# Dynamics Control of Pendulums Driven Spherical Robot 

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#### Abstract

The pendulum driven spherical robot exhibit unique dynamic behavior which designed for reconnaissance and unstructured hostile environment exploration. Unfortunately spherical type robot maneuvering is nonlinear to get the desired part, internal propulsion mechanism and motion control need to be reconciled. This is a mechanical challenge to get in balancing and weight distribution. The robot has three DOFs and single inputs, of which the nature is a nonlinear and under actuated system with non-holonomic dynamic constraints. The enhanced construction of two sections on pendulums joint offers novel motion principle of spherical robot, which is moving simultaneously actuated by eccentric moment and inertial moment generated by this pendulum. Meanwhile the mobility is enhanced when the robot behaves dynamically. The dynamic model of linear motion is formulated on the basis of Lagrange equation, and a smooth trajectory planning method is proposed for linear motion. A feedback controller is build to ensure the accurate trajectory planning. Turning in place motion is an indispensable element of omni directional locomotion which can enhance the mobility of spherical robots. The dynamic model is derived using the theory of moment of momentum, and a stick-slip principle is analyzed. The two motion control methods are validated by both simulations and prototype experiments.


## Introduction

Conventionally, mobile robot is considered to be wheeled robot or legged robot. The wheeled robot has a great mobility on relatively flat terrains and the legged robot has potentially greater mobility on tough terrain [1]. In recent years, spherical robot has become the potential platform for research. This make spherical robot can be used in hostile environments, such as rescue in disaster and military reconnaissance, especially planetary exploration. According to the differences of actuator, the representative spherical robots are grouped into six types: rotor type, car type, mobile masses type, gyroscope type, memory alloy type and pendulum type; each type has advantage and disadvantage respectively [2]. The researches on spherical robot in recent years mainly focus on trajectory planning and motion control [3], rather than motion principle and mechanism design.

## System Description

The mechanical construction of the robot is shown in Fig. 1, which is mainly composed of two motors, two pendulums, springs, linear bearings, two guides and the outer shell.


Fig. 1. Mechanical structure of the robot
The outer shell is sphere, whose diameter of 270 mm . While in irregular motion, such as passing over bumps, the attitude of the robot is easy to be observed. Meanwhile, the pitch angle is restricted to a certain range by the sphere shell, which ensures the robot can tolerate side slopes without rolling over. A novel two pendulums type is adopted. The robot is driven by two pendulums placed diametrically opposite on the major axis of the sphere shell, which are actuated by two direct current (DC) gear motors. The robot is motivated by eccentric moment, inertia force and inertia moment simultaneously. The total weight of the masses is 340 gram which accounts for $45.6 \%$ of the total mass of robot. Compared with the traditional pendulum type, the improved type can afford more eccentric moment and inertia forces which make the robot has featured high speed, the ability of slope climbing and obstacle overcoming. And in the future works, the masses will be controlled to resonate with the springs in order to let the robot jump while in trap.

## Dynamic model of linear motion

In linear motion, the robot can be projected onto side view, which is modeled as a pendulum inside a rigid sphere unrelated to the $s$ sphere shell as shown in Fig. 2.


Fig. 2. Planar simplified model of linear motion.

The dynamics of the robot is derived under the following assumptions: (i) there is mo slip between the shell and floor. (ii) the two pendulums rotate synchronously without angle difference. As shown in Fig. 2, the coordinate $y_{o}, z_{o}$ is considered to be reference coordinates fixed to the ground, and the coordinate $y, z$ is fixed to the robot. The two pendulums are rotated by motors to a $\theta$ angle to change the position of center of the sphere. Additionally the inertia forces in $y, z$ plane perpendicular to the pendulums are generated. The sphere rolls about the $x$ axis driven by the eccentric moment and inertia moment from the two pendulums, so the center of the sphere moves straightly along the $y_{o}$ axis. The locomotion should be maintained by the continuous rotation of the inner pendulums [4]. The Lagrange equations are used to derive the dynamic equations of linear motion. The angle created by second vertical pendulums is referred as the tilt angle $\theta$. The angle the sphere rotates through with respect to the reference coordinates is the body angle $\alpha$. So the angle the motor shafts rotates through is $\theta+\alpha$. The tilt angle $\theta$ and distance of the center of the sphere $y$ are considered to be the generalized coordinates in Lagrange equations, and the torque of the motors $\tau$ is the generalized force.

The kinetic energy of the sphere except for pendulums respect to ground can be expressed as
$T_{1}=\frac{1}{2} M y^{2}+\frac{1}{2} J \alpha^{2}$
where $M$ is the mass of the sphere, $J=M R^{2}$ is the moment of inertia of the sphere, $R$ is the sphere radius, $\dot{y}=\dot{\alpha} R$ is the forward speed of the center of the sphere.

The kinetic energy of the pendulums can be expressed as
$T_{2}=\frac{1}{2} m(\dot{y}+r \dot{\theta} \sin \theta)^{2}+\frac{1}{2} m(r \dot{\theta} \cos \theta)^{2}$
where $m$ is the mass of the two pendulums, $r$ is the radius of the pendulums.
With respect to the center of the ball considered to be the point of zero potential, the potential energy of the system is
$V=-m g r \cos \theta$
The Lagrange function can be express as
$L=T_{1}+T_{2}-V$
According to Lagrange equations
$\frac{d}{d t}\left(\frac{\partial L}{\partial q_{j}}\right)-\frac{\partial L}{\partial q_{j}}=Q_{j}$
The dynamics of the linear motion can be written as
$\left\{\begin{array}{c}(m+2 M) \ddot{y}+m r \ddot{\theta} \cos \theta-m r \dot{\theta}^{2} \sin \theta=\frac{\tau}{R} \\ m \ddot{y} r \cos \theta+m r^{2} \ddot{\theta}+m g r \sin \theta=\tau\end{array}\right.$
This dynamic function is not integrable, but provides the basis for trajectory planning that can make the system controllable from an initial configuration to a desired configuration

## Smooth trajectory planning of linear motion

The smooth trajectory planning from an initial configuration to an expected final configuration is essential to the robot, which can enhance the performance of linear motion. The dynamic function given by (6) can be rearranged as

$$
\begin{align*}
y & =f(\theta, \dot{\theta}, \ddot{\theta}) \\
& =\frac{m g \sin \theta+m r \sin \theta R \dot{\theta}^{2}+\left(m r^{2}-m r \cos \theta R\right) \ddot{\theta}}{(m+2 M) R-m r \cos \theta} \tag{7}
\end{align*}
$$

The coefficients of the terms related to $\theta$ in (7) are constant terms. This indicates that in linear motion, in order to keep a desired uniform speed ( $\ddot{y}=0$ ) of the robot, the pendulums should be maintained on the vertical position $(\dot{\theta}=0, \ddot{\theta}=0, \sin \theta=0)$. The nonzero tilt angle of the pendulums may activate the robot to accelerate. It is also implied that the desired velocity of the robot in linear motion can be obtained by given the proper tilt angle, that is, the position of the robot is controllable.

Result


Fig. 3(a) Water environment
(b) Climbing up
(c) Uneven ground.

In field test, the pendulum driven spherical mobile robots have exposed on tough terrain. These demonstration show that this platform able to a cross puddle ground as shown in Fig. 3(a). Fig. 3(b) shows it can climb up slope, this prove gear motor accumulator physically deliver higher torques. In additional, it also Traverse obstacles and uneven ground as shown in Fig. 3(c). Visibly the results demonstrate that this platform has it own superiority. The stick-slip principle is verified in experiment of turning in place motion. The two pendulums are controlled to the desired path. The robot can only rotate about vertical axis without rolling in theory, so the tilt angle is the angle the motor shafts rotate through. As shown in Fig. 2, the robot can continuously turn in place by the repeated input of the two stages of stick-slip principle. The robot can orient to the desired direction through the regulation of the amplitude of the tilt angle of the two pendulums.


Fig. 4(a) The result for linear floor track

(b) Uneven rainforests ground

The displacement and time of the robot in experiment are shown in Fig. 4(a) and 4(b). The displacement of the sphere is measured by the angular rate sensor, and the data of displacement is obtained by integration of the data of velocity. There is slide different between experimental and desired which is noise on the curves may come from the sensitivity of the angular rate sensor and the condition of the ground. It can be concluded that the motion state of the sphere coincides with the smooth trajectory planning [5].

## Conclusion

A spherical robot of new drive mechanism is studied in this paper. The linear motion and turning in place motion control of a two pendulums driven spherical robot are studied in this paper. The emphasis of linear motion control is placed on the movement to a desired configuration. The smooth trajectory planning method for linear motion based on normal distribution function is proposed. A feedback controller is constructed for the accurate trajectory planning. In future recommendation the climbing up stair is potentially area of expansion.

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