the expected loss from not buying the good at the current clock price. The expected loss from not buying amounts to the difference of the expected true valuation of the bidder and $\hat{v}_j + x^*(c_j)$. This difference is at least as large as the efficiency loss. Or more precisely: The efficiency loss from misallocation is at most $x_j - x^*(c_j)$ with $x_j > x^*(c_j)$.[15] However, $x^*(c_j)$ is such that

$$c_j = \int_{x^*(c_j)}^{\Delta} (x_j - x^*(c_j))dL(x).$$

It follows that c_j is larger than the expected efficiency loss from misallocation. Hence, the descending auction is more efficient ex-ante than the first-price auction in which all bidder acquire information. We summarize this finding in the following proposition.

Proposition 7. *If in the first-price auction all bidders acquire information in equilibrium, the descending auction is more efficient than the first-price auction.*

Proof. The proof is relegated to the appendix. □

3.3.3 Comparison of revenue

For the following discussion suppose $B \in \mathcal{B}$ is the subset of types that acquire information in either auction format. If in the sealed-bid auction, all bidders with a type in B acquire information, the resulting valuation distribution will be a mean-preserving spread of the original distribution $F_{\hat{v}}$ of expected valuations. Such a

[15] The maximal efficiency loss occurs when bidder i stops the clock right before bidder j acquired information, i.e. if v_i is equal to $\hat{v}_j + x^*(c_j)$.

mean-preserving spread has two consequences: first, as the distribution is less concentrated, bidders will compete less intensely, i.e., for every given valuation the bidding function has a lower value compared to an auction without information acquisition. Second, a mean-preserving spread leads to a shift of probability mass to the extremes which makes it more likely that a bidder has a higher valuation. So compared to a situation without information acquisition the revenue might go up or down. Now in a descending auction where all types in B acquire information, the resulting consequence for the distribution is a mean-preserving spread plus an additional shift in mass to lower values. This is due to the fact that types who learn that their true valuation v is above $\hat{v} + x^*(c)$ bid as if their valuation is $\hat{v} + x^*(c)$. This weakens both effects just mentioned and it is a priori not clear which auction format generates more revenue.

We start the analysis of the revenue by showing that every thing else equal the first-price auction generates higher prices than the descending auction in the following sense: For any given set of types that acquire information with positive probability, the distribution of prices in the first-price auction first-order stochastically dominates the distribution of prices in the descending auction.

Proposition 8. *For all $B \in \mathcal{B}$, $T_s(B)$ dominates $T(B)$ in the sense of first-order stochastic dominance.*

Proof. $T_s(B)(v) \leq T(B)(v)$ for all $v \in [-\Delta, \bar{v} + \Delta]$ by inspection. □

Having fixed B the distribution of bids in a first-price auction with information acquisition is the same as in a first-price auction with the initial distribution of valuations $T_s(B)$ and no information acquisition. The same holds for the descending auction with $T(B)$ instead of $T_s(B)$. In this case, the expected price is the expectation of the second-highest order statistic of n draws from $T_s(B)$ (or $T(B)$ respectively).[16] It follows that if $T_s(B)$ first-order stochastically dominates $T(B)$,

[16]This follows from an application of the revenue equivalence theorem (see Krishna, 2009, p. 66).

then the second-order statistic of n draws from $T_s(B)$ first-order stochastically dominates the second-order statistic of n draws from $T(B)$. Using this observation and *Proposition 8* we can state the following result.

Proposition 9. *If all bidders acquire information in equilibrium the first-price auction generates strictly more revenue than the descending auction.*

Proof. Define $\bar{B} := [0, \bar{v}] \times [\underline{c}, \bar{c}]$. The revenue is the second-order statistic of n draws from $T_s(\bar{B})$ (or $T(\bar{B})$ respectively). From Lemma 8 it follows that the first-price auction generates strictly more revenue than the descending auction. □

It is an interesting feature of the descending auction that it generates less revenue than the first-price auction even though it is more efficient. From this we can conclude that the descending auction must generate more bidder surplus than the first-price auction.

Corollary 1. *If all bidders acquire information in equilibrium, the descending auction generates more bidder surplus than the first-price auction:*

If the sets of types that acquire information with positive probability in equilibrium coincide in both auction formats, the first-price auction generates more revenue than the descending auction. However, if the sets do not coincide in equilibrium, it is not clear which format will generate more revenue. As the results in Section 3.3.1 suggest more information is acquired in the descending auction, then the dominance result of *Proposition 8* does not hold anymore. The distribution of valuations will be more spread out in the descending auction causing a rise in revenue if the number of bidders is large.[17] Hence, the revenue result may be reversed. For example Miettinen (2010) finds in a framework that is a discrete limit case of the present model that the descending auction generates more revenue if the number of bidders is large.

[17]As stated above, a spread in the distribution of valuations leads to lower bids but more extreme valuations. The second effect predominates the first if the number of bidders grows large.

3.4 Conclusion

If bidders can acquire information during the auction, the descending auction might induce more bidders to acquire information as bidders can condition their decision whether to buy information or not on the price level reached. Even when the incentives to acquire information coincide in both formats, i.e., the same types acquire information in equilibrium, the descending auction is more efficient as information is only acquired if the costs of information acquisition is lower than the gain in allocative efficiency. However, as the savings in spending on information only benefit the bidder and the possible misallocation mostly hurts the seller, the descending auction generates less revenue than the first-price sealed-bid auction and bidders enjoy a higher bidder surplus.

Our informational structure is similar to the informational structure in Rezende (2005) who analyzes the English and the second-price auction. Thus, we can easily comment on how the descending and the first-price auction compare to the English and second-price auction in terms of revenue. Rezende (2005) finds that the English auction generates more revenue than the second-price sealed-bid auction – at least if the number of bidders is large. Now, in the second-price sealed-bid auction, the same types acquire information as in the first-price sealed bid auction.[18] Due to revenue equivalence, the second-price auction generates the same revenue and achieves the same efficiency as the first-price auction. Together with the revenue superiority of the English auction over the second-price auction, it follows that if the costs of information acquisition is sufficiently low, the English auction generates more revenue than the descending auction.

Nevertheless, it is not clear whether the English or the descending auction is more efficient. If the costs of information acquisition is small, all bidders acquire information with positive probability in both auctions. In the English auction, all but the bidder with the most favorable combination of expected valuation and costs of

[18]This follows from the straight forward comparison between the equilibrium conditions derived in this chapter and in Rezende (2005)

information acquisition acquire information with probability 1. In the descending auction *only* the bidder with the most favorable combination of expected valuation and costs of information acquisition acquires information with probability 1. As we have seen above, the costs of information is always higher than the gain in allocative efficiency in equilibrium. Hence, depending to the realizations of the valuations the bidder acquire too much information in the English auction and the descending auction is more efficient in those cases.

3.5 Appendix

Proof of *Proposition 3*

Proof. (i) is obvious. Concerning (iv), it is also obvious that a bidder who learns that her true valuation is v at some clock price larger than $\beta(v)$ will accept the clock price in the descending auction once it reaches $\beta(v)$. A similar argument holds for (iii): a bidder who learns her true valuation will accept the clock price immediately, if the valuation is such that a bidder with this true valuation would have accepted the clock price before. It remains to analyze case (ii), i.e., we have to determine when a bidder with expected valuation $\hat{v}$ will acquire information. From the reasoning in the example it follows that a bidder will not acquire information before the clock price has reached $\beta(\hat{v} + \Delta)$. Suppose the clock price in the auction has reached $p = \beta(\hat{v} + \Delta)$ and a bidder with expected valuation $\hat{v}$ plans to acquire information when the clock price reaches $\beta(\hat{v} + x')$ for some x' with $-\Delta \leq x' \leq \Delta$. If, at this point, this bidder learns that her true valuation is $v = \hat{v} + x$ with $x \geq x'$, she will accept the clock price immediately (result (iii)). If she learns that $x < x'$, she will wait until $p = \beta(\hat{v} + x)$ before accepting the clock price if the auction has not ended before (result (iv)). Thus, her expected profit is given by

$$\frac{H(\hat{v}+x')}{H(\hat{v}+\Delta)}\left(\int_{x'}^{\Delta}(\hat{v}+x-\beta(\hat{v}+x'))dL(x)+\right.$$
$$\left.\int_{-\Delta}^{x'}\frac{H(\hat{v}+x)}{H(\hat{v}+x')}(\hat{v}+x-\beta(\hat{v}+x))dL(x)-c\right). \quad (3.13)$$

The first term is the probability of reaching the clock price $\beta(\hat{v}+x')$ while the other terms are the expected profit conditional on reaching this clock price minus the costs of information. Maximizing this expression with respect to x' entails two terms for the borders of the integrals which cancel each other out. Thus, the maximization problem is equivalent to maximizing the expression

$$H(\hat{v}+x')\left((1-L(\bar{x}))\left(\hat{v}+E[x|x\geq\bar{x}]-\beta(\hat{v}+x')\right)-c\right)$$

with respect to x', where $\bar{x}$ has to be set equal to the optimal x' after the optimization. This expression has the same structure as the maximization problem in (3.1), so the optimal x' is given by $x^*(c) = E[x|x \geq x^*(c)] - c/(1-L(x^*(c)))$. Reformulating this expression we get:

$$c=\int_{x^*(c)}^{\Delta}(x-x^*(c))dL(x). \quad (3.14)$$

As the right-hand side is monotonously decreasing in $x^*(c)$, this expression determines $x^*(c)$ uniquely. □

Proof of *Proposition 4*

Proof. To make use of the Schauder fixed point theorem, we have to establish that $T \circ R$ is a continuous compact map from a convex subset of the normed vector space of continuous functions from $[-\Delta, \bar{v}+\Delta]$ to $[0,1]$ onto itself. More precisely, we will restrict our attention to the set $\mathcal{H} = \left\{H \in \bar{\mathcal{F}}\,|H(\epsilon) > 0\ \forall\epsilon > 0\right\}$ and to the restriction of $T \circ R$ to $\mathcal{H}$. As $\mathcal{H}$ is convex, we have to show that $T \circ R$ is continuous on $\mathcal{H}$ and that the image of $T \circ R$ is contained in a compact subset of $\mathcal{H}$. Hence, if $T \circ R$ has a fixed point in $\mathcal{F}$, it will be in $\mathcal{H}$. To see that $cl(T \circ R(\mathcal{F})) \subset \mathcal{H}$, fix an $\epsilon > 0$. If all types $(\hat{v}, c)$ with $\hat{v} \leq \epsilon$ remain uninformed with probability one, then for any $H \in \mathcal{F}$, $T \circ R(H)(\epsilon) \geq F_{\hat{v}}(\epsilon)$. Note that $F_{\hat{v}}(\epsilon) > 0$ as $F_{\hat{v}}$ is absolutely continuous and its density is bounded away from zero. If on the other hand, a subset of types $A \subset \{(\hat{v}, c) \,|\hat{v} \leq \epsilon\}$ with $\text{Prob}(A) > 0$ acquires information, then

$$
\begin{aligned}
T \circ R(H)(\epsilon) \geq L(0)\text{Prob}(A \,|\{(\hat{v}, c) \,|\hat{v} \leq \epsilon \,\}) \\
+ \text{Prob}(A^c \,|\{(\hat{v}, c) \,|\hat{v} \leq \epsilon \,\})F_{\hat{v}}(\epsilon) \geq \min\{L(0), F_{\hat{v}}\} > 0
\end{aligned}
$$

as all distributions are absolutely continuous. It follows $cl(T \circ R) \subset \mathcal{H}$. To apply Schauder's fixed point theorem, it remains to show that $T \circ R_{\mathcal{H}}$ is continuous and its image is relatively compact. We start by establishing that $T(\mathcal{B})$ is relatively compact. A set is relatively compact if it is a subset of a compact set. We apply the Arizola-Ascoli Theorem (see, e.g., Rudin (1987)), which states that a subset of continuous functions on $[-\Delta, \bar{v}+\Delta]$ is compact if and only if it is pointwise bounded, closed, and equicontinuous. As all distribution functions are pointwise bounded in the uniform norm, it remains to verify that $T(\mathcal{B})$ is equicontinuous. Fix an $\epsilon > 0$. To show equicontinuity, it is sufficient to show that for all $H \in T(\mathcal{B})$ and for all $v \in [-\Delta, \bar{v} + \Delta]$, there exists a $\delta > 0$ such that $H(v + \delta) - H(v) < \epsilon$. For any

$H \in T(\mathcal{B})$, it holds

$$\begin{aligned}
H(v+\delta) - H(v) &= \int_B \text{Prob}\left[v \leq \hat{v} + \min\left\{x, x^*(c)\right\} \leq v + \delta \,|\hat{v}, c\right] dF_c dF_{\hat{v}} \\
&+ \int_{B^c} \text{Prob}\left[v \leq \hat{v} \leq v + \delta \,|\hat{v}, c\right] dF_c dF_{\hat{v}} \\
&= \text{Prob}[B]\text{Prob}\left[v \leq \hat{v} + \min\left\{x, x^*(c)\right\} \leq v + \delta \,|B\right] \\
&+ \text{Prob}[B^c]\text{Prob}\left[v \leq \hat{v} \leq v + \delta \,|B^c\right] \\
&\leq \text{Prob}[\hat{v} + \min\left\{x, x^*(c)\right\} \in [v+\delta, v]] + \text{Prob}[\hat{v} \in [v+\delta, v]] \\
&\leq \text{Prob}[\hat{v} + x^*(c) \in [v+\delta, v]] + \text{Prob}[\hat{v} + x \in [v+\delta, v]] \\
&+ \text{Prob}[\hat{v} \in [v+\delta, v]]
\end{aligned}$$

From (3.14) we know that $x^*(c)$ is a monotone function of c. As the distribution of c is absolutely continuous, it follows that the distribution of $x^*(c)$ is absolutely continuous. Hence, as a convolution of absolutely continuous distributions, $\hat{v}+x^*(c)$ and $\hat{v}+x$ are also distributed with absolutely continuous distribution functions. It follows that there exists a $\delta > 0$ such that

$$\begin{aligned}
&\text{Prob}[\hat{v} + x^*(c) \in [v+\delta, v]] \leq \frac{\epsilon}{3}, \\
&\text{Prob}[\hat{v} + x \in [v+\delta, v]] \leq \frac{\epsilon}{3}, \\
&\qquad \text{and } \text{Prob}[\hat{v} \in [v+\delta, v]] \leq \frac{\epsilon}{3}
\end{aligned}$$

for all $v \in [-\Delta, \bar{v} + \Delta]$. As δ only depends on ϵ and not on H, we can conclude that $T(\mathcal{B})$ is equicontinuous. To see that $T \circ R_{\mathcal{H}}$ is continuous, take a sequence $(H_k)_{k\in\mathbb{N}} \in \mathcal{H}$ with $\lim_{k\to\infty} H_k = H$ uniformly. It follows $\lim_{k\to\infty} r_{H_k^{n-1}} = r_{H^{n-1}}$ pointwise. Expression (3.3) can be rewritten as

$$\begin{aligned}
T(R(H))(v) = \int \mathbf{1}_{\{r_{H^{n-1}}(\hat{v},c)\geq 0\}}(\hat{v}, c)\Big[&\text{Prob}\left[\hat{v} + \min\left\{x, x^*(c)\right\} \leq v \,|\hat{v}, c\right] \\
&- \mathbf{1}_{\{\hat{v}\leq v\}}(\hat{v}, c)\Big] + \mathbf{1}_{\{\hat{v}\leq v\}}(\hat{v}, c) dF_c dF_{\hat{v}}.
\end{aligned}$$

From the fact that the indicator function ($\mathbf{1}_{\{\}}$) is bounded by one together with the properties of the uniform norm, we obtain that

$$
\begin{aligned}
\|T(R(H_k))(v) - T(R(H))(v)\|_\infty = \\
&\Bigg\| \int \Big(\mathbf{1}_{\{r_{H_k^{n-1}}(\hat{v},c)\geq 0\}}(\hat{v},c) - \mathbf{1}_{\{r_{H^{n-1}}(\hat{v},c)\geq 0\}}(\hat{v},c) \Big) \\
&\Big[\text{Prob}\left[\hat{v} + \min\{x, x^*(c)\} \leq v \,|\hat{v}, c\right] - \mathbf{1}_{\{\hat{v}\leq v\}}(\hat{v},c) \Big] dF_c dF_{\hat{v}} \Bigg\|_\infty \\
\leq &\Bigg\| \int \Big(\mathbf{1}_{\{r_{H_k^{n-1}}(\hat{v},c)\geq 0\}}(\hat{v},c) - \mathbf{1}_{\{r_{H^{n-1}}(\hat{v},c)\geq 0\}}(\hat{v},c) \Big) \\
&\Big\| \Big[\text{Prob}\left[\hat{v} + \min\{x, x^*(c)\} \leq v \,|\hat{v}, c\right] - \mathbf{1}_{\{\hat{v}\leq v\}}(\hat{v},c) \Big] \Big\|_\infty dF_c dF_{\hat{v}} \Bigg\|_\infty \\
\leq &\Bigg\| \int \Big(\mathbf{1}_{\{r_{H_k^{n-1}}(\hat{v},c)\geq 0\}}(\hat{v},c) - \mathbf{1}_{\{r_{H^{n-1}}(\hat{v},c)\geq 0\}}(\hat{v},c) \Big) dF_c dF_{\hat{v}} \Bigg\|_\infty .
\end{aligned}
$$

Independent of v, $\int \left\| \Big(\mathbf{1}_{\{r_{H_k^{n-1}}(\hat{v},c)\geq 0\}}(\hat{v},c) - \mathbf{1}_{\{r_{H^{n-1}}(\hat{v},c)\geq 0\}}(\hat{v},c) \Big) \right\|_\infty dF_c dF_{\hat{v}}$ converges to zero whenever $\text{Prob}\left[\{r_{H^{n-1}}(\hat{v},c) = 0\}\right] = 0$. It follows that the convergence is uniform as long as bidders are indifferent with respect to buying information with probability zero. Hence, to complete the proof, we have to show that $\text{Prob}\left[\{r_{H^{n-1}}(\hat{v},c) = 0\}\right] = 0$ for all $H \in \mathcal{H}$. For this purpose, we show that $H(\hat{v} + x^*(c))r_{H^{n-1}}(\hat{v},c)$ is strictly increasing in $x^*(c)$ (and thus strictly decreasing in c) whenever $r_{H^{n-1}}(\hat{v},c) = 0$. It then follows that for each $\hat{v}$, there is at most one c such that $r_{H^{n-1}}(\hat{v},c) = 0$. Thus, $\text{Prob}\left[\{r_{H^{n-1}}(\hat{v},c) = 0\}\right] = 0$. Indeed,

$$
\begin{aligned}
\frac{\partial}{\partial x^*} \left((1 - L(x^*)) \int_{\hat{v}}^{\hat{v}+x^*} H(s)ds - \int_{-\Delta}^{x^*} \left(\int_{\hat{v}+x}^{\hat{v}} H(s)ds \right) dL(x) \right) \\
= (1 - L(x^*))H(\hat{v} + x^*). \quad (3.15)
\end{aligned}
$$

If $x^*(c) \leq 0$, then $r_{H^{n-1}}(\hat{v},c) < 0$. If $x^*(c) > 0$, then the right-hand side of equation (3.15) is strictly positive for all $H \in \mathcal{H}$. We have established that $T \circ R$ is a continuous compact map from the convex and closed subset $\mathcal{H}$ of $\bar{\mathcal{F}}$ onto itself. By Schauder's fixed point theorem there exists a fixed point of $T \circ R$ and thereby an equilibrium of the descending auction. $\square$

Proof of *Proposition 5*

Proof. To establish the result, we have to establish that $r_H(\hat{v}, c) \geq r_H^s(\hat{v}, c)$, i.e. we have to show that

$$\frac{1}{H(\hat{v}+x^*(c))}\left[(1-L(x^*(c)))\int_{\hat{v}}^{\hat{v}+x^*(c)} H(s)ds - \int_{-\Delta}^{x^*(c)}\left(\int_{\hat{v}+x}^{\hat{v}} H(s)ds\right)dL(x)\right] \\ \geq \int_{-\Delta}^{\Delta}\int_{\hat{v}}^{\hat{v}+x} H_s(s)dsdL(x) - c.$$

or equivalently

$$(1-L(x^*(c)))\int_{\hat{v}}^{\hat{v}+x^*(c)} H(s)ds + \int_{-\Delta}^{x^*(c)}\left(\int_{\hat{v}}^{\hat{v}+x} H(s)ds\right)dL(x) \\ \geq \int_{-\Delta}^{\Delta}\int_{\hat{v}}^{\hat{v}+x} H_s(s)dsdL(x) - c.$$

Observe that

$$(1-L(x^*(c)))\int_{\hat{v}}^{\hat{v}+x^*(c)} H(s)ds = \int_{x^*(c)}^{\Delta}\left(\int_{\hat{v}}^{\hat{v}+x^*(c)} H(s)ds\right)dL(x) \\ = \int_{x^*(c)}^{\Delta}\left(\int_{\hat{v}}^{\hat{v}+x} H(s)ds\right)dL(x) - \int_{x^*(c)}^{\Delta}\left(\int_{\hat{v}+x^*(c)}^{\hat{v}+x} H(s)ds\right)dL(x).$$

Since $0 \leq H \leq 1$, the last term

$$\int_{x^*(c)}^{\Delta}\left(\int_{\hat{v}+x^*(c)}^{\hat{v}+x} H(s)ds\right)dL(x) < \int_{x^*(c)}^{\Delta}\left(\int_{\hat{v}+x^*(c)}^{\hat{v}+x} ds\right)dL(x) = c.$$

Hence the result. □

Proof of *Proposition 6*

Proof. Ad (i): if no information is acquired in the first-price auction, the distribution of valuations is just $F^*(v) = F_{\hat{v}}(v)$ and the value of information is given by

$$\int_{-\Delta}^{\Delta}\int_{\hat{v}}^{\hat{v}+x} F_{\hat{v}}(s)^{n-1} ds dL(x) \leq F_{\hat{v}}(\hat{v}+\Delta)^{n-1}\int_{0}^{\Delta} x dL(x) - \int_{-\Delta}^{0}\int_{\hat{v}+x}^{\hat{v}} F_{\hat{v}}(s)^{n-1} ds dL(x).$$

The inequality is due to the fact that $F_{\hat{v}}(v)^{n-1}$ is increasing. Define $\underline{c}$ as

$$\underline{c} := \max_{\hat{v}} F_{\hat{v}}(\hat{v}+\Delta)^{n-1}\int_{0}^{\Delta} x dL(x) - \int_{-\Delta}^{0}\int_{\hat{v}+x}^{\hat{v}} F_{\hat{v}}(s)^{n-1} ds dL(x).$$

From the fact that $F_{\hat{v}}$ is absolutely continuous and the fact that $F_{\hat{v}}(\Delta) < 1$ it follows that

$$\underline{c} < \int_{0}^{\Delta} x dL(x).$$

Hence, there exists a $\tilde{c}$ such that $x^*(c) > 0$ for all $c \in [\underline{c}, \tilde{c}]$. It follows that $r_{F_{\hat{v}}(v)^{n-1}}(0, \underline{c}) > 0$, i.e., the best reply of a bidder with type $(0, \underline{c})$ is to acquire information in the descending auction. As $F_{\hat{v}}$ is absolutely continuous and $[\underline{c}, \tilde{c}]$ is of strictly positive length, it follows that $\text{Prob}[\left\{(\hat{v}, c) \middle| r_{F_{\hat{v}}(v)^{n-1}}(\hat{v}, c) > 0\right\}] > 0$, i.e. a set of types with positive measure is strictly better off by acquiring information in the descending auction. Hence, not acquiring information is not an equilibrium of the Dutch auction.

Ad (ii): if no information is being acquired in an equilibrium of the Dutch auction, the distribution of valuations is just $F^*(v) = F_{\hat{v}}(v)$ and $r_{F_{\hat{v}}(v)^{n-1}} < 0$ for all $v \in [0, \bar{v}]$ and all $c \in [\underline{c}, \bar{c}]$. From *Proposition* 5 it follows that $r^s_{F_{\hat{v}}(v)^{n-1}} < 0$. Hence, not acquiring information is also an equilibrium of the sealed-bid auction.

Ad (iii): if all bidder acquire information in an equilibrium of the first-price auction, the distribution of valuations is $F_{\hat{v}} * L(v)$ and $r^s_{F_{\hat{v}} * L(v)^{n-1}} \geq 0$ for all $v \in [0, \bar{v}]$ and all $c \in [\underline{c}, \bar{c}]$. From *Proposition* 5 it follows that $r_{F_{\hat{v}} * L(v)^{n-1}} \geq 0$. Hence, every bidder acquires information with positive probability in an equilibrium of the descending auction. □

Proof of *Proposition 7*

Proof. Take any realizations of the types $(\hat{v}_1, c_1), ..., (\hat{v}_n, c_n)$ and the true valuations $v_1, ..., v_n$. Order the bidders such that $\hat{v}_1 + x^*(c_1) \geq ... \geq \hat{v}_n + x^*(c_n)$. The efficiency loss in the first-price auction in the case where every bidder acquires information is just the sum of the costs $\sum_{i=1}^n c_i$. Now consider the descending auction. Suppose bidder 1 to bidder $m \leq n$ acquire information in equilibrium. The allocation will be efficient for these m bidders in the sense that if the object is bought by one of the bidders, it will be the bidder with $v_i \geq \max_{j\in\{1..m\}} v_j$. If $m = n$, then the efficiency is the same as in the first-price auction. If $m < n$ an allocative inefficiency may arise if a bidder $i \in \{1, ..m\}$ stops the clock but a bidder $j \in \{m + 1, ..., n\}$ has a higher true valuation for the object. This can only happen if $\hat{v}_j + x^*(c_j) < v_i$ and $v_j > v_i$. The expected efficiency loss is then at most $\hat{v}_j + \text{Prob}[x_j > x^*(c_j)]E[x_j|x_j > x^*(c_j)] - (\hat{v}_i + x_i) \leq \hat{v}_j + \text{Prob}[x_j > x^*(c_j)]E[x_j|x_j > x^*(c_j)] - \hat{v}_j - x^*(c_j) \leq \text{Prob}[x_j > x^*(c_j)]E[x_j - x^*(c_j)|x_j > x^*(c_j)] = c_j$. Hence, the efficiency loss from misallocating the good is less than the costs of information acquisition for the bidder that remained uninformed. It follows that the efficiency in the descending auction is at least as large as in the first-price auction. □

4 Regret and excess information acquisition in auctions

Information acquisition is an important feature in most auctions, where one's exact private valuation is usually unknown ex-ante. We conduct the first experiment testing a risk-neutral expected surplus maximization model with this feature. Varying the auction format and the costs of information acquisition we find excess information acquisition. Moreover, bidders remaining uninformed place bids significantly below the optimal bid. Our results can be accommodated if we assume that bidders anticipate regret.

4.1 Introduction

Typically, by investing resources bidders can gain a better understanding of their valuation for the object being auctioned. For example, in real-estate auctions, bidders can acquire expertise about the value of the property. However, to place a bid it is not necessary to obtain perfect information about one's valuation. If the cost of acquiring information is large, bidders may prefer to place bids based on partial information.

This chapter analyzes bidders' behavior in second-price and English auctions if information about a bidder's own private valuation is costly. We conduct an experiment in which we vary the cost of information and analyze the bidding and information acquisition behavior of the subjects. We find that in both auction formats, bidders acquire too much information. Moreover, bidders who choose to remain uniformed place bids significantly below the optimal bid. Both observations contradict the theoretical predictions that we derive from a risk-neutral expected surplus maximization model. This contradiction cannot be explained by risk-aversion of the subjects but rather by assuming that bidders anticipate the feeling of regret if they pay too much for the auctioned object.

Firms that participate in auctions spend a considerable amount of time and money to collect information about the value of the auctioned object. Collection of information about one's valuation is not only relevant for mergers and acquisitions where due diligence is a well established part of the process.[1] Most auction environments involve information acquisition. For example, in spectrum auctions, information is usually acquired by means of technical research about the infrastructure, internal reports on future revenues, or costs of setting up a new network. A similar logic applies in a procurement context. In preparing a bid for a procurement auction, suppliers may spend a considerable amount of resources to estimate their cost of delivering this project. Even simple bidding on Ebay for objects of personal value may require costly information acquisition in terms of cognitive cost.[2]

All of the described situations have in common that perfect information about one's own valuation is not required to participate in the auction. If cost of information acquisition is high, it may be optimal to bid based on a coarse estimation of the valuation. However, even if such a bid is optimal ex-ante, it most likely will not be optimal ex-post after the true valuation has been learned. With positive probability the price paid in the auction will exceed the actual realized value for the object. This discrepancy between the ex-ante optimal and the ex-post optimal decision may cause regret. If the bidder anticipates the feeling of regret, this concern will be

[1] Due diligence is costly and increases the precision of buyers' information about their valuation.

[2] The theoretical works on auctions with information acquisition largely focus on the comparison of different auction formats in terms of information acquisition strategies, revenues and efficiency. For auction with interdependent valuations, see Matthews (1984), Hausch and Li (1993), Bergemann, Shi, and Välimäki (2009), Persico (2000), and Hernando-Veciana (2009). For auctions with independent private values Lee (1985), Guzman and Kolstad (1997), Shi (2011), Engelbrecht-Wiggans (1988), Parkes (2005), Rasmusen (2006), Rezende (2005), Compte and Jehiel (2000), and Compte and Jehiel (2007) are relevant.

reflected in his bidding and information acquisition decision.[3]

The typical large data sets from auction platforms cannot help to explain the effects of information acquisition as these costs usually materialize outside the auction itself and hence cannot be observed (not even ex post). Therefore, we use a laboratory experiment designed along the lines of the rational choice model of auctions with information acquisition developed by Compte and Jehiel (2007).[4] The objective of this study is to understand the effects of information costs on acquisition and bidding behavior.

Our experiment analyzes two standard auction mechanisms, i.e., a second-price sealed-bid and an English auction with independent private values. Both formats are augmented with the opportunity to buy information about one's valuation. Information acquisition takes place *before* the second-price auction and at any time *during* the English auction. Prior to their information acquisition, subjects only know the distribution of their valuations but not their precise value.

To test the predictions concerning the value of information we analyze two cases: If the cost of information acquisition is low, the model predicts that in both formats subjects should acquire information with positive probability. Contrary to that, with high costs, no information acquisition should be observed in both formats.

Based on the data, we provide three new and robust insights into bidders' behavior in such auctions. First and most evidently, subjects acquire information excessively in the sense that information is acquired in more than 50% of the auctions if the cost of information is high. Second, in terms of the bidding strategies we find that

[3] Anticipated regret in auctions was brought forward as a possible explanation for overbidding in first-price auctions see: Engelbrecht-Wiggans (1989), Engelbrecht-Wiggans and Katok (2007), Engelbrecht-Wiggans and Katok (2008) and Filiz-Ozbay and Ozbay (2005) and the references therein.

[4] Compte and Jehiel (2000, 2007) compare second-price auctions to English auctions and allow for the bidders to observe the number of remaining competitors in the English auction. The equilibrium bidding and information acquisition strategies obtained by Compte and Jehiel (2000) are very intuitive and do not demand for sophisticated reasoning. Essentially the decisions of the uninformed bidders boil down to comparing two random variables. Hence, their setup is well suited for a first experimental investigation of bidders' behavior in auctions with information acquisition.

bidders who remain uninformed bid significantly below the optimal bid. Finally, in the English auction, subjects buy information prematurely and thus their timing of their information acquisition is not optimal.

We proceed by showing that excess information acquisition and underbidding can be explained by incorporating regret into the initial model. If the bidder is fully informed about his valuation for the object, the optimal bids in the second-price auction and the English auction are also optimal ex-post.[5] Hence, no regret is experienced by the bidders. If, however, the valuation is unknown, the bid placed by the bidder may either lead to a negative pay-off or the failure to win the object even when the price is below the realized valuation. Both of these situations may cause regret. If the bidder anticipates the feeling of regret, his willingness to pay for information increases. If – on the other hand – the bidder remains uninformed, our feedback procedure only induces the regret suffered from overpaying.[6] In this case it is optimal for a regret sensitive subject to shade the bid below the optimal bid.

Auctions with information acquisition are a relatively new branch in the literature. Hence, there are only few papers explicitly concerned with information acquisition in the context of auctions. To the best of our knowledge, there is only one experimental study that is broadly related to this subject: Davis, Katok, and Kwasnica (2011) experimentally compare bidder behavior in an English auction and a sequential mechanism. The bidders have to invest in information about their valuation before entering the mechanism. Contrary to our study, bidders are not able to participate if they do not invest in information, which makes investing more valuable. Similar to our findings, Davis, Katok, and Kwasnica (2011) find that irrational subjects overinvest in information and enter the mechanisms more often than predicted by theory.

[5]This is due to the fact that both auction formats are solvable in (weakly) dominant strategies.

[6]Filiz-Ozbay and Ozbay (2005) and Engelbrecht-Wiggans and Katok (2007) show that the influence of regret on equilibrium bidding depends strongly on the feedback given to bidders. Our subjects learn their true valuation only if they acquire information or win the auction. In any other case the valuation is not revealed to the subjects. This feedback procedure only enforces regret due to overpaying but not the regret due to loosing the auction.

4.2 Theory

In what follows, we proceed along the lines of Compte and Jehiel (2000). N risk-neutral bidders are competing in an auction for an indivisible object. Each bidder i assigns a value of v_i to the object. The valuation is independently and identically distributed on $[0, 1]$ according to the absolutely continuous distribution $F(v)$. Before the auction starts, all bidders but bidder 1 observe their private valuations. Bidder 1 is only informed about the distribution of the valuations. We consider two different auctions.

In the *second-price auction,* bidder 1 decides before the auction starts whether to learn his true valuation at price $c \in R_+$. After the information acquisition decision, all bidders simultaneously submit a bid for the object. The bidder with the highest bid wins the auction and pays the second highest bid to the auctioneer.

In the *English auction,* a price clock starts at 0 and continuously increases. At each price p, bidder 1 may decide whether to learn his true valuation at a cost $c \in \mathbb{R}_+$ and whether to stay in the auction or not. All other bidders only decide on whether to leave the auction at price p. Each dropping out decision is commonly observed by all bidders. The last bidder remaining in the auction receives the object at the price at which the last opponent dropped out.

Compte and Jehiel (2000) characterize the equilibria of the second-price and English auction with information acquisition by risk-neutral expected-profit maximizers.

Second-price auction

In equilibrium it is a weakly dominant strategy for the informed bidders to bid their valuations. If bidder 1 remains uniformed, his best reply to the bidding strategies of the other bidders is to bid $E[v_1]$. Hence, he will acquire information before the auction starts if the expected utility of acquiring information is higher than the expected utility of not acquiring information:[7]

$$E\Bigg[\max\{v_1, v^{(1)}\} - v^{(1)}\Bigg] - c \geq E\Bigg[\max\{E[v_1], v^{(1)}\} - v^{(1)}\Bigg]. \tag{4.1}$$

This yields the following predictions about the information acquisition and bidding behavior of bidder 1 in a second-price auction:

Fact 1. *In the second-price auction there exists a cutoff* $\tilde{c} = E\Bigg[\max\{v_1, v^{(1)}\} - v^{(1)}\Bigg] - E\Bigg[\max\{E[v_1], v^{(1)}\} - v^{(1)}\Bigg]$ *such that bidder* 1 *acquires information if* $c \leq c_s^e$. *He refrains from information acquisition whenever* $c > \tilde{c}$. *If bidder* 1 *remains uniformed in the second-price auction, he bids his expected valuation.*

English auction

Again bidders who are informed about their valuation v_i have the weakly dominant strategy to drop out whenever $p = v_i$.

To make an ex-post optimal decision whether to drop out at the current price p, it is sufficient for bidder 1 to know whether his true valuation is below the current price to avoid buying at an unfavorable price. As long as more than one competitor is still in the auction, the probability of winning the object is 0. Hence, in this case, it is a weakly dominant strategy not to buy information and to observe how strong the competition is. As soon as only one competitor is left in the auction, bidder 1 has to trade off the cost of information acquisition, the probability of winning, and the risk of buying at an unfavorable price. To formalize this trade off, define

[7] Let $v^{(1)}$denote the highest order statistic of $N-1$ independent draws from F.

$$H(p,c) := E\left[\max\{v_1, v_2\} - v_2 | v_2 \geq p\right] - c$$
$$K(p) := E\left[\max\{E[v_1], v_2\} - v_2 | v_2 \geq p\right]$$

$$p^{**}(c) := \sup\{p \in [0,1] | H_e(p,c) \geq K_e(p)\}, \text{ and} \tag{4.2}$$
$$p^{*}(c) := \inf\{p \in [0,1] | E[\max(p, v_1) - v_1] - c \geq 0\}. \tag{4.3}$$

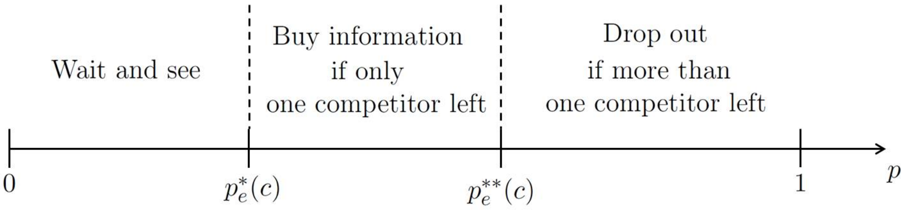

Figure 4.1: Structure of the information acquisition and drop out decisions of the uninformed bidder.

$p^{**}(c)$ is the highest p such that the expected payoff from buying information at p exceeds the expected payoff from not buying information. $p^*(c)$ is the lowest p such that the information cost is lower than the expected loss from buying at a price above valuation. The equilibrium behavior of bidder 1 can then be characterized as follows:

Fact 2. *In the English auction there exist $p^*(c)$ and $p^{**}(c)$* given by (4.2) and (4.3) *such that:*

If there are at least two other competing bidders left in the auction at price p, bidder 1 does not acquire information and stays in the auction as long as $p < \max\{p^*(c), E[v_1]\}$. If only one other competing bidder is left in the auction, two cases are relevant:

(i) *If* $p^*(c) > p^{**}(c)$, *bidder* 1 *never acquires information and drops out as soon as the price reaches* $E[v_1]$.

(ii) *If* $p^*(c) < p^{**}(c)$, *no information has been acquired by bidder* 1 *and*

a) $p \in [p^*(c), p^{**}(c)]$, *bidder* 1 *acquires information, drops out immediately if* $p > v_1$, *or stays in the auction if* $p \leq v_1$*;*

b) $p > p^{**}(c)$, *bidder* 1 *drops out at;*

c) $p < p^*(c)$, *bidder* 1 *stays in the auction.*

The information acquisition strategy is depicted in Figure 4.1. A formal proof can be found in Compte and Jehiel (2000).

4.3 Experimental design, procedures, and predictions

For our experimental design, we follow the basic structure of the already characterized theoretical model. Accordingly, in every auction group, only one player does not know his valuation ex ante. The other players always have perfect information about their valuations. We choose groups of four players and, as we are only interested in the behavior of the one uninformed player per group and the informed players have a salient dominant strategy, we opt to implement the three remaining players as bidding robots. These robots are programmed to always bid their true valuation, which is also explained to the human subjects in the instructions.

The uninformed human bidder only knows that his valuation is uniformly distributed between 0 and 100 ECU. Moreover, he knows that the valuations of the three bidding robots were drawn from the same distribution. The main innovation of our experiment is the investigation of information acquisition in auctions. Hence, we offer the human participants with unknown valuation to buy information about their valuation at a certain cost. This cost parameter is varied between the treatments in order to test our theoretical predictions: low costs ($c = 2$) and high cost ($c = 8$).

Furthermore, we use the second-price price auction as a static and the English auction as a dynamic format. The object is not specified and profits for the auction are calculated as one's valuation minus the final price and if applicable minus the cost for information acquisition. This gives the 2×2 design for the four experimental treatment variations shown in Table 4.1.

Table 4.1: Experimental treatments

	Information costs	
Auction format	Low cost	High cost
Second-price auction	$c = 2$	$c = 8$
English auction	$c = 2$	$c = 8$

In the second-price auction, information acquisition is only possible prior to the auction, i.e., before submitting the individual sealed-bid offer. During the English auction, information can be acquired throughout the auction. To this end, we implemented a pause button, enabling subjects to pause the price clock at any time to buy information. Thus, we can rule out any time pressure effects, shaping the decision of buying information or quitting the auction. The price clock increases by 1 ECU every two seconds, which is similar to other experiments on English auctions (Levin, Kagel, and Richard, 1996). Overall, every auction format is repeated for 20 rounds and valuations for all players are redrawn for every round.

Concerning feedback, we provide the subjects with the information whether they won the auction, at what price the auction was won, what their final bid was, and what the subject has won or lost in this round including the information costs. Hence, learning one's valuation remains costly. If the human bidder wins the auction, we additionally give feedback regarding his valuation, which is then known anyway, and makes it easier to check the profit calculations. As it is crucial for us that subjects assess the real costs of information acquisition, we stress in the instructions that all losses in experimental money must also be covered in real money after the experiment.

The experimental sessions took place in the Cologne Laboratory for Economic Research (CLER) in April 2011. We had 30 subjects per treatment and 120 subjects overall participating. The average payment was 13.20€ including a guaranteed show-up fee of 2.50€. On average each subject participated in the experiment for 75 minutes. For the recruitment procedure, we used the online recruitment system ORSEE (Greiner, 2004) and the experiment itself was programmed with z-Tree (Fischbacher, 2007).

Predictions

The hypotheses for the behavior in our experiment are directly computed from the equilibrium characterization in Section 4.2. Subtracting the expected profit from uninformed bidding from the expected profit of informed bidding yields:

$$\tilde{c} = E\left[\max\{v_1, v^{(1)}\} - v^{(1)}\right] - E\left[\max\{E[v_1], v^{(1)}\} - v^{(1)}\right] \approx 3.4.$$

Hence, the subjects should always acquire information if the cost of information acquisition is below 3.4 and always refrain from doing so if the cost is above 3.4. This leads to the following hypothesis:

Hypothesis 1. *In the low-cost treatment of the second-price auction ($c = 2$), all subjects should acquire information prior to bidding. In the high-cost treatment of the second-price auction ($c = 8$), all subjects should refrain from information.*

Concerning the bidding strategy, we can state the following hypothesis:

Hypothesis 2. *If bidder 1 acquires information, he bids his valuation, i.e. $b = v_1$. If bidder 1 remains uninformed in the second-price auction, he bids his expected valuation, i.e. $b = E[v_1] = 50$.*

We summarize the predictions for the second-price auction in Table 4.2.

Table 4.2: Predictions for information acquisition in the second-price auction.

Treatment	**Information acquisition**	**Bidding**
second-price auction ($c = 2$)	always	$b = v_1$
second-price auction ($c = 8$)	never	$b = E[v_1] = 50$

In the English auction bidder 1 will acquire information whenever only one competitor is left in the auction and the price p is above $p^*(c)$ and below $p^{**}(c)$. If $p^{**}(c) > p^*(c)$, information acquisition takes place with probability 0 in equilibrium. Hence, for the English auction we expect no information acquisition in the high-cost treatment as $p^{**}(8) > p^*(8)$. In the low-cost treatment, we can make a more sophisticated prediction regarding the timing of information acquisition:

Hypothesis 3. *In the low-cost treatment of the English auction ($c = 2$), bidder 1 never acquires information as long as more than one other bidder is active in the auction. If only one other bidder is left in the auction, bidder 1 will acquire information if $p^*(2) = 20 < p < p^{**}(2) = 67$. If $p > 67$, he will exit. In the high-cost treatment of the English auction ($c = 8$), bidder 1 never acquires information.*

Concerning the bidding strategy, we can state the following:

Hypothesis 4. *If bidder 1 learns his true valuation, he drops out of the auction as soon as the price has reached his valuation. If bidder 1 remains uninformed, he drops out at $E[v_1] = 50$.*

Table 4.3: Predictions for information acquisition in the English auction.

	Information acquisition		**Bidding**	
Treatment	$p^*(c)$	$p^{**}(c)$	*informed*	*uninformed*
English auction ($c = 2$)	20	67	$b = v_1$	$b = 50$
English auction ($c = 8$)	never	never	–	$b = 50$

We summarize the predictions for the English auction in Table 4.3.

4.4 Experimental analysis and results

4.4.1 Excess information acquisition

First of all, we analyze the frequencies with which subjects harness information acquisition. For the high-cost treatments, where the theoretical prediction is that information is never bought, the data in Figure 4.2 shows that subjects nevertheless buy information in 59% and 51% of all auctions. This shows excess information acquisition compared to the theoretical prediction in both formats.[8]

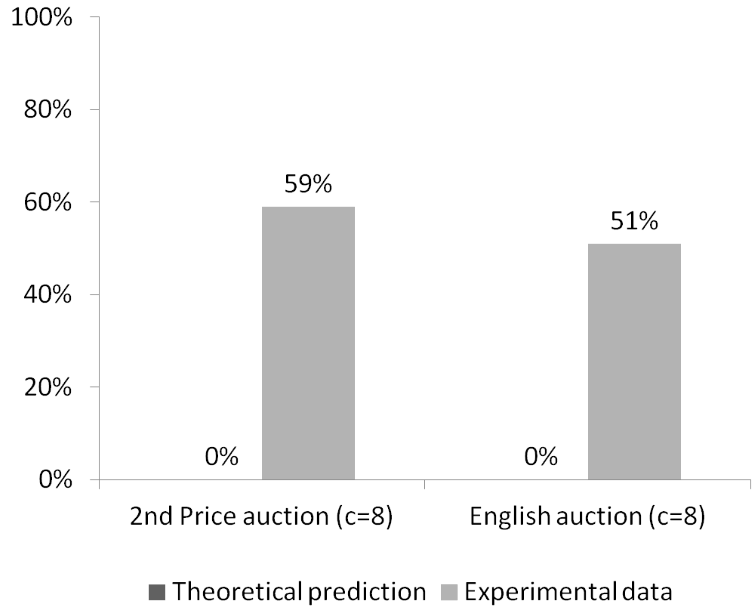

Figure 4.2: Frequency in information acquisition ($c = 8$).

Keeping in mind that the optimal threshold for not buying information at all is at information costs of 3.4 ECU and that acquiring information yields a negative expected profit for costs above 4.9 ECU, the subjects are treating these costs differently than in our rational choice model. In the high-cost treatment, we deliberately choose prohibitively high costs of $c = 8$ in order to make the decision of not buying information easy. This strong effect of excess information acquisition offers a clear

[8]T-test: p-value < 0.0001.

indication that the subjects assess the situation differently and overestimate the benefits of additional information. In Section 4.5 of this chapter, we will discuss a model extension assessing this strong effect.

Result 1. *Information is acquired excessively in both auction formats if the cost of information acquisition is high.*

Having established the first effect, we compare the data from the two low-cost treatments. Here the effect of excess information acquisition is also found in the dynamic format.[9] However, in the static format, where the rational model revealed that buying information is always optimal, we find less information acquisition than predicted.[10] Overall, none of the treatments yields the information acquisition strategies, which one would expect based on the fully rational, risk-neutral model. In three of the four treatments, we observe excess information acquisition, in one treatment the opposite.

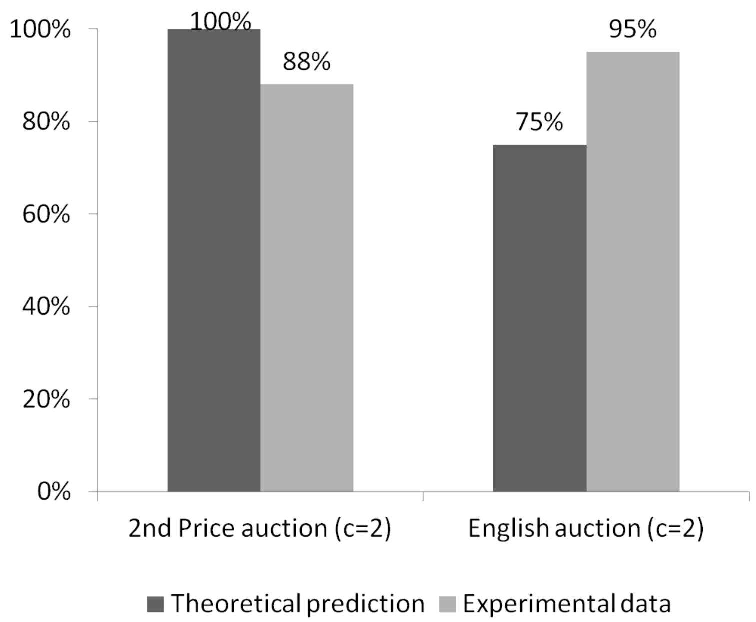

Figure 4.3: Frequency of information acquisition

[9]T-test: p-value <0.0001.
[10]T-test: p-value <0.0001.

4.4.2 Premature information acquisition

For the dynamic format (i.e., the English auction) in our experiment, the theory does not only make predictions about the optimal frequencies of information acquisition but also about the optimal timing of doing so. Therefore, we first analyze the average price clock at time of information acquisition and compare it with the optimal value p^* in Table 4.4.

Table 4.4: Price clock at information acquisition.

Treatment	**Average clock price at information acquisition**	**Prediction**
English ($c = 2$)	2 ECU	20 ECU
English ($c = 8$)	12 ECU	never

Here we find strongly premature information acquisition. In the low-cost treatment, where subjects should optimally buy at a price of 20 ECU, given our realization of the random variable, subjects on average buy almost immediately after the start of the auction at a price of 2 ECU. This means that subject acquire information after observing the competitors behavior for at most four seconds although they know that the auction could be paused at any time. Furthermore, as long as there are at least two competitors remaining, the probability of the auction terminating at the next price step is virtually zero. Hence, the pivotal information acquisition decision in the dynamic format should factor in the additional information about the number of competitors. This information allows the subjects to learn about the valuations of the competitors at no cost and no risk.

Table 4.5 corroborates the initial finding of premature information acquisition as subjects fail to wait for the previous bidders to drop out before they decide on whether information should be bought. In the low-cost treatment, where information should be bought in 75% of the auctions when only one bidder is remaining, only 2.6% of information acquisitions can be classified as optimal in that sense. Interest-

Table 4.5: Number of opponents at information acquisition (English auction).

Treatment	**Information acquisition with 1 opponent remaining**	**Prediction**	**Information acquisition with ≥ 2 opponents remaining**	**Prediction**
English ($c = 2$)	2.6 %	75 %	97,4 %	0 %
English ($c = 8$)	20.2 %	0 %	79.8 %	0 %

ingly, in the high cost treatments, where information acquisition should never occur according to our model, subjects who nevertheless engage in information acquisition do so after observing the competition longer and hence factoring in previous drop outs.[11]

Result 2. *Information is acquired prematurely in the English auction.*

4.4.3 Underbidding behavior

Next, we consider the bidding strategies employed by the subjects. First of all, we analyze the bidding strategies, where the bidder chooses to buy information and thus had been perfectly informed. Here we can calculate the deviations between the bids and valuations for all auction rounds where a subject had information.[12] We define valuation bidding as bidding one's valuation plus 1 or minus 1, if informed. Hence, underbidding is every bid under the valuation minus 1 and overbidding every bid above the valuation plus 1.

[11] This finding is consistent with the price clock data from the high-cost treatment, where the average clock price at information acquisition is 12 ECU.

[12] The English auction ends once the second last bidder dropped out so that we can not observe the full bidding strategy of a winner. Hence, the estimated average bid in the English auction is merely a lower bound on the actual average bid.

Table 4.6: Bidding given information has been acquired.

Treatment	**Underbidding**	**Valuation bidding**	**Overbidding**
2nd price ($c = 2$)	24%	55.2%	20.8%
2nd price ($c = 8$)	27.8%	51.7%	20.5%
English ($c = 2$)	34.2%	62.8%	2.9%
English ($c = 8$)	41.7%	45.5%	12.8%

Table 4.6 shows that we do not find that overbidding is as strong as in previous studies, whereas underbidding is a predominant pattern in subjects' behavior. This finding differs from many the standard results received in second-price auctions (Cooper and Fang, 2008; Kagel and Levin, 1993).

Result 3. *Bidding strategies of informed bidders exhibit an underbidding effect.*

Additionally, we consider the bidding strategies without information acquisition. As in the low-cost treatment, almost all subjects always buy the available information, this data is not reliable as the remaining few observations are probably subject to a strong selection effect. However, for both high-cost treatments, we have approximately 50% rates for information acquisition, which is robust with respect to learning or timing effects. Therefore we can report the mean bids for these treatments.

Table 4.7: Mean bids without information in the high-cost treatment.

Treatment	**Mean Bid**	**Prediction**
2nd Price (c=8)	40.6 (1.74)	50
English (c=8)	36.7 (1.64)	50

Given our parametrization, the optimal bid in the second-price auction should be 50 and in the English auction without information acquisition the optimal bid is also equal to 50. Again, the experimental results clearly depart from these predictions. In both treatments, we find severe underbidding, which is in stark contrast to our theoretical prediction:[13]

[13] A regression analysis of the main findings can be found in the appendix.

Result 4. *Bidding strategies of uninformed bidders exhibit an underbidding effect.*

4.5 Alternative hypothesis: regret avoidance

Our hypotheses for the experiment are derived from a model with risk-neutral expected-profit-maximizing bidders. As we have seen, the experimental results contradict the predictions derived in Section 4.2. As presented in Section 4.4, the main deviations from the predictions are that bidders acquire more information than predicted and that uninformed bidder bid significantly below $E[v_1]$. In what follows we explore an alternative theory to explain the experimental results: regret avoidance.[14]

A bidder who acquires information and bids his valuation in the second-price auction bids always optimally – not only from the ex-ante point of view but also from the ex-post point of view.[15] By contrast, without acquiring information, the bidding decision may turn out suboptimal ex-post.

Regret enters the bidding decision along two dimensions: winners regret and losers regret. For example, suppose the bidder does not acquire information and places a bid of $E[v_1] = 50$ ECU in the second-price auction. At the end of the auction, he learns that the second highest bid is 40 ECU but his valuation is 10 ECU. In this case, the bidder loses 30 ECU. If he had acquired information and bid his valuation, this loss could have been avoided. The fact that the ex-ante best bid is no longer the best bid ex-post will make the bidder regret his decision. In particular, if the placed bid is above the realized valuation, there is a chance that the bidder may purchase the object at a price higher than his valuation.

On the other hand, suppose that at the end of the auction the same bidder learns that the second highest bid is 60 ECU but his valuation is 90 ECU. In this case,

[14]Risk-aversion is usually the first explanation that is brought forward if experimental results differ from expected-profit-maximizing behavior. In Appendix 4.7 we argue that while risk-aversion can explain the underbidding effect of uninformed bidders it fails to account for the deviation in the information acquisition strategy.

[15]In the English auction, dropping out when the price reaches a bidder's valuation is ex-post optimal for any realization of the other bidders valuations.

the bidder would have won 30 ECU if he had acquired information and placed a bid equal to his valuation. Again, the best bid ex-ante is no longer the best bid ex-post and the bidder will regret his decision. In particular, if the placed bid is below the actual valuation, there is a chance that one of the other bidders wins the object at a price that is below the valuation of the bidder.

Filiz-Ozbay and Ozbay (2005) and Engelbrecht-Wiggans and Katok (2007) show that the influence of regret on equilibrium bidding depends strongly on the feedback given to bidders. In our set-up we do not reveal the true valuation to bidders who do not acquire information and lose the auction. Hence, losing bidders cannot observe whether the bidding decision was optimal ex-post. Thus, regret from loosing the auction should *not* influence their bidding decision.

In what follows, we focus on the effect of the regret from winning the auction on the information acquisition and bidding decision and adapt the model of Filiz-Ozbay and Ozbay (2005) to our situation.[16]

4.5.1 Second-price auction

Suppose a bidder did not acquire information and placed a bid based on his expected valuation in the subsequent second price auction. Formally winner regret in a second-price auction translates to the following form of utility for bidder i:

$$u_i(v_i, b^{(1)}) := \begin{cases} v_i - b^{(1)} & \text{if } v_i \geq b^{(1)} \text{ and } i \text{ wins,} \\ v_i - b^{(1)} - r(b^{(1)} - v_i) & \text{if } v_i < b^{(1)} \text{ and } i \text{ wins,} \\ 0 & \text{if } i \text{ loses,} \end{cases} \tag{4.4}$$

where $b^{(1)}$ is the highest bid of the competitors and $r(\cdot) : \mathbb{R} \to \mathbb{R}_+$is the regret

[16]Anticipated regret in auctions was originally brought forward as an explanation for overbidding in first-price auctions (see, e.g., Filiz-Ozbay and Ozbay, 2005, or Engelbrecht-Wiggans and Katok, 2007).

function which is assumed to be non-negative and non-decreasing. For the informed bidders, it is a weakly dominant strategy to bid their valuation. Hence, those bidders never experience regret. If the uninformed bidder remains uniformed, his best reply b^* to the bidding strategies of the informed bidders is the solution to[17]

$$\max_b \int_0^b \int_0^1 (v_1 - v^{(1)}) - \chi_{\{v_1 \leq v^{(1)}\}}(v_1) r(v^{(1)} - v_1) dF(v_1) dF^{N-1}(v^{(1)}). \tag{4.5}$$

The first-order condition for this problem amounts to

$$\left(\int_0^{b^*} (v_1 - b^*) - r(b^* - v_1) dF(v_1) + \int_{b^*}^1 (v_1 - b^*) dF(v_1) \right) (N-1) f(b^*) F^{N-2}(b^*) = 0. \tag{4.6}$$

If r is strictly positive on a subset of $[0, 50]$ with Lebesgue measure larger than 0, it directly follows that $b^* < E[v_1]$ and that

$$\int_0^{b^*} \int_0^1 (v_1 - v^{(1)}) - \chi_{\{v_1 \leq v^{(1)}\}} r(v^{(1)} - v_1) dF(v_1) dF^{N-1}(v^{(1)}) \leq E\left[\max\left\{ E[v_1], v^{(1)} \right\} - v^{(1)} \right]. \tag{4.7}$$

If bidder 1 acquires information, he bids his valuation and his bid is ex-post optimal which means he does not experience regret. Hence, he will acquire information before the auction starts if the expected utility of acquiring information is higher than the expected utility of remaining uninformed:

[17] $\chi_M(\cdot)$ denotes the indicator function with $\chi_M(x) = 1$ if $x \in M$ and $\chi_M(x) = 0$ otherwise.

$$E\left[\max\{v_1, v^{(1)}\} - v^{(1)}\right] - c \geq \int_0^{b^*}\int_0^1 (v_1 - v^{(1)}) - \chi_{\{v_1 \leq v^{(1)}\}} r(v^{(1)} - v_1) dF(v_1) dF^{N-1}(v^{(1)}). \quad (4.8)$$

Comparing inequation (4.8) and (4.1) using inequality (4.7) yields the following information acquisition and bidding behavior of bidder 1 in a second-price auction with regret:

Fact 3. *In the second-price auction with regret, there exists a cutoff $c^r > c^e$ such that bidder 1 acquires information if $c \leq c^r$. He refrains from information acquisition whenever $c > c^r$. If bidder 1 remains uniformed in the second-price auction with regret, he bids $b^* < E[v_1]$.*

This implies that there exists a range of information cost such that bidder 1 acquires information in the second-price auction if and only if he anticipates the feeling of regret. Moreover, if the feeling of regret is anticipated, bidders who remain uniformed bid below expected valuation. Hence, with the appropriate definition of $r(\cdot)$, a theory of regret accounts for the departure from risk-neutral expected-surplus maximization in the second-price auction as described in Result 1 and Result 4.

4.5.2 English auction

Formally, the following form of utility for bidder i accounts for winner regret in an English auction :

$$u_i(v_i, b^{(1)}) := \begin{cases} v_i - p & \text{if } v_i \geq p \text{ and } i \text{ wins,} \\ v_i - p - r(p - v_i) & \text{if } v_i < p \text{ and } i \text{ wins,} \\ 0 & \text{if } i \text{ loses,} \end{cases} \quad (4.9)$$

where p is the price at which the last competitor left the auction and $r(\cdot) : \mathbb{R} \to \mathbb{R}_+$is the regret function as defined above. As before, for bidders who are informed about their valuation v_i it is a weakly dominant strategy to drop out whenever $p = v_i$. Hence, those bidders never experience regret.

With the same argument as in Section 4.2, it remains weakly dominant for the uninformed bidder to consider information acquisition if and only if one competitor is left in the auction. If bidder 1 remains uniformed, his optimal drop out point $b^* < E[v_1]$ is the solution to problem (4.5) and is given by equation (4.6). We proceed in the same manner as in Section 4.2. To formalize the trade-off between the cost of information acquisition, the probability of winning, and the risk of buying at an unfavorable price, define:

$$\begin{aligned} H_r(p,c) &:= E[\max\{v_1, v_2\} - v_2 | v_2 \geq p] - c, \\ K_r(p) &:= \int_0^{b^*} \int_0^1 (v_1 - v_2) - \chi_{\{v_1 \leq v_2\}} r(v_2 - v_1) dF(v_1) dF(v_2 | v_2 \geq p), \end{aligned}$$

$$p_r^{**}(c) := \sup\{p \in [0,1] | H_r(p,c) \geq K_r(p)\}, \text{ and} \tag{4.10}$$

$$p_r^{*}(c) := \inf\{p \in [0,1] | E[\max(p, v_1) - v_1 + r(\max(p, v_1) - v_1)] - c \geq 0\}. \tag{4.11}$$

Therein v_2 denotes the valuation of the last remaining competitor. Comparing equations (4.10) and (4.11) to equation (4.2) and (4.3) using inequality (4.7) yields the following predictions about the information acquisition and bidding behavior of bidder 1 in an English auction with regret:

Fact 4. *In the English auction with regret, there exists $p_r^*(c) < p^*(c)$, $p_r^{**}(c) > p^{**}(c)$,* and $b^* < E[v_1]$ given by equation (4.10) and (4.11) *such that:*

If there are at least two other competing bidders left in the auction at price p*, bidder* 1 *does not acquire information and stays in the auction as long as* $p < \max\{p_r^*(c), b^*\}$*. If only one other competing bidder is left in the auction, two cases are relevant:*

(i) If $p_r^*(c) > p_r^{**}(c)$*, bidder* 1 *never acquires information and drops out as soon as the price reaches* $E[v_1]$

(ii) If $p_r^*(c) < p_r^{**}(c)$*, no information has been acquired by bidder* 1*, and*

a) $p \in [p_r^*(c), p_r^{**}(c)]$*, bidder* 1 *acquires information, drops out immediately if* $p > v_1$*, or stays in the auction if* $p \leq v_1$*;*

b) $p > p_r^{**}(c)$*, bidder* 1 *drops out at ;*

c) $p < p_r^*(c)$, bidder 1 stays in the auction.

The information acquisition strategy in the English auction with regret has the same structure as the strategy in the English auction without regret as depicted in Figure 4.1. However, as $p_r^*(c) < p^*(c)$ and $p_r^{**}(c) > p^{**}(c)$, the range of prices on the price-clock for which the uninformed bidder acquires information in equilibrium is greater for any c if regret is an issue. Hence, there exists a range of information costs $[\underline{c}, \bar{c}]$ such that $p_e^*(c) \geq p^{**}(c)$ and $p_r^*(c) < p_r^{**}(c)$ for all $c \in [\underline{c}, \bar{c}]$. This implies that for $c \in [\underline{c}, \bar{c}]$, bidder 1 acquires information with positive probability in the English auction if and only if he anticipates the feeling of regret. Moreover, if the feeling of regret is anticipated, bidders who remain uniformed drop out of the auction before the price reaches their expected valuation. Hence, with the appropriate definition of $r(\cdot)$, a theory of regret accounts for the excessive information acquisition as described in Result 1 and the underbidding effect of the uniformed bidders as described in Result 4.

Assuming bidders' regret also explains Result 2. Due to $p_r^*(c) < p^*(c)$, we should observe earlier dates of information acquisition in the English auction. This would explain why bidders buy information prematurely.

It remains to show why subjects acquire information if more than one competitor is left in the auction. One possible explanation is the following: suppose the regret concerns are of such high magnitude that $p_r^{**}(c)$ is close to 100. In this case, to avoid regret the bidder would acquire information for sure at some point in the auction given that the price reaches $p_r^*(c)$. Hence, the bidders are indifferent between acquiring information right away or after all but one competitors have dropped out. In the experiment, the valuations of the other bidders are drawn from a discrete distribution. Therefore, all remaining competitors may drop out at the same price with positive probability. In this case, the bidder strictly prefers to buy information as soon as the price reaches $p_r^*(c)$ even if more than one competitor is left in the auction.

4.5.3 Estimation of the regret function

Our theoretical analysis has shown that regret avoidance explains the experimental results for general regret functions $r(\cdot)$. If we assume a linear form

$$r(x) := \begin{cases} \alpha x & \text{if } x < 0, \\ 0 & \text{otherwise,} \end{cases}$$

we can use the experimental data to estimate the regret function. Solving equation (4.6) for $N = 4$ with valuations uniformly distributed on $[0, 100]$ yields

$$b^* = \frac{\sqrt{1+\alpha} - 1}{\alpha} 100. \tag{4.12}$$

When $\alpha \to 0$, the optimal bid in the second-price auction without information approaches $b^* = 50$, the risk-neutral expected-profit-maximizing bid. If α increases, b^* decreases, i.e., the more regret is experienced by a subject the lower is his bid if he remains uninformed.

Our experimental results for the high-cost treatment of the second-price auction suggest that the average bid amounts to 40.6 if the bidder remains uninformed. Solving equation (4.12) with $b^* = 40.6$ for α yields $\alpha = 1.25$.[18] The regret coefficient in our experiment is therefore in line with previous results on regret in auctions. For example, Filiz-Ozbay and Ozbay (2005) find a regret coefficient of 1.23 and Engelbrecht-Wiggans and Katok (2008) find a regret coefficient of 0.623.

Summing up, we have shown that the theory of anticipated regret accounts for the results of the experiment. Moreover, an estimation with the experimental data demonstrates that the regret coefficient is within a reasonable range.

4.6 Conclusion

In this chapter, we used a risk-neutral expected-profit-maximization model for auctions with the opportunity for information acquisition as a starting point. On this basis, we designed the first laboratory experiment to study information acquisition strategies and bidding behavior in such an auction where a bidder's private valuation is unknown ex-ante. First, we find excessive information acquisition in two different formats, i.e., the second-price sealed-bid and the English auction. Second, we find significant underbidding behavior across both formats for uninformed bidders.

On the basis of our robust results, we extend the basic model by assuming that the subjects are averse to regret and take this into account when they decide on information acquisition and bidding. We find that regret aversion can serve as an explanation of the observed behavior.

Two normative implications from our study are: first, if bidders do not acquire information they bid significantly below their expected valuation. Hence, information acquisition should be encouraged by the auction designer. Second, bidders willingness to pay for information is surprisingly high. Hence, an auction designer who

[18] We cannot use the data from the English auction for a precise estimation of the regret parameter. This is due to the fact that the estimated average bid in the English auction is merely a lower bound on the actual average bid. However, we can estimate the upper bound of the regret coefficient which amounts to $\alpha = 1.97$.

controls the information (for example through due diligence fees) can price information aggressively.

4.7 Appendix

Further analysis

The robustness of our main findings is further corroborated by the results of regression analysis. For the result of excess information acquisition, we run separate logit regressions with random effects for the two formats. These regressions confirm that the round of the auction does not affect the information acquisition behavior. Also, the outcome of the previous round does not significantly influence the information acquisition behavior, as one might argue. For the bidding behavior, we run standard linear regressions as reported in Table 4.8. Here the dummy variable “high_cost” takes the low-cost treatments as a baseline for comparison and shows significantly lower bids for the high-cost treatments. Next, the dummy variable “dynamic_format” takes the second-price auction as a baseline and confirms that the average bids in the dynamic format are significantly lower. The variable “valuation” takes the valuation associated with each bid and shows a highly significant relationship between valuation and bid. Therefore, we can exclude arbitrary behavior or simple mistakes as an explanation for the intriguing underbidding effect.

Moreover, we integrate a dummy variable “buy_info” into the regression model and find that bids are significantly higher when subjects have acquired information about their valuation. This does not conflict with the fact that we find underbidding for both cases (with and without information acquisition). However, it indicates that the underbidding effect must be much more prevailing without prior information acquisition. Finally, the control variable “round”, associating bids and the current round, is not significant. This further proves that there is no trend or learning effect in our data.

Table 4.8: Linear regression (robust standard errors). Dependent variable: bid.

Variables	**Model 1**
Intercepts	7.50***
	(2.167)
High_Cost	-1.67**
	(0.771)
Dynamic_Format	-9.45***
	(0.823)
Valuation	0.61***
	(0.017)
Buy_Info	16.34***
	(1.304)
Round	0.07
	(0.071)
N observations	2400
Prob>F	0.000
MSE	20.14
R^2	0.456

Risk aversion

In what follows, we show that assuming risk aversion fails to explain the excessive information acquisition observed in the high-cost treatment of the second-price auction. We can show that risk-averse bidders would acquire less information than predicted by the risk-neutral model. As a consequence, we reject risk aversion as an explanation for the observed data.

Suppose bidder 1 is risk averse with a concave utility function $u(x)$. We start by showing that if bidder 1 decides not to buy information, his bid in a second-price auction b^* will be lower than $E[v_1]$. To see this, consider the maximization problem of bidder 1 once he decided not to buy information:

$$\max_b \int_0^b \int_0^{100} u(v_1 - v^{(1)}) dv_1 dv^{(1)}.$$

The first-order condition for this problem is:

$$\int_0^{100} u(v_1 - b^*)dv_1 = Eu(v_1 - b^*) = 0.$$

With the parameters from the experiment, it holds that $E(v_1 - 50) = 0$. By definition of risk aversion, all risk-averse individuals dislike zero-mean risks. Hence, $Eu(v_1 - 50) \leq 0$ and therefore $b^* < 50$.

Given the equilibrium behavior of the informed bidders and the choice of b^*, the decision whether to buy information or not is the choice between two random variables $\tilde{x} - c$ and $\tilde{y}$. If bidder 1 decides to buy information, his pay-off is:

$$\tilde{x} - c = \max(v_1 - v^{(1)}, 0) - c.$$

If bidder 1 decides not to acquire information, the pay-off is:[19]

$$\tilde{y} = \chi_{\left\{v^{(1)} \leq b^*\right\}}(v_1 - v^{(1)}).$$

The maximal for which a risk-neutral bidder would acquire information is defined by $c^* = E[\tilde{x}] - E[\tilde{y}]$. To demonstrate that risk-averse bidders may refrain from information acquisition in situations where a risk-neutral bidder would have acquired information, we show that $E[u(\tilde{x} - c^*)] - E[u(\tilde{y})] \leq 0$.

Define $\tilde{x}_0 := \tilde{x} - E[\tilde{x}]$, $\tilde{y}_0 := \tilde{y} - E[\tilde{y}]$ and denote by $\pi(\omega, u, \tilde{x}_0)$ the risk premium of $\tilde{x}_0$ at a wealth level of ω. It follows:

[19] χ denotes the indicator function.

$$
\begin{aligned}
& E[u(\tilde{x}-c^*)] - E[u(\tilde{y})] \leq 0 \\
\Leftrightarrow & E[u(\tilde{x}-E[\tilde{x}]+E[\tilde{y}])] - E[u(\tilde{y})] \leq 0 \\
\Leftrightarrow & E[u(\tilde{x_0}+E[\tilde{y}])] - E[u(\tilde{y})] \leq 0 \\
\Leftrightarrow & E[u(E[\tilde{y}]-\pi(E[\tilde{y}],u,\tilde{x}_0))] - E[u(E[\tilde{y}]-\pi(E[\tilde{y}],u,\tilde{y}_0))] \leq 0 \\
\Leftrightarrow & \pi(E[\tilde{y}],u,\tilde{x}_0)) \geq \pi(E[\tilde{y}],u,\tilde{y}_0).
\end{aligned}
\tag{4.13}
$$

For small risks, we can use the Arrow-Pratt approximation. Hence, inequality (4.13) holds true whenever $\mathrm{Var}[\tilde{y}_0] \leq \mathrm{Var}[\tilde{x}_0]$. For the parametrization of the experiment, we get $\mathrm{Var}[\tilde{y}_0] \leq 1.354$ and $\mathrm{Var}[\tilde{x}_0] = 1.\bar{6}$,whenever $b^* < 50$.

We can conclude that risk-averse bidders have a smaller willingness to pay for information than risk-neutral bidders in the second-price auction. Accordingly, the maximal willingness to pay for information of risk-neutral bidders is such that $E[\tilde{x} - c^*] = E[\tilde{y}]$, i.e., the expected values of the relevant random variables are equal but the pay-off from not buying information and bidding b^* is less volatile. Risk-averse subjects then prefer not to buy information. Summing up, risk aversion cannot explain the experimental data.

Instructions

Welcome and thank you for participating in today's experiment. Please read the following instructions thoroughly. These are the same for all participants. Please do not hesitate to ask if you have any questions. However, we ask you to raise your hand and wait for us to come and assist you. We also ask you to restrain from communicating with other participants from now on until the end of the experiment. Please ensure that your mobile phone is switched off. Violating these rules can result in an exclusion from this experiment. You will be able to earn money during this experiment. The amount of your payout depends on your decisions. Each participant will receive his payout individually in cash at the end of the experiment. You will receive 2.50 € as a show-up fee for your presence as well as the sum of payouts from each round. Possible losses will at the end of the experiment be set against the show-up fee (if you accumulated losses on top of that, you will be required to pay these in cash at the end of the experiment). During the experiment payouts will be stated in the currency "ECU" (Experimental Currency Unit). 10 ECU are equivalent to 1 Euro (10 ECU = 1 EUR). The experiment consists of 20 payout relevant rounds.

Course of a Round (Treatment: 2nd Price Auction)

During this experiment you will take part in an auction of a fictional good. You will be bidding in a group of four with three other participants. These three participants are per-programmed bid robots. Their exact functioning will be described in more detail in the following.

Information prior to the Auction:

The fictional good is of different value for each bidder. Therefore prior to each round the valuation is determined for each participant. This valuation is between 0 and 100 ECU and each number has the same probability. However, during this auction you do not have any information about your valuation at first. Nevertheless, at the cost of 2 ECU/ 8ECU you can at any time acquire knowledge of your exact valuation. By contrast, the bid robots know their exact valuation of the fictional good. Their valuation, just as your own valuation, is between 0 and 100 ECU and each number has the same probability. The three bid robots will always have different valuations.

Profits and Losses during the Auction:

All bidders simultaneously make an offer for the fictional good. The bidder with the highest offer wins the auction. The price for the fictional good is set at the amount of the second highest bid. The winner of the auction has to pay this price for the good. If multiple bidders make the same offer during one round, then the winner is randomly determined. (Please note: You will not be able to revoke an offer or buy any information, once an offer has been submitted.) The payout for the winner of an auction is calculated from his previously, randomly determined valuation of the good minus the price at the end of the auction. (Please note: You will incur a loss if your offer is higher than your valuation of the good. Losses will at the end of the experiment be set against the show-up fee. However, if you accumulate losses on top of that, you will be required to pay these in cash at the end of the experiment.) Additionally, if you have bought information at the cost of 2 ECU/ 8 ECU, then this amount will be deducted from your profit or entered as loss.

Feedback after an Auction Round:

At the end of an auction round you will be informed, whether you won the fictional good with your bid. Additionally, you will be informed about the second highest bid and therefore the price of the fictional good as well as your individual profit for this round.

Course of a Round (Treatment: English Auction)

During this experiment you will take part in an auction of a fictional good. You will be bidding in a group of four with three other participants. These three participants are per-programmed bid robots. Their exact functioning will be described in more detail in the following.

Information Prior to the Auction:

The fictional good is of different value for each bidder. Therefore prior to each round the valuation is determined for each participant. This valuation is between 0 and 100

ECU and each number has the same probability. However, during this auction you do not have any information about your valuation at first. Nevertheless, at the cost of 2 ECU/ 8ECU you can at any time acquire knowledge of your exact valuation. By contrast, the bid robots know their exact valuation of the fictional good. Their valuation, just as your own valuation, is between 0 and 100 ECU and each number has the same probability. The three bid robots will always have different valuations.

Profits and Losses during the Auction:

The auction begins at 0 ECU for the fictional good. The bid will increase every 2 seconds by 1 ECU. A price clock indicates the current bid in ECU during the auction. You will also be able to see at any time of the auction how many bidders are still active and you will be able to buy information on your exact valuation. You can pause the price clock at wish by clicking the button "Pause/ Continue". All participants automatically continue bidding until they leave the auction round by clicking the button "Quit" on their screen. The auction ends automatically once only one bidder is left active. The last active bidder wins the auction and has to pay the last price on the price-clock, i.e. the price when the second last bidder dropped out. If multiple bidders quit simultaneously, then the winner of this round is randomly determined. The payout for the winner of an auction is calculated from his previously, randomly determined valuation of the good minus the price at the end of the auction. (Please note: You will incur a loss if your offer is higher than your valuation of the good. Losses will at the end of the experiment be set against the show-up fee. However, if you accumulate losses on top of that, you will be required to pay these in cash at the end of the experiment.) Additionally, if you have bought information at the cost of 2 ECU/ 8 ECU, then this amount will be deducted from your profit or entered as loss.

Feedback after each Auction Round:

At the end of an auction round you will be informed, whether you won the fictional good with your bid. Additionally, you will be informed about the second highest bid and therefore the price of the fictional good as well as your individual profit for this round.

End of Experiment

All auction rounds of this experiment are payout relevant. After completion of all 20 auction rounds, your payouts for each round as well as your overall result will be presented to you in a summary on your screen. After that we will ask you fill in a short questionnaire concerning the experiment. Please raise your hand, if you have any further questions.

Screenshots

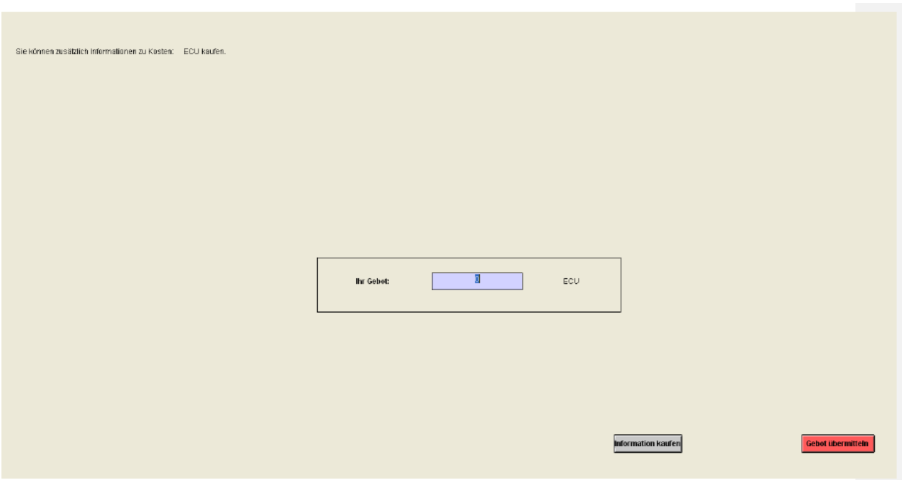

Figure 4.4: Screenshot example (treatment: second-price auction).

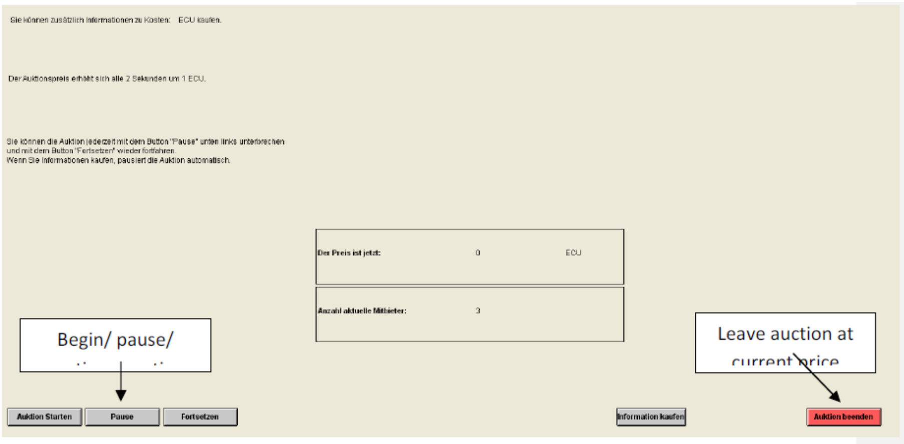

Figure 4.5: Screenshot example (treatment: English auction).

5 Auctions vs. negotiations: The case of favoritism

We compare two commonly used mechanisms in procurement: auctions and negotiations. The execution of the procurement mechanism is delegated to an agent of the buyer. The agent has private information about the buyer's preferences and may collude with one of the sellers. We provide a precise definition of both mechanisms and show – contrary to conventional wisdom – that an intransparent negotiation yields a higher buyer surplus than a transparent auction for a range of parameters. In particular, for small expected punishments there exists a lower and an upper bound on the number of sellers such that the negotiation yields a higher buyer surplus with a probability arbitrary close to 1 in the parameter space. Moreover, if the expected punishment is small, the negotiation is always more efficient and generates a higher surplus for the sellers.

5.1 Introduction

Auctions are believed to be transparent procurement mechanisms and hence less prone to favoritism than private negotiations. For instance, Paul Klemperer (2000) argues that "..., allocation by bureaucrats leads to the perception - if not the reality - of favoritism and corruption. In fact some governments have probably chosen beauty contests [over auctions] precisely because they create conditions for favoring "national champions" over foreign competitors. This is unlikely to benefit consumers and taxpayers."[1]

[1] More recently Subramanian (2010) states the following: "Auctions are more transparent processes than private negotiations, so if transparency is important, an auction is better. This is

The perception that auctions are transparent mechanisms stems from the fact that auctions are executed publicly, whereas negotiations are conducted privately. Hence, in an auction all relevant parameters and procedures have to be defined *before* the bidders submit their offers and it is apparent whether the implemented procedures have been followed. In a negotiation – on the other hand – it is impossible to reconstruct the decision process and only the final decision becomes public.

However, public scrutiny does not imply that auctions are favoritism proof, as the parameters and procedures of an auction may be chosen in a way that benefits one of the sellers *before* the auction has even started. Moreover, even though a negotiation is conducted privately, the final outcome of the process has to be rationalized to the public *after* all offers have been collected. Thus, some public scrutiny cannot be avoided in a negotiation.[2]

This chapter focuses on the definition and comparison of auctions and negotiations in the presence of favoritism. For both processes we consider a procurement setting with sellers that are horizontally differentiated with respect to the specification of the procured project. Buyer surplus depends not only on the final price but also on the implemented specification. The buyer has to delegate the execution of either process to an agent who privately observes the specification preference of the buyer and colludes with one – exogenously chosen – seller.[3] The agent maximizes the

the reason that most public procurement contracts [...] are done through auctions, particularly when the government is looking to defuse criticisms of corruption or favoritism." Moreover, Martin Wolf (2000) argues that "it [the auction] is the fairest [mechanism] because it ensures that the economic value goes to the community, while eliminating the favoritism and corruption inherent in bureaucratic discretion."

[2] This argument easily generalizes to private auctions and negotiation. Even though, private procurement is not conducted publicly the managers still have to answer to the shareholders of the procuring company.

[3] The assumption that the agent colludes with one specific seller resembles many real-life situations in public procurement. For example, Laffont and Tirole (1991) argue: "There has been much concern that the auction designer may prefer or collude with a specific buyer. And indeed most military or governmental markets acquisition regulations go to a great length to impose rules aimed at curbing favoritism. Similarly, the European Economic Comission, alarmed by the abnormally large percentage (above 95% in most countries) of government contracts awarded to domestic firms is trying to design rules that would foster fairer competition between domestic and foreign suppliers and would fit better than recent experience with the aim of fully opening borders ..."

surplus of his preferred agent. At the end of either process the buyer observes his true specification with a small probability and punishes the agent if the process has been manipulated.

We start our analysis by arguing that the main difference between auctions and negotiations in terms of transparency is that in an auction public scrutiny is imposed *before* the agent collects the offers of the sellers, whereas in the negotiation public scrutiny is imposed *after* collecting the offers. Hence, public scrutiny in an auction restricts the choice of the process, whereas in the negotiation public scrutiny merely places restrictions on the final decision of the agent. In our set-up, the manipulation power of the agent stems from the fact that the preferred specification of the buyer is private knowledge to the agent. Thus, public scrutiny in the auction implies that the implemented procedure has to be optimal given some feasible specification.[4] In the negotiation, public scrutiny implies that in the end the winning seller must have offered the lowest price at some feasible specification.[5] How this price was achieved is not salient to the public.

We proceed by precisely defining the resulting mechanisms and comparing them in terms of revenue and efficiency. We find that forcing the auction to be a publicly observable mechanism implies that if there is no manipulation, the auction yields a larger revenue for the buyer than the negotiation. Interestingly, if both the auction and the negotiation are manipulated, the buyer is still better off with the auction, as the optimal auction discriminates against the (manipulated) specification. However, this does not imply that the auction performs better in general. One of our main insights is that the decision whether to manipulate the auction is different from the decision whether to manipulate the negotiation. In the auction, the decision to manipulate has to be taken before the bidders submit their offers, whereas in the negotiation, the decision to manipulate can be taken after the bidders have submitted their offers. Hence, if the expected punishment is low, the agent always

[4] In this case the agent can claim that this specification is the true specification of the buyer and that the procedure is optimal.

[5] In this case the agent can claim that this is the true specification of the buyer.

manipulates the auction, whereas in the negotiation, the decision to manipulate depends on the realized costs and specifications of the sellers.

To get some intuition for this result, recall that in the negotiation the agent can observe the offers of the sellers before public scrutiny forces him to reveal the specification on which his allocation decision is based. Thus, the preferred specification of the buyer is only distorted if the favorite seller can benefit from the distortion ex-post. It follows that if the favorite seller turns out to be relatively weak, the specification is set optimally and the project is allocated efficiently among the honest sellers. In the auction, the details of the process have to be set prior to collecting the offers. Therefore, the auction is manipulated whenever the favorite seller can profit from manipulation ex-ante. Thus, if the expected punishment is low, the preferred specification is distorted even if the favorite seller is relatively weak.

Three different cases are relevant for the comparison of the revenue. First, if the number of sellers and the expected punishment is low, either of the processes may generate the higher revenue depending on the initial specifications of the sellers. Second, if the expected punishment decreases and the number of sellers increases, the negotiation outperforms the auction with probability close to one in the specification space. Third, if for any fixed expected punishment the number of sellers grows very large, the auction is not manipulated and therefore yields the optimal revenue.

Beyond the ranking of revenues, we find that if the expected punishment is low the negotiation is always more efficient than the auction. Interestingly, the favorite seller always prefers the negotiation over the auction mechanism. Thus, only the regular sellers may profit if an auction is used.

A setting in which the specification matters are spectrum auctions. Before the introduction of auctions, beauty contests were widely used for the allocation of spectrum licenses.[6] One of the reasons to move from beauty contests to auctions

[6]Classifying beauty contest as negotiations in the broader sense of this work seems reasonable, because the exact criteria of the decision in a beauty contest are not stated in advance but rather found in the process: “In beauty contests (also known as comparative tender), a committee

for the allocation of spectrum was the suspicion that beauty contests had been manipulated to favor domestic firms.[7] Given that favoritism is probable, we argue that auctions are not favoritism proof and can be influenced by manipulation of the specification of the project. Consider for example the German spectrum auction in 2010. Even though the spectrum was allocated by a simultaneous ascending auction, the required specifications in terms of coverage and implementation speed were set by the agency in charge (BNetzA) prior to the auction.[8] Among other specifications, the rules required the winner of a license to provide 80% coverage within four years.[9] These requirements significantly influenced the cost structures of the involved bidders.

Relation to the literature

One of the main contributions of this chapter is that it brings together two strands of literature: the literature on favoritism in auctions, and the literature on the comparison of auctions and negotiations.

In most cases favoritism enters auctions through two different channels. First, the auctioneer can favor a seller by allowing him to adjust his bid in a first-price auction after observing all of the competing bids ("right of first refusal" or bid rigging). Second, the auctioneer can manipulate the quality assessment of his favorite seller. In the former case the final allocation will be inefficient and the revenue of the buyer diminishes (Burguet and Perry, 2007; Menezes and Monteiro, 2006; Lengwiler and

typically sets a number of criteria, possibly with different weightings. Candidates' offers are then evaluated by a jury that selects the plan that has the best "mix" of those criteria, usually the highest weighting. [...] one of the criteria in a beauty contest can be a monetary one." See Prat and Valletti (2003).

[7]Prominent examples of suspected favoritism in beauty contests are the spectrum allocation processes in France in 1994 and in South Korea in 1992. See McMillan (1995) or Prat and Valletti (2003) and the references therein.

[8]The Bundesnetzagentur (Federal Network Agency) is in charge of regulating the German electricity, gas, telecommunications, postal and railway markets.

[9]See the "Präsidentenkammerentscheidung - Vergabeverfahren Mobilfunk" from October 12, 2009. http://www.bundesnetzagentur.de/DE/DieBundesnetzagentur/Beschlusskammern/1BK-Geschaeftszeichen-Datenbank/BK1-GZ/2009/2009_001bis100/BK1-09-002/BK1-09-002_E_BKV.html?nn=53804.

Wolfstetter, 2010). In our model, the auction is undertaken under public scrutiny. Thus, such a form of bid rigging can not occur. The later case is analyzed in Laffont and Tirole (1991), Burguet and Che (2004), and Celentani and Ganuza (2002). We take a different approach in assuming that the agent may misrepresent the preferences of the buyer rather than the quality assessment of the seller, which means that favoritism not only distorts the mechanism for the favorite bidder but may also distort the allocation among the honest bidders.

The second strand of literature is concerned with the comparison of auction and negotiation. Bulow and Klemperer (1996) show in their seminal article that a simple auction with one additional bidder leads to higher revenues than the best mechanism without this bidder. The result by Bulow and Klemperer (1996) is often used to argue in favor of auctions. However, in case the number of bidders is not an issue, the best designed mechanism will be better than the simple auction. In addition, if one extends the model to allow for common values, the result no longer holds. Bulow and Klemperer (2009) compare a standard English auction to a negotiation that is defined as a sequential procedure, where in each round a new bidder might enter the negotiation, and then competes head on with any bidder left from previous rounds. In case he wins this competition, he can make a jump bid in order to deter further entry. Bulow and Klemperer (2009) show that in this context, the auction fares better in terms of revenue although the negotiation is more efficient. This is due to the fact that entrants have to incur costs to learn their true valuation. Thus, bidders may prevent further entry with pre-emptive bids thereby capturing most of the efficiency gains.

In our set-up, the negotiation also is the more efficient mechanism: the gain in efficiency is due to the fact that the negotiation is less likely to be manipulated and the optimal specification for the buyer is more likely to be implemented. Hence, contrary to Bulow and Klemperer (2009), the buyer is able to capture most of the efficiency gain and thus may benefit from the negotiation.[10]

[10]Other approaches to the comparison of auctions and negotiations include McAdams and Schwarz

The major challenge in comparing auctions and negotiations is to find a precise definition for each of the mechanisms. The sparse literature on this subject uses different approaches to tackle this issue. We argue that one of the main differences between both formats is the timing at which the precise rules are set and show that, contrary to previous works, negotiations can outperform auctions.

The rest of the chapter is organized as follows. In Section 5.2, we set up the model and discuss the modeling choices. In Section 5.3, we derive the equilibria of the mechanisms in question. In Sections 5.4 and 5.5, we provide a comparison of both mechanisms in terms of revenue, efficiency, and sellers surplus. Section 5.6 contains a robustness check of the results. Section 5.7 concludes.

5.2 The Model

Suppose one indivisible project has to be procured from N risk neutral sellers. Let $i \in \{1, \ldots, N\}$ index the sellers. Each of the sellers has a privately known cost c_i of delivering the project. It is common knowledge that c_i is distributed with c.d.f. F on support $[0, \bar{c}]$. The sellers are horizontally differentiated with respect to the specifications of the project. This is captured for seller i by a given location q_i along the specification space $[\underline{q}, \bar{q}]$. The location of each seller is known by the buyer. Seller i incurs a cost of $|q_i - q|$ to move his specification from q_i to some q. If a seller is selected to deliver the project at a price p and specification $\hat{\theta}$ the value to the buyer is $V - |\hat{\theta} - \theta| - p$ with $V \in \mathbb{R}_+$.[11] The parameter $\theta \in [\underline{q}, \bar{q}]$ represents the desired specification of the buyer and is not observed by the buyer prior to the procurement process.

The buyer has to delegate the execution of the procurement mechanism to an agent

(2006), Fluck, John, and Ravid (2007) or Manelli and Vincent (1995).

[11] Assuming that the costs of moving the specification are given by some convex function $c_i(|q - q_i|)$ for each seller i and that the value to the buyer is $V(|\theta - \hat{\theta}|)$ for some concave function V does not change our results qualitatively.

who can privately observe the parameter θ prior to procuring the project.[12] The auctioneer colludes with one of the sellers and may favor this seller by misrepresenting θ by announcing some $\hat{\theta}$ to the buyer. In what follows, let seller 1 be the seller in question.[13] We define and compare two different procurement mechanisms – auctions and negotiations:

Auction

An auction is conducted under full public scrutiny, i.e., all relevant dimensions of the auction have to be made publicly available prior to its start. Hence, in an auction the agent has to set all relevant parameters and procedures of a specific auction format before the sellers submit their offers.[14] Moreover, public scrutiny implies that even if the buyer is not aware of his preferred specification θ, once the auction format has been set, auction experts can point out whether the proposed auction format is optimal given some feasible specification $\hat{\theta}$. Thus, in the context of public procurement, it is reasonable to assume that the agent has to implement the optimal auction given some $\hat{\theta} \in [\underline{q}, \bar{q}]$.[15]

The timing of the auction is the following:

[12] For example, we can think of the buyer being the public and the agent being a bureaucrat in charge of running a public procurement. In this case, it is easy to make sense of the assumption that the agent is better informed about the preferences of the buyer than the buyer himself. See Arozamena and Weinschelbaum (2009), Burguet and Perry (2007), Celentani and Ganuza (2002), or Laffont and Tirole (1991) for an exhaustive description of such situations.

[13] We assume that the favorite bidder is exogenously given. This assumption is a good approximation for many situations in public procurement where the agent may have a well established relationship with the domestic firm.

[14] The public procurement directive of the European Union states concerning (electronic) auctions: "The electronic auction shall be based [...] on prices and/or values of the features of the tenders, when the contract is awarded to the most economically advantageous tender. The specifications shall contain [...] the quantifiable features (figures and percentages) whose values are the subject of the electronic auction and the minimum differences when bidding. [...] The invitation shall state the mathematical formula to be used to determine automatic rankings, incorporating the weighting of all the award criteria." (See the *"Directive 2004/18/EC of the European Parliament and of the Council of 31 March 2004 on the coordination of procedures for the award of public works contracts, public supply contracts and public service contracts"*).

[15] Allowing the agent to implement some other auction will reinforce our results in favor of the negotiation.

(i) The agent privately observes θ.

(ii) The agent publicly commits to the revenue-optimal auction given some $\hat{\theta} \in [\underline{q}, \bar{q}]$.

(iii) The sellers submit bids to the agent and the winning bidder is determined.[16]

(iv) The winning bidder is required to invest $|q_i - \hat{\theta}|$ to meet the specifications of the project.

(v) The buyer observes θ with probability ϵ and punishes the agent by imposing a fine D if $\theta \neq \hat{\theta}$.[17]

Negotiation

The negotiation is conducted privately by the agent and the process cannot be publicly observed. Thus, in a negotiation the agent is not bound by the requirement to set all the relevant parameters and procedures in advance. He is rather free to choose his decision criteria at any time during the process. Even though the negotiation is conducted privately, the agent has to publicly rationalize his final decision. Hence, some public scrutiny cannot be avoided. This places two restrictions on the decision of the agent.

First, the agent cannot prevent any of the bidders from submitting offers. This is due to the fact that in public procurement the contracting authority has "obligations regarding information [...]. This takes the form of publishing information notices [...]" prior to the start of the procurement process.[18] Hence, all relevant sellers are aware that the project is being procured and could appeal against the exclusion

[16] Bidders are committed to their offers.

[17] We assume that the agent and the favorite seller form a perfect coalition. Thus, it does not matter who is bearing the punishment or how the additional surplus from corruption is divided. Moreover, we assume that the agent observes c_1 and that there is no information problem between the agent and seller 1. See Celentani and Ganuza (2002) for a discussion of how these assumptions are a good approximation to many situations in real-life procurement.

[18] See the above mentioned *"Directive 2004/18/EC"* on public procurement.

of their offers.[19] In what follows, we will explore two different ways of how this restriction reflects on the decision of the agent. We start by assuming that the agent has to inform the sellers whether he rejects their offer because it is not the best offer. If he rejects an offer, the seller in question may resubmit a better offer. We will relax this assumption in Section 5.6, where the agent does not have to inform the bidders whether their offers were rejected and simply picks one of the submitted offers as the winner.

Second, in the end of the process, the agent has the obligation to reveal the winner of the process and the final agreement to the buyer. Moreover, the sellers that did not win the project may request a statement by which means their offer is inferior to the offer of the winner.[20] In our set-up the price that a seller receives for implementing a certain specification is the only relevant decision dimension. Hence, this kind of public scrutiny places a restriction on the decision of the agent in the sense that the final winning offer has to be the lowest of all submitted offers for the implemented specification.

Other than these two restrictions the agent is not bound by any rules during the negotiation process. Thus, the negotiation takes the following form:

(i) The agent privately observes θ.

(ii) Each seller submits an offer function $p_i(q)$ (with $q \in [\underline{q}, \bar{q}]$) to the agent.[21]

(iii) The agent compares the offers of the sellers and informs each seller privately whether his offer was rejected.

[19]Without this minimal restriction the problem at hand would become trivial, as the agent could simply exclude all bidders and award the contract to his favorite bidder at the reservation value.

[20]For example, the public procurement directive of the European Union states: "Each contracting authority shall provide information, as soon as possible, on the decisions reached concerning the award of a contract, including grounds for not awarding it. [...] On the request of the economic operator concerned [the contacting authority should provide information on] any unsuccessful candidate of the reasons for rejecting them; any tenderer who has made an admissible tender of the relative advantages of the tender selected, as well as the name of the economic operator chosen." (See the above mentioned *"Directive 2004/18/EC"* on public procurement).

[21]$p_i(q)$ is the price for which bidder i will deliver specification q. The offer is only observed by the agent and bidder i. Moreover, bidders are committed to their offers.

(iv) A bidder whose offer was rejected may submit a new offer. If he submits a new offer (iii) and (iv) are repeated.

(v) After collecting the offers, the agent chooses the winning bidder and sets the final specification $\hat{\theta} \in [\underline{q}, \bar{q}]$.

(vi) Public scrutiny implies that if bidder i is the winning bidder, $p_i(\hat{\theta}) \leq \min_{i \neq j} p_j(\hat{\theta})$ has to hold.

(vii) The winning bidder is paid $p_i(\hat{\theta})$ and required to invest $|q_i - \hat{\theta}|$ to meet the specifications of the project.

(viii) The buyer observes θ with probability ϵ and punishes the agent by imposing a fine D if $\theta \neq \hat{\theta}$.

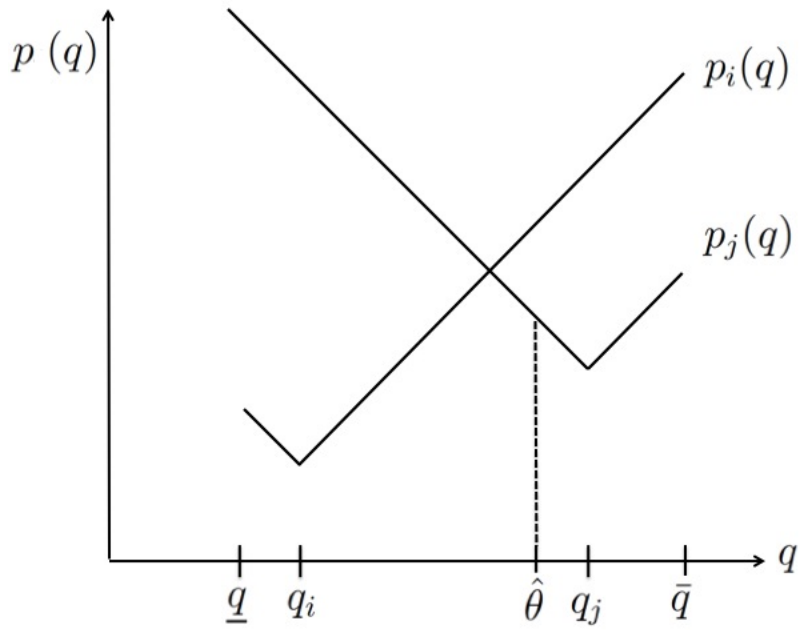

Figure 5.1: *With the appropriate choice of $\hat{\theta}$ the agent can declare bidder j as the winning bidder.*

To illustrate the public scrutiny requirement suppose that the final offer of bidder j is $p_j(q)$ while bidder i makes a final offer $p_i(q)$ – as depicted in Figure 5.1. As argued above, at the end of the process, the final agreement and $\hat{\theta}$ have to be revealed to the buyer and the loosing sellers. If the agent announces $\hat{\theta}$ as the buyer's preferred specification, he can claim that bidder j has the lowest standing offer. If the final offers are as depicted in Figure 5.2, there is no announcement of $\hat{\theta}$ such that the agent can claim that bidder j has the lowest standing offer without violating $p_j(\hat{\theta}) \leq \min_{i \neq j} p_i(\hat{\theta})$.

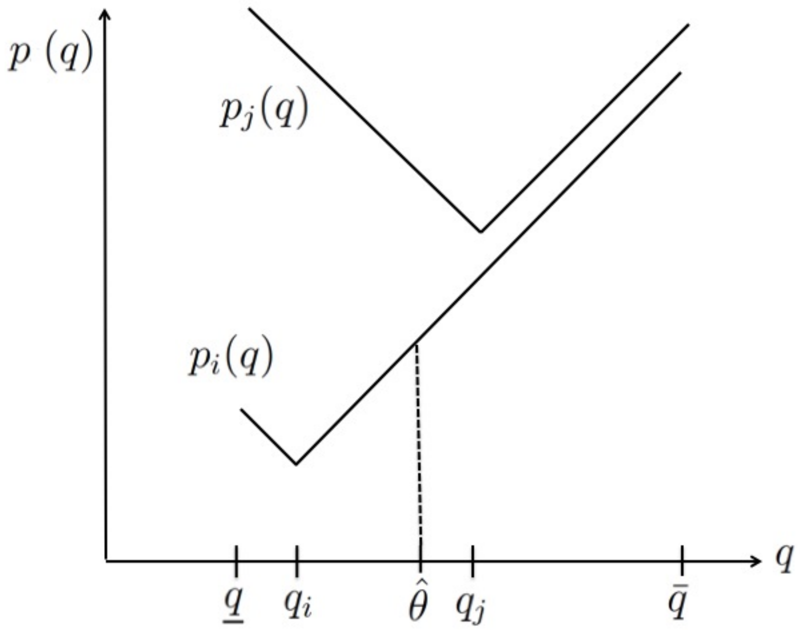

Figure 5.2: *There is no choice of $\hat{\theta}$ such that the agent can declare bidder j as the winning bidder.*

5.3 Equilibria of the mechanisms

In this section, we derive the equilibria for the auction and the negotiation.

5.3.1 Equilibrium of the auction

First, we derive the revenue-optimal auction that implements specification $\hat{\theta}$. To simplify the exposition, we make a standard assumption that ensures that it is always optimal to procure the object:

Assumption 1. *The following holds true for all $c \in [\underline{c}, \bar{c}]$:*

(i) $V - |q - \theta| - c - F(c)/f(c) \geq 0$ *for all* $q, \theta \in [\underline{q}, \bar{q}]$

(ii) $\psi(c) := c + F(c)/f(c)$ *is strictly increasing in* c.

Assumption 1 is satisfied if $F(c)/f(c)$ is non-decreasing and V is sufficiently large. We use the revelation principle and restrict our attention to direct revelation mechanisms: $g_i(\boldsymbol{c})$ denotes the awarding rule - the probability of winning the project for firm i; $t_i(\boldsymbol{c})$ denotes the expected payment to firm i if the vector of announced costs is $\boldsymbol{c} = (c_1, \ldots, c_N)$.[22] The optimal auction can be described as follows:

Lemma 2. *Suppose Assumption 1 holds true. The optimal auction that implements* $\hat{\theta}$ *is fully characterized by the awarding rule:*

$$\begin{aligned} g_i^{\hat{\theta}}(\boldsymbol{c}) = 1 \quad &\textit{if} \quad V - c_i - |q_i - \hat{\theta}| - \frac{F(c_i)}{f(c_i)} > V - c_j - |q_j - \hat{\theta}| - \frac{F(c_j)}{f(c_j)} \quad \forall j \neq i \\ g_i^{\hat{\theta}}(\boldsymbol{c}) = 0 \quad &\quad \textit{otherwise.} \end{aligned} \tag{5.1}$$

The expected surplus of seller i *is given by*

$$U_i(\hat{\theta}, c_i) = \int_{c_i}^{\bar{c}} \int g_i^{\hat{\theta}}(s, \boldsymbol{c}_{-i}) dF^{N-1}(\boldsymbol{c}_{-i}) ds. \tag{5.2}$$

The expected profit of the buyer in terms of his true desired specification θ *is given by*

$$\Pi_a(N) := E_c \left[\sum_{i=1}^{N} g_i^{\hat{\theta}}(\boldsymbol{c}) \left(V - |\theta - \hat{\theta}| - c_i - |q_i - \hat{\theta}| - \frac{F(c_i)}{f(c_i)} \right) \right]. \tag{5.3}$$

Proof. Immediate from Krishna (2009, p. 70) or Naegelen (2002). □

[22] The specification q_i is known to the buyer. Hence, it suffices to restrict our attention to direct mechanisms that ask the sellers to report their cost c_i.

Sellers with a specification q_i that is close to $\hat{\theta}$ have a relative cost advantage. If all sellers are treated equally, those sellers would bid less aggressively and thereby lower the revenue. Hence, the optimal awarding rule discriminates against those sellers and thereby elicits more aggressive bidding.[23]

The optimal auction can be implemented as a first- or second-score auction.[24] Hence, it is meaningful to speak about auctions in the context of this chapter. We are only interested in the resulting buyer and seller surplus. Thus, we will refrain from deriving the exact scoring rules and just state the following lemma:

Lemma 3. *Let b_i^f denote the bid of firm i in a first-score auction and b_i^s the bid of firm i in a second-score auction. There exist scoring rules $W^f(q_i, b_i^f)$ for the first-score auction and $W^s(q_i, b_i^s)$ for the second-score auction such that in equilibrium the buyer and seller surplus coincides with the surplus in the optimal auction.*

Proof. Immediate from Naegelen (2002). □

The agent colludes with seller 1. Hence, the equilibrium in the auction mechanism is fully characterized by $\hat{\theta}$ that maximizes the expected utility of seller 1 from participating in the auction minus the expected punishment in case the manipulation is detected. From expressions (5.1) and (5.2) it follows that maximizing the expected utility is equivalent to maximizing the winning probability of seller 1. The winning probability of seller 1 is maximized for $q_1 = \arg\max_{\hat{\theta}} V - c - |q_1 - \hat{\theta}| - F(c)/f(c)$. We summarize this finding in the following:

Corollary 2. *In the auction the agent will set $\hat{\theta} = q_1$ if $U_1(q_1, c_1) - U_1(\theta, c_1) \geq \epsilon D$. Otherwise the agent will set $\hat{\theta} = \theta$.*

[23] To illustrate this discrimination, suppose that $F(c) = c$ and $N = 2$. In the revenue optimal auction, seller 1 wins whenever $2c_1 + |q_1 - \hat{\theta}| < 2c_2 + |q_2 - \hat{\theta}|$, where as in an efficient mechanism seller 1 wins whenever $c_1 + |q_1 - \hat{\theta}| < c_2 + |q_2 - \hat{\theta}|$. Thus, the specification advantage matters less.

[24] In a first-score auction, each seller transmits a bid b_i^f. The seller with the highest score $W^f(q_i, b_i^f)$ is selected as a winner and receives a payment equal to his bid. In a second-score auction, each seller transmits a bid b_i^s. The seller with the highest score $W^s(q_i, b_i^s)$ is selected as a winner and receives a payment p^* such that $W^s(q_i, p^*) = W^s(q_j, b_j^s)$ where j is the bidder with the highest rejected score.

5.3.2 Equilibrium of the negotiation

We start the analysis of the negotiation by characterizing the rejection strategy of the agent and the offer strategy of seller 1 that maximizes their joint surplus. In a second step we will discuss the offer strategy of the honest sellers. Observe that the agent has to objectives when maximizing the joint surplus: Give seller 1 the project whenever possible and avoid punishment whenever not possible. The first objective entails that seller 1 should receive project whenever he can underbid the lowest offer of the other sellers at some specification. The second objective entails that the agent prefers to set the true specification as the final specification and thus to keep the bidder with the lowest offer on the true specification in the process. To characterize the optimal behavior of the agent we need one more definition:

Definition 1. *We call a seller active if his offer was rejected and he resubmitted a new offer or if his offer has not been rejected. Define the set of active honest sellers as* $A \subseteq \{2, ...N\}$.

The following proposition summarizes the equilibrium behavior of the agent.

Proposition 10. *The following strategy maximizes the ex-post joint surplus of seller* 1 *and the agent;*

(i) If $|A| > 2$, seller 1 offers $p_1(q) \equiv V$ and the agent rejects all offers but the offer of bidder $j = \arg\min_{i \in A} p_i(\theta)$.

(ii) If $|A| = 1$,

a) seller 1 *offers* $p_1(q_1) = \min_{i \neq 1} p_i(q_1)$ *and the agent rejects all offers but the offer of seller* 1 *if*

$$\min_{i \neq 1} p_i(q_1) - \epsilon D > \min_{i \neq 1} \{p_i(\theta) - |\theta - q_1|\} \text{ and } c_1 < \min_{i \neq 1} p_i(q_1) - \epsilon D.$$

b) seller 1 *offers* $p_1(\theta) = \min_{i \neq 1} p_i(\theta)$ *and the agent rejects all offers but the* *offer of seller* 1 *if*

$$\min_{i\neq 1} p_i(q_1) - \epsilon D < \min_{i\neq 1}\{p_i(\theta) - |\theta - q_1|\} \text{ and } c_1 + |\theta - q_1| \leq \min_{i\neq 1} p_i(\theta).$$

c) *seller* 1 *offers* $p_1(\theta) = c_1 + |q_1 - \theta|$ *and the agent rejects all offers but the offer of bidder* j *with* $j = \arg\min_{i\neq 1} p_i(\theta)$ *otherwise.*

If at the end of the process $\min_{i\neq 1} p_i(q_1) - \epsilon D > \min_{i\neq 1}\{p_i(\theta) - |\theta - q_1|\}$ and $c_1 < \min_{i\neq 1} p_i(q_1) - \epsilon D$ *the agent sets* $\hat{\theta} = q_1$ *as the final specification. Otherwise the agent sets* $\hat{\theta} = \theta$.

Proof. Suppose that at some point during the negotiation the honest bidders have submitted offer functions $p_i(q)$, $i \in \{2, ..., N\}$.

ad (i): As long as $|A| > 2$ it is optimal for seller 1 to submit an offer function $p_1(q) \equiv V$ and for the agent to reject all offers but the lowest offer at the true specification of the buyer, i.e. the offer of bidder $j = \arg\min_{i\in A} p_i(\theta)$. This ensures, that whenever the agent realizes that seller 1 cannot win, he can pick bidder j as the winner and θ as the final specification to avoid punishment.

ad (ii): As soon as $|A| = 1$, three cases are relevant. First, as long as manipulation is favorable at the end of the process and seller 1 has a chance to win, i.e., as long as $\min_{i\neq 1} p_i(q_1) - \epsilon D > \min_{i\neq 1}\{p_i(\theta) - |\theta - q_1|\}$ and $c_1 < \min_{i\neq 1} p_i(q_1) - \epsilon D$, it is optimal for seller 1 to submit an offer function with $p_1(q_1) = \min_{i\neq 1} p_i(q_1)$ and for the agent to reject the offer of the last active seller.[25] Second, if the agent realizes that manipulation is not worthwhile but seller 1 can still win the project, i.e., if $\min_{i\neq 1} p_i(q_1) - \epsilon D < \min_{i\neq 1}\{p_i(\theta) - |\theta - q_1|\}$ but $c_1 + |\theta - q_1| \leq \min_{i\neq 1} p_i(\theta)$, it is optimal for seller 1 to submit some offer function with $p_1(\theta) = \min_{i\neq 1} p_i(\theta)$ and for the agent to reject the offer of the last seller. Third, in any other case, seller 1 has

[25] If the offers of the other sellers are such that $\min_{i\neq 1} p_i(q) - |q_1 - q| > \min_{i\neq 1} p_i(q_1)$ for some $q \in [\underline{q}, \bar{q}]$, it would be optimal to manipulate with $\hat{\theta} = q$ at the end of the process. However, we will show below that this cannot be an equilibrium outcome and hence – for the sake of clarity of exposition – we do not include this case in the discussion.

no chance of winning. To avoid punishment it is optimal for the agent to declare the last seller in A as the winner of the project and set θ as the final specification. The offer of the last bidder $j \in A$ satisfies $j = \arg\min_{i \neq 1} p_i(\theta)$ because of (i). This strategy of the agent ensures that bidder 1 wins whenever he can offer the lowest price at some specification and that punishment can be avoided whenever seller 1 fails to win. $\square$

It remains to characterize the behavior of the honest sellers and the equilibrium outcome. Observe first that a seller whose offer was rejected has no chance to win the project if he does not lower his offer. The expected surplus of this seller is then zero. If, however, $p_i(q) > c_i + |q - q_i|$ for some $q \in [\underline{q}, \bar{q}]$ and he submits a lower offer, he receives an expected surplus of at least zero.[26] If, contrary to that, $p_i(q) < c_i + |q - q_i|$ for some $q \in [\underline{q}, \bar{q}]$, the seller receives the project, and in the end the agent sets $\hat{\theta} = q$ as the final specification, the surplus of this seller will be negative. Hence, if $p_i(q)$ has been rejected and $p_i(q) > c_i + |q - q_i|$ for some $q \in [\underline{q}, \bar{q}]$, not submitting a new offer is weakly dominated by lowering $p_i(q)$ at some $q \in [\underline{q}, \bar{q}]$. Similarly, if $p_i(q) = c_i + |q - q_i|$ for all $q \in [\underline{q}, \bar{q}]$, lowering $p_i(q)$ at any $q \in [\underline{q}, \bar{q}]$ is weakly dominated by not submitting a new offer. Moreover, public scrutiny implies that in order for seller j to win $p_j(\hat{\theta}) \leq \min_{i \neq j} p_i(\hat{\theta})$ has to hold if the agent sets $\hat{\theta}$ as the final specification. It follows directly that any (undominated) equilibrium yields the same allocation for any final specification $\hat{\theta}$: For all sellers whose offers were rejected, it is a weakly dominant strategy to lower their offers until for every specification their offer curve is equal to the cost of delivering the project at this specification.

Hence, if the agent sets the final specification $\hat{\theta}$, the winning seller will receive at most the lowest cost of the other sellers at this specification. Thus, it is only favorable for him to win if his cost of delivering $\hat{\theta}$ is the lowest among all sellers. We summarize this finding in the following proposition.

[26] The surplus is strictly positive if the negotiation stops at a price $p_i(q) > c_i + |q - q_i|$ and the agent sets $\hat{\theta} = q$.

Proposition 11. *In any equilibrium of the negotiation in undominated strategies each bidder i lowers his offer until his offer is the lowest standing offer or $p_i(q) = c_i + |q - q_i|$ for all $q \in [\underline{q}, \bar{q}]$. Thus, for any final $\hat{\theta} \in [\underline{q}, \bar{q}]$, seller j wins the project iff $c_j + |\hat{\theta} - q_j| < \min_{i \neq j} c_i + |\hat{\theta} - q_i|$.*

Hence, any undominated equilibrium of the negotiation is efficient in the following sense: Given a final $\hat{\theta}$, the negotiation selects the seller who can deliver the project at specification $\hat{\theta}$ at the lowest cost.

Combining Proposition 10 with Proposition 11 yields that the agent only manipulates final specification if the costs of seller 1 plus the expected punishment are below of the minimal costs of all other sellers at specification q_1. In all other cases the final specification is not manipulated and the true specification of the buyer is implemented. Hence, the surplus loss from misspecification to the buyer is $|q_1 - \theta|$ and the cost of seller 1 is c_1 whenever seller 1 wins the project and the specification is manipulated. Hence, the virtual surplus to the buyer is $V - |\theta - q_1| - c_1 - F(c_1)/f(c_i)$ whenever seller 1 wins the project and the specification is manipulated. Whenever the specification is not manipulated, there is no surplus loss to the buyer but the cost to the winning seller amounts to $c_i + |q_i - \theta|$. In this case the virtual surplus to the buyer is $(V - c_i - |q_i - \theta| - F(c_i)/f(c_i))$ if the specification is not manipulated and seller i wins the project.

We can rewrite the equilibrium outcome of the negotiation - characterized by Lemma 11 and Proposition 10 - in terms of an awarding rule of a direct revelation mechanism:

Lemma 4. *The equilibrium outcome of the negotiation is equivalent to the outcome of a direct revelation mechanism characterized by the following awarding rule $g^n(c)$:*

$$
\begin{aligned}
g_1^n(c) &= 1 \quad \textit{if} \quad c_1 \leq \min_{j \neq 1} \{c_j + \max\{|q_j - q_1| - \epsilon D,\ |q_j - \theta| - |q_1 - \theta|\}\} \\
g_1^n(c) &= 0 \qquad\quad \textit{otherwise;} \\
g_i^n(c) &= 1 \quad \textit{if} \quad c_i + |q_i - \theta| \leq \min_{j \neq i} \{c_j + |q_j - \theta|\} \ \textit{and}\ \{c_j + |q_1 - q_j|\} - \epsilon D \leq c_1,\ i \neq 1 \\
g_i^n(c) &= 0 \qquad\quad \textit{otherwise.}
\end{aligned}
$$

The expected surplus of seller i is given by

$$U_i^n(c_i) = \int_{c_i}^{\bar{c}} \int g_i^n(s, \boldsymbol{c}_{-i}) dF^{N-1}(\boldsymbol{c}_{-i}) ds.$$

The expected profit of the buyer in terms of his true desired specification θ is given by

$$\begin{aligned}\Pi_n(N) := E_{\mathbf{c}} \Bigg[g_1(c) \left(V - |\theta - q_1| - c_1 - \frac{F(c_1)}{f(c_1)} \right) \\ + \sum_{i=2}^{N} g_i^n(\mathbf{c}) \left(V - c_i - |q_i - \theta| - \frac{F(c_i)}{f(c_i)} \right) \Bigg] \\ = E_{\mathbf{c}} \left[\sum_{i=1}^{N} g_i^n(\mathbf{c}) \left(V - c_i - |q_i - \theta| - \frac{F(c_i)}{f(c_i)} \right) \right]. \quad (5.4)\end{aligned}$$

5.4 Revenue

We will show that if the number of sellers is rather small, the negotiation may outperform the auction depending on $q = (q_1, \ldots, q_N)$, ϵD, and θ. Then, as long as the expected gains from manipulation in the auction are positive, the negotiation becomes more profitable with an increasing number of sellers and outperforms the auction for most of the parameter values. If the expected gains from manipulation in the auction turn negative, the auction always outperforms the negotiation in terms of revenue.

As noted in Section 5.3, the optimal auction discriminates against sellers with a specification close to $\hat{\theta}$. The negotiation, however, selects the seller who can deliver $\hat{\theta}$ at the lowest cost but leaves him with more rent. Whenever both mechanisms are manipulated, manipulation gives seller 1 an advantage by moving $\hat{\theta}$ to his specification q_1. Because of the mentioned discrimination, this advantage is less valuable in the auction. No such discrimination takes place in the negotiation, and seller

1 can fully benefit from the manipulation. Hence, the auction generates a higher buyer surplus if both mechanisms are manipulated.[27] However, even if the expected punishment is arbitrary small, the negotiation is not always manipulated. This is due to the fact that in the negotiation, the agent observes the offers of the other sellers before choosing the final $\hat{\theta}$. Whenever the realization of c_1 is such that seller 1 cannot benefit from manipulation ex-post, the agent chooses not to manipulate the preferred specification.[28] In this case, the winner delivers the efficient specification and the unmanipulated negotiation generates in most cases more buyer surplus than the manipulated auction.

As long as the expected punishment is sufficiently small, manipulation remains optimal in the auction, whereas the probability that the agent manipulates the negotiation approaches zero with an increasing number of sellers. Hence, the negotiation becomes more profitable. If N becomes very large, however, and the expected punishment is larger than the expected gains from manipulation, manipulation in the auction will no longer be profitable. In this case, the auction yields the optimal surplus.

Summing up, whenever both mechanisms are manipulated or the auction is not manipulated, the auction generates a higher revenue. If the auction is manipulated but not the negotiation, the negotiation in most cases yields a higher revenue. With small expected punishments, the latter case becomes more likely with an increasing number of sellers.

For a meaningful comparison of both mechanisms along the specification space, we assume that each q_i is drawn from a continuous distribution F_q on $[\underline{q}, \bar{q}]$ and asses the probability over q that the revenue from the auction mechanism $(\Pi_a(N))$ exceeds the revenue from the negotiation $(\Pi_n(N))$. We will show that if ϵ is sufficiently small, there exists a lower and an upper bound on the number of sellers such that the probability that the auction generates more revenue than the negotiation becomes

[27] Similarly, if no mechanism is manipulated, the auction generates a higher buyer surplus.

[28] This is the case whenever $c_1 > \min_{j\neq 1} c_j + \max\{|q_j - q_1| - \epsilon D,\ |q_j - \theta| - |q_1 - \theta|\}$. The probability of this event approaches 1 if N becomes large.

arbitrarily small (smaller than any $\delta \in (0,1)$). Moreover, for any $\epsilon > 0$ there exist a lower bound on the number of sellers such that the auction generates more revenue than the negotiation with probability 1. However, this lower bound approaches infinity if the expected punishment approaches 0.[29]

Proposition 12. *For each $\delta \in (0,1)$, there exist an $\epsilon > 0$ and $N_1(\delta,\epsilon) \leq N_2(\delta,\epsilon) < N_3(\epsilon)$ in $\mathbb{N}$ such that*

(i) if $N_1(\delta,\epsilon) \leq N \leq N_2(\delta,\epsilon)$, the surplus of the buyer in the negotiation is higher than in the auction with high probability, i.e., $Prob_q[\Pi_n(N) > \Pi_a(N)] > 1-\delta$.

(ii) If $N \geq N_3(\epsilon)$, the surplus of the buyer in the auction is higher than in the negotiation with probability one, i.e., $Prob_q[(\Pi_a(N) > \Pi_n(N)] = 1$.

Moreover, $\lim_{\epsilon\to 0} N_1(\epsilon,\delta) < \infty$ *and* $\lim_{\epsilon\to 0} N_3(\epsilon) = \infty$.

Proof. For any $\epsilon > 0$, define $N_3(\epsilon)$ such that iff $N \geq N_3(\epsilon)$, $\text{Prob}_q[U_1(q_1,\bar{c}) - U_1(\theta,\bar{c}) > \epsilon D] = 0$. As $\lim_{N\to\infty} U_1(q_1,\bar{c}) - U_1(\theta,\bar{c}) = 0$ for all $q \in [\underline{q},\bar{q}]^N$, $N_3(\epsilon)$ is finite for any fixed ϵ.

Observe next that $U_1(q_1,\bar{c}) - U_1(\theta,\bar{c}) = 0$ whenever $|q_1-\theta| - |q_i-\theta| = |q_1-q_i|$ for all $i \in \{2,\ldots,N\}$ and $U_1(q_1,\bar{c}) - U_1(\theta,\bar{c}) > 0$ otherwise. It follows that

$$\lim_{N\to\infty} \text{Prob}_q[U_1(q_1,\bar{c}) - U_1(\theta,\bar{c}) > 0] = 1$$

and hence

$$\lim_{N\to\infty}\lim_{\epsilon\to 0} \text{Prob}_q[U_1(q_1,\bar{c}) - U_1(\theta,\bar{c}) > \epsilon D] = 1.$$

.

[29] Note that the probability that the auction mechanism will generate more revenue than the negotiation will never be equal to zero as long as $\epsilon > 0$. This is due to the fact that if $\theta \leq q_1 \leq \min_{i\neq 1} q_i$ or $\theta \geq q_1 \geq \max_{i\neq 1} q_i$, $U_1(q_1,c_1) - U_1(\theta,c_1) = 0$ for any realization of c_1 and the favorite bidder cannot gain from the manipulation of the auction mechanism. The auction then generates the optimal revenue.

Thus, if the expected punishment converges to 0 and the number of bidders is sufficiently high, the agent manipulates the auction and sets $\hat{\theta} = q_1$ with probability one, i.e. $\lim_{\epsilon \to 0} N_3(\epsilon) = \infty$. It follows that

$$\lim_{N\to\infty} \lim_{\epsilon\to 0} \text{Prob}_q \Bigg[\Pi_a(N) = V - |\theta - \hat{\theta}| - E_c \left[\sum_{i=1}^{N} g_i^{q_1}(\mathbf{c}) \left(c_i + |q_i - \hat{\theta}| + \frac{F(c_i)}{f(c_i)} \right) \right] = V - |\theta - q_1| \Bigg] = 1. \quad (5.5)$$

The agent manipulates the negotiation if and only if $c_1 \leq \min_{j\neq 1} c_j + |q_j - q_1| - \epsilon D$. It follows that $\lim_{N\to\infty} \text{Prob}[c_1 \leq \min_{j\neq 1} c_j + |q_j - q_1| - \epsilon D] = 0$ for all $q \in [\underline{q}, \bar{q}]$. Thus,

$$\lim_{N\to\infty} \lim_{\epsilon\to 0} \text{Prob}_q \Bigg[\Pi_n(N) = V - |a - \hat{\theta}| - E_c \left[\sum_{i=1}^{N} g_i^{n}(\mathbf{c}) \left(c_i + |q_i - \hat{\theta}| + \frac{F(c_i)}{f(c_i)} \right) \right] = V \Bigg] = 1. \quad (5.6)$$

Hence,

$$\lim_{N\to\infty} \lim_{\epsilon\to 0} \text{Prob}_q \left[\Pi_n(N) > \Pi_a(N) \right] = 1. \quad (5.7)$$

For any $\epsilon > 0$ define $N_1(\epsilon, \delta)$ as the (possibly infinite) infinum of N such that

$$\text{Prob}_q \left[\Pi_n(N) > \Pi_a(N) \right] > 1 - \delta.$$

Together with the fact that F_q is continuous, equation (5.7) implies that $\lim_{\epsilon\to 0} N_1(\delta, \epsilon) < \infty$. Hence, $N_1(\epsilon, \delta)$ defines a convergent family of natural numbers. Thus, there exists a $\bar{\epsilon}$ such that $N_1(\epsilon, \delta) = \lim_{\epsilon\to 0} N_1(\delta, \epsilon)$ for all $\epsilon \leq \bar{\epsilon}$. Summing up, there exists an $\epsilon > 0$ such that $N_1(\epsilon, \delta) < \infty$, $N_3(\epsilon) > N_1(\epsilon, \delta)$, and therefore there also must exist a $N_2(\epsilon, \delta)$ with the desired properties. $\square$

Proposition 12 is inconclusive about the ranking of the revenue of both mechanisms if N is small. The following example illustrates that for small N, the revenue can be higher in each of the formats with positive probability.

Example 1. Let $N = 2$, $c \backsim U[0,1]$, and ϵ be close to zero. In this case, the agent manipulates the auction with probability one, seller 1 receives the object whenever $c_1 \leq |q_1 - q_2|/2 + c_2$, and the implemented specification is $\hat{\theta} = q_1$. The agent manipulates the negotiation whenever $c_1 \leq |q_1 - q_2| + c_2$. In this case, seller 1 receives the object and the implemented specification is $\hat{\theta} = q_1$. If $c_1 > |q_1 - q_2| + c_2$, the agent does not manipulate the negotiation, seller 2 receives the object, and the implemented specification is $\hat{\theta} = \theta$. The expected surplus of the buyer in the auction can be calculated using expression (5.3). It amounts to

$$\Pi_a(2) = V - \frac{2}{3} - |q_1 - \theta| - \frac{1}{2}|q_1 - q_2| + \frac{|q_1 - q_2|^2}{4}. \tag{5.8}$$

The expected surplus of the buyer in the negotiation can be calculated using expression (5.4). It amounts to

$$\Pi_s(2) = V - \frac{2}{3} - \frac{1}{2}(|q_1 - \theta| + |q_2 - \theta|) - |q_2 - q_1|(|q_1 - \theta| - |q_2 - \theta|) - |q_1 - q_2|^2. \tag{5.9}$$

Hence, the auction generates a higher surplus whenever the right hand side of expression(5.8) is larger than the right hand side of expression (5.9). Figure 5.3 illustrates that buyer surplus can be better in the auction or the negotiation depending on the chosen parameters. Applying the terminology of Proposition 12 it follows that if q_i is distributed with a continuous distribution function F_q with full support on $[\underline{q}, \bar{q}]$, $0<\text{Prob}_q[\Pi_a(2) > \Pi_s(2)]<1$ holds. Moreover, depending on F_q, $\text{Prob}_q[\Pi_a(2) > \Pi_n(2)]$ can be arbitrary close to zero or one.

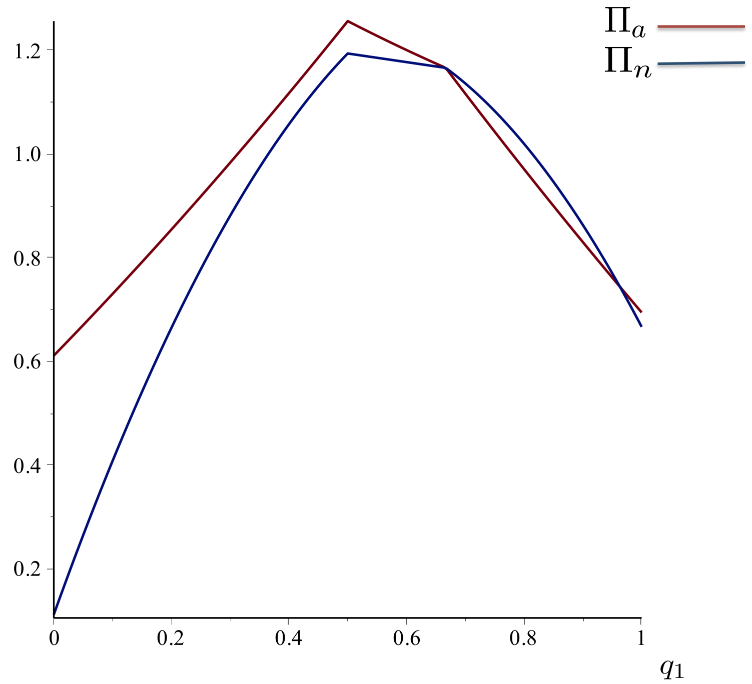

Figure 5.3: Buyer surplus for $V = 2$, $N = 2$, $\theta = 1/2$, $q_2 = 2/3$, and $q_1 \in [0, 1]$.

5.5 Efficiency

In this section, we will show that if the expected punishment is sufficiently small, the negotiation is more efficient with probability one. This result holds independent of whether the auction or the negotiation leads to larger revenue. This is due to the fact that the negotiation allocates the project to the seller who can deliver the – possibly manipulated – specification at the lowest cost.

For the comparison of efficiency of both formats four cases are relevant: (i) Both mechanisms are manipulated, (ii) the auction is manipulated but not the negotiation, (iii) the negotiation is manipulated but not the auction and (iv) both mechanisms are not manipulated. However, if the expected punishment is sufficiently low, the auction is always manipulated and the third and the fourth case are not relevant for our comparison.[30]

[30]Nevertheless, if both mechanism are not manipulated the negotiation is the more efficient mechanism.

If both, the auction and the negotiation, are manipulated (case (i)), the efficiency loss from the misspecification is the same ($|q_1 - \theta|$) in both mechanisms. However, as stated above, the agent will manipulate the negotiation if and only if seller 1 can deliver specification q_1 at the lowest price of all sellers. Hence, allocating the object to seller 1 – given that specification q_1 has to be delivered – is efficient. Thus, the negotiation is more efficient than the auction if both mechanisms are manipulated. If the auction is manipulated but not the negotiation, the negotiation is the fully efficient mechanism and thus more efficient than the auction.

Summing up, if the expected punishment is small the auction is always manipulated. Moreover, the negotiation is more efficient with probability one whenever both mechanisms are manipulated or the negotiation is not manipulated. Hence, the negotiation is more efficient than the auction.

Proposition 13. *As long as $U_1(q_1, \bar{c}) - U_1(\theta, \bar{c}) \geq \epsilon D$ the negotiation is more efficient than the auction .*

Proof. If $U_1(q_1, \bar{c}) - U_1(a, \bar{c}) \geq \epsilon D$, the agent manipulates the auction. The ex-post efficiency from the auction is given by

$$V - |q_1 - \theta| - c_i - |q_i - q_1| \tag{5.10}$$

and $q_i \neq q_1$ with positive probability. If the agent does not manipulate the negotiation, the negotiation yields the ex-post fully efficient outcome: $V - \min_{i \in \{1,\dots,N\}} \{c_i + |q_i - \theta|\}$. If the agent manipulates the negotiation,

$$c_1 < \min_{i \in \{2,\dots,N\}} c_i + |q_i - q_1| \tag{5.11}$$

has to hold. The ex-post efficiency of the negotiation is given by

$$V - |q_1 - \theta| - c_1. \tag{5.12}$$

Comparing expression (5.10) and (5.12) and using expression(5.11) yields the result. $\square$

We have shown that for small ϵ the negotiation is always more efficient than the auction, and that for small N there exist parameter values such that the auction generates a higher revenue. From this it directly follows that there exist parameter values such that the sellers receive a higher surplus in the negotiation. However, most of this surplus is captured by seller 1. The following proposition demonstrates that if the expected punishment is small, seller 1 prefers the negotiation over the auction.

Proposition 14. *There exists an $\hat{\epsilon}$ such that seller 1 prefers the negotiation over the auction for all $\epsilon \leq \hat{\epsilon}$.*

Proof. Either $|q_1 - \theta| - |q_i - \theta| = |q_1 - q_i|$ or there exists a $\epsilon_1 > 0$ such that $U_1(q_1, \bar{c}) - U_1(\theta, \bar{c}) > \epsilon D$ for all $\epsilon \leq \epsilon_1$. In both cases, Lemma 2 can be used to write the expected utility of seller 1 as

$$\begin{aligned} U_1^a(q_1, c_1) := \int_{c_1}^{\bar{c}} \int g_i^{q_1}(c_1, c_{-1}) dF^{N-1}(c_{-1}) dc_1 \\ = \int_{c_1}^{\bar{c}} \prod_{i=2}^{N} (1 - F(\psi^{-1}(-|q_i - q_1| + \psi(c_1))) dc_1. \quad (5.13) \end{aligned}$$

Observe that

$$\begin{aligned} -|q_i - q_1| + c_1 &< \psi^{-1}(-|q_i - q_1| + \psi(c_1)) \\ \Leftrightarrow \psi(-|q_i - q_1| + c_1) &< -|q_i - q_1| + \psi(c_1) \\ \Leftrightarrow -|q_i - q_1| + c_1 + \frac{F(-|q_i - q_1| + c_1)}{f(-|q_i - q_1| + c_1)} &< -|q_i - q_1| + c_1 + \frac{F(c_1)}{f(c_1)}. \end{aligned}$$

The last inequality is true as we assumed that $F(c_1)/f(c_1)$ is increasing. Hence, there exists a $\epsilon_2 > 0$ such that $-|q_i - q_1| + c_1 + \epsilon D < \psi^{-1}(-|q_i - q_1| + \psi(c_1))$ for all $\epsilon \leq \epsilon_2$.

The expected surplus of seller 1 in the negotiation can be written as

$$U_1^n(c_1) = \int_{c_1}^{\bar{c}} \int g_i^n(c_1, c_{-1}) dF^{N-1}(c_{-1}) dc_1$$
$$\geq \int_{c_1}^{\bar{c}} \prod_{i=2}^{N} (1 - F(-|q_i - q_1| + c_1 + \epsilon D) dc_1. \quad (5.14)$$

Take $\hat{\epsilon} = \min\{\epsilon_1, \epsilon_2\}$. It follows that $U_1^n(q_1, c_1) \geq U_1^a(q_1, c_1)$ for all $\epsilon \leq \hat{\epsilon}$. □

Whether the honest sellers appropriate a higher surplus is uncertain. Most of the additional surplus that is captured by seller 1 in the negotiation is due to the fact that the negotiation does not discriminate against sellers with a favorable specification q_i. Hence, he is able to capture all of the additional surplus from the manipulation in the negotiation. Whether the honest sellers prefer the negotiation over the auction depends therefore on how close their specification q_i is to the specification q_1 of seller 1.

5.6 Robustness

In deriving the negotiation procedure in Section 5.2 we have assumed that public scrutiny places two restrictions on the agent: no exclusion of offers and lowest offer at implemented specification wins. Observe that if the latter restriction is relaxed, the comparison of auctions and negotiations becomes meaningless, as in the negotiation the agent could simply give the project to his favorite bidder at price V and discard all the other offers. A similar argument applies if the agent is not obligated to take at least one offer from each seller. Hence, the obligation to take at least one offer from each seller and to award the project to the seller with the lowest offer at the implemented specification are in a sense minimal.

We modify the negotiation procedure from Section 5.2 by allowing the agent to award the project after collecting at least one offer from each seller. In particular the agent is not required to inform the sellers whether their offer was rejected until after the project was awarded. As the agent – to benefit his preferred seller – always prefers higher offers to lower offers, he will never inform one of the honest sellers whether his offer was rejected and thereby give him no chance to improve his offer. Hence, essentially, each seller submits exactly one offer.

(i) The agent privately observes θ.

(ii) Sellers submit an offer function $p_i(q)$, $q \in [\underline{q}, \bar{q}]$.

(iii) After collecting the offers, the agent chooses the winning bidder and sets the final specification $\hat{\theta}$.

(iv) The winning bidder is paid $p_i(\hat{\theta})$ and required to invest $|q_i - \hat{\theta}|$ to meet the specifications of the project.

(v) The buyer observes θ with probability ϵ and punishes the agent by imposing a fine D if $\theta \neq \hat{\theta}$.

As before ,the winning bid has to satisfy $p_i(\hat{\theta}) \leq \min_{j \neq i} p_j(\hat{\theta})$.

The strategy that maximizes joint surplus of the agent and seller 1 is straightforward:

(i) whenever $\min_{j \neq 1} p_j(q_1) - c_1 > \epsilon D$ the agent sets $\hat{\theta} = q_1$ and seller 1 offers $p_1(q_1) = \min_{j \neq 1} p_j(q_1)$;

(ii) whenever $\min_{j \neq 1} p_j(q_1) - c_1 < \epsilon D$ the agent sets $\hat{\theta} = \theta$ and seller 1 offers $p_1(\theta) = \max\{\min_{j \neq 1} p_j(\theta), c_1 + |\theta - q_1|\}$.

For the honest bidders, the problem of choosing an optimal offer for each possible $\hat{\theta}$ is essentially the same as choosing bids in a family of asymmetric first-price auctions with a stochastic reserve price.[31] An equilibrium for this game is known to exist.[32] However, a closed-form solution for the bidding strategies is hard to derive.

Nevertheless, due to the fact that in equilibrium $p_i(q) > c_i + |q_i - q|$ and $\lim_{N \to \infty} p_i(q) = c_i + |q_i - q|$ has to hold for all $i \neq 1$, the revenue result from Section 5.4 also holds for the negotiation at hand: if N is sufficiently small and the expected punishment

[31]The bid of the corrupt seller 1 resembles a stochastic reserve price.
[32]See Athey (2001).

is sufficiently low, the auction and the negotiation are both manipulated with a high probability. Manipulation then gives seller 1 a specification advantage over the other sellers. However, this advantage is less valuable in the auction as it discriminates against sellers with such an advantage. The allocation is less distorted than in the negotiation in which seller 1 can fully benefit from the manipulation. Hence, the auction with favoritism may generate a higher buyer surplus for small N. However, if the number of sellers grows but the expected punishment remains small, the outcome of the negotiation converges to the outcome characterized in Section 5.3 as $\lim_{N\to\infty} p_i(q) = c_i + |q_i - q|$. In this case, we know from Proposition 12 that the revenue from the negotiation exceeds the revenue from the auction with high probability. Hence, the negotiation generates a higher revenue than the auction mechanism if N grows. If N becomes so large that manipulation of the auction is not optimal, the auction is the optimal mechanism and generates a higher revenue than the negotiation. We summarize this finding in the following:

Corollary 3. *The negotiation generates a higher revenue than the auction if ϵ is sufficiently small and N is sufficiently large. If N is very large, the auction generates a higher revenue than the negotiation.*

5.7 Conclusion

We have shown that – contrary to common wisdom – the transparency of an auction does not render it favoritism proof. If the agent of the buyer is able to manipulate the specification of the procured project, an intransparent negotiation is more efficient and may generate more buyer surplus. This is due to the fact that in the auction, public scrutiny forces the agent to decide whether to manipulate the process *before* sellers submit their offers. In the negotiation on the other hand, *after* observing the offers of the sellers, the agent may still decide not to manipulate if he realizes that his preferred seller is not able to win the project.

If no manipulation takes place, the auction is the revenue-optimal mechanism. Moreover, if the specification is manipulated in both procedures, the auction is the revenue optimal mechanism that implements the manipulated specification. In those cases, the auction will outperform the negotiation. However, if the auction is manipulated but not the negotiation, the negotiation may generate more surplus. This difference in manipulation is due to the fact that the auction is manipulated whenever the expected punishment is low. The negotiation, on the other hand, may not be manipulated even if the expected punishment is low because after observing the offers of the honest sellers, the agent may realize that his preferred seller has no chance of winning the project. This becomes more likely if the number of sellers increases.

This chapter sheds light on the question whether auctions or negotiations should be used when designing a public procurement mechanism. We have argued that a seemingly straightforward reasoning that auctions – because of their transparency – should be preferred in the presence of favoritism does not apply. Whether an auction should be used over a negotiation depends on the number of participating sellers and the buyers' ability to detect deviations from his preferred specification.

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