

Discrete Time Sliding Mode Control Using Multirate Output Feedback To Reduce Chattering

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Abstract—This paper presents a multirate output feedback based discrete time sliding mode for different reaching law. The proposed algorithm used past control input for switching function and output samples to design controller. It has been shown that chattering problem can be reduced by proper selection of the reaching law parameter. The performance and reliability of the proposal algorithm will be determined by performing extensive simulation works using simulink

Keywords—component; Discrete Sliding Mode, Multirate Output Feedback

I. INTRODUCTION

Sliding mode plays a dominant role in variable structure system (VSS). The design of Sliding Mode Control(SMC) system consists of two stages. The first stage selects a set of desired switching plane structures such that the sliding mode in their intersection exhibits the desired dynamics. The second is to enforce the system motion to reach and stay in the desired switching manifold[1]. Once the system reaches the switching plane, the structure of feedback loop is adaptively altered to slide the system state along the switching manifold.

SMC in both continuous and discrete time forms has become popular method for various applications. Digital Sliding Mode Controller (DSMC) design has been a topic of study during the past few years due to recent use of computers for control purpose. The concept of discrete-time sliding mode was proposed in [2-3]. In DSMC, the measurement and control signal are performed only at after regular intervals of time and control signal is held constants in between this time.

Multirate output feedback (MROF) concept is sampling the control input and sensor output of a system at different rates[4]. MROF can guarantee closed loop stability, a feature not assured by static output feedback [5] while retaining the structural simplicity of static output feedback. Considering the DSMC with MROF, some studies and research were undertaken [6-9]. SMC generally suffers from the well-known problem, namely, chattering. Analytical design methods were proposed by various researchers to reduce the effects of

chattering. The chattering phenomenon is generally perceived as motion which oscillates about the sliding manifold [10]. Many researcher used a linear reaching law proposed by Gao et al [11] and power rate reaching law to design MROF. An attempt has been done to investigate discrete power rate reaching law, integrate with MROF for chattering reduction and elimination [7]. In this paper a new discrete reaching law, which the saturation function replacing sign function proposed by [12] is used to reduce chattering and push the system approach to zero gradually.

The paper is structured as follows. Section II presents review of multirate output feedback. Section III contains the main contributions, discrete time reaching law proposed by [12] integrate with multirate output feedback which able to reduce chattering. An examples and some simulation results are provided in section IV. Finally, concluding remarks are made in section V.

II. MULTIRATE OUTPUT FEEDBACK

Feedback control strategy that used different samples rates for the control input and sensor output known as MROF, that also can guarantee closed loop system stability.

Consider the discrete system

$$\begin{aligned} x(k+1) &= \Phi_{\tau}x(k) + \Gamma_{\tau}u(k) \\ y(k) &= Cx(k) \end{aligned} \quad (1)$$

where $x(k) \in R^n$, $u(k) \in R$, $\Phi_{\tau}(k) \in R^{n \times n}$, $\Gamma_{\tau}(k) \in R^{n \times 1}$, τ is the sampling period, assume that the pair $(\Phi_{\tau}, \Gamma_{\tau})$ is controllable and the pair (Φ_{τ}, C) is observable.

The control signal is held constant during each sampling interval τ . In this work, the output is sampled at a faster rate at η second and the control input is computed at a slower rate of $\tau = \eta N$, where N is an integer. The ν denote the observability index of (Φ_{τ}, C) . N is chosen to be greater or equal to ν . The advantages of MROF are the error between the computed state and the actual state of the system goes to zero once a multirate sampled output measurement is available[4].

Consider the discrete time system having at time $t = k\tau$, the previous N fast output samples

$$y_k = \begin{bmatrix} y(k\tau - \tau) \\ y(k\tau - t + \eta) \\ \vdots \\ y(k\tau - \eta) \end{bmatrix} \quad (2)$$

Then the multirate output sampled system can be represented as

$$x(k+1) = \Phi_\tau x(k) + \Gamma_\tau u(k) \quad (3)$$

$$y_{k+1} = C_0 x(k) + D_0 u(k) \quad (4)$$

Where the C_0 and D_0 are as defined in [13].

$$C_0 = \begin{bmatrix} C \\ C\Phi \\ \vdots \\ C\Phi^{N-1} \end{bmatrix} \quad (5)$$

$$D_0 = \begin{bmatrix} 0 \\ C\Gamma \\ \vdots \\ C\sum_{j=0}^{N-2} \Phi^j \Gamma \end{bmatrix} \quad (6)$$

III. DISCRETE SLIDING MODE CONTROL DESIGN

According to the concept of discrete-time sliding mode, systems will show sliding mode behavior if their motions can reach a manifold in the state space in finite time and thereafter be confined to the manifold [2]. A reaching law for the sliding mode control of a discrete time system in the following form was introduced by [12]

$$s(k+1) = (1 - q\tau)s(k) - \varepsilon\tau \text{sat}\left(\frac{s(k)}{\sigma}\right) \quad (7)$$

where ε , q and σ are design parameter with constraints $\varepsilon > 0, q > 0, 1 - q\tau > 0$, τ is the sampling period, $\sigma > 0$ and

$$\text{sat}\left(\frac{s(k)}{\sigma}\right) = \begin{cases} 1 & s(k) > \sigma \\ \frac{s(k)}{\sigma} & |s(k)| \leq \sigma \\ -1 & s(k) < -\sigma \end{cases} \quad (8)$$

Also the condition below must be satisfy [12]

$$\frac{1 - q\tau}{\tau} < \frac{\varepsilon}{\sigma} < \frac{2 - q\tau}{\tau} \quad (9)$$

The switching function define as

$$s(k) = c^T x(k) \quad (10)$$

Hence

$$s(k+1) = c^T x(k+1) \quad (11)$$

From Equ. (1) and Equ. (11)

$$\begin{aligned} s(k+1) &= c^T [\Phi_\tau x(k) + \Gamma_\tau u(k)] \\ &= c^T \Phi_\tau x(k) + c^T \Gamma_\tau u(k) \end{aligned} \quad (12)$$

Comparing Eqn. (7) and Eqn. (12) give the control law :

$$u(k) = -(c^T \Gamma_\tau)^{-1} \left[c^T \Phi_\tau x(k) + (1 - q\tau)s(k) + \varepsilon\tau \text{sat}\left(\frac{s(k)}{\sigma}\right) \right] \quad (13)$$

Form equation (3) and (4)

$$x(k+1) = \Phi_\tau C_0^{-1} y_{k+1} + [\Gamma_\tau - \Phi_\tau C_0^{-1} D_0] u(k) \quad (14)$$

Therefore:

$$x(k) = \Phi_\tau C_0^{-1} y_k + [\Gamma_\tau - \Phi_\tau C_0^{-1} D_0] u(k-1) \quad (15)$$

$$\begin{aligned} s(k) &= c^T x(k) \\ &= c^T \Phi_\tau C_0^{-1} y_k + c^T [\Gamma_\tau - \Phi_\tau C_0^{-1} D_0] u(k-1) \end{aligned} \quad (16)$$

$$\begin{aligned} u(k) &= -(c^T \Gamma_\tau)^{-1} [c^T \Phi_\tau (\Phi_\tau C_0^{-1} y_k + \\ &(\Gamma_\tau - \Phi_\tau C_0^{-1} D_0) u(k-1)) + (1 - q\tau)s(k) \\ &+ \varepsilon\tau \text{sat}\left(\frac{s(k)}{\sigma}\right)] \end{aligned} \quad (17)$$

IV. NUMERICAL EXAMPLES AND SIMULATION RESULT

To illustrate the effectiveness of the proposed algorithm, simulation result for the following discrete time system is considered [12]

$$\Phi_\tau = \begin{bmatrix} 1 & 0.01 \\ 0 & 0.7 \end{bmatrix}$$

$$\Gamma_\tau = \begin{bmatrix} 0.01 \\ 1 \end{bmatrix}$$

The switching surface is design as

$$s(k) = 8x_1(k) + x_2(k)$$

The initial value is set as

$$x_1(0) = 1, x_2(0) = 1$$

Then the control law $u(k)$ obtain as

$$u(k) = -0.9259[[8.0005.265]y_k + 0.665u(k-1) + 0.94s(k) + 0.01\varepsilon \text{sat}\left(\frac{8x_1(k) + x_2(k)}{0.2}\right)]$$

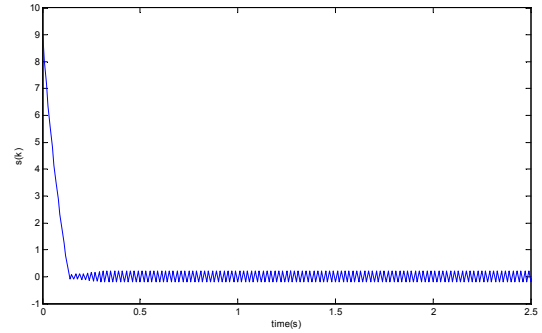
A desirable reaching mode response can be obtained by appropriate choice of parameters q, d and ε . The parameters of the controller chosen as $q = 6, \tau = 0.01, N = 2, d = 0.2$. The response of the system with the control law of Eqn. (17) shows in Figure 1 and Figure 2. Chattering is a well-known phenomenon of sliding mode control and it appears in Figure 1, for the case $\varepsilon = 40$ which not satisfies condition (9). Proper selection of $\varepsilon = 20$ that follow condition (9) can effectively reduced and eliminated chattering phenomena as shows in Figure 2.

V. CONCLUSION

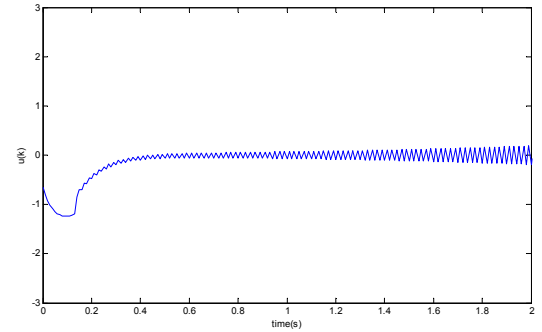
A new algorithm for Discrete time sliding mode controller algorithm using multirate output feedback for reaching law designed by [12] has been proposed. The proposed controller used only output information to compute switching function and sliding mode control. The proposed algorithm can reduce chattering effectively using output information that measured at a faster rate than the input update. Simulation study shown the response of the system can be adjusted with appropriate selection of the reaching law parameter is potentially reducing the chattering problem.

ACKNOWLEDGMENT

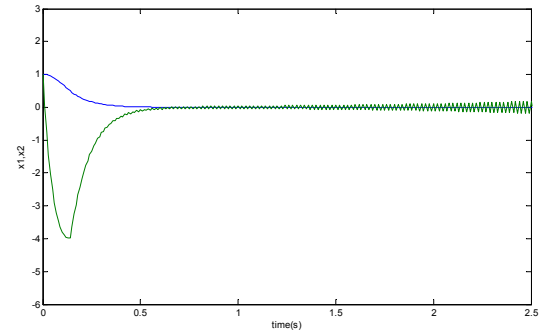
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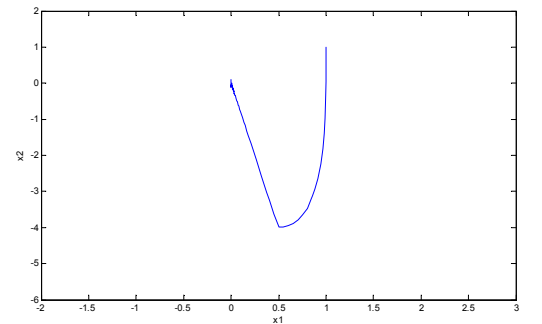
a) Switching Surface



b) control Input



c) System State

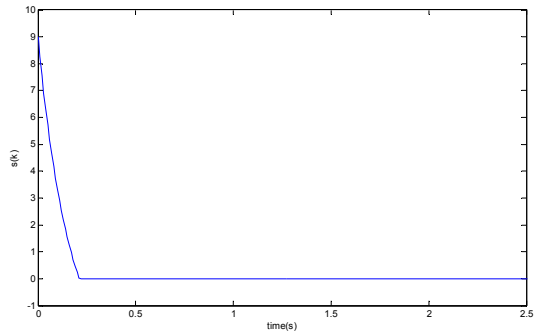


d)Phase plot

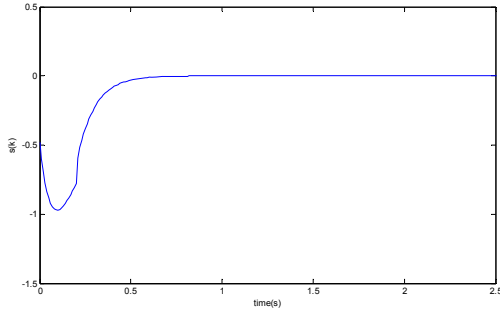
Figure 1: Response of the system with $\varepsilon = 40$

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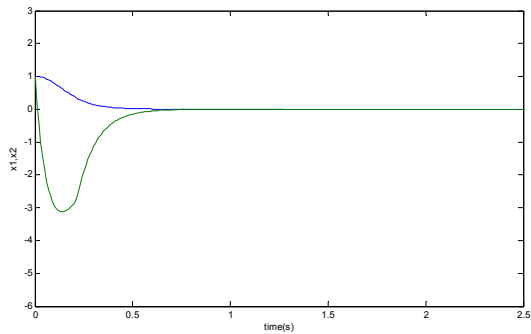
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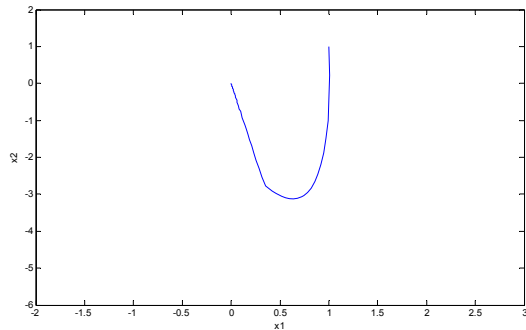
a) Switching Surface



b) control Input



c) System State



d)Phase plot

Figure 2: Response of the system with $\varepsilon = 20$