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Linear Fractional Programming for Fuzzy Random based Possibilistic Programming Problem

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Abstract—**The uncertainty in real-world decision making originates from several sources, i.e., fuzziness, randomness, ambiguous. These uncertainties should be included while translating real-world problem into mathematical programming model though handling such uncertainties in the decision making model increases the complexities of the problem and make the solution of the problem hard. In this paper, a linear fractional programming is used to solve multi-objective fuzzy random based possibilistic programming problems to address the vague decision maker's preference (aspiration) and ambiguous data (coefficient), in a fuzzy random environment. The developed model plays a vital role in the construction of fuzzy multi-objective linear programming model, which is exposed to various types of uncertainties that should be treated properly. An illustrative example explains the developed model and highlights it's effectiveness.**

Keywords-component; possibilistic programming; fractional programming; fuzzy random data; vagueness and ambiguity

I. INTRODUCTION

Real world applications and decision making faces many evaluations that depends on the human judgment which is usually based on intuition and experience; it makes the problem more complicated to express parameter's value in the mathematical model. Additionally, given the imprecise situation, the uncertainties should be handled properly to ensure that the developed mathematical model retain the uncertainty in finding the solution. Consequently, probability and possibility theories are widely used as it is capable to treat the random and fuzzy information respectively. Fuzzy sets [1] play a significant role and are useful to represent uncertain and imprecise information in the mathematical programming that reflect the uncertainties. The application of fuzzy set and possibility theories to decision-making allows decisions based on imprecise information. In linear programming, possibility theory can be used whereby imprecise parameters consisted in the problem formulation [2]. There are various approaches have been used to model uncertainty and imprecision on mathematical programming problems, such as, stochastic programming model (i.e. [3]) and fuzzy programming model (i.e. [4]), which plays a pivotal role to deal with uncertainties, especially when the mathematical programming model's

parameters and goals cannot be estimated precisely from real situations.

The possibilistic programming differs from fuzzy mathematical programming in a sense of the uncertainty is characterized in the coefficients of goals and/or constraints of the mathematical programming model and these imprecise coefficients are restricted by possibilistic distributions [5]. In the possibility theory, an impression is expresses in terms of a possibility distribution [6]. A stochastic and possibilistic linear programming considers two different sources of uncertainty in model parameters that is randomness and fuzziness, respectively. Since the invention of possibilistic programming [7], the possibility theory has become more important in the decision making field and several methods have been developed to solve possibilistic programming problems (see [6], [8]). Additionally, possibilistic linear programming has also been applied in multi-objective programming problem, in which all the parameters are fuzzy, because in real world problems we may face cases where the expert knowledge is uncertain to specify the parameters as real numbers and cases where parameters fluctuate in certain ranges [9].

Fuzziness such as the ambiguity of received information and vagueness in decision maker's goal are common in decision making which influence by human evaluations [5]. When the mathematical model contains uncertain information, that is, the coefficients and goal are fuzzy or not known exactly, the uncertainties should be treated before further solve the mathematical model. Thus, the uncertainties that are included in decision making raises the complexities of problem modeling. It is difficult to find the solution properly, as the uncertainties cannot be described precisely using numerical values. Moreover, in practical systems, coefficient values should rather be taken for uncertain values. The uncertainties occurs in probabilistic or/and vague situations such as predictions of future profits, incomplete historical data or/and replacement of decision makers, which result in ambiguous situations with uncertain information. That is, in the most real-world situations, it is realistic to consider the estimated value of the coefficients as imprecise rather values than precise ones. A possible range of the coefficient can be represented by a fuzzy set, which is regarded as a possibility distribution. Thus, the mathematical programming models for decision support must explicitly consider such issues, and the treatment of

the inherent uncertainty associated with the model coefficients is essential.

The focus is placed on the solutions from various models introduced by many scholars, whereby the initial setting of the model (i.e., determining the model coefficients) are not discussed or understood the experts gave previously. Yet, several discusses on the implication of inappropriate model development. Apparently, the development of mathematical programming models is crucial to obtain the feasible and proper solution. In this paper, it is motivated to emphasize the development of the mathematical programming model which treats fuzzy random data, and further treats ambiguity and fuzziness in the decision making process using possibility theory. Finally the developed model is solved using linear fractional programming. Real world problem is first translates into a fuzzy model where the coefficients are deduced by fuzzy random regression approach [10]. The methodology presented in this paper emphasizes its precious advantages to translate real world problem that exposed to various types of uncertainties into a mathematical programming model and further treats the inherent uncertainties includes in the problem model to find the proper solution.

The remainder of this paper is organized as follows. Section 2 describes a fuzzy random based possibilistic programming problem and Section 3 explains the treatments to the uncertainties used in this paper. Section 4 explains the solution of the possibilistic programming evaluation model with the linear fractional programming approach. The methodology herein is applied to an illustrative example in production planning presented in Section 5. Section 6 draws conclusions.

II. A FUZZY RANDOM BASED POSSIBILISTIC PROGRAMMING

A possibilistic programming [5] is formulated as follows:

$$Y_i \underline{\Delta} \sum_{j=1}^{n} \alpha_{ij} x_j \overset{\sim}{>} g_i, i = 1, \ldots, m,$$

$$x_j \geq 0, j = 1, \ldots, n, \qquad (1)$$

where α_{ij} is a possibilistic, $A_{ij} = \langle a_{ij}, d_{ij} \rangle$ is triangular fuzzy number with center a_{ij} and width d_{ij}, and $\overset{\sim}{>} g_i$ is a corresponding fuzzy goal G_i, defined by a fuzzy set with linear membership function $G_i = (g_i, d_i[$.

Consider that m fuzzy constraints or fuzzy goals $Y_i \overset{\sim}{>} g_i$ are divided into two groups, namely strongly desired to be satisfied and weakly desired to be satisfied, and is denoted by notations V_1 and V_2 respectively [6], [13]. Using $V_1 \cap V_2 = \emptyset$ and $V_1 \cup V_2 = \{1, 2, \ldots, m\}$, and

introducing a parameter h, the possibilistic linear programming problem is formulated as follows:

$$max \quad h$$

$$s.t. \quad N_{Y_i}(G_i) \geq h, \quad i \in V_1,$$

$$\Pi_{Y_i}(G_i) \geq h, \quad i \in V_2, \qquad (2)$$

$$\widetilde{a}_{ij} x_j \geq b, \quad i = 1, \ldots, m,$$

$$x_j \geq 0, \quad j = 1, \ldots, n,$$

$$0 \leq h \leq 1.$$

That is, the problem is reduced to the non-linear programming problem [6] as follows:

$$max \quad h$$

$$s.t. \quad \sum_{j=1}^{n} a_{ij} x_j - h \sum_{j=1}^{n} d_{ij} x_j \geq g_i - (1-h) d_1, \quad i \in V_1,$$

$$\sum_{j=1}^{n} a_{ij} x_j + (1-h) \sum_{j=1}^{n} d_{ij} x_j \geq g_i + (1-h) d_1, \quad i \in V_2, (3)$$

$$\widetilde{a}_{ij} x_j \geq b, \quad i = 1, \ldots, m,$$

$$x_j \geq 0, \quad j = 1, \ldots, n,$$

$$0 \leq h \leq 1.$$

Problem (3) can be solved using the simplex method and bisection method with respect to h.

However, estimating the coefficients of objective functions in mathematical model such as (3) is sometimes not easy in real situations especially when the values are unavailable or difficult to obtain. Thus, mathematical analysis such as regression model can be used to obtain the model coefficient using practical data. To simplify the explanation, we restrict ourselves to describe a concise introduction to fuzzy random regression (FRR) approach [12] to estimate the model coefficient and develop the fuzzy random objective function.

The FRR model is constructed based on fuzzy random variables and its confidence intervals. The interval is obtained by the expectation and variance of a fuzzy random variable [13] as follows:

$$I[e_X, \sigma_X] \underline{\Delta} \left[E(X) - \sqrt{var\ (X)}, \quad E(X) + \sqrt{var\ (X)} \right] \qquad (4)$$

Hence, the FRR model with $1 \times \sigma$ – confidence intervals is expressed as follows:

$$\min_{A} \quad J(C) = \sum_{k=1}^{K} \left(\widetilde{C}_k^r - \widetilde{C}_k^l \right)$$

$$\widetilde{C}_k^r \geq \widetilde{C}_k^l,$$

$$Y_i = \widetilde{C}_i\ I[e_{X_{i1}}, \sigma_{X_{i1}}] + \cdots + \widetilde{C}_K I[e_{X_{i1}}, \sigma_{X_{i1}}] \underset{\widetilde{h}}{\supseteq} I[e_{Y_i}, \sigma_{Y_i}] \qquad (5)$$

$$for \quad i=1, \cdots, N \quad k=1, \cdots, K.$$

Based on fuzzy random regression approach ([11], [12]), the fuzzy objective model is written as follows:

$$\widetilde{Y}_i = \overline{\widetilde{A}} \times I\!\left[e_{X_i}, \sigma_{X_i}\right] = \left(\frac{\left[\overline{\widetilde{a}}_k^l + \overline{\widetilde{a}}_k^r\right]}{2}, \overline{\widetilde{a}}_k^l, \overline{\widetilde{a}}_k^r\right)_T I\!\left[e_{X_i}, \sigma_{X_i}\right] \quad (6)$$

where fuzzy random coefficients $\overline{\widetilde{A}}_k = \left[\overline{\widetilde{a}}_k^l, \overline{\widetilde{a}}_k^r\right]$, were deduced by fuzzy random regression method.

III. TREATING THE UNCERTAINTIES

Two types of uncertainties were dealt in this discussion. First, fuzzy random regression is used to treat the fuzziness and randomness that co-exist in the data used to estimate the model's coefficient. And, secondly, a necessity measure is used to treat the vagueness in the goal target and the ambiguity in the model's coefficients.

A. Treating the Fuzziness and Randomness

A symmetric triangular fuzzy random $\overline{\widetilde{A}}_k$ is then denotes as $\overline{\widetilde{A}}_k = \left\langle \overline{\widetilde{a}}_k, \overline{\widetilde{d}}_k \right\rangle$ with center $\overline{\widetilde{a}}_k$ and width $\overline{\widetilde{d}}_k$. Fuzzy random objective containing one-sigma confidential-interval is rewritten as follows:

$$\widetilde{Y}_i = \overline{\widetilde{A}}_k \times I\!\left[e_{X_i}, \sigma_{X_i}\right] = \left\langle \overline{\widetilde{a}}_k, \overline{\widetilde{d}}_k^r \right\rangle_T I\!\left[e_{X_i}, \sigma_{X_i}\right] \quad (7)$$

Therefore, by using fuzzy random based objective function (7), possibilistic programming (3) is rewritten into the following expression:

$$max\ h$$

$$s.t.\ \sum_{j=1}^{n}\overline{\widetilde{a}}_{ij}x_j - h\sum_{j=1}^{n}\overline{\widetilde{d}}_{ij}x_j \geq g_i - (1-h)\overline{\widetilde{d}}_1,\ \ i \in V_1,$$

$$\sum_{j=1}^{n}\overline{\widetilde{a}}_{ij}x_j + (1-h)\sum_{j=1}^{n}\overline{\widetilde{d}}_{ij}x_j \geq g_i + (1-h)\overline{\widetilde{d}}_1,\ \ i \in V_2, \quad (8)$$

$$\widetilde{a}_{ij}x_j \geq b,\ \ i = 1,\ldots,m,$$

$$x_j \geq 0,\ \ j = 1,\ldots,n,$$

$$0 \leq h \leq 1.$$

From (8), $\overline{\widetilde{A}}_k = \left\langle \overline{\widetilde{a}}_k, \overline{\widetilde{d}}_k \right\rangle$ are the possibilistic variables restricted by fuzzy numbers $\widetilde{A} = \left\langle \widetilde{a}, d_{\widetilde{a}_i} \right\rangle$ and $\overline{\widetilde{A}}_k = \left\langle \overline{\widetilde{a}}_k, \overline{\widetilde{d}}_k \right\rangle$, respectively.

That is, possibilistic programming model (8) are initially developed by considering fuzzy random situation that estimates the coefficients of the model.

To solve possibilistic programming problem (8), the constraints and objective function are treated using necessity and possibility measure. It is assumed that the decision maker specifies the possibility and necessity aspiration levels with respect to objective function values. The possibility aspiration level is the objective function value the decision maker would like to keep a chance to achieve. The necessity aspiration level is the objective function value the decision maker would like to achieve certainly. In this paper, we restrict ourselves to explain necessity measure in the treatment of constraints and objective function. Necessity measure evaluates to what extent the decision maker's aim can be achieved certainly.

B. Dealing the constraints

It is important to treat the constraint that is described in ambiguous coefficient in the mathematical model. The treatment is prepared by giving the interpretation to the constraint so as the constraint in the model is closely translates the meaning of decision maker's desire. Using the necessity measure, the certainty of decision maker intention to the constraint is indicated.

Let $\mathbf{v}^\eta \in [0,1]^m$ be a necessity aspiration degree that a decision maker is aspired to achieve certainly. The constraints $\overline{\widetilde{\mathbf{A}}}\mathbf{x} \leq \overline{\widetilde{\mathbf{b}}}$ can be treated as follows:

$$Nec\!\left(\!\left(\overline{\widetilde{\mathbf{A}}}\mathbf{x} \leq \overline{\widetilde{\mathbf{b}}}\right) \geq \mathbf{v}^\eta\right) \quad (9)$$

Note that, this is the case where the decision maker feels that a certainty degree is not less than v^Ω. The symmetric fuzzy number is written as $A = \left\langle \sum_{j=1}^{n} x_j a_j, \sum_{j=1}^{n}|x_j|d_j \right\rangle$. From (5.5), let us assume that s is less than v^η, and we obtain,

$$s = \sum_{j=1}^{n} x_j a_j + v^\eta\!\left(\sum_{j=1}^{n}|x_j|d_j\right) \quad (10)$$

Thus, expression (5.6) is the treated constraint which considers the certainty degree of decision maker's intention to the problem constraint.

C. Dealing the objectives

In a fuzzy mathematical programming problem, each objective function value is not always a real number. The objective function value is frequently only restricted by a possibility distribution $\alpha(x)$. Therefore, the meaning of the objective should be interpreted.

There are several ways to deal with the objective [14]. In this model, the fuzzy goal (decision maker's target value) is included in the objective function and is treated as a constraint by using a modality approach.

Let us consider that the decision maker wants to maximize the certainty degree that the event is not smaller than g^η, and is modeled as $max\ Nec(\alpha x \geq g^\eta)$.

Using additional variable h, the following model expresses the decision maker's intention (aspiration).

$$max \quad h$$
$$s.t \quad Nec\left(\left(\alpha x \ge g^{\eta}\right) \ge h\right) \quad (11)$$

Problem (11) is equivalent to the following model.

$$max \quad h$$
$$s.t \quad \left(\frac{\sum_{j=1}^{n} x_j \alpha_j}{\sum_{j=1}^{n} |x_j| d_j}\right) \ge h \quad (12)$$

IV. Weighted Linear Fractional Programming for Multi-Objective Fuzzy Random-Possibilistic Programming Problem

Let us consider the following multi-objective linear programming problem:

$$opt \quad Z = \left[\widetilde{\mathbf{C}}_i \mathbf{x}\right]^T = \left[\mathbf{Z}_i(\mathbf{x})\right]^T, \quad (13)$$
$$\text{subject to:} \quad \mathbf{x} \in X, X = \left\{\mathbf{x} \in R^n : \widetilde{\mathbf{A}}\mathbf{x} \widetilde{\le} \widetilde{\mathbf{b}}, \mathbf{x} \ge \mathbf{0}\right\}$$

where opt indicates the objective that is to maximize or to minimize. $\widetilde{\mathbf{c}}_i \in R^n$ are the coefficients of objectives, $\widetilde{\mathbf{A}} = \left(\widetilde{a}_{ij}\right)_{m \times n}$ are the coefficients of constraints, and $\widetilde{\mathbf{b}}_i \in R^m$ are fuzzy resource. $\widetilde{\le}$ are used to exemplifies the vague aspiration of decision maker towards the objective.

The multiple objective possibilistic problem is formulate as the following model:

$$max \quad \sum_{i=1}^{p} h_i^{\eta}$$
$$\text{subject to:} \quad \sum_{i=1}^{p} \left(\frac{\sum_{j=1}^{n} x_j \alpha_j}{\sum_{j=1}^{n} |x_j| d_j}\right) \ge h \quad (14)$$

Using a modality optimization to solve a multi-objective possibilistic programming problem results in fractional programming[14]. Fractional programming solution is important as various problems consider the optimization of a ratio between physical and/or economic linear functions [15], among others. Thus, the modality optimization takes the advantages of fractional programming in finding the problem's solution.

A multi-objective fuzzy-random based possibilistic programming problem (FR-PPP) model (13) is rewritten by using the treated constraints (10) and objectives (12) as follows:

$$max \quad \sum_{i=1}^{p} \left(\frac{\sum_{j=1}^{n} x_j \alpha_j}{\sum_{j=1}^{n} |x_j| d_j}\right)$$
$$\text{subject to} \quad \sum_{i=1}^{p} \sum_{j=1}^{n} x_j a_j + v^{\eta}\left(\sum_{j=1}^{n} |x_j| d_j\right), \quad (15)$$
$$x_i \ge 0$$

Problem (15) is a linear fractional programming problem with multiple objectives. General form of multi-objective linear fractional programming problem is as follows:

$$opt \quad Z(x) = \left[\frac{N_1 x}{D_1 x}, \cdots, \frac{N_p x}{D_p x}\right]^T, \quad (16)$$
$$\text{s.t.} \quad x \in X, X = \left\{x \in R^n : \widetilde{A}x \widetilde{\le} \widetilde{b}, x \ge 0\right\}$$

The variable change technique [16], which turns a linear fractional problem into a linear program, is used to solve problem (15).

The compatibility of a value of j of $P \le \left(N_i x, N_i^0\right)$ may be given as in [17] by the following function.

$$C_j^{N_i} = \begin{cases} 0 & \text{if } N_i x < p_i^j \\ \frac{N_i x - p_i^j}{N_i^0 - p_i^j} & \text{if } p_i^j \le N_i x \le N_i^0, j = 1,2,3, i = 1, \ldots, p. \\ 0 & \text{if } N_i x > N_i^0 \end{cases} \quad (17)$$

Similarly, the compatibility of a value of j of $P \ge \left(D_i x, D_i^0\right)$ is given by

$$C_j^{D_i} = \begin{cases} 0 & \text{if } D_i x > s_i^j \\ \frac{s_i^j - D_i x}{s_i^j - D_i^0} & \text{if } D_i^0 \le D_i x \le s_i^j, j = 1,2, i = 1, \ldots, p. \\ 0 & \text{if } D_i x < D_i^0 \end{cases} \quad (18)$$

Let us consider relative importance w_i for $\mu_i^{N_i} = C_j^{N_i}$ and w_i' for $\mu_i^{D_i} = C_j^{D_i}$ such that $w_i > 0, w_i' > 0$ and $\sum_{i=1}^{p} w_i + w_i' = 1$. Thus we obtain the simple additive weighting model to solve the multi-objective linear fractional programming problem (15) as follows:

$$opt \quad V(\mu) = \sum_{i=1}^{p} \left(w_i \mu_i^{N_i} + w_i' \mu_i^{N_i}\right),$$
$$\text{such that:} \quad \mu_i^{N_i} = C_j^{N_i}, \mu_i^D = C_j^D, \quad (19)$$
$$Ax \le b, \mu_i^{N_i} \le 1, \mu_i^{D_i} \le 1,$$
$$\mu_i^{N_i} \ge 0, \mu_i^{D_i} \ge 0, x \ge 0, i = 1, \ldots, p,$$

where $V(\mu)$ is the achievement function.

The proposed multi-objective possibilistic evaluation algorithm is simplified as follows:
1. Describe the problem and build the initial model using fuzzy random regression model (Nureize and Watda, 2010).
2. Treats the constraints.
3. Treats the objectives
4. Develop a multi-objective possibilistic programming model as Equation (15).

5. Solve the linear fractional programming model (19) for the solution.

V. Experimentation

The production planning problems in industrial estate are investigated with two decision variables and two functional objectives under four system constraints. The problem is then modeled as follows:

$$\begin{aligned}
&\text{find} \quad x \\
&\text{to satisfy} \\
&\quad \text{max profit } Z_1 = \langle 0.860, 0.100 \rangle x_1 + \langle 1.100, 0.100 \rangle x_2, \\
&\quad \text{max production } Z_2 = \langle 1.126, 0.020 \rangle x_1 + \langle 0.000, 0.000 \rangle x_2, \quad \Big\}(a) \\
&\text{subject to} \\
&\quad \text{raw material } F_1 = \langle 3.75, 0.06 \rangle x_1 + \langle 0.91, 0.08 \rangle x_2 \leq 87.75, \\
&\quad \text{labor } F_2 = \langle 0.65, 0.55 \rangle x_1 + \langle 0.90, 0.09 \rangle x_2 \leq 4.42, \quad \Big\}(b) \\
&\quad \text{mills } F_3 = \langle 17.35, 0.85 \rangle x_1 + \langle 2.16, 0.27 \rangle x_2 \leq 95.20, \\
&\quad \text{capital } F_4 = \langle 0.87, 0.65 \rangle x_1 + \langle 0.98, 0.65 \rangle x_2 \leq 20.15,
\end{aligned} \quad (20)$$

Based on Step 2 that is dealing with the constraints, let us assume that the decision maker decides that certainty degree not less than $v^\eta = 0.7$ is high enough for the system constraint (b) in the Problem (20). To satisfy the decision maker aim, analyze the constraints under expression. The treated constraints based on decision maker aim are as follows:

$$\begin{aligned}
&Nec\big(\big(\langle 3.75, 0.06 \rangle x_1 + \langle 0.91, 0.08 \rangle x_2 \leq 87.75\big) \geq v^\eta\big), \\
&Nec\big(\big(\langle 0.65, 0.55 \rangle x_1 + \langle 0.90, 0.09 \rangle x_2 \leq 4.42\big) \geq v^\eta\big), \\
&Nec\big(\big(\langle 17.35, 0.85 \rangle x_1 + \langle 2.16, 0.27 \rangle x_2 \leq 95.20\big) \geq v^\eta\big), \\
&Nec\big(\big(\langle 0.87, 0.65 \rangle x_1 + \langle 0.98, 0.65 \rangle x_2 \leq 20.15\big) \geq v^\eta\big),
\end{aligned} \quad \bigg\}(b) \quad (21)$$

For the objective part, let us assume that the decision maker aims to maximize the certainty degree of profit is not smaller than 5.0 million dollars, and to maximize the certainty degree of production volume is not smaller than 5.2 million tones. The decision maker aims are modeled as follows:

$$\begin{aligned}
&Nec\big(0.86x_1 + 1.10x_2 \geq 5.0\big) \\
&Nec\big(1.126x_1 + 0.00x_2 \geq 5.2\big)
\end{aligned} \quad (22)$$

The treated problem (20) is rewritten as follows:

$$\max \quad \left\{ \frac{0.86x_1 + 1.1x_2 - 5.0}{0.1x_1 + 0.1x_2}, \frac{1.126x_1 + 0x_2 - 5.2}{0.02x_1 + 0x_2} \right\} \quad (23)$$

$$\begin{aligned}
\text{subject to}: \quad &3.79x_1 + 0.95x_2 \leq 87.75, \\
&1.03x_1 + 0.96x_2 \leq 4.42, \\
&17.94x_1 + 2.34x_2 \leq 95.20, \\
&1.32x_1 + 1.43x_2 \leq 20.15, \\
&x_i \geq 0.
\end{aligned}$$

The equivalent linear programming for problem (23) is as follows:

$$\begin{aligned}
\max \quad : \quad &V(\mu) = 0.258\mu_{N_1} + 0.241\mu_{N_2} + 0.072\mu_{D_1} + 0.127\mu_{D_2} \\
\text{subject to}: \quad &0.45\mu_{N_1} - 0.860x_1 - 1.100x_2 = -5.3, \\
&0.37\mu_{N_{21}} - 1.126x_1 + 0.000x_2 = -5.8, \\
&2.00\mu_{D_1} + 0.100x_1 + 0.100x_2 = 2, \\
&3.00\mu_{D_2} + 0.020x_1 + 0.000x_2 = 2, \\
&\text{system constraints (b)}, \\
&\mu_{N_i} \leq 1, \mu_{D_i} \leq 1, \mu_{N_i} \geq 0, \mu_{D_i} \geq 0, x_i \geq 0.
\end{aligned} \quad (24)$$

The mathematical programming problem (24) was solved by proposed method that is a multi-objective fuzzy random based possibilistic programming approach using linear fractional programming approache. The constraints and objectives that contain fuzzy random coefficient were re-treated using necessity measure to exemplify decision maker's intention so as to make the mathematical programming for the respective model is as close as a decision maker's aim. The optimal solution of the problem model (20) is $(x_1, x_2) \approx (5.35, 1.03)$ whose objective value is $V(\mu) = 0.87$, with $\mu_{N_1} = 1.00$, $\mu_{N_2} = 0.62$, $\mu_{D_1} = 0.68$ and $\mu_{D_2} = 0.96$.

Taking the central value of the coefficient for problem (24), we get the following crisp multi-objective linear programming problem.

$$\begin{aligned}
\text{find} \quad &: x \\
\text{to satisfy} \quad &: \text{maximize} \, 0.86x_1 + 1.10x_2, \\
&\quad \text{maximize} \, 1.126x_1 + 0.02x_2, \\
\text{subject to} \quad &: 3.75x_1 + 0.91x_2 \leq 87.75, \\
&\quad 0.65x_1 + 0.90x_2 \leq 4.42, \\
&\quad 17.35x_1 + 2.16x_2 \leq 95.20, \\
&\quad 0.87x_1 + 0.98x_2 \leq 20.15.
\end{aligned} \quad (25)$$

Following Zimmermann max-min operator approach [18], we solve problem (25) with optimal solutions $\lambda = 0.98$ and $(x_1, x_2) = (5.37, 0.92)$.

Even though the solution of the crisp mathematical model problem (25) is slightly similar to the proposed method solution for the problem (23), it is not considering the decision maker's aspiration and fuzziness and randomness in its model evaluation.

VI. Conclusions

This paper explains a a fractional programming to solve a fuzzy random based possibilistic programming problem. Possibilistic optimization in solving multi-criteria decision making is important for ensuring that the developed mathematical model is able to retain and account for uncertainties until solving the model and, furthermore, permits the existence of imprecise situations in multi-criteria decision making to be handled appropriately.

It is pointed out that the new formulation of possibilistic multi-objective programming problem is presented. The existing possibilistic programming approach is extended to address two important issues. First, the model coefficients of mathematical possibilistic programming are capable of handling fuzzy random information. Second, the fuzzy random based possibilistic programming is extended to solve multi-objective problem. That is, the new possiilistic multi-objective evaluation scheme is capable of providing double treatment of uncertainties in its problem solution.

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