MODELLING AND ANALYSIS OF COMPLEX ELECTROMAGNETIC PROBLEMS USING FDTD SUBGRIDDING IN HYBRID COMPUTATIONAL METHODS

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Development of Hybridised Method of Moments, Finite-Difference Time-Domain Method and Subgridded Finite-Difference Time-Domain Method for Precise Computation of Electromagnetic Interaction with Arbitrarily Complex Geometries

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Abstract

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Keywords

Computational Electromagnetics; Method of Moments (MoM); Finite-Difference Time-Domain (FDTD); Quasi-static method; Hybrid computational method; Subgridding; Principle of equivalent sources; Perfectly Matched Layer (PML); Antennas.

The main objective of this research is to model and analyse complex electromagnetic problems by means of a new hybridised computational technique combining the frequency domain Method of Moments (MoM), Finite-Difference Time-Domain (FDTD) method and a subgridded Finite-Difference Time-Domain (SGFDTD) method. This facilitates a significant advance in the ability to predict electromagnetic absorption in inhomogeneous, anisotropic and lossy dielectric materials irradiated by geometrically intricate sources. The Method of Moments modelling employed a two-dimensional electric surface patch integral formulation solved by independent linear basis function methods in the circumferential and axial directions of the antenna wires. A similar orthogonal basis function is used on the end surface and appropriate attachments with the wire surface are employed to satisfy the requirements of current continuity. The surface current distributions on structures which may include closely spaced parallel wires, such as dipoles, loops and helical antennas are computed. The results are found to be stable and showed good agreement with less comprehensive earlier work by others.

The work also investigated the interaction between overhead high voltage transmission lines and underground utility pipelines using the FDTD technique for the whole structure, combined with a subgridding method at points of interest, particularly the pipeline. The induced fields above the pipeline are investigated and analysed.

FDTD is based on the solution of Maxwell’s equations in differential form. It is very useful for modelling complex, inhomogeneous structures. Problems arise when open-region geometries are modelled. However, the Perfectly Matched Layer (PML) concept has been employed to circumvent this difficulty. The establishment of edge elements has greatly improved the performance of this method and the computational burden due to huge numbers of time steps, in the order of tens of millions, has been eased to tens of thousands by employing quasi-static methods.

This thesis also illustrates the principle of the equivalent surface boundary employed close to the antenna for MoM-FDTD-SGFDTD hybridisation. It depicts the advantage of using hybrid techniques due to their ability to analyse a system of multiple discrete regions by employing the principle of equivalent sources to excite the coupling surfaces. The method has been applied for modelling human body interaction with a short range RFID antenna to investigate and analyse the near field and far field radiation pattern for which the cumulative distribution function of antenna radiation efficiency is presented. The field distributions of the simulated structures show reasonable and stable results at 900 MHz. This method facilitates deeper investigation of the phenomena in the interaction between electromagnetic fields and human tissues.

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Acronyms

1-D	One-dimension
2-D	Two-dimension
3-D	Three-dimension
3G	Third Generation
ABC	Absorbing Boundary Condition
AC	Alternating Current
ADI-FDTD	Alternating Direction Implicit Finite-Difference Time-Domain
BAN	Body Area Networks
CDF	Cumulative Distribution Function
CFL	Courant-Friedrich-Levy
DC	Direct Current
DOP	Direction of Propagation
EMC	Electromagnetic Compatibility
FDTD	Finite-Difference Time-Domain
FEM	Finite-Element Method
FSS	Frequency Selective Surface
GA	Genetic Algorithm
GTD	Geometrical Theory of Diffraction
HF	High Frequency
HSG	Huygens Subgridding
IFA	Incident-field Array
MIMO	Miltiple Input Multiple Output
MoM	Method of Moment
MRI	Magnetic Resonance Imaging
MSMoM	Multistructure Method of Moment
MTR	Multitemporal Resolution
NAG	Numerical Algorithms Group
NMHA	Normal Mode Helical Antenna
OHTL	Overhead High Voltage Transmission Lines
PBG	Photonic Bandgap
PDF	Probability Density Function
PEC	Perfect Electric Conductor
PEE	Partial Eigenfunction Expansion
PML	Perfectly Matched Layer
Q-factor	Quality Factor
RCS	Radar Cross Section
RF	Radio Frequency
RFID	Radio Frequency Identification

RSS	Received Signal Strength
SAR	Specific Absorption Rate
SGFDTD	Subgridded Finite-Difference Time-Domain
TE	Transverse Electric
TE_z	Transverse electric relative to the *z*-direction (only H_z, E_x, and E_y fields are present)
TEM	Transverse Electromagnetic
TM	Transverse Magnetic
TM_z	Transverse magnetic relative to the *z*-direction (only E_z, H_x, and H_y fields are present)
UMTS	Universal Mobile Telecommunications System
UTD	Uniform Geometrical Theory of Diffraction
UWB	Ultra Wideband
VIE	Volume Integral Equation
VHF	Very High Frequency
WBSN	Wireless Body Sensor Network
WLAN	Wireless Local Area Network

Symbols

E	Electric field intensity
H	Magnetic field intensity
B	Magnetic flux density
D	Electric flux density
σ^*	Magnetic resistivity
σ	Electric conductivity
μ	Magnetic permeability
ε	Electric permittivity
I_j	Basis function for surface current
$\mathbf{J}_j$	Surface current
$\mathbf{E}_i$	Incident electric field strength
L	Integro-differential operator
A	Vector potentials
ϕ	Scalar potentials
$\mathbf{W}_m$	Testing functions
Z	Impedance matrix
V	Excitation vector
$g(R)$	Free-space Green function
R	Distance between observation and source points on wire surface
A	Radius of helix wire
B	Radius of helix wire
P	Pitch distance between helix wire turns
α	Azimuth angle of circumferential cross-section wire
δ	Pitch angle
γ	Axial length of curvilinear patch
$\hat{a}_\gamma$	Unit vector in axial surfaces of wire antenna
$\hat{a}_{cs}$	Unit vector in circumferential surfaces of wire antenna
$\hat{a}_r$	Unit vectors in radial directions on the end surface of wire antenna
$\hat{a}_{ce}$	Unit vectors in circumferential directions on the end surface of wire antenna
$f(\gamma)$	Triangular basis functions into the axial direction
$P_{absorbed}$	Absorbed power
$P_{radiated}$	Radiated power

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Chapter 1

Introduction

1.1 Background and Motivations

Over the past years, a number of numerical and analytical approaches to Maxwell's time-dependent curl equations were broadly used with the increases in computer memory capacity and relentless advances in computational imitating efficiency. Consequently, the demand for efficient field modelling tools in electromagnetic scattering problems is ceaselessly expanding. In general, computational electromagnetic techniques have been applied to vast areas including the study of the radiation, scattering and penetration of electromagnetic wave with 3-D objects, in problems related to telecommunication, electromagnetic compatibility (EMC), microwave devices, waveguide structures and medical diagnosis.

Apparently, many considerations need to be taken into account when deciding to choose the most suitable numerical technique in order to solve a specific problem. Some of the main factors include the geometry of the scattering and radiating objects, computer requirements, the domain of interest whether time-domain or frequency-domain, and the absorbing boundary conditions (ABC). The material characteristics and its geometry play important roles in choosing the approach used to mathematically model the properties of the electromagnetic interaction. In biomedical application for instance, the object is inhomogeneous, anisotropy, penetrable body which cannot be easily included in the formulations. However, the ability of FDTD algorithm to simulate and investigate the inhomogeneous, anisotropy media makes it very useful and effective technique thus far. Typically, there are two main categories of numerical algorithms approaches applied by researches. They are namely the frequency-domain integral formulation using the Method of Moments (MoM), and the time-domain differential formulation by means of the Finite-Difference Time-Domain (FDTD) method and the Finite Element Method (FEM). Integro-differential equation method is generally global in nature so that the initial and boundary conditions must be imposed as the algorithm continues. In contrast, differential-equation technique is typically local in nature so that the initial and boundary conditions are both directly included in the computational algorithm. The former technique usually postulates extensive analytical pre-processing whereas the later demands negligible analytical pre-processing. The methods mentioned above can be applied to certain specific geometries of concern. Consequently, the differential-equation formulations are increasingly well-known due to the fact that it can solve any type of geometries in the problem space of the computational domain.

1.2 Development History of FDTD

First introduced by Yee in 1966 [1], FDTD is widely used to solve electromagnetic scattering problems due to its muscular characteristics including:

- Simplicity: The second-order accurate central finite-difference approximations for spatial and temporal derivatives of the electric and magnetic vector field components is directly used to solve Maxwell's equations explicitly in the absence of linear algebra.

- Fidelity: Wideband and narrowband applications can be easily implemented by applying different type of time pulse shape such as Gaussian and sinusoidal wave respectively.
- Robustness: Numerical dispersions in FDTD computations can be enclosed to model very large variety of electromagnetic scattering problems accurately. Furthermore, the FDTD algorithm can be easily implemented on parallel computers for faster simulation time.
- Effectiveness: Problems involving nonlinear media can be inherently alleviated in a straightforward manner in the time-domain compared with those in the frequency-domain technique.
- Versatility: It can intrinsically be used to model inhomogeneous, anisotropy materials such as biological tissues, geophysical strata and shielding metal structures.

Generally, the algorithm used by Yee was described by the electric field component which was spatially and temporally offset from the magnetic field component to acquire the update equations. These equations were used in a leap-frog manner to propagate the electric and magnetic fields ahead in time. The equations provide the present fields in terms of the past fields all over the computational domain. After Yee’s publication, the approach was widely used with different endeavour [2-6].

The boundaries of the computational domain in FDTD need to be carefully treated when simulating problems in open regions. Spurious reflections will generally occur from the termination of the grid. The problem can be solved by means of the well known method called the absorbing boundary condition (ABC). It is generally meant to absorb any outgoing propagating waves without ideally producing spurious reflections. The ABCs was first proposed in 1971 by Merewether [7] to solve the open region difficulties. The development chronicle to magnify the practicability study of the technique was continued in the literature by [8-12] which were based of nonmaterial type. In contrast, Berenger presented a new idea in 1994 called the perfectly matched layer (PML) ABC which was based on material category [13]. The state of the art of Berenger’s PML contributes to notably better precision when compared to the other ABCs in the written works [14, 15] for broad assortment of applications.

The main handicap of FDTD lies in the truth that only consistent grids can be used. Accordingly, the geometry resemblance in FDTD is restricted to staircase-shaped boundaries which lead to a large number of computer memory requirements and the CPU time particularly when dealing with curvature geometries with fine features [16]. The total number of cells in the computational domain grows significantly due to a global fine mesh. Another FDTD weakness is the presence of error due to numerical dispersion [17, 18]. In this case, many scientists were prompted to examine the subgridding scheme as an approach to parry the problem. A variety of methods have been proposed to boost the efficiency of FDTD technique such as non-uniform meshing [19], sub-cellular technique [20], non-orthogonal meshing [21], alternative direction implicit (ADI) method [22-24], higher-order technique [25, 26], hybrid method [27-29] and subgridding method [30-34]. In FDTD subgridding technique, the smaller size components in a structure is filled with fine grids and the remaining of the space is represented by coarse grids. The fields on the boundary between coarse and fine grids are basically unknown in nature. They are predicted by using spatial and temporal interpolations. The regions of the coarse and fine grids are computed by the FDTD method and are kept in time step which satisfy the Courant stability condition. Consequently, the stable subgridding algorithm can refine the mesh locally. Hence, the accuracy of the solution can be improved without increasing the computational efforts significantly.

1.3 Development History of MoM

The MoM is basically a general procedure for solving linear equations. The “moments” in its name is due to the process of taking moments by multiplying the suitable weighting functions and integrating. In other words, it is essentially the technique of weighted residuals applicable for solving both the differential and integral equations. The advantages of MoM are accuracy, versatility and the potential to compute the near and far zone parameters. Furthermore, the method proved its ability to solve real complex antenna geometry in both frequency and time domain. The use of MoM and related matrix methods has become widespread in electromagnetic areas since the published paper of Richmond in 1965 [35] by generating a system of linear equations for the unknown current density and enforcing the boundary conditions at discrete points in the scattering body. Afterwards, he developed a point-matching solution for scattering by conducting bodies of arbitrary shape [36]. In 1967, Harrington documented the mathematical concept of MoM by which the functional equations of the field theory were reduced to matrix equations [37]. Later, he published a book on MoM which was a step forward towards the development of the numerical techniques [38].

The prime drawback of using MoM lies beneath its rectangular and triangular basis function. Their usage to examine the problem of electromagnetic scattering by dielectric objects with high dielectric constant leads to spurious charges. This problem was alleviated by means of solenoidal basis function [39, 40]. Wilton and Govind [41] made an effort to circumvent the error currents and anomaly behaviour of the solution near the edges by means of triangle expansion functions with suitable singular pulses at the edges.

The MoM has been favourably employed in variety of electromagnetic applications such as scattering problems [42-45], synthesis of slotted waveguide array antenna [46], field analysis in circular-loop antenna [47, 48], the solution for patch antenna using volume integral equation (VIE) [49], VHF propagation modelling [50] and microwave tomography system [51].

1.4 State of the Art of the Original Contribution

The dominant focus of the research work is the modelling and analysis of the complex electromagnetic problems by means of subgridded FDTD (SGFDTD) scheme to be employed in several applications. In order to achieve the research goal, the basic idea is to use the hybridisation of SGFDTD with MoM in which the tools for electromagnetic field modelling problems can be designed with more accuracy and efficiency. The surface kernel solution of MoM technique is derived. The method is used to predict the surface current distributions on structures with closely spaced parallel wires, such as dipoles, loops and helical antennas. Next, the present work is devoted to mathematical modelling and implementing SGFDTD in 2-D Cartesian coordinate keeping minimum reflection at the boundary. This method is applied to the interaction between overhead transmission lines to the underground pipeline for validation purposes. The SGFDTD formulation is then embedded inside the hybrid MoM-FDTD method. The full code with the adaptation of subgridding inside the hybrid MoM-FDTD design problems is written in Fortran 90 as a platform. The hybridisation of MoM-FDTD-SGFDTD code is used to analyse and investigate the applications in electromagnetic problems for validation such as the interaction between EM fields to the human body.

1.5 Overview of the Thesis

Chapter 1 postulates historical background and literature survey of FDTD and MoM techniques used to solve electromagnetic scattering issues. It should be noted that a more detailed review of existing literature is reported at the beginning of each chapter with separate references at the end.

Chapter 2 unfolds the theoretical concept of FDTD principles including the derivation of the magnetic and electric field update equations, parameters that control the stability and accuracy, plane wave source modelling concept by applying the equivalent surface, and finally the implementation of Berenger’s PML absorbing boundary condition.

Chapter 3 presents the surface kernel solution of the Method of Moments. The surface current distributions on structures with closely spaced parallel wires, such as dipoles, loops and helical antennas, are computed by using the method of moments with a general surface patch formulation. The modelling method employed a two-dimensional electric surface patch integral equation formulation solved by independent piecewise-linear basis function methods in the circumferential and axial directions of the wire.

Chapter 4 explains the modelling and analysis of quasi-static FDTD subgridding technique in two-dimensional approach. The method has been applied to model biological cell with floquet theorem. The interaction between overhead transmission lines and underground pipeline at power-line frequency is also modelled for validation. FDTD technique is used for the whole structure spatial problem combined with subgrid method at the pipeline. The soil in the common corridor has been designed as arbitrarily inhomogeneous.

Chapter 5 describes the hybridisation MoM-FDTD-SGFDTD computational method. The modelling on multiple-region hybrid techniques with frequency-domain MoM and time-domain FDTD and subgridding are suggested and investigated. The method is validated for near field and far field applications particularly on the interaction between electromagnetic fields and human body in which the RFID antenna is located and moved at several positions in front and back of inhomogeneous human body model.

Chapter 6 summarises the overall conclusions and recommendations for further work on related topics.

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Chapter 2

FDTD Technique for Field Truncation

2.1 Introduction

Over the past few years, finite-difference time-domain (FDTD) method [1] have become increasingly prevalent in the computational electromagnetic problems due to its simplicity, efficiency, robustness and versatility scheme for highly complex configuration in the computational domain. Generally, FDTD technique is the most well-known numerical method for the solution of problems in electromagnetic simulation ranging from RF to optical frequencies. It is considered to be one of the most powerful numerical techniques for solving partial differential equations of any kind. In addition, it can be utilized to solve the spatial as well as the temporal distributions of electric and magnetic fields in various media. In principle, FDTD is a method that divides the solution domain into finite discrete points and then replaces the partial differential equation with a set of difference equations. It has successfully been applied to many problems of propagation, radiation and scattering of electromagnetic waves such as antenna, radar, wireless communication system, high speed electronic, photonic, radiography, x-ray crystallography, bio-electromagnetic and geophysical imaging. A good measure of its success lies in the fact that thousand of papers on the subject have been published in journals and international symposium, apart from the books and tutorials devoted to it. Moreover, much specific and general purpose commercial software is available on the market which further extends its appeal globally. Furthermore, three books are used as the main references to the recent FDTD research written by Taflove and Hagness [1], Taflove [2] and Kunz and Luebbers [3].

2.2 FDTD Updating Stencils

A pioneering way of describing the electromagnetic phenomena was introduced by James Clerk Maxwell in 1865 [4]. Later in 1873, he published an article called “Treatise on Electricity and Magnetism” in which the discoveries of Coulomb, Oersted, Ampere and Faraday were united into four refined constructed mathematical equations known as Maxwell’s equations. The differential time domain Maxwell’s equations in a linear medium are given by:

$$\frac{\partial B}{\partial t} = -\nabla \times E - J_m \tag{2.1}$$

$$\frac{\partial D}{\partial t} = \nabla \times H - J_e \tag{2.2}$$

B is the magnetic flux density in Wb/m^2, D is the electric flux density in C/m^2, E is the electric field in V/m and H is the magnetic field in A/m. J_m is the magnetic current density in V/m^2 and is defined to relate any magnetic loss to the field. J_e is the electric current density in A/m^2 and is defined to relate any electric loss to the field. J_m and J_e are respectively given by:

$$J_m = \sigma^* \times H \tag{2.3}$$

$$J_e = \sigma \times E \tag{2.4}$$

σ^* is magnetic resistivity in Ω/m and σ is the electric conductivity in S/m. In materials with field-independent, direction-independent and frequency-independent electric and magnetic properties, the following proportions apply:

$$B = \mu \times H \tag{2.5}$$

$$D = \varepsilon \times E \tag{2.6}$$

μ is the magnetic permeability in H/m and ε is the electric permittivity in F/m. Inserting (2.3) and (2.5) to (2.1) and dividing by μ gives:

$$\frac{\partial H}{\partial t} = \frac{1}{\mu}\left(-\nabla \times E - \sigma^* H\right) \tag{2.7}$$

$\sigma^* H$ is the magnetic losses which may exist inside the medium. Inserting (2.4) and (2.6) to (2.2) and dividing by ε gives:

$$\frac{\partial E}{\partial t} = \frac{1}{\varepsilon}\left(\nabla \times H - \sigma E\right) \tag{2.8}$$

σE is the electric losses which may exist inside the medium. In Cartesian coordinates, equations (2.7) and (2.8) yield the following six scalar equations:

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} - \sigma^* H_x\right) \tag{2.9}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} - \sigma^* H_y\right) \tag{2.10}$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} - \sigma^* H_z\right) \tag{2.11}$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma\, E_x\right) \tag{2.12}$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma\, E_y\right) \tag{2.13}$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma\, E_z\right) \tag{2.14}$$

A space point in a rectangular grid is defined from Yee's original notation [5] as:

$$(i, j, k) = (i\Delta x, j\Delta y, k\Delta z) \tag{2.15}$$

Let F denote any function of discrete space and time given by:

$$F(i\Delta x, j\Delta y, k\Delta z, n\Delta t) \equiv F^{n}_{i,j,k} \tag{2.16}$$

Δx, Δy and Δz are the grid space increments in x, y and z directions respectively, and Δt is the time increment. Using a central finite-difference approximation, space and time derivatives of F can be written as:

$$\frac{\partial F}{\partial x} = \frac{F^{n}_{i+1/2,j,k} - F^{n}_{i-1/2,j,k}}{\Delta x} + O(\Delta x)^2 \tag{2.17}$$

$$\frac{\partial F}{\partial t} = \frac{F^{n+1/2}_{i,j,k} - F^{n-1/2}_{i,j,k}}{\Delta t} + O(\Delta t)^2 \tag{2.18}$$

In equation (2.17), $O(\Delta x)^2$ is the error term that represents all the remaining terms in a Taylor series expansion. It is known as a central finite difference scheme in space with second-order accuracy. Similarly, (2.18) is second-order accurate in time. Applying Yee's finite-difference scheme to (2.9) gives:

$$\frac{H_x\big|_{i,j,k}^{n+1/2} - H_x\big|_{i,j,k}^{n-1/2}}{\Delta t} = \frac{1}{\mu_{i,j,k}}\left(\begin{array}{l}\dfrac{E_y\big|_{i,j,k+1/2}^{n} - E_y\big|_{i,j,k-1/2}^{n}}{\Delta z} - \\ \dfrac{E_z\big|_{i,j+1/2,k}^{n} - E_z\big|_{i,j-1/2,k}^{n}}{\Delta y} - \sigma_{i,j,k}^{*} H_x\big|_{i,j,k}^{n}\end{array}\right) \tag{2.19}$$

The $H_x\big|_{i,j,k}^{n}$ field component in (2.19) is evaluated at time step *n*. However, the value of $H_x\big|_{i,j,k}^{n}$ at time step *n* is not available and hence the following interpolated approximation is used:

$$H_x\big|_{i,j,k}^{n} = \frac{H_x\big|_{i,j,k}^{n+1/2} + H_x\big|_{i,j,k}^{n-1/2}}{2} \tag{2.20}$$

By substituting equation (2.20) in (2.19), leaving $H_x\big|_{i,j,k}^{n+1/2}$ on the left hand side and passing the all remaining terms to the right, assuming cubical FDTD cells are used, the finite difference updating equation for the magnetic and electric field components can be derived as:

$$H_x\big|_{i,j,k}^{n+1/2} = D_{a,H_x}\big|_{i,j,k} H_x\big|_{i,j,k}^{n-1/2} + D_{b,H_x}\big|_{i,j,k}\left(\begin{array}{l}E_y\big|_{i,j,k+1/2}^{n} - E_y\big|_{i,j,k-1/2}^{n} \\ + E_z\big|_{i,j-1/2,k}^{n} - E_z\big|_{i,j+1/2,k}^{n}\end{array}\right) \tag{2.21}$$

$$H_y\big|_{i,j,k}^{n+1/2} = D_{a,H_y}\big|_{i,j,k} H_y\big|_{i,j,k}^{n-1/2} + D_{b,H_y}\big|_{i,j,k}\left(\begin{array}{l}E_z\big|_{i+1/2,j,k}^{n} - E_z\big|_{i-1/2,j,k}^{n} \\ + E_x\big|_{i,j,k-1/2}^{n} - E_x\big|_{i,j,k+1/2}^{n}\end{array}\right) \tag{2.22}$$

$$H_z\Big|_{i,j,k}^{n+1/2} = D_{a,H_z}\Big|_{i,j,k} H_z\Big|_{i,j,k}^{n-1/2} + D_{b,H_z}\Big|_{i,j,k} \begin{pmatrix} E_x\Big|_{i,j+1/2,k}^{n} - E_x\Big|_{i,j-1/2,k}^{n} \\ + E_y\Big|_{i-1/2,j,k}^{n} - E_y\Big|_{i+1/2,j,k}^{n} \end{pmatrix} \tag{2.23}$$

$$E_x\Big|_{i,j,k}^{n+1} = C_{a,E_x}\Big|_{i,j,k} E_x\Big|_{i,j,k}^{n} + C_{b,E_x}\Big|_{i,j,k} \begin{pmatrix} H_z\Big|_{i,j+1/2,k}^{n+1/2} - H_z\Big|_{i,j-1/2,k}^{n+1/2} \\ + H_y\Big|_{i,j,k-1/2}^{n+1/2} - H_y\Big|_{i,j,k+1/2}^{n+1/2} \end{pmatrix} \tag{2.24}$$

$$E_y\Big|_{i,j,k}^{n+1} = C_{a,E_y}\Big|_{i,j,k} E_y\Big|_{i,j,k}^{n} + C_{b,E_y}\Big|_{i,j,k} \begin{pmatrix} H_x\Big|_{i,j,k+1/2}^{n+1/2} - H_x\Big|_{i,j,k-1/2}^{n+1/2} \\ + H_z\Big|_{i-1/2,j,k}^{n+1/2} - H_z\Big|_{i+1/2,j,k}^{n+1/2} \end{pmatrix} \tag{2.25}$$

$$E_z\Big|_{i,j,k}^{n+1} = C_{a,E_z}\Big|_{i,j,k} E_z\Big|_{i,j,k}^{n} + C_{b,E_z}\Big|_{i,j,k} \begin{pmatrix} H_y\Big|_{i+1/2,j,k}^{n+1/2} - H_y\Big|_{i-1/2,j,k}^{n+1/2} \\ + H_x\Big|_{i,j-1/2,k}^{n+1/2} - H_x\Big|_{i,j+1/2,k}^{n+1/2} \end{pmatrix} \tag{2.26}$$

It can be seen that the coefficients on the left hand side are referred to as Yee's updating coefficients. The electric field coefficients are given by:

$$C_a\Big|_{i,j,k} = \left(1 - \frac{\sigma_{i,j,k}\Delta t}{2\varepsilon_{i,j,k}}\right) \Bigg/ \left(1 + \frac{\sigma_{i,j,k}\Delta t}{2\varepsilon_{i,j,k}}\right) \tag{2.27}$$

$$C_{b_p}\Big|_{i,j,k} = \left(\frac{\Delta t}{\varepsilon_{i,j,k}\Delta_p}\right) \Bigg/ \left(1 + \frac{\sigma_{i,j,k}\Delta t}{2\varepsilon_{i,j,k}}\right) \tag{2.28}$$

The magnetic updating coefficients can be written as:

$$D_a\Big|_{i,j,k} = \left(1 - \frac{\sigma^*_{i,j,k}\Delta t}{2\mu_{i,j,k}}\right) \Bigg/ \left(1 + \frac{\sigma^*_{i,j,k}\Delta t}{2\mu_{i,j,k}}\right) \tag{2.29}$$

$$D_{b_p}\Big|_{i,j,k} = \left(\frac{\Delta t}{\mu_{i,j,k}\Delta_p}\right) \Bigg/ \left(1 + \frac{\sigma^*_{i,j,k}\Delta t}{2\mu_{i,j,k}}\right) \tag{2.30}$$

The subscript p can be x, y or z and Δ_p is the cell size in the p-direction. Assuming the structure under investigation contains different types of material such as dielectric or magnetic, electric and magnetic field updating coefficients can be easily calculated from equations (2.27) to (2.30) before the FDTD time stepping algorithm starts. The orientation of the fields in Figure 2.1 is known as the FDTD lattice or Yee cell. The magnetic and electric fields are located on the faces and the edges of the cube respectively. Each electric field vector component is surrounded by four circulating magnetic field vector components and vice versa. Both the electric and magnetic field vector components are located half a cell from each other. In addition, this arrangement permits easy implementations of the central finite difference approximations and the integral form of the Faraday's law and the Ampere's law. The system of difference equations is solved at the nodes. Figure 2.2 illustrates the typical relationship between field components within a quarter of a cell and on a plane distinctly helpful when handling boundary conditions of a closed region.

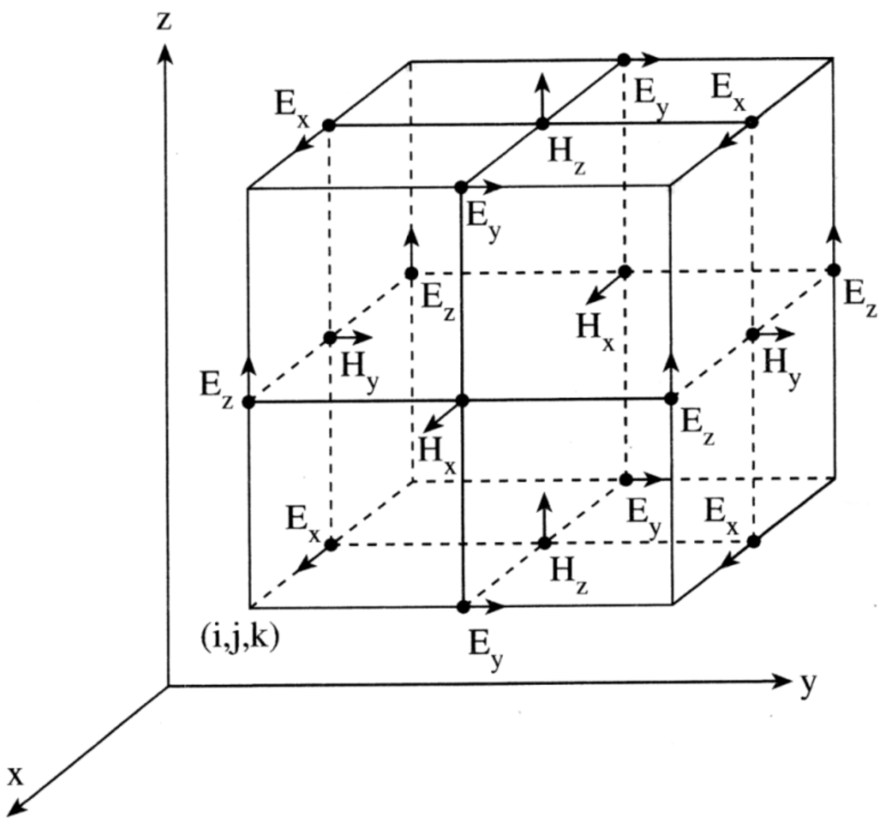

Figure 2.1: The electric and magnetic field components distribution on the FDTD lattice [6].

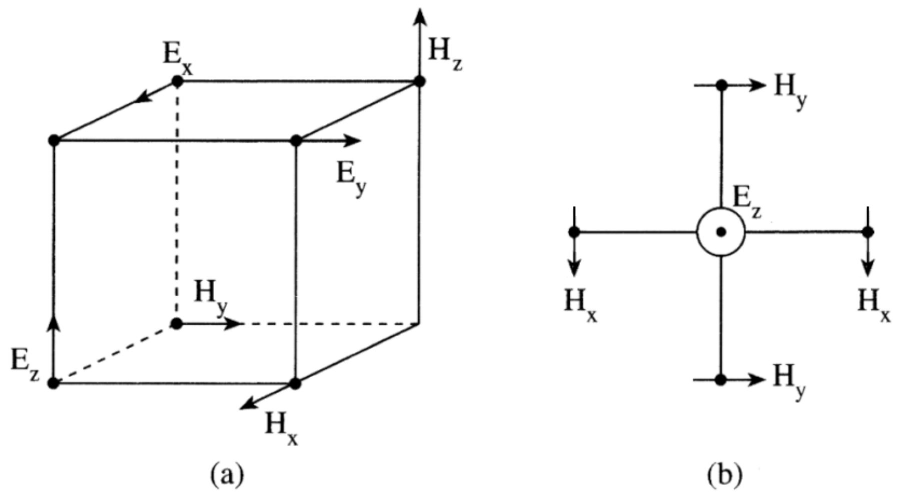

Figure 2.2: Relationship between field components: (a) within a quarter of a unit cell, (b) on a plane [6].